Seminar Hasil

13 Januari 2015

TESIS

PEMODELAN REGRESI DERET FOURIER DAN SPLINE *TRUNCATED* DALAM REGRESI NONPARAMETRIK MULTIVARIABEL (APLIKASI: DATA KEMISKINAN DI PROPINSI PAPUA)

Ni Putu Ayu Mirah Mariati

1313 201 019

Dosen Pembimbing :

Prof. Dr. Drs. I Nyoman Budiantara, M.Si

PROGRAM MAGISTER
PROGRAM PASCA SARJANA STATISTIKA
FAKULTAS MATEMATIKA DAN ILMU PENGETAHUAN ALAM
INSTITUT TEKNOLOGI SEPULUH NOPEMBER SURABAYA
2015

Estimasi Spline

Estimasi
Deret Fourier

HASIL DAN PEMBAHASAN

Model Regresi
Nonparametrik
Spline *Truncated*

Model Regresi
Nonparametrik
Deret Fourier

Perbandingan
antara Spline
Truncated dan
Deret Fourier

Estimasi Spline *Truncated*

Diberikan model regresi nonparametrik multivariabel:

$$y_i = \sum_{j=1}^{p} f_j\left(x_{ji}\right) + \varepsilon_i; i = 1, 2, \ldots, n$$

Kurva regresi f_j dihampiri dengan fungsi spline multivariabel , dengan:

$$f_j\left(x_{ji}\right) = \sum_{v=1}^{m} \beta_{vj} x_{ji}^{v} + \sum_{k=1}^{r} \beta_{j(k+m)} \left(x_{ji} - K_{jk}\right)_+^m$$

$$= \beta_{1j} x_{ji}^1 + \ldots + \beta_{mj} x_{ji}^m + \beta_{j(1+m)} \left(x_{ji} - K_{j1}\right)_+^m + \ldots + \beta_{j(r+m)} \left(x_{ji} - K_{jr}\right)_+^m$$

$$\sum_{j=1}^{p} f_j\left(x_{ji}\right) = \sum_{j=1}^{p} \left(\beta_{1j} x_{ji}^1 + \ldots + \beta_{mj} x_{ji}^m + \beta_{j(1+m)} \left(x_{ji} - K_{j1}\right)_+^m + \ldots + \beta_{j(r+m)} \left(x_{ji} - K_{jr}\right)_+^m\right)$$

$$= \left(\beta_{11} x_{1i}^1 + \ldots + \beta_{m1} x_{1i}^m + \beta_{1(1+m)} \left(x_{1i} - K_{11}\right)_+^m + \ldots + \beta_{1(r+m)} \left(x_{1i} - K_{1r}\right)_+^m\right) + \ldots +$$

$$\left(\beta_{1p} x_{pi}^1 + \ldots + \beta_{mp} x_{pi}^m + \beta_{p(1+m)} \left(x_{pi} - K_{p1}\right)_+^m + \ldots + \beta_{j(r+m)} \left(x_{pi} - K_{pr}\right)_+^m\right)$$

Dapat dijadikan bentuk Matriks

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_{11}^1 & \cdots & x_{11}^m & (x_{11}-K_{11})_+^m & \cdots & (x_{11}-K_{1r})_+^m \\ x_{12}^1 & \cdots & x_{12}^m & (x_{12}-K_{11})_+^m & \cdots & (x_{12}-K_{1r})_+^m \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ x_{1n}^1 & \cdots & x_{1n}^m & (x_{1n}-K_{11})_+^m & \cdots & (x_{1n}-K_{1r})_+^m \end{bmatrix} \begin{bmatrix} \beta_{11} \\ \vdots \\ \beta_{m1} \\ \beta_{1(1+m)} \\ \vdots \\ \beta_{1(r+m)} \end{bmatrix} + \dots +$$

$$\begin{bmatrix} x_{p1}^1 & \cdots & x_{p1}^m & (x_{p1}-K_{p1})_+^m & \cdots & (x_{p1}-K_{pr})_+^m \\ x_{p2}^1 & \cdots & x_{p2}^m & (x_{p2}-K_{p1})_+^m & \cdots & (x_{p2}-K_{pr})_+^m \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ x_{pn}^1 & \cdots & x_{pn}^m & (x_{pn}-K_{p1})_+^m & \cdots & (x_{pn}-K_{pr})_+^m \end{bmatrix} \begin{bmatrix} \beta_{1p} \\ \vdots \\ \beta_{mp} \\ \beta_{p(1+m)} \\ \vdots \\ \beta_{p(r+m)} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

Dapat dituliskan menjadi:

$$\underset{\sim}{y} = X\left(K_{11},\dots,K_{1r} \vdots \cdots \vdots K_{p1},\dots,K_{pr}\right)\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}$$

Dengan :

$$\underset{\sim}{y} = \left[y_1, \ldots, y_n \right]'$$

$$X\left(K_{11}, \ldots, K_{1r} \vdots \ldots \vdots K_{p1}, \ldots, K_{pr}\right) = \left[A_1 \;\vdots\; \cdots \;\vdots\; A_p \right]$$

$$A_1 = \begin{bmatrix} x_{11}^1 & \cdots & x_{11}^m & \left(x_{11} - K_{11}\right)_+^m & \cdots & \left(x_{11} - K_{1r}\right)_+^m \\ x_{12}^1 & \cdots & x_{12}^m & \left(x_{12} - K_{11}\right)_+^m & \cdots & \left(x_{12} - K_{1r}\right)_+^m \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ x_{1n}^1 & \cdots & x_{1n}^m & \left(x_{1n} - K_{11}\right)_+^m & \cdots & \left(x_{1n} - K_{1r}\right)_+^m \end{bmatrix} \quad A_p = \begin{bmatrix} x_{p1}^1 & \cdots & x_{p1}^m & \left(x_{p1} - K_{p1}\right)_+^m & \cdots & \left(x_{p1} - K_{pr}\right)_+^m \\ x_{p2}^1 & \cdots & x_{p2}^m & \left(x_{p2} - K_{p1}\right)_+^m & \cdots & \left(x_{p2} - K_{pr}\right)_+^m \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ x_{pn}^1 & \cdots & x_{pn}^m & \left(x_{pn} - K_{p1}\right)_+^m & \cdots & \left(x_{pn} - K_{pr}\right)_+^m \end{bmatrix}$$

$$\underset{\sim}{\beta} = \left(\underset{\sim}{\beta}_1, \ldots, \underset{\sim}{\beta}_p \right)'$$

$$\underset{\sim}{\beta}_1 = \left(\beta_{11}, \ldots, \beta_{m1}, \beta_{1(1+m)}, \ldots, \beta_{1(r+m)} \right)', \ldots, \underset{\sim}{\beta}_p = \left(\beta_{1p}, \ldots, \beta_{mp}, \beta_{p(1+m)}, \ldots, \beta_{p(r+m)} \right)'$$

$$\underset{\sim}{\varepsilon} = \left(\underset{\sim}{\varepsilon}_1, \ldots, \underset{\sim}{\varepsilon}_n \right)'$$

Estimator parameter $\underset{\sim}{\beta}$ didapat dari menyelesaikan optimasi:

$$\underset{\beta \varepsilon R^{p(m+r)}}{Min} \left\{ n^{-1} \left\| y - X\beta \right\|^2 \right\} = \left\{ \left(\underset{\sim}{y} - X\left(K_{11},...,K_{1r} \vdots \cdots \vdots K_{p1},...,K_{pr}\right)\underset{\sim}{\beta} \right)' \left(\underset{\sim}{y} - X\left(K_{11},...,K_{1r} \vdots \cdots \vdots K_{p1},...,K_{pr}\right)\underset{\sim}{\beta} \right) \right\}$$

Derivatif Parsial

$$\mathbb{Q}\left(\underset{\sim}{\beta}\right) = \left\{ \left(\underset{\sim}{y} - X\left(K_{11},...,K_{1r} \vdots \cdots \vdots K_{p1},...,K_{pr}\right)\underset{\sim}{\beta} \right)' \left(\underset{\sim}{y} - X\left(K_{11},...,K_{1r} \vdots \cdots \vdots K_{p1},...,K_{pr}\right)\underset{\sim}{\beta} \right) \right\}$$

$$= \underset{\sim}{y}'\underset{\sim}{y} - \underset{\sim}{\beta}'X'\left(K_{11},...,K_{1r} \vdots \cdots \vdots K_{p1},...,K_{pr}\right)\underset{\sim}{y} - \left(\underset{\sim}{y}'X\left(K_{11},...,K_{1r} \vdots \cdots \vdots K_{p1},...,K_{pr}\right)\underset{\sim}{\beta} \right)$$

$$+ \underset{\sim}{\beta}'X'\left(K_{11},...,K_{1r} \vdots \cdots \vdots K_{p1},...,K_{pr}\right)X\left(K_{11},...,K_{1r} \vdots \cdots \vdots K_{p1},...,K_{pr}\right)\underset{\sim}{\beta}$$

$$= \underset{\sim}{y}'\underset{\sim}{y} - 2\underset{\sim}{\beta}X'\left(K_{11},...,K_{1r} \vdots \cdots \vdots K_{p1},...,K_{pr}\right)\underset{\sim}{y}$$

$$+ \underset{\sim}{\beta}'X'\left(K_{11},...,K_{1r} \vdots \cdots \vdots K_{p1},...,K_{pr}\right)X\left(K_{11},...,K_{1r} \vdots \cdots \vdots K_{p1},...,K_{pr}\right)\underset{\sim}{\beta}$$

Untuk menyelesaikan optimasi dengan menggunakan derivatif parsial, misalkan:

$$\frac{\partial \mathbb{Q}(\underset{\sim}{\beta})}{\partial \underset{\sim}{\beta}} = -2X'\left(K_{11},\ldots,K_{1r} \vdots \cdots \vdots K_{p1},\ldots,K_{pr}\right)\underset{\sim}{y} + 2X'\left(K_{11},\ldots,K_{1r} \vdots \cdots \vdots K_{p1},\ldots,K_{pr}\right)X\left(K_{11},\ldots,K_{1r} \vdots \cdots \vdots K_{p1},\ldots,K_{pr}\right)\underset{\sim}{\beta}$$

$$0 = -2X'\left(K_{11},\ldots,K_{1r} \vdots \cdots \vdots K_{p1},\ldots,K_{pr}\right)\underset{\sim}{y} + 2X'\left(K_{11},\ldots,K_{1r} \vdots \cdots \vdots K_{p1},\ldots,K_{pr}\right)X\left(K_{11},\ldots,K_{1r} \vdots \cdots \vdots K_{p1},\ldots,K_{pr}\right)\underset{\sim}{\hat{\beta}}$$

$$X'\left(K_{11},\ldots,K_{1r} \vdots \cdots \vdots K_{p1},\ldots,K_{pr}\right)X\left(K_{11},\ldots,K_{1r} \vdots \cdots \vdots K_{p1},\ldots,K_{pr}\right)\underset{\sim}{\hat{\beta}} = X'\left(K_{11},\ldots,K_{1r} \vdots \cdots \vdots K_{p1},\ldots,K_{pr}\right)\underset{\sim}{y}$$

