# Design Of Position Estimation Algorithm Of Navigation And Trajectory System For Unmanned Underwater Vehicle Its Auv-01 Using Ensemble Kalman Filter (Enkf) Method

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Abstract -Unmanned submarine is one of the unmanned water vehicle that are controlled automatically and included inAutonomous (AUV)Underwater Vehicle groups. underwater surface operating area, AUV required a navigation system that can be maneuvered 6 degree of freedom (multi DOF) which is projected in the 3x2D projection planes (XY, XZ, and YZ plane) and able to estimate position exactly match the desired trajectory. The navigation system itself is the coordination of planning, sensing and control in trajectory from initial position to the target without a collision or ability to avoid existing obstacle. The result of Ensemble Kalman Filter algorithm implementation during the simulation using 2 or 3 measurement data from 100~400 ensemble shows that in estimating position of AUV trajectory are proficient to produce a relatively small RMS Error (0~1) in each state.

**Keywords** :navigation system, autonomous underwater vehicle, path, ensemble kalman filter, trajektory, multi DOF.

# 1. INTRODUCTION

Unmanned underwater vehicle is now widely used in several fields. Its ability to be controlled, positioning that can also to be monitored from long distance even able to be programmed to move itself by a particular track is very beneficial to humans. However, there are many difficulties to be overcome before unmanned underwater vehicle is driven as desired, such as internal disturbance (interaction between the compass with the magnetic field generated by the motor) and external disturbance (turbulence of ocean current that flow through the submarine). Thus required estimating the right position so the submarine can perform the desired function.

Ensemble Kalman Filter (EnKF) is a development of the Kalman Filter (KF) algorithm which has the common function that is estimate some cases using model and measurement system. The differences between the KF and EnKF was first introduced by Rudolph E. Kalman (1960) which the KF algorithm can only be implemented on a linear

dynamic model while EnKF algorithm can be used both linear and nonlinear dynamics. EnKF algorithm works by generating a specific ensemble to calculate the mean and covariance variable error each state. In this research, EnKF method used to estimate the position of the submarine according its trajectory with better accuracy.

# 2. LITERATURE REVIEW

#### 2.1 AUV Kinematics

Two important things needed to analyze the Autonomous Underwater Vehicle (AUV) are Earth Fixed Frame (EFF) and Body Fixed Frame (BFF) (see picture 2.1) [4,8]. AUV movement has six degrees of freedom (6 DOF) with three degrees of freedom for the translation motion and the rest three degrees of freedom for rotational motion on the axis x, y and z. General description of the AUV in 6 DOF as follows [8]:

• Position and Euler Angle

$$\eta = [\eta_1^T, \eta_2^T]^T, \ \eta_1 = [x, y, z]^T \eta_2 = [\phi, \theta, \psi]^T$$

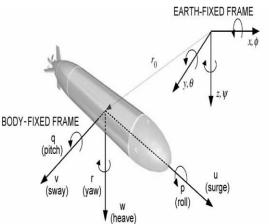
• Linier and Anguler Velocity

$$v = [v_1^T, v_2^T]^T$$
,  $v_1 = [u, v, w]^T$ ,  $v_2 = [p, q, r]^T$ 

• Forces and Moments

$$\tau = [\tau_1^T, \tau_2^T]^T, \ \tau_1 = [X, Y, Z]^T, \ \tau_1 = [K, M, N]^T$$
Table 1 AUV Coordinates

DOF	KETERA NGAN	GAYA / MOMEN	KEC LINIER / ANGULA R	POSISI / SUDUT EULER
1	surge	X	u	X
2	Sway	Y	v	Y
3	Heave	Z	w	Z
4	Roll	K	p	Φ
5	Pitch	M	q	Θ
6	Yaw	N	r	$\Psi$



Picture 2.1 Six Degrees of Freedom Submarine Motion[13] In the AUV dynamics, existing external forces that affect the movement are:

$$\tau = \tau_{hidrostati} + \tau_{addedmass} + \tau_{drag} + \tau_{lift} + \tau_{control}...(1)$$
The AUV general equation of motion in 6 DOF

consists of three equation of translation motion and three equation of rotational motion, which as follows [4,8]:

SURGE:

$$m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] \approx X$$
  
...(2.a)

$$m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})] \approx Y$$
...(2.b)

HEAVE:

HEAVE:  

$$m[\dot{w}-uq+vp-z_G(p^2+q^2)+x_G(rp-\dot{q})+y_G(rq+\dot{p})]\approx Z$$
  
...(2.c)

ROLL:

$$\begin{split} I_{x}\dot{p} + & (I_{z} - I_{y})qr - (\dot{r} + pq)I_{xz} + (r^{2} - q^{2})I_{yz} + (pr - \dot{q})I_{xy} + \\ & m \big[ y_{G}(\dot{w} - uq + vp) - z_{G}(\dot{v} - wp + ur) \big] \approx K \dots (2.d) \end{split}$$
 PITCH:

$$I_{y}\dot{q} + (I_{x} - I_{z})rp - (\dot{p} + qr)I_{xy} + (p^{2} - r^{2})I_{zx} + (qp - \dot{r})I_{yz} + m[z_{G}(\dot{u} - vr + wq) - x_{G}(\dot{w} - uq + vp)] \approx M \dots (2.e)$$
YAW:

$$I_{z}\dot{r} + (I_{y} - I_{x})pq - (\dot{q} + rp)I_{yz} + (q^{2} - p^{2})I_{xy} + (rq - \dot{p})I_{zx} + m[x_{G}(\dot{v} - wp + ur) - y_{G}(\dot{u} - vr + wq)] \approx N$$
(2.f)

In above equation can be write in more simple form,

$$M_{RB}\dot{v} + C_{RB}(v)v = \tau$$
 ...(3)  
where  $v = [u, v, w, p, q, r]^T$  as linear and anguler  
velocity vector,  $\tau = [X, Y, Z, K, M, N]^T$  as external  
forces and moments working on the AUV,  $M_{RB}$  as  
inertia matrix and  $C_{RB}$  as coriolis and centripetal  
matrix.

# 2.2 External Forces and Moments

#### 1. Hidrostatic Forces

Every objects in the water will have a hydrostatic forces consisting of gravity and buoyancy forces.

2.

ydrodynamics Forces

Hydrodynamic force components consists of added mass force, drag force, and lift force.

hrust Force

When the AUV using fin to control its balance would required a constant velocity. Propeller generating force and moment to the surge direction only due to single propeller was used. The generated force and moment can be written as follows [4,8]:

$$F_{t} = \frac{1}{2} \rho D^{4} K_{T}(J) \left| \omega_{p} \right| \omega_{p} \qquad \dots (4)$$

$$M_{t} = \frac{1}{2} \rho D^{5} K_{T}(J) \left| \omega_{p} \right| \omega_{p} \qquad \dots (5)$$

With  $\rho$  as the density fliud, D as propeller diameter,  $\omega_p$  as anguler velocity,  $K_T$  as thrust force coefficient,  $J = \frac{V_a}{\omega_v D}$  as advance number,  $V_a = (1 - \alpha)u$  as the propeller advance speed,  $\alpha$  as wake fraction number

#### 2.3 Forces and Moments Total

is about 0,1~0,4.

By combining the equations of hydrostatic force, lift force, added mass force, drag force, thrust force, and diagonal of inertia tensor  $(I_o)$  assuming is zero than the total forces and moments obtained from all model as follows:

Translation along x direction:

$$m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X_{res} + X_{|u|u}u|u| + X_u\dot{u} + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop}$$
...(6.a)

Translation along y direction:

$$m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(pq + \dot{r})] = Y_{res} + Y_{|v|v}v|v| + Y_{r|r|}r|r| + Y_{v}\dot{v} + Y_{r}\dot{r} + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r$$
...(6.b)