Sehingga, Estimator $\underset{\sim}{\hat{\beta}}$

$$\underset{\sim}{\hat{\beta}} = \left(X'\left(K_{11},\ldots,K_{1r} \vdots \cdots \vdots K_{p1},\ldots,K_{pr}\right)X\left(K_{11},\ldots,K_{1r} \vdots \cdots \vdots K_{p1},\ldots,K_{pr}\right)\right)^{-1} X'\left(K_{11},\ldots,K_{1r} \vdots \cdots \vdots K_{p1},\ldots,K_{pr}\right)\underset{\sim}{y}$$

$$\underset{\sim}{\hat{\beta}} = \left(\underset{\sim}{\hat{\beta}}_1',\ldots,\underset{\sim}{\hat{\beta}}_p'\right)'.$$

Estimator kurva regresi $\hat{\underset{\sim}{f}}(x)$

$$\hat{\underset{\sim}{f}}(x)=\left[\hat{\underset{\sim}{f}}(x_1)',\hat{\underset{\sim}{f}}(x_2)',\ldots,\hat{\underset{\sim}{f}}(x_p)'\right]$$

$$=\left(X'(K_{11},\ldots,K_{1r}\vdots\cdots\vdots K_{p1},\ldots,K_{pr})X(K_{11},\ldots,K_{1r}\vdots\cdots\vdots K_{p1},\ldots,K_{pr})\right)^{-1}X'(K_{11},\ldots,K_{1r}\vdots\cdots\vdots K_{p1},\ldots,K_{pr})\underset{\sim}{y}$$

$$\hat{f}_j(x_{ji})=\sum_{v=1}^{m}\hat{\beta}_{vj}x_{ji}^{v}+\sum_{k=1}^{r}\hat{\beta}_{j(k+m)}(x_{ji}-K_{jk})_{+}^{m}$$

$$\sum_{j=1}^{p}\hat{f}_j(x_{ji})=\sum_{j=1}^{p}\left(\sum_{v=1}^{m}\hat{\beta}_{vj}x_{ji}^{v}+\sum_{k=1}^{r}\hat{\beta}_{j(k+m)}(x_{ji}-K_{jk})_{+}^{m}\right)$$

$$=\sum_{j=1}^{p}\sum_{v=1}^{m}\hat{\beta}_{vj}x_{ji}^{v}+\sum_{j=1}^{p}\sum_{k=1}^{r}\hat{\beta}_{j(k+m)}(x_{ji}-K_{jk})_{+}^{m}$$

Jadi, estimator kurva regresi $\hat{\underset{\sim}{f}}(x)$

dimana $\hat{\beta}_{vj}$ dan $\hat{\beta}_{j(k+m)}$
Diperoleh dari:

$$\hat{\underset{\sim}{\beta}}=\left(\hat{\underset{\sim}{\beta}}_1',\ldots,\hat{\underset{\sim}{\beta}}_p'\right)'$$

$$\hat{\underset{\sim}{\beta}}_1=\left(\beta_{11},\ldots,\beta_{m1},\beta_{1(1+m)},\ldots,\beta_{1(r+m)}\right)',\ldots,\hat{\underset{\sim}{\beta}}_p=\left(\beta_{1p},\ldots,\beta_{mp},\beta_{p(1+m)},\ldots,\beta_{p(r+m)}\right)'$$

Estimasi Deret Fourier

Diberikan model regresi nonparametrik multivariabel

$$y_i = \sum_{j=1}^{q} f_j\left(x_{ji}\right) + \varepsilon_i,$$

Dihampiri dengan fungsi Deret Fourier

$$f_j\left(x_{ji}\right) = b_j x_{ji} + \frac{1}{2}\alpha_{0j} + \sum_{k=1}^{K}\alpha_{kj}\cos kx_{ji},\ j = 1,2,\ldots,q.$$

Diperoleh dari optimasi

$$\underset{\underset{\sim}{\beta}\in R^{q(K+2)}}{Min}\left\{\sum_{i=1}^{n}\left(y_i - \sum_{j=1}^{q}\left(b_j x_{ji} + \frac{1}{2}\alpha_{0j} + \sum_{k=1}^{K}\alpha_{kj}\cos kx_{ji}\right)\right)^2\right\} = \underset{\underset{\sim}{\beta}\in R^{q(K+2)}}{Min}\left\{\mathbb{Q}\left(\underset{\sim}{\beta}\right)\right\}$$

$$\mathbb{Q}\left(\underset{\sim}{\beta}\right) = \sum_{i=1}^{n}\left(y_i - \sum_{j=1}^{q}\left(b_j x_{ji} + \frac{1}{2}\alpha_{0j} + \alpha_{1j}\cos x_{ji} + \ldots + \alpha_{Kj}\cos Kx_{ji}\right)\right)^2$$

$$= \sum_{i=1}^{n}\left(y_i - \left(b_1 x_{1i} + \frac{1}{2}\alpha_{01} + \alpha_{11}\cos x_{1i} + \ldots + \alpha_{K1}\cos Kx_{1i}\right) - \ldots + \right.$$

$$\left. - \left(b_q x_{qi} + \frac{1}{2}\alpha_{0q} + \alpha_{1q}\cos x_{qi} + \ldots + \alpha_{Kq}\cos Kx_{qi}\right)\right)^2$$

Dapat Ditulis menjadi:

$$Q\left(\underset{\sim}{\beta}\right)=\left(\underset{\sim}{y}-X(K)\underset{\sim}{\beta}\right)'\left(\underset{\sim}{y}-X(K)\underset{\sim}{\beta}\right)$$

Dengan

$$\underset{\sim}{y}=\left[y_1,y_2,\ldots,y_n\right]',$$

$$\underset{\sim}{\beta}=\begin{bmatrix} b_1 & \frac{1}{2}\alpha_{01} & \alpha_{11} & \cdots & \alpha_{K1} & \vdots & \cdots & \vdots & b_q & \frac{1}{2}\alpha_{0q} & \alpha_{1q} & \cdots & \alpha_{Kq} \end{bmatrix}'$$

$$X(K)=\begin{bmatrix} x_{11} & 1 & \cos x_{11} & \cdots & \cos Kx_{11} & \vdots & \cdots & \vdots & x_{q1} & 1 & \cos x_{q1} & \cdots & \cos Kx_{q1} \\ x_{12} & 1 & \cos x_{12} & \cdots & \cos Kx_{12} & \vdots & \cdots & \vdots & x_{q2} & 1 & \cos x_{q2} & \cdots & \cos Kx_{q2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & & & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{1n} & 1 & \cos x_{1n} & \cdots & \cos Kx_{1n} & \vdots & \cdots & \vdots & x_{qn} & 1 & \cos x_{qn} & \cdots & \cos Kx_{qn} \end{bmatrix}$$

$$\mathbb{Q}\left(\underset{\sim}{\beta}\right)=\left(\underset{\sim}{y}'-\underset{\sim}{\beta}'X'(K)\right)\left(\underset{\sim}{y}-X(K)\underset{\sim}{\beta}\right)$$

$$=\underset{\sim}{y}'\underset{\sim}{y}-\underset{\sim}{y}'X(K)\underset{\sim}{\beta}-\underset{\sim}{\beta}'X'(K)\underset{\sim}{y}+\underset{\sim}{\beta}'X'(K)X(K)\underset{\sim}{\beta}$$

$$=\underset{\sim}{y}'\underset{\sim}{y}-\left(X'(K)\underset{\sim}{\beta}'\underset{\sim}{y}\right)'-\underset{\sim}{\beta}'X'(K)\underset{\sim}{y}+\underset{\sim}{\beta}'X'(K)X(K)\underset{\sim}{\beta}$$

$$=\underset{\sim}{y}'\underset{\sim}{y}-2\underset{\sim}{\beta}'X'(K)\underset{\sim}{y}+\underset{\sim}{\beta}'X'(K)X(K)\underset{\sim}{\beta}$$

Menurunkan secara parsial

$$\frac{\partial\mathbb{Q}\left(\underset{\sim}{\beta}\right)}{\partial\underset{\sim}{\beta}}=-2X'(K)\underset{\sim}{y}+2X'(K)X(K)\underset{\sim}{\beta}$$

Disamakan dengan nol

$$-2X'(K)\underset{\sim}{y}+2X'(K)X(K)\underset{\sim}{\hat{\beta}}=0.$$

Estimator $\underset{\sim}{\beta}$

$$\hat{\underset{\sim}{\beta}}(K) = \left(X'(K)X(K) \right)^{-1} X'(K)\underset{\sim}{y}$$

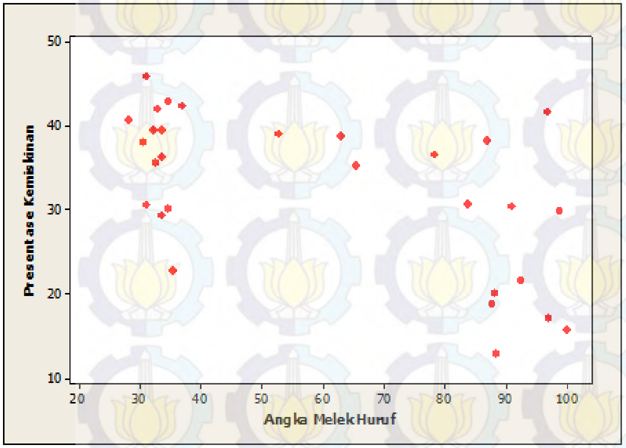
$$= \left[\hat{b}_1 \quad \frac{1}{2}\hat{\alpha}_{01} \quad \hat{\alpha}_{11} \quad \cdots \quad \hat{\alpha}_{K1} \quad \vdots \quad \cdots \quad \vdots \quad \hat{b}_q \quad \frac{1}{2}\hat{\alpha}_{0q} \quad \hat{\alpha}_{1q} \quad \cdots \quad \hat{\alpha}_{Kq} \right]'$$