Translation along z direction : 
$$m[\dot{w}-uq+vp-z_G(p^2+q^2)+x_G(rp-\dot{q})+y_G(rq+\dot{p})]=Z_{res}+Z_{|w|w}w|w|+Z_{q|q|}q|q|+Z_{w}\dot{w}+Z_{q}\dot{q}+Z_{uq}uq+Z_{vp}vp+Z_{rp}rp+Z_{uw}uw+Z_{uu\delta_s}u^2\delta_s$$
 ...(6.c)

Rotation along x direction:

$$I_x \dot{p} + (I_z - I_y)qr + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = K_{res} + K_{p|p|}p|p| + K_p \dot{p} + K_{prop}$$
(6 d)

Rotation along y direction:

$$\begin{split} I_{y}\dot{q} + & (I_{x} - I_{z})rp + m[z_{G}(\dot{u} - vr + wq) - \\ & x_{G}(\dot{w} - uq + vp)] = M_{res} + M_{w|w|}w|w| + \\ & M_{q|q|}q|q| + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uq}uq + M_{vp}vp + \\ & M_{rp}rp + M_{uw}uw + M_{uu\delta_{s}}u^{2}\delta_{s} \\ & \dots (6.e) \end{split}$$

Rotation along z direction:

$$I_z \dot{r} + (I_y - I_z)pq + m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = N_{res} + N_{v|v|}v|v| + N_{r|r|}r|r| + N_{\phi}\dot{v} + N_{r}\dot{r} + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv + N_{uu\delta_r}u^2\delta_r$$

...(6.f)

AUV type [4] only using a single propeller on the tail of AUV where it will producing a thrust  $X_{prop}$  and additional moment force  $K_{prop}$ .

#### 2.4 Ensemble Kalman Filter Method

Ensemble Kalman Filter (EnKF) method is a modification estimation method of Kalman Filter algorithms that can be used to estimate the linear and nonlinear model system. EnKF method was introduced by Evensen in 1994 by generating a number of ensemble at prediction phase to estimate the covariance errors.

Lewis (2006) gives the form of EnKF algorithms in estimating the nonlinear dynamic systems and linear measurements, as found in Table 2.

Table 2. Ensemble Kalman Filter Algorithm

System and Measurement Model:

$$\begin{aligned} x_{k+1} &= f(x_k, u_k) + w_k \\ z_k &= Hx_k + v_k \\ w_k &\sim N(0, Q_k), v_k \sim N(0, R_k) \end{aligned}$$

Initialization:

- Generate N ensemble according to the initial guesses

$$x_{0,i} = [x_{0,1} \quad x_{0,2} \quad x_{0,3} \quad \cdots \quad x_{0,N}]$$

- Determine the initial value :

$$\widehat{x}_k = \widehat{x}_k^* = \frac{1}{N} \sum_{i=1}^N x_{0,i}$$

Time update : 
$$\widehat{x}_{k,i}^- = f(\widehat{x}_{k-1}, u_{k-1}) + w_{k,i} w_{k,i} \sim N(\mathbf{0}, Q_k)$$

- Estimation : 
$$\widehat{x}_k^- = \frac{1}{N} \sum_{i=1}^N \widehat{x}_{k,i}^-$$

- Covariance error : 
$$P_{k}^{-} = \frac{1}{N-1} \sum_{i=1}^{N} (\widehat{x}_{k,i} - \widehat{x}_{k}^{-}) (\widehat{x}_{k,i} - \widehat{x}_{k}^{-})^{T}$$
Measurement update :

Measurement update:

$$\mathbf{z}_{k,i} = \mathbf{z}_k + \mathbf{v}_{k,i}, \mathbf{v}_{k,i} \sim N(\mathbf{0}, \mathbf{R}_k)$$

$$\begin{aligned} & z_{k,i} = z_k + v_{k,i}, v_{k,i} \sim N(0, R_k) \\ & \text{- Kalman gain:} \\ & K_k = P_k^- H^T (H P_k^- H^T + R_k)^{-1} \end{aligned}$$