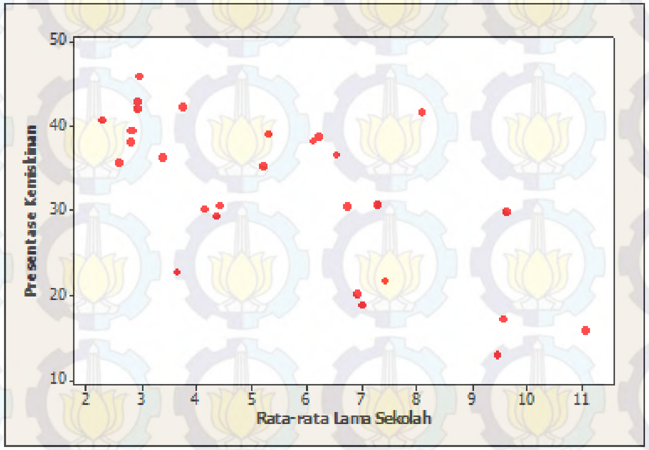
Estimator f_j

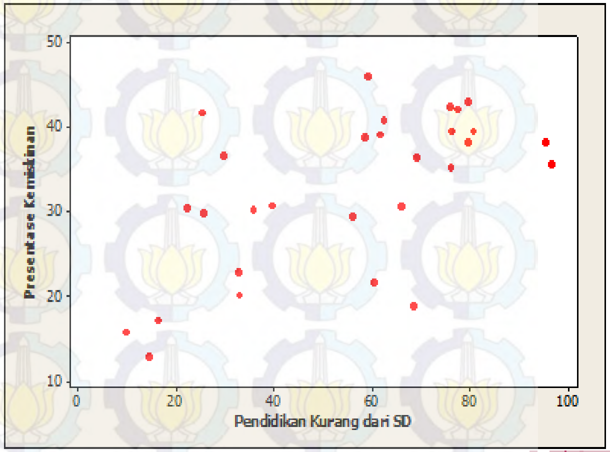
$$\hat{f}_j\left(x_{ji}\right) = \hat{b}_j x_{ji} + \frac{1}{2}\hat{\alpha}_{0j} + \sum_{k=1}^{K} \hat{\alpha}_{kj} \cos kx_{ji}.$$

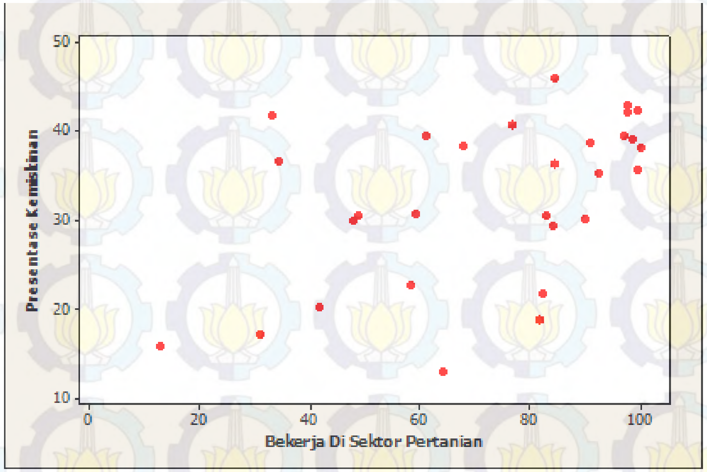
$$= \sum_{j=1}^{q} \left(\hat{b}_j x_{ji} + \frac{1}{2}\hat{\alpha}_{0j} + \sum_{k=1}^{K} \hat{\alpha}_{kj} \cos kx_{ji} \right)$$

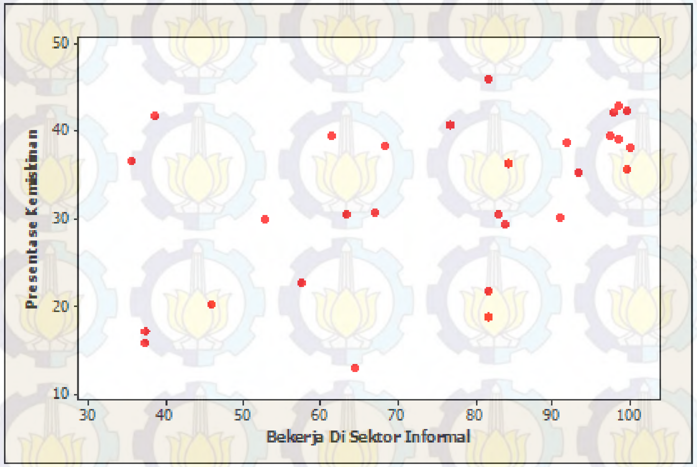
Scaterplot Presentase Kemiskinan dengan Variabel











MODEL REGRESI NONPARAMETRIK SPLINE *TRUNCATED*

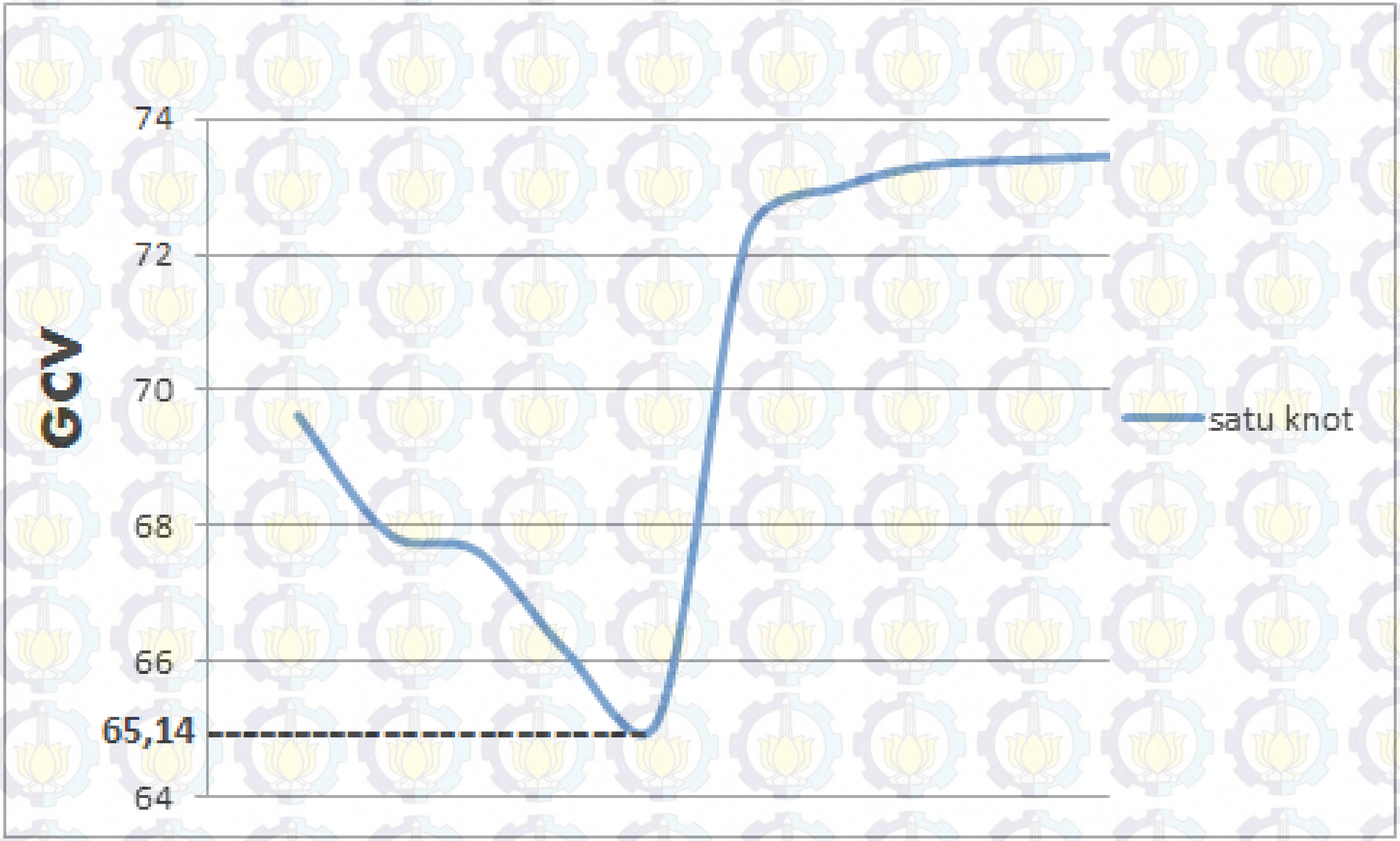
Pemilihan Titik Knot Optimum dengan Satu Titik Knot

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 (x_1 - K_1)_+ + \hat{\beta}_3 x_2 + \hat{\beta}_4 (x_2 - K_2)_+ +$$
$$\hat{\beta}_5 x_3 + \hat{\beta}_6 (x_3 - K_3)_+ + \hat{\beta}_7 x_4 + \hat{\beta}_8 (x_4 - K_4)_+ + \hat{\beta}_9 x_5 + \hat{\beta}_{10} (x_5 - K_5)_+$$

GCV MINIMUM

No	x_1	x_2	x_3	x_4	x_5	GCV
1	39,80	3,73	24,07	27,03	46,05	66,12
2	41,26	3,91	25,84	28,81	47,36	67,62
3	38,33	3,55	22,30	25,25	44,73	67,89
4	91,05	9,99	86,04	89,32	92,10	69,63
5	85,20	9,27	78,96	82,20	86,84	65,14
6	89,59	9,81	84,27	87,54	90,79	72,29
7	86,66	9,45	80,73	83,98	88,16	72,98
8	42,72	4,09	27,61	30,59	48,68	73,31
9	88,12	9,63	82,50	85,76	89,47	73,39
10	36,87	3,37	20,53	23,47	43,42	73,45

GCV Satu Knot



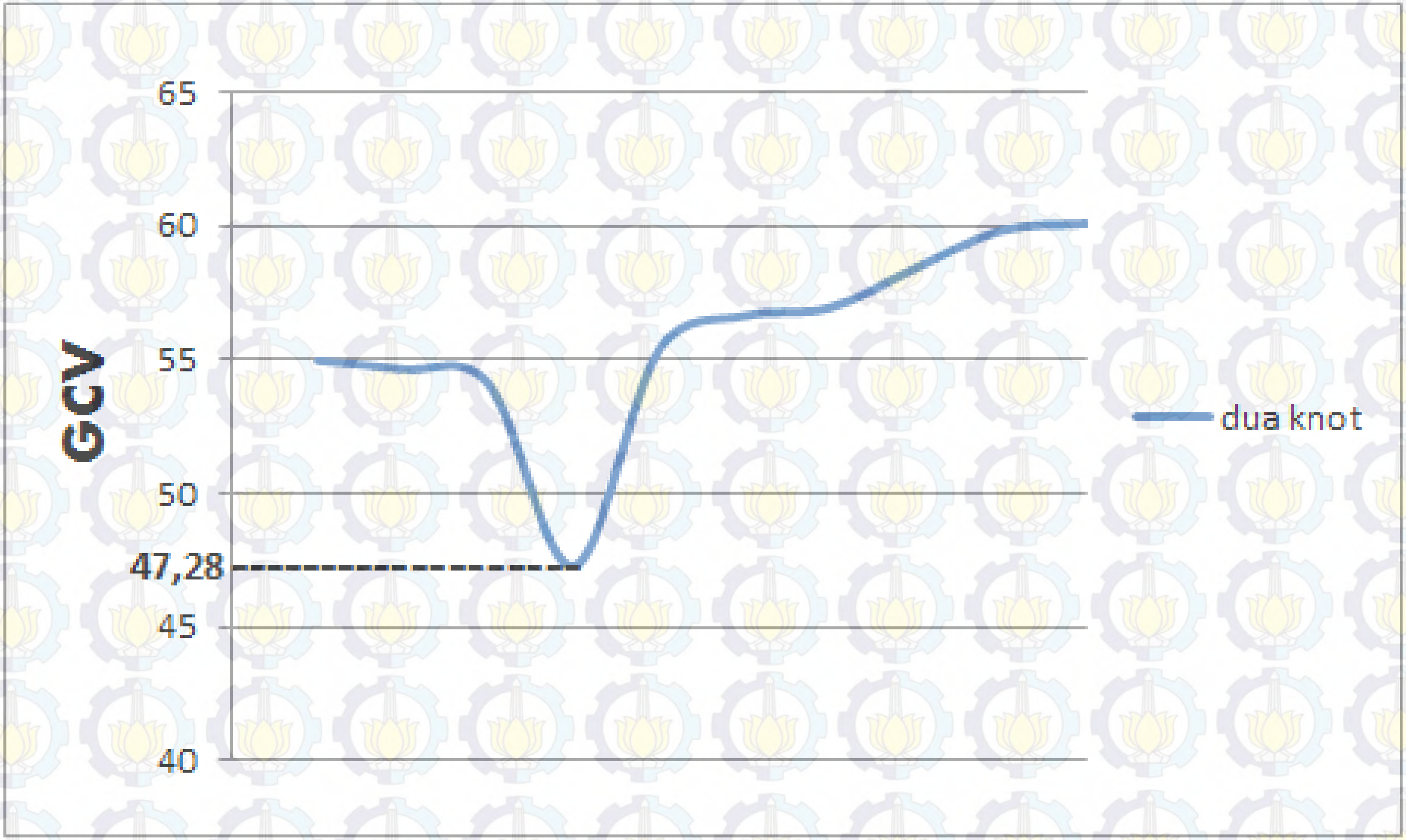
Pemilihan Titik Knot Optimum dengan Dua Titik Knot

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 (x_1 - K_1)_+ + \hat{\beta}_3 (x_1 - K_2)_+ + \hat{\beta}_4 x_2 + \hat{\beta}_5 (x_2 - K_3)_+ + \hat{\beta}_6 (x_2 - K_4)_+ +$$
$$\hat{\beta}_7 x_3 + \hat{\beta}_8 (x_3 - K_5)_+ + \hat{\beta}_9 (x_3 - K_6)_+ + \hat{\beta}_{10} x_4 + \hat{\beta}_{11} (x_4 - K_7)_+ + \hat{\beta}_{12} (x_4 - K_8)_+ +$$
$$\hat{\beta}_{13} x_5 + \hat{\beta}_{13} (x_5 - K_9)_+ + \hat{\beta}_{14} (x_5 - K_{10})_+$$

No	x1	x2	x3	x4	x5	GCV
1	70,55	7,48	61,25	64,40	73,68	54,10
	72,01	7,66	63,02	66,18	75,00	
2	82,27	8,91	75,42	78,64	84,21	54,63
	85,20	9,27	78,96	82,20	86,84	
3	70,55	7,48	61,25	64,40	73,68	54,96
	73,48	7,84	64,79	67,96	76,31	
4	80,80	8,74	73,64	76,86	82,89	47,28
	85,20	9,27	78,96	82,20	86,84	
5	79,34	8,56	71,87	75,08	81,58	55,37
	85,20	9,27	78,96	82,20	86,84	
6	72,01	7,66	63,02	66,18	75,00	56,63
	73,48	7,84	64,79	67,96	76,31	
7	35,40	3,19	18,76	21,69	42,10	56,93
	38,33	3,55	22,30	25,25	44,73	
8	77,87	8,38	70,10	73,30	80,26	58,40
	85,20	9,27	78,96	82,20	86,84	
9	76,41	8,20	68,33	71,52	78,95	59,81
	85,20	9,27	78,96	82,20	86,84	
10	48,58	4,80	34,70	37,71	53,94	60,08
	50,05	4,98	36,47	39,49	55,26	

GCV MINIMUM

GCV Dua Knot



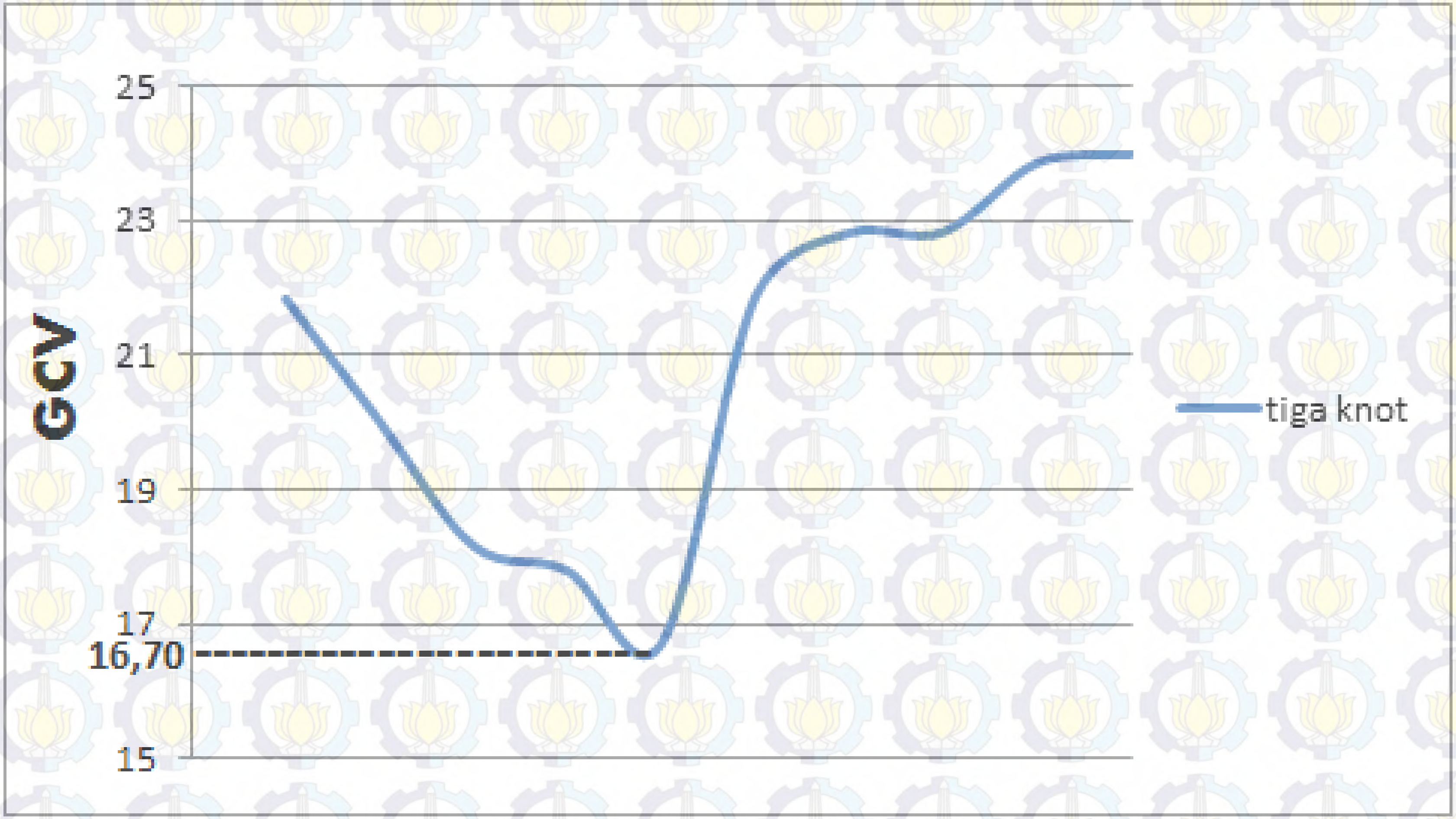
Pemilihan Titik Knot Optimum dengan Tiga Titik Knot

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 (x_1 - K_1)_+ + \hat{\beta}_3 (x_1 - K_2)_+ + \hat{\beta}_4 (x_1 - K_3)_+ +$$
$$\hat{\beta}_5 x_2 + \hat{\beta}_6 (x_2 - K_4)_+ + \hat{\beta}_7 (x_2 - K_5)_+ + \hat{\beta}_8 (x_2 - K_6)_+ +$$
$$\hat{\beta}_9 x_3 + \hat{\beta}_{10} (x_3 - K_7)_+ + \hat{\beta}_{11} (x_3 - K_8)_+ + \hat{\beta}_{12} (x_4 - K_9)_+ +$$
$$\hat{\beta}_{13} x_4 + \hat{\beta}_{14} (x_4 - K_{10})_+ + \hat{\beta}_{15} (x_4 - K_{11})_+ + \hat{\beta}_{16} (x_4 - K_{12})_+ +$$
$$\hat{\beta}_{17} x_5 + \hat{\beta}_{18} (x_5 - K_{13})_+ + \hat{\beta}_{19} (x_5 - K_{14})_+ + \hat{\beta}_{20} (x_5 - K_{15})_+ +$$

No	x_1	x_2	x_3	x_4	x_5	GCV
1	61,76	6,41	50,63	53,73	65,79	17,79
	74,94	8,02	66,56	69,74	77,63	
	98,38	10,88	94,89	98,22	98,68	
2	41,26	3,91	25,84	28,81	47,36	18,16
	63,23	6,59	52,40	55,51	67,10	
	67,62	7,13	57,71	60,84	71,05	
3	58,83	6,05	47,09	50,17	63,15	20,01
	60,30	6,23	48,86	51,95	64,47	
	89,59	9,81	84,27	87,54	90,79	
4	61,76	6,41	50,63	53,73	65,79	21,83
	76,41	8,20	68,33	71,52	78,95	
	98,38	10,88	94,89	98,22	98,68	
5	61,76	6,41	50,63	53,73	65,79	16,70
	73,48	7,84	64,79	67,96	76,31	
	98,38	10,88	94,89	98,22	98,68	