- Estimation : 
$$\widehat{x}_{k,i} = \widehat{x}_{k,i}^{T} + K_{k}(\mathbf{z}_{k,i} - H\widehat{x}_{k,i}^{T})$$

$$\widehat{x}_{k} = \frac{1}{N} \sum_{i=1}^{N} \widehat{x}_{k,i}$$

- Covariance error :

$$P_k = [1 - K_k H] P_k^-$$

# 2.5 Finite Different Methods

If 
$$u = u(x)$$
 is expanded in a Taylor series, then:  

$$u(x+h) = u(x) + h \frac{\partial}{\partial x} u(x) + \frac{h^2}{2!} \frac{\partial^2}{\partial x^2} u(x) + \cdots \qquad (7)$$

$$u(x-h) = u(x) - h \frac{\partial}{\partial x} u(x) + \frac{h^2}{2!} \frac{\partial^2}{\partial x^2} u(x) - \cdots \qquad (8)$$

$$u(x-h) = u(x) - h \frac{\partial}{\partial x} u(x) + \frac{h^2}{2!} \frac{\partial^2}{\partial x^2} u(x) - \cdots$$
 (8)

Some of the numerical scheme of finite different method, that is:

1.

orward Finite Different Method

From equation (7) and (8) are obtained: 
$$\frac{\underline{u(x+h)-u(x)}}{h} \approx \frac{\partial u}{\partial x} \qquad ...(9)$$

if using a finite different notation with U(x = ih) in the form above, then:

$$\frac{\partial u}{\partial x} = \frac{u_{i+1} - u_i}{\Delta x} \qquad \dots (10)$$

From equation (7) and (8) are obtained:

ackward Finite Different Method

$$\frac{u(x-h)-u(x)}{h} \approx \frac{\partial u}{\partial x} \qquad ...(11)$$

if using a finite different notation with U(x = ih) in the form above, then:

$$\frac{\partial u}{\partial x} = \frac{u_i - u_{i-1}}{\Delta x} \qquad \dots (12)$$

enter Finite Different Method

From equation (7) and (8) are obtained:

$$\frac{u(x+h)-u(x-h)}{2h} \approx \frac{\partial u}{\partial x} \qquad \dots (13)$$

if using a finite different notation with U(x = ih) in the form above, then:

$$\frac{\partial u}{\partial x} = \frac{u_{i+1} - u_{i-1}}{2\Delta x} \qquad \dots (14)$$

 $\frac{\partial x}{\partial x} = \frac{1}{2\Delta x}$ In this research, forward finite different was used to estimate position of submarine.

#### 3. RESEARCH METHODS

Research conducted using a simulation in the navigation and trajectory system planning of unmanned underwater vehicle ITS AUV 01 by using Matlab software. This research consists of several stages as follows:

- 1. Literature review on planning for ITS AUV 01 specification of dimensions and dynamics of motion.
- 2. Obtain the equation of motion in 6 DOF of AUV.
- 3. Examine the EnKF method, finite different method, and dynamic of motion model of AUV before implementing in the form of Matlab program.
- 4. Implementation of EnKF to the 6 DOF equation of motion of AUV.
- 5. Analyzing the result of simulation program.
- 6. Draw the conclusions of the analysis and simulation result.

# 4. ANALYSIS AND DISCUSSION

### 4.1 Discretization Model

AUV translation equations of motion should be converted in the form discretization because EnKF algorithm can be implemented only for discrete systems. Changes in variables state over the time is approximated by using a forward finite different method. Thus, obtained:

$$\dot{u} = \frac{du}{dt} \approx \frac{u_{k+1} - u_k}{\Delta t} \qquad ...(15.a)$$

$$\dot{v} = \frac{dv}{dt} \approx \frac{v_{k+1} - v_k}{\Delta t} \qquad F \qquad ...(15.b)$$

$$\dot{w} = \frac{dw}{dt} \approx \frac{w_{k+1} - w_k}{\Delta t} \qquad ...(15.c)$$

$$\dot{v} = \frac{dv}{dt} \approx \frac{v_{k+1} - v_k}{\Lambda t} \qquad \qquad F \qquad \dots (15.b)$$

$$\dot{w} = \frac{dw}{dt} \approx \frac{w_{k+1} - w_k}{\Delta t} \qquad \dots (15.c)$$