GCV MINIMUM

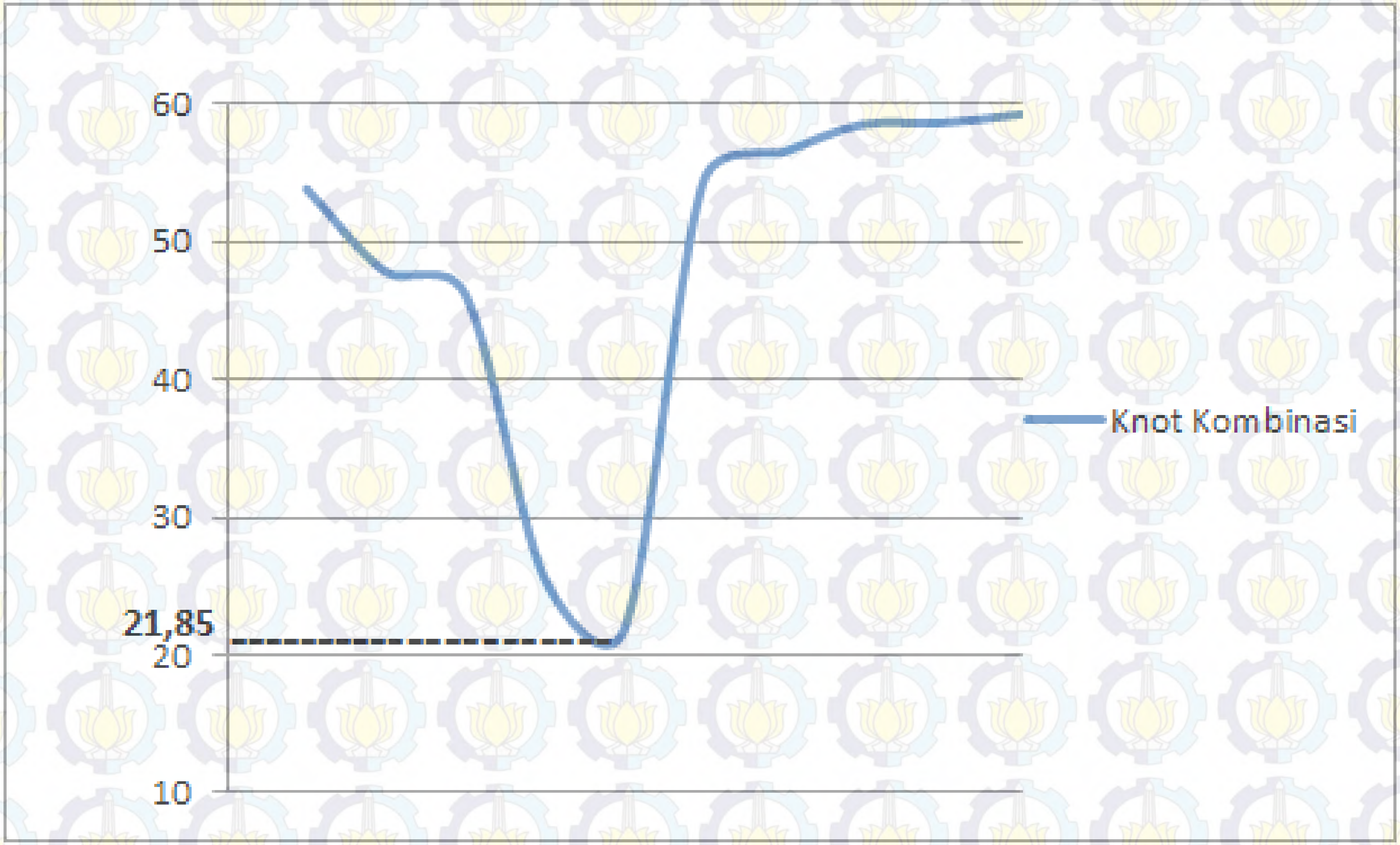
GCV Tiga Knot



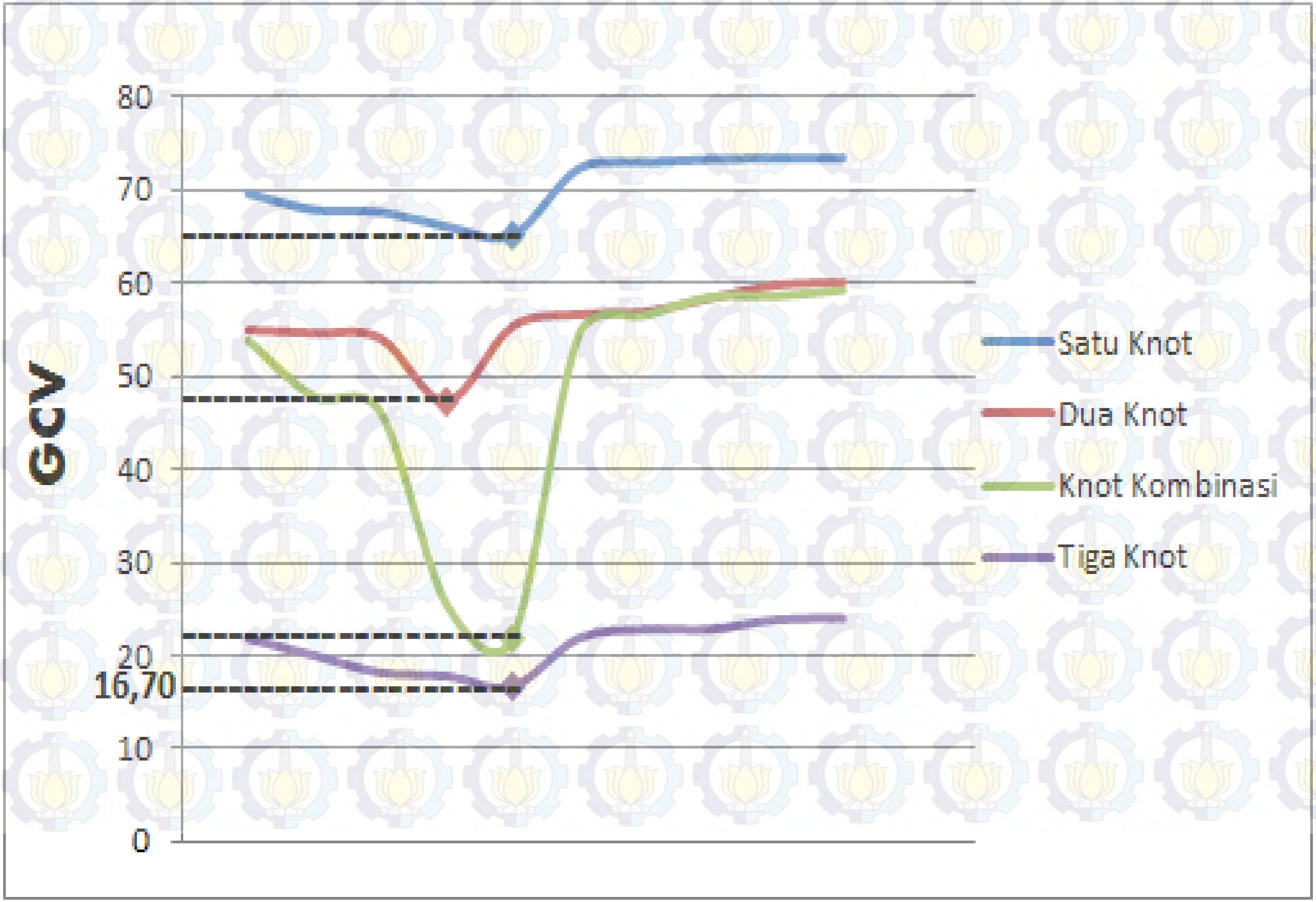
Pemilihan Titik Knot Optimum dengan Kombinasi Knot

No	Variabel	Variasi Titik Knot	Titik-Titik Knot	GCV
1	x_1	2	K_1=80,80; K_2=85,20	53,84
	x_2	1	K_3 = 9,27	
	x_3	2	K_5 = 73,64; K_6 =78,96	
	x_4	2	K_7 = 76,86; K_8 =82,20	
	x_5	2	K_9 = 21.00; K_{10} =78.90	
2	x_1	2	K_1 =80,80; K_2 =85,20	47,80
	x_2	1	K_3 =9,27	
	x_3	1	K_4 =78,96	
	x_4	2	K_5 =76,86; K_6 =82,20	
	x_5	2	K_7 =21.00; K_8 =78.90	
3	x_1	2	K_1 =80,80; K_2=85,20	46,17
	x_2	2	K_3 =8,74; K_4 =9,27	
	x_3	1	K_5 =78,96	
	x_4	2	K_6 =76,86; K_7 =82,20	
	x_5	2	K_8 =21.00; K_9 =78.90	
4	x_1	3	K_1 =61,76; K_2 =73,48; K_3 =98,38	25,52
	x_2	3	K_4 =6,41; K_5 =7,84; K_6 =10,88	
	x_3	2	K_7 =73,64; K_8 =78,96	
	x_4	3	K_9 =53,73; K_{10} =67,96; K_{11} =98,22	
	x_5	3	K_{12} =65,79; K_{13} =76,31; K_{14} =98,68	
5	x_1	3	K_1 =61,76; K_2 =73,48; K_3 =98,38	21,85
	x_2	3	K_4 =6,41; K_5 =7,84; K_6 =10,88	
	x_3	1	K_7 =78,96	
	x_4	3	K_8 =53,73; K_9 =67,96; K_{10} =98,22	
	x_5	3	K_{11} =65,79; K_{12} =76,31; K_{13} =98,68	

GCV Kombinasi Knot



GCV Satu, Dua, Tiga dan Kombinasi Knot



Model Terbaik dengan Tiga Titik Knot

$$\hat{y} = 145{,}11 + 0{,}64\, x_1 - 0{,}83\,(x_1 - 61{,}76)_+ - 0{,}39\,(x_1 - 73{,}48)_+ + 218{,}58\,(x_1 - 98{,}38)_+ +$$

$$-8{,}89\, x_2 + 13{,}97\,(x_2 - 6{,}41)_+ - 27{,}72\,(x_2 - 7{,}84)_+ - 2182{,}53\,(x_2 - 10{,}88)_+ +$$

$$+0{,}11\, x_3 + 0{,}41\,(x_3 - 50{,}63)_+ - 1{,}87\,(x_3 - 64{,}79)_+ + 8{,}31\,(x_3 - 94{,}89)_+ +$$

$$-5{,}82\, x_4 + 9{,}44(x_4 - 53{,}73)_+ - 1{,}63(x_4 - 67{,}96)_+ - 45{,}01(x_4 - 98{,}22)_+ +$$

$$+3{,}08\, x_5 - 10\,(x_5 - 65{,}79)_+ + 5{,}95\,(x_5 - 76{,}31)_+ + 70{,}24\,(x_5 - 98{,}68)_+ .$$

R^2 =98,46%

Pengujian Signifikansi Parameter

Serentak

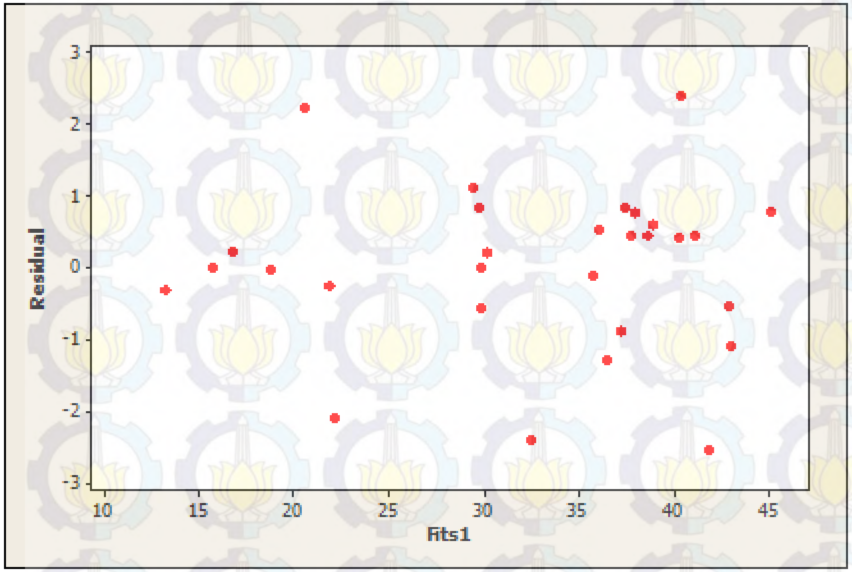
Sumber	Df	Sum of Square	Mean Square	F_{hitung}	P-value
Regresi	20	2363,076	118,1538	25.64944	0.000035
Error	8	36,85189	4,606486		
Total	28	2399,928			

Kurang dari nilai α (0.05). Dapat disimpulkan bahwa H_0 ditolak, maka minimal terdapat satu parameter yang signifikan terhadap variabel respon

Pengujian Signifikansi Parameter secara Individu

Variabel	Parameter	Koefisien	*p-value*	Keputusan
	β_0	145,11	0,000	Signifikan
x_1	β_1	0,64	0,001	Signifikan
	β_2	-0,83	0,098	Tidak Signifikan
	β_3	-0,39	0,427	Tidak Signifikan
	β_4	218,58	0,000	Signifikan
x_2	β_5	– 8,90	0,000	Signifikan
	β_6	13,97	0,001	Signifikan
	β_7	–27,72	0,001	Signifikan
	β_8	–2182,53	0,000	Signifikan
x_3	β_9	0,11	0,506	Tidak Signifikan
	β_{10}	0,41	0,263	Tidak Signifikan
	β_{11}	–1,87	0,018	Signifikan
	β_{12}	8,31	0,117	Tidak Signifikan
x_4	β_{13}	– 5,82	0,000	Signifikan
	β_{14}	9,44	0,000	Signifikan
	β_{15}	–1,63	0,084	Tidak Signifikan
	β_{16}	– 45,01	0,039	Signifikan
x_5	β_{17}	3,08	0,000	Signifikan
	β_{18}	–10,00	0,000	Signifikan
	β_{19}	5,95	0,000	Signifikan
	β_{20}	70,24	0,031	Signifikan

Pengujian Asumsi Residual

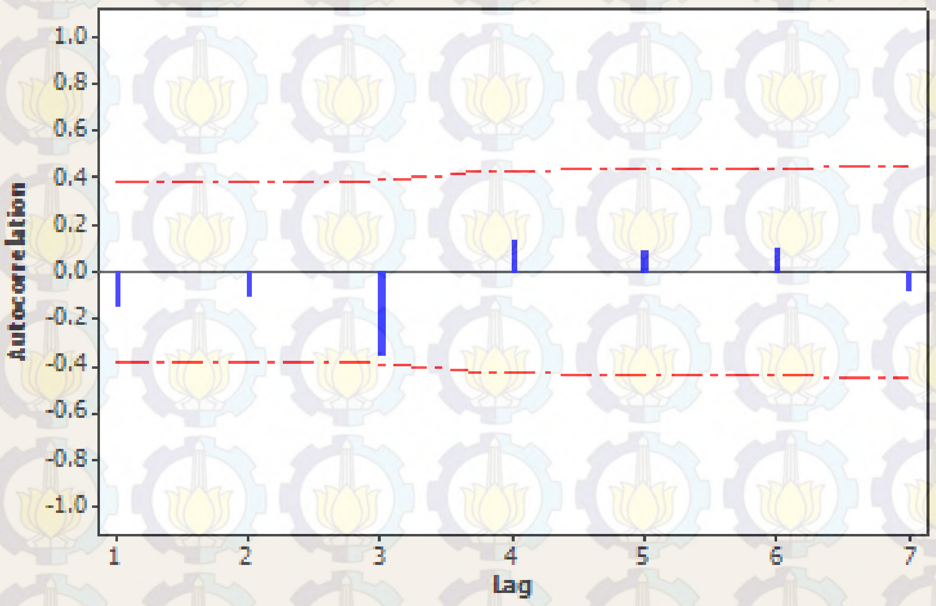


Identik

lebih besar dari nilai α(0.05). Sehingga dapat diputuskan bahwa H_0 gagal ditolak

Sumber	Df	*Sum of Square*	*Mean Square*	F_{hitung}	*P-value*
Regresi	20	10,68937	0,5344683	0,7323337	0,7299152
Error	8	5,838522	0,7298152		
Total	28	16,52789			

Pengujian Asumsi Residual



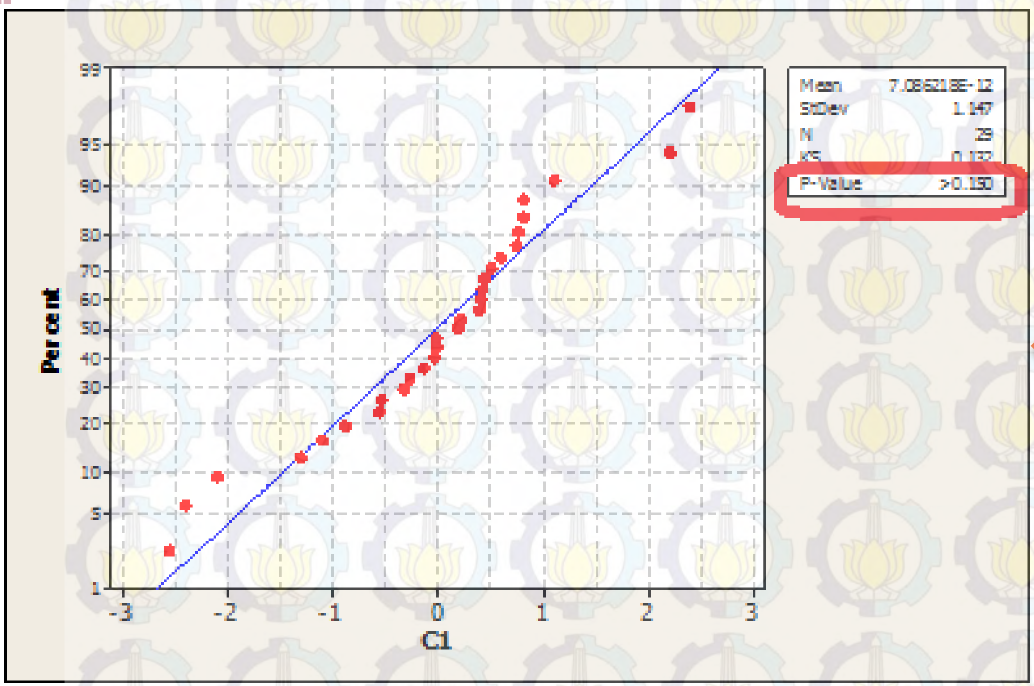
Idependen

Tidak ada lag yang keluar

H_0 tidak ditolak, maka residual telah memenuhi asumsi independen

Pengujian Asumsi Residual

Normal



lebih dari nilai α(0.05). Maka H_0 tidak ditolak, sehingga residual telah berditribusi normal.

MODEL REGRESI NONPARAMETRIK DERET FOURIER

Regresi Nonparametrik Deret Fourier untuk K=1

$$\hat{y}_i = \hat{\beta}_0 + \hat{b}_1 x_{1i} + \hat{\alpha}_{11} \cos x_{1i} + +\hat{b}_2 x_{2i} + \hat{\alpha}_{12} \cos x_{2i} + \hat{b}_3 x_{3i} + \hat{\alpha}_{13} \cos x_{3i}$$

$$+\hat{b}_4 x_{4i} + \hat{\alpha}_{14} \cos x_{4i} + \hat{b}_5 x_{5i} + \hat{\alpha}_{15} \cos x_{5i}$$

$$= 21,88 + 0,01x_{1i} - 1,76\cos x_{1i} - 0,76x_{2i} - 0,59\cos x_{2i} + 0,24x_{3i} + 0,99\cos x_{3i} +$$

$$-0,37x_{4i} - 9,30\cos x_{4i} + 0,35x_{5i} + 4,53\cos x_{5i}.$$

Parameter Osilasi (K)	GCV
1	214,27

Nilai GCV

Regresi Nonparametrik Deret Fourier untuk K=2

$$\hat{y}_i = \hat{\beta}_0 + \hat{b}_1 x_{1i} + \hat{\alpha}_{11}\cos x_{1i} + \hat{\alpha}_{21}\cos 2x_{1i} + \hat{b}_2 x_{2i} + \hat{\alpha}_{12}\cos x_{2i} + \hat{\alpha}_{22}\cos 2x_{2i} +$$

$$+\hat{b}_3 x_{3i} + \hat{\alpha}_{13}\cos x_{3i} + \hat{\alpha}_{23}\cos 2x_{3i} + \hat{b}_4 x_{4i} + \hat{\alpha}_{14}\cos x_{4i} + \hat{\alpha}_{24}\cos 2x_{4i} +$$

$$+\hat{b}_5 x_{5i} + \hat{\alpha}_{15}\cos x_{5i} + \hat{\alpha}_{25}\cos 2x_{5i}.$$

$$= 16,67 - 0,10x_{1i} - 2,31\cos x_{1i} + 1,39\cos 2x_{1i} + 0,85x_{2i} + 0,24\cos x_{2i} +$$

$$+1,39\cos 2x_{2i} + 0,34x_{3i} + 0,59\cos x_{3i} + 1,26\cos 2x_{3i} - 0,38x_{4i} +$$

$$-10,02\cos x_{4i} - 2,49\cos 2x_{4i} + 0,31x_{5i} + 3,85\cos x_{5i} + 1,77\cos 2x_{5i}.$$