Since only the components u, v, and w are used from the AUV equation of dynamics then the result od discretization as follows:

$$\begin{bmatrix} u_{k+1} \\ v_{k+1} \\ w_{k+1} \end{bmatrix} = \begin{bmatrix} \left( \frac{(X_{res} + X_{u}u_{k} + X_{yroy})}{(m - X_{u})} \right) \Delta t + u_{k} \\ \left( \frac{(Y_{res} + Y_{v}v_{k} + (Y_{uvl} + Y_{uvl})u_{k}v_{k} + Y_{uuS_{r}}u_{k}^{2} \delta_{r})}{(m - Y_{v})} \right) \Delta t + v_{k} \\ \left( \frac{(Z_{res} + Z_{w}w_{k} + (Z_{ubvl} + Z_{uvrf})u_{k}w_{k} + Z_{uuS_{s}}u_{k}^{2} \delta_{s})}{(m - Z_{w})} \right) \Delta t + w_{k} \end{bmatrix}$$

$$(16)$$

Elaboration of  $X_{res}$ ,  $Y_{res}$ ,  $Z_{res}$  forces are contained in the equation of hydrostatic force,  $X_u$ ,  $Y_v$ ,  $Z_w$ ,  $Y_{uvl}$ ,  $Y_{uvl}$ ,  $Z_{uwl}$ ,  $Z_{uwl}$ ,  $Y_{uudr}$ , and  $Z_{uuds}$  forces are found in [13]. While the  $X_{prop}$  force in the equation of thruster.

# 4.2 Addition of Stochastic Factors

Equation of motion of AUV model is still in the form of deterministic so required added stochastic factors in the form of noise in each equation. Thus the stochastic model used in the following forms:

$$x_{k+1} = f(x_k, u_k) + w_k$$
 ...(17)  
 $z_k = Hx_k + v_k$  ...(18)

where  $f(x_k, u_k)$  as a nonlinear function. As noted in chapter 2 that the noise was used consisting of system noise  $w_k$  and measurement noise  $v_k$ . In this case, both noise are usually taken in the normally distributed form and have zero mean value during the calculation by Matlab software.

# 4.3 Implementation of ITS AUV Model in the EnKF Algorithm

The steps carried out include:

- a. Defining the component of X along with the initial value for each component are the position of surge  $(u_0)$ , sway  $(v_0)$ , dan heave  $(w_0)$ .
- b. Determine the model and measurement system based on the model used.
- c. In the initialization phase, noise was given and make the average value for each state from the generating of initial ensemble along with finding the ensemble errors.
- d. In the prediction phase, calculate the predicted values with the addition of noise system  $w_k$  where  $w_k \sim N(0, Q_k)$ . Next is to find the mean value of the ensemble, the value of ensemble error and the ensemble covariance error.
- e. In the correction phase, started by calculating the measurement data along with the addition of measurement noise  $v_k$  where  $v_k \sim N(0, R_k)$ .

#### 4.4 Simulation dan Analysis

The simulation was conducted using the parameters variation of number of measurement data (2 or 3) and the number of ensemble (100, 200, 300, and 400) which each simulation performed a total of 1000x iterations. In addition, simulation conducted for each level of ensemble as much as ten times.

# 3 Parameter

a. 100 ensemble

Data from first simulation providing a value of RMS Error for the state  $u=0.0011479;\ v=0.11528;$  and w=0.098743.

#### b. 200 ensemble

Data from the third simulation providing a value of RMS Error for the state  $u=0.01333;\ v=0.09373;\ and\ w=0.10636.$ 

#### c. 300 ensemble

Data from the  $6^{th}$  simulation providing a value of RMS Error for the state u=0.01481; v=0.09204; and w=0.10688.