Nilai GCV

Parameter Osilasi (K)	GCV
2	84,73

Regresi Nonparametrik Deret Fourier untuk K=3

$$\hat{y}_i = \hat{\beta}_0 + \hat{b}_1 x_{1i} + \hat{\alpha}_{11} \cos x_{1i} + \hat{\alpha}_{21} \cos 2x_{1i} + \hat{\alpha}_{31} \cos 3x_{1i}$$

$$+\hat{b}_2 x_{2i} + \hat{\alpha}_{12} \cos x_{2i} + \hat{\alpha}_{22} \cos 2x_{2i} + \hat{\alpha}_{32} \cos 3x_{2i}$$

$$+\hat{b}_3 x_{3i} + \hat{\alpha}_{13} \cos x_{3i} + \hat{\alpha}_{23} \cos 2x_{3i} + \hat{\alpha}_{33} \cos 3x_{3i}$$

$$+\hat{b}_4 x_{4i} + \hat{\alpha}_{14} \cos x_{4i} + \hat{\alpha}_{24} \cos 2x_{4i} + \hat{\alpha}_{34} \cos 3x_{4i}$$

$$+\hat{b}_5 x_{5i} + \hat{\alpha}_{15} \cos x_{5i} + \hat{\alpha}_{25} \cos 2x_{5i} + \hat{\alpha}_{35} \cos 3x_{5i}$$

Nilai GCV

Parameter Osilasi (K)	GCV
3	18,79

K	GCV	R^2	MSE
1	214,27	69,21%	25,48
2	84,73	72,61%	22,67
3	18,79	89,20%	8,94

Dengan Nilai GCV Minimum dan R^2 Maksimum maka Model yang digunakan yaitu Regresi Deret Fourier untuk K=3

Estimasi Parameter Deret Fourier untuk K=3

Estimasi	Parameter	Estimasi	Parameter	Estimasi	Parameter
$\hat{\beta}_0$	16,88	$\hat{\alpha}_{22}$	-4,95	$\hat{\alpha}_{14}$	-13,58
$\hat{b}_1$	-0,27	$\hat{\alpha}_{32}$	-0,04	$\hat{\alpha}_{24}$	-9,10
$\hat{\alpha}_{11}$	-0,02	$\hat{b}_3$	0,41	$\hat{\alpha}_{34}$	-9,48
$\hat{\alpha}_{21}$	0,80	$\hat{\alpha}_{13}$	6,35	$\hat{b}_5$	0,14
$\hat{\alpha}_{31}$	-3,66	$\hat{\alpha}_{23}$	-2,99	$\hat{\alpha}_{15}$	4,79
$\hat{b}_2$	3,69	$\hat{\alpha}_{33}$	0,49	$\hat{\alpha}_{25}$	10,28
$\hat{\alpha}_{12}$	-1,36	$\hat{b}_4$	-0,32	$\hat{\alpha}_{35}$	4,55

$$\hat{y}_i = 16,88 - 0,27 x_{1i} - 0,02 \cos x_{1i} + 0,80 \cos 2x_{1i} - 3,66 \cos 3x_{1i} +$$

$$+3,69 x_{2i} - 1,36 \cos x_{2i} - 4,95 \cos 2x_{2i} - 0,04 \cos 3x_{2i} +$$

$$+0,41 x_{3i} + 6,35 \cos x_{3i} - 2,99 \cos 2x_{3i} + 0,49 \cos 3x_{3i} +$$

$$-0,32 x_{4i} - 13,58 \cos x_{4i} - 9,10 \cos 2x_{4i} - 9,48 \cos 3x_{4i} +$$

$$+0,14 x_{5i} + 4,79 \cos x_{5i} + 10,28 \cos 2x_{5i} + 4,55 \cos 3x_{5i}.$$

HASIL PERBANDINGAN

Model	GCV	R^2	MSE
Spline Truncated	16,70	98,46%	4,61
Deret Fourier	18,79	89,20%	8,94

KESIMPULAN DAN SARAN

Kesimpulan

Saran

Kesimpulan

1.

Estimator kurva regresi nonparametrik multivariabel Spline *Truncated* diperoleh dari optimasi:

$$\underset{\underset{\sim}{\beta}\in R^{q(K+2)}}{Min}\left\{\sum_{i=1}^{n}\left(y_i-\sum_{j=1}^{p}f_j\left(x_{ji}\right)\right)^2\right\}$$

Optimasi ini menghasilkan estimator untuk kurva regresi Spline *Truncated*:

$$\hat{\mu}\left(x_{1i},x_{2i},\ldots,x_{pi}\right)=\sum_{j=1}^{p}\hat{f}_j\left(x_{ji}\right)=\sum_{j=1}^{q}\sum_{v=1}^{m}\hat{\beta}_{vj}x_{ji}^{v}+\sum_{j=1}^{q}\sum_{k=1}^{r}\hat{\beta}_{j(k+m)}\left(x_{ji}-K_{jk}\right)_{+}^{m}.$$

dimana $\hat{\beta}_{vj}$ dan $\hat{\beta}_{j(k+m)}$, $v=1,2,\ldots,m; k=1,2,\ldots,r,$ dan $j=1,2,\ldots,p$

Diperoleh dari:

$$\underset{\sim}{\hat{\beta}}=\left(\underset{\sim}{\hat{\beta}}_1{}',\ldots,\underset{\sim}{\hat{\beta}}_p{}'\right)'$$

$$\underset{\sim}{\hat{\beta}}_1=\left(\beta_{11},\ldots,\beta_{m1},\beta_{1(1+m)},\ldots,\beta_{1(r+m)}\right)',\ldots,\underset{\sim}{\hat{\beta}}_p=\left(\beta_{1p},\ldots,\beta_{mp},\beta_{p(1+m)},\ldots,\beta_{p(r+m)}\right)'$$

2.

Estimator kurva regresi nonparametrik multivariabel Deret Fourier diperoleh dari optimasi:

$$\underset{\underset{\sim}{\beta} \in R^{q(K+2)}}{Min} \left\{ \sum_{i=1}^{n} \left(y_i - \sum_{j=1}^{q} f_j \left(x_{ji} \right) \right)^2 \right\}$$

Optimasi ini menghasilkan estimator Deret Fourier:

$$\hat{\mu}\left(x_{1i}, x_{2i}, ..., x_{qi} \right) = \sum_{j=1}^{q} \hat{f}_j \left(x_{ji} \right) = \sum_{j=1}^{q} \left(\hat{b}_j x_{ji} + \frac{1}{2} \hat{\alpha}_{0j} + \sum_{k=1}^{K} \hat{\alpha}_{kj} \cos k x_{ji} \right)$$

dengan $\hat{b}_j, \hat{\alpha}_{0j}, \hat{\alpha}_{kj}; j = 1, 2, ..., q; k = 1, 2..., K.$ diberikan oleh persamaan:

$$\underset{\sim}{\hat{\beta}}(K) = \begin{bmatrix} \hat{b}_1 & \frac{1}{2}\hat{\alpha}_{01} & \hat{\alpha}_{11} & \cdots & \hat{\alpha}_{K1} & \vdots & \cdots & \vdots & \hat{b}_q & \frac{1}{2}\hat{\alpha}_{0q} & \hat{\alpha}_{1q} & \cdots & \hat{\alpha}_{Kq} \end{bmatrix}'$$

$$= \left(X'(K) X(K) \right)^{-1} X'(K) \underset{\sim}{y}.$$

3.

Model regresi nonparametrik Spline *Truncated* terbaik adalah dengan tiga titik knot. Berikut adalah model terbaik yang telah diperoleh.

$$\hat{y} = 145{,}11 + 0{,}64\, x_1 - 0{,}83\, (x_1 - 61{,}76)_+ - 0{,}39\, (x_1 - 73{,}48)_+ + 218{,}58\, (x_1 - 98{,}38)_+ +$$
$$-8{,}90\, x_2 + 13{,}97\, (x_2 - 6{,}41)_+ - 27{,}72\, (x_2 - 7{,}84)_+ - 2182{,}53\, (x_2 - 10{,}88)_+ +$$
$$+0{,}11\, x_3 + 0{,}41\, (x_3 - 50{,}63)_+ - 1{,}87\, (x_3 - 64{,}79)_+ + 8{,}31\, (x_3 - 94{,}89)_+ +$$
$$-5{,}82\, x_4 + 9{,}44 (x_4 - 53{,}73)_+ - 1{,}63 (x_4 - 67{,}96)_+ - 45{,}01 (x_4 - 98{,}22)_+ +$$
$$+3{,}08\, x_5 - 10\, (x_5 - 65{,}79)_+ + 5{,}95\, (x_5 - 76{,}31)_+ + 70{,}24\, (x_5 - 98{,}68)_+ .$$

4.

Model regresi nonparametrik Deret Fourier terbaik adalah dengan K=3. Berikut adalah model yang terbaik berdasarkan data kemiskinan di Provinsi Papua.

$$\hat{y}_i = 16{,}88 - 0{,}27 x_{1i} - 0{,}02 \cos x_{1i} + 0{,}80 \cos 2x_{1i} - 3{,}66 \cos 3x_{1i} +$$
$$+3{,}69 x_{2i} - 1{,}36 \cos x_{2i} - 4{,}95 \cos 2x_{2i} - 0{,}04 \cos 3x_{2i} +$$
$$+0{,}41 x_{3i} + 6{,}35 \cos x_{3i} - 2{,}99 \cos 2x_{3i} + 0{,}49 \cos 3x_{3i} +$$
$$-0{,}32 x_{4i} - 13{,}58 \cos x_{4i} - 9{,}10 \cos 2x_{4i} - 9{,}48 \cos 3x_{4i} +$$
$$+0{,}14 x_{5i} + 4{,}79 \cos x_{5i} + 10{,}28 \cos 2x_{5i} + 4{,}55 \cos 3x_{5i} .$$

5.

Berdasarkan pemodelan yang telah dilakukan menggunakan Spline Truncated dan Deret Fourier pada kasus kemiskinan di provinsi Papua maka dapat disimpulkan bahwa model Spline Truncated lebih baik daripada Deret Fourier. Nilai GCV dari Spline Truncated adalah 16,70 sedangkan nilai GCV dari Deret Fourier adalah 18,79. Sehingga nilai GCV Spline Truncated lebih minimum dibandingkan dengan Deret Fourier. Nilai R^2 pada Spline Truncated yaitu sebesar 98,46% sedangkan R^2 nilai pada Deret Fourier sebesar 89,20% dan nilai MSE Spline Truncated lebih kecil dibandingkan dengan Deret Fourier.

6.

Variabel-variabel yaitu angka melek huruf, rata-rata lama sekolah, berpendidikan kurang dari SD, bekerja di sektor pertanian dan bekerja di sektor informal memiliki pengaruh yang signifikan terhadap presentase kemiskinan di Provinsi Papua.

Saran

1. Perlu dilakukan lebih lanjut penelitian nonparametrik Deret Fourier untuk lebih dari satu respon yang kemudian dibandingkan dengan Spline *Truncated* dengan smoothing menggunakan *Penalized Least Square* (PLS).
2. Bagi pemerintah Provinsi Papua diharapkan agar lebih memperhatikan variabel – variabel yang memberikan suatu nilai tambah untuk peningkatan derajat kesejahteraan yang dapat dilihat dari hasil penelitian ini yakni mengenai faktor penduduk miskin.

DAFTAR PUSTAKA

- **Asrini, Luh Juni. (2012), “Regresi Semiparametrik Deret Fourier”, *Prosiding Seminar Nasional FMIPA Universitas Negeri Surabaya*, hal. 77-80.**
- **Asrini, Luh Juni and Budiantara, I.,N. (2014), “Fourier Series Semiparametric Regression Models (Case Study: The Production of Law Land Rice Irrigation in Central Java)”. *ARPN Journal of Engineering and Applied Sciences*, Vol. 9, hal. 1501-1506.**
- **Billier, C., and Fahrmeir, L. (2000), “Bayesian varying-coefficient models using adaptive regression Spline, Statistical Modeling”, http://citeseer.ist.psu.edu/biller00bayesian. html.**
- **Bilodeau, M. (1992), “Fourier Smoother and Additive Models”, *The Canadian Journal of Statistics*, Vol.3, hal.257-269.**
- **Budiantara, I., N. (2000), *Estimator Spline dalam Regresi Nonparametrik dan Semiparametrik,* Disertasi, Universitas Gajah Mada. Yogyakarta.**
- **Budiantara, I., N. (2007), “Inferensi Statistik untuk Model Spline”, *Jurnal Mat-Stat*, Vol. 7, hal.1-14.**
- **Budiantara, I.,N. (2009), *Spline dalam Regresi Nonparametrik dan Semiparametrik: Sebuah Pemodelan Statistika Masa Kini dan Masa Mendatang*, Institut Teknologi Sepuluh Nopember Surabaya.**

DAFTAR PUSTAKA

- Chernichovsky,D. dan Meesok, O.,A. (1985), *Urban-rural Food and Nutrition Consumption Pattern in Indonesia*, PHN Technical Note 85-5, July 1985, World Bank.
- Craven, P. and Wahba, G. (1979), “Smoothing Noisy Data with Spline Functions”, *Numerische Mathematics*, Vol. 31, hal.377-403.
- Crainiceanu, C. M. Ruppert, D., and Wand, M.P. (2004), “Bayesian Analysis for Penalized Spline Regression Using Wenbugs”, *Statistical Software,* Vol.14, No.14, hal.6-14.
- Damayanti,Y. (2013), *Pemodelan Penduduk Miskin di Jawa Timur Menggunakan Metode Geographically Weighted Regression (GWR)*, Tugas Akhir, Jurusan Statistika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Institut Teknologi Sepuluh Nopember, Surabaya.
- Draper, N.R., and Smith, H. (1992), *Analisis Regresi Terapan*, PT Gramedia Pustaka Utama Jakarta.
- Ekasari, Dewi F. (2012), *Pemodelan SEM dengan Generalized Structured Component Analysis (GSCA) (Studi Kasus: Penentuan Struktur Model Kemiskinan Kabupaten/Kota di Provinsi Jawa Tengah)*, Tugas Akhir, Jurusan Statistika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Institut Teknologi Sepuluh Nopember, Surabaya.
- Eddy S.S., dan Agung. (2010), “Faktor-faktor yang Mempengaruhi Kemiskinan secara Makro di Lima Belas Provinsi Tahun 2007”, *Jurnal dan Organisasi Manajemen,* Vol.6, No.2, hal.89-100.

DAFTAR PUSTAKA

- Eubank,R.L. (1988), *Spline Smoothing and Nonparametric Regression*, Mercel Dekker, New York.
- Eubank, R. L. (1999), *Nonparametric Regression and Spline Smoothing,* Second Edition, New York, Marcal Dekker, Inc.
- Eubank, R. L., & Thomas, W. (1993), "Detecting Heterocedasticity in Nonparametric Regression". *Journal of the American Statistical Association*, 387-392.
- Fadillah. (2010), *Analisis Regresi Jumlah Penduduk Miskin dengan Faktor – Faktor yang Mempengaruhinya di Jawa Timur*, Tugas Akhir, Jurusan Statistika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Institut Teknologi Sepuluh Nopember, Surabaya.
- Fatriana, M. (2014). *Pemodelan Regresi Nonparametrik Spline Truncated dan Aplikasinya pada Angka Kelahiran Kasar di Surabaya*, Tugas Akhir, Jurusan Statistika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Institut Teknologi Sepuluh Nopember, Surabaya.
- Green, P.,J. and Silverman, B.,W. (1994), *Nonparametric Regression and Generalized Linear Models*, Chapman & Hall, London.
- Gujarati, D. N. (2004), *Basic Econometric 4th edition*. New York: The Mc Gra Hill Companies.
- Haerdle, W. (1990), *Applied Nonparametric Regression*, Cambrige University Press: New York.

DAFTAR PUSTAKA

- **Holmes, C.,C., and Mallick B.,K. (2003), "Generalized Nonlinear Modelling With Multivariate Free-Knot Regression *Spline*", *Journal of the American Statistical Association*, Vol.98, hal.462.**
- **Howell, J. R,. (2007), *Analysis Using Smoothing Splines As Implemented in L M E O In R.* Bringham Young University.**
- **Huang, Z. (2003), "Local Asymptotic for Polynomial Spline Regression", *The Annals of Statistics,* Vol.31, No.5, hal.1600-1635.**
- **Kohn, R., Ansley C. F., Tharm, D. (1991), "The Performance of Cross Validation and Maximum Likelihood Estimators of Spline Smooting Parameters", *Journal of The American Statistical Assosiations*, Vol.86, hal.1042-1050.**
- **Kim, Y.J and Gu, C. (2004), "Smoothing Spline Gaussian Regression: More Sealable Computation Via Efficient Approximation", *Royal Statistical Society.* Series B, Vol. 66, No.2, hal.337-356.**
- **Lee, T. C. M. (2004), "Improved Smoothing Spline Regression by Combining Estimatics of Different Smoothness", *Statistics and Probability Letters,* Vol. 67, hal.133-140.**
- **Li, L. (1986), *Nonlinear Waveled-Based Nonparametric Curve Estimation with Consored Data and Inference on Long Memory Processes,* Midingan State University.**

DAFTAR PUSTAKA

- **Mubarak, Reza. (2012).** ***Analisis Regresi Spline Multivariabel untuk Pemodelan Kematian Penderita Demam Berdarah Dengeu (DBD) di Jawa Timur*****. Tugas Akhir, Jurusan Statistika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Institut Teknologi Sepuluh Nopember, Surabaya.**
- **Pane, Rahmawati., Budiantara, I.N., Zain, Ismaini., dan Otok, Bambang Widjanarko. (2014), "Parametric and Nonparametric Estimators in Fourier Series Semiparametric Regression and Their Characteristics".** ***Applied Mathematical Sciences*****, Vol.102, No.8, hal. 5053-5064.**
- **Pintowati,W dan Bambang W.,O. (2012). "Pemodelan Kemiskinan di Provinsi Jawa Timur dengan Pendekatan Multivariate Adaptive",** ***Jurnal Sains dan Seni ITS*****, Vol.1, No.1.**
- **Prahutama, Alan. (2013), "Model Regresi Nonparametrik dengan Pendekatan Deret Fourier pada Kasus Tingkat Pengangguran Terbuka di Jawa Timur",** ***Prosiding Seminar Nasional Statistika Undip*****, Vol. 10, hal. 69-76.**
- **Prasetyawan, Iwan F. (2011).** ***Penentuan Matriks Pembobot yang Optimum pada Pemodelan Geographically Weighted Regression (GWR) (Studi Kasus: Penyusunan Model Kemiskina di Jawa Tengah*****, Tugas Akhir, Jurusan Statistika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Institut Teknologi Sepuluh Nopember, Surabaya.**
- **Rencher, A, C. (2002),** ***Methods of Multivariate Analysis.Second Edition,*** **John Wiley &Sons, Inc.New York.**

DAFTAR PUSTAKA

- **Rencher, Alvin., dan Schaalje, G.B. (2008), *Linear Models in Statistics 2nd Edition*, John Willey and Sons Inc., New Jersey.**
- **Rusdarti dan Karolina. (2013),"Faktor-faktor yang Mempengaruhi Tingkat Kemiskinan di Provinsi Jawa Tengah", *Jurnal Economia*,Vol.9, No.1.**
- **Samsodin, Mohamad. (2012), *Regresi Spline Polynomial Truncated Multirespon untuk Pemodelan Indikator Kemiskinan di Provinsi Jawa Timur*, Tugas Akhir, Jurusan Statistika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Institut Teknologi Sepuluh Nopember, Surabaya.**
- **Searle, S.R. (1971), *Linear Models*, John Willey and Sons Inc., New York.**
- **Semiati, Rini. (2010), *Regresi Nonparametrik Deret Fourier Birespon*, Tesis, Jurusan Statistika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Institut Teknologi Sepuluh Nopember, Surabaya.**
- **Seruni P.,S., Putu, dan Sutrisna, Ketut. (2014), "Pengaruh PDRB per Kapita, Pendidikan dan Produktivitas Tenaga Kerja terhadap Kemiskinan di Provinsi Bali", *E-Jurnal EP Unud*, Vol.3, No.10, hal. 431-439**
- **Shao, J. (1993), "Linear Model Selection by Cross Validation", *Journal of the American Statistical Association*, Vol. 88, hal 486-494.**
- **Surya,L. (2013), Faktor – Faktor yang Mempengaruhi Persentase Penduduk Miskin di Jawa Timur Menggunakan Regresi Semiparametrik Spline, Tugas Akhir, Jurusan Statistika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Institut Teknologi Sepuluh Nopember, Surabaya.**

DAFTAR PUSTAKA

- **Tjahjono, Eko. (2009), *Estimator Deret Fourier Terbobot pada Regresi Nonparametrik*, Tesis, Jurusan Statistika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Institut Teknologi Sepuluh Nopember, Surabaya.**
- **Tripena, A. and Budiantara, I N., (2007), "Fourier Estimator in Nonparametric Regression", *International Conference On Natural Sciences and Applied Natural Scienes*, Ahmad Dahlan University, Yogyakarta.**
- **Tripena, A. (2013), "Estimator Deret Fourier untuk Estimasi Kurva Regresi Nonparametrik Birespon", *Magistra*, Vol. 84, hal. 1-15.**
- **Venter, J.H., and Snyman, J.L.J. (1995), "A Note on the Generalized Cross Validation Criterion in Linear Model Selection", *Biometrika*, Vol.82, hal.215-219.**
- **Wahba, G. (1985), "A Comparizon of GVC and GML for Choosing The Smoothing Parameter in Generalized Spline Smoothing Problem", The *Annal of Statistic*, Vol.13, hal.1378-1402.**
- **Wahba, G. (1990), "Spline Models For Observational Data", *SIAM, CBMS-NSF Regional Conference Series in Applied mathematics*, Philadelphia.**
- **Walpole. (1995), *Pengantar Statistika*. Jakarta: PT. Gramedia Utama.**
- **Wei, W. W. (2006), *Time Series Analisis: Univariate and Multivariate*. USA: Pearson Education Inc.**

TERIMA KASIH