#### d. 400 ensemble

Data from the  $8^{th}$  simulation providing a value of RMS Error for the state  $u=0.01286;\ v=0.09404;$  and w=0.09609.

#### 2 Parameter

#### a. 100 ensemble

Data from first simulation providing a value of RMS Error for the state  $u=0.08453;\,v=0.09141;\,\text{and}\,w=0.09319.$ 

#### b. 200 ensemble

Data from the  $4^{th}$  simulation providing a value of RMS Error for the state u=0.21771; v=0.09522; and w=0.1009.

#### c. 300 ensemble

Data from the first simulation providing a value of RMS Error for the state  $u=0.13319;\ v=0.08834;$  and w=0.09036.

#### d. 400 ensemble

Data from the  $6^{th}$  simulation providing a value of RMS Error for the state  $u=0.06912;\ v=0.10496;$  and w=0.10582.

# \* RMSE Gabungan

Table 3 The Combination of Mean Value of RMS Error

Mean Value of RMSE											
100		200		300		400					
2	3	2	3	2	3	2	3				
0,4657915	0,0131269	0,5267845	0,0133642	0,6248045	0,0134163	0,5636125	0,0131367				
0,1773087	0,0981181	0,1639702	0,0981352	0,1807688	0,0944913	0,171614	0,0984648				
0,1786218	0,1003879	0,183757	0,1007494	0,1583816	0,0973879	0,155444	0,0988708				
	2 0,4657915 0,1773087	2 3 0,4657915 0,0131269 0,1773087 0,0981181	100     2       2     3     2       0,4657915     0,0131269     0,5267845       0,1773087     0,0981181     0,1639702	100         200           2         3         2         3           0,4657915         0,0131269         0,5267845         0,0133642           0,1773087         0,0981181         0,1639702         0,0981352	100         200         3(           2         3         2         3         2           0,4657915         0,0131269         0,5267845         0,0133642         0,6248045           0,1773087         0,0981181         0,1639702         0,0981352         0,1807688	100         200         300           2         3         2         3         2         3           0,4657915         0,0131269         0,5267845         0,0133642         0,6248045         0,0134163           0,1773087         0,0981181         0,1639702         0,0981352         0,1807688         0,0944913	100         200         300         44           2         3         2         3         2         3         2           0,4657915         0,0131269         0,5267845         0,0133642         0,6248045         0,0134163         0,5636125           0,1773087         0,0981181         0,1639702         0,0981352         0,1807688         0,0944913         0,171614				

From the simulation results obtained did not proving the hypothesis which stated the relationship between the number of ensemble used by the error value earned is inversely proportional which the greater the number of ensemble is used, the smaller the error value is obtained. But it supports the hypothesis that stated the more number of ensemble used, the more accurate the results obtained from the error.

#### 5. CONCLUSIONS AND ADVICES

From chapter 4.4, some conclusions are obtained:

1. Ensemble Kalman Filter (EnKF) method has been implemented successfully for navigation system by using the parameter 2 and 3 measurement data and the number of ensemble from 100 to 400 in the estimation position of the translation of motion. It

- is seen from the size of RMS Error relatively small for each state.
- 2. From the result of average RMS Error values in table 3, the hypothesis which stated the greater the number of ensemble then the smaller its RMS Error value obtained was not proven. But it supports the hypothesis that stated the more number of ensemble used, the more accurate the results obtained from the error. Due to the effect of using normal distribution at its noise during the simulation so the values and graphs obtained are randomized and without control system in this program lead to uncontrollable noise.

In this research, the problems which examined are still far from perfect. So its possible to develop more extensive of its studies. In order to obtain better results, the authors recommend that:

- 1. When deriving dynamics equation of AUV, are expected to calculate all the components of force so the adjustment of parameters (include the assumptions are used) during the implementation of a method producing the actual conditions
- 2. EnKF method can also be applied to estimating the position when the AUV perform rotational motion such as pitching, yawing and rolling.

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