Design of Autonomous Underwater Vehicle Motion Control Using Sliding Mode Control Method

Kresna Oktafianto
Mathematics Dept.
PGRI Ronggolawe
University Tuban
UNIROW, INDONESIA
krsn.ktfianto@gmail.com

Teguh Herlambang
Electrical Eng. Dept.
Nahdlatul Ulama
Surabaya University
UNUSA, INDONESIA
teguh.ftkits@gmail.com

Mardlijah
Mathematics Dept.
Institut Teknologi Sepuluh
Nopember
Surabaya, INDONESIA
mardlijah@matematika.its.ac.id

Hendro Nurhadi*
Mechanical Eng. Dept
Institut Teknologi Sepuluh
Nopember
Surabaya, INDONESIA
hdnurhadi@me.its.ac.id
(*Corresponding author, IEEE member)

Abstract— **This paper presents a study of the Autonomous Underwater Vehicle (AUV). Nonlinear model of AUV which has six degrees of freedom being linearized using Jacobian matrix. In this paper, Sliding Mode Control law as a method is applied Autonomous Underwater Vehicle and the simulation obtained a stable performance.**

Keywords—AUV; Linearization; Nonlinear; SMC.

I. INTRODUCTION

One of the technologies in the scope of shipping that is underwater robot, known as Autonomous Underwater Vehicle (AUV). AUV has an important role for a country that has a ocean region greater than land region. AUV widely used for ocean exploration, contourmapping and as a means of defense under the sea. AUV work independently means without direct control by humans. AUV has six degrees of freedom, namely surge, sway, heave, roll, pitch and yaw are shown in Figure 1. Figure 1 also described that movement of AUV is influenced by Earth Fixed Frame (EFF) and Body Fixed Frame (BFF). EFF is used to determine position and direction of the movement of the AUV, which is the x-axis direct to the north, the y-axis to the east and the z-axis to the center of the earth, while BFF is used to determine speed and acceleration of the AUV with the point of origin is at the center of gravity.[2][3]

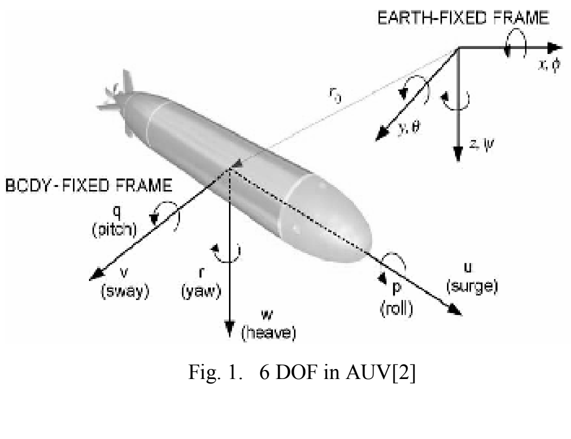


Fig. 1. 6 DOF in AUV[2]

Many AUV control research using Proportional Integral Derivative Method [1], but in this paper SMC method being used.

II. MATHEMATIC MODELING OF AUV

Figure 1 has been described on the EFF and BFF, whereas movement of the maneuvering on AUV usually moves in 6 DOF consisting of 3 DOF for translational and 3 DOF motion to rotational motion. Table 1 shows the notations and symbols on the AUV. A general description of motion in the AUV which has 6 DOF expressed as vectors, namely:[2]

The position vector and Euler angles:

$$\eta = \begin{bmatrix}\eta_1^T \\ \eta_2^T\end{bmatrix}, \eta_1 = \begin{bmatrix}x \\ y \\ z\end{bmatrix} \text{ and } \eta_2 = \begin{bmatrix}\phi \\ \theta \\ \psi\end{bmatrix}$$

Linear and angular velocity vector:

$$v = \begin{bmatrix}v_1^T \\ v_2^T\end{bmatrix}, v_1 = \begin{bmatrix}u \\ v \\ w\end{bmatrix} \text{ and } v_2 = \begin{bmatrix}p \\ q \\ r\end{bmatrix}$$

Force and Moment:

$$\tau = \begin{bmatrix}\tau_1^T \\ \tau_2^T\end{bmatrix}, \tau_1 = \begin{bmatrix}X \\ Y \\ Z\end{bmatrix} \text{ and } \tau_2 = \begin{bmatrix}K \\ M \\ N\end{bmatrix}$$

TABLE I. NOTATION OF MOTION AUV[1][2]

DOF	Motion	Force/Moment	Linear/Angular Velocity	Positions/ Euler Angles
1	Surge	X	u	x
2	Sway	Y	v	y
3	Heave	Z	w	z
4	Roll	K	p	ϕ
5	Pitch	M	q	θ
6	Yaw	N	r	ψ

Description:

η: The position vector and direction of the EFF

v: Linear and angular velocity vector at BFF

τ: Force and moment working on the AUV on BFF

In addition AUV also has external forces that affect the movement of AUV, among others:

$$\tau = \tau_{hidrostatic} + \tau_{addedmass} + \tau_{drag} + \tau_{lift} + \tau_{control}$$

By combining the equation of hydrostatic force, lift *added mass,* drag, *thrust* and assuming a diagonal tensor of inertia (I_o) is zero then the total force and moment of models obtained from the following:[2]

Surge :

$$m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X_{res} + X_{|u|u}u|u| + X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop}$$

Sway :

$$m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(pq + \dot{r})] = Y_{res} + Y_{|v|v}v|v| + Y_{r|r|}r|r| + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r$$

Heave :

$$m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = Z_{res} + Z_{|w|w}w|w| + Z_{q|q|}q|q| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq}uq + Z_{vp}vp + Z_{rp}rp + Z_{uw}uw + Z_{uu\delta_s}u^2\delta_s$$

Roll:

$$I_x\dot{p} + (I_z - I_y)qr + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = K_{res} + K_{p|p|}p|p| + K_{\dot{p}}\dot{p} + K_{prop}$$

Pitch:

$$I_y\dot{q} + (I_x - I_z)rp + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] = M_{res} + M_{w|w|}w|w| + M_{q|q|}q|q| + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uq}uq + M_{vp}vp + M_{rp}rp + M_{uw}uw + M_{uu\delta_s}u^2\delta_s$$

Yaw :

$$I_z\dot{r} + (I_y - I_z)pq + m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = N_{res} + N_{v|v|}v|v| + N_{r|r|}r|r| + N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv + N_{uu\delta_r}u^2\delta_r \quad (1)$$

Equation (1) can be summarized in matrix form as follows [1]:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m - Y_{\dot{v}} & 0 & -mz_G & 0 & mx_G - Y_{\dot{r}} \\ 0 & 0 & m - Z_{\dot{w}} & my_G & -mx_G - Z_{\dot{q}} & 0 \\ 0 & -mz_G & my_G & I_x - K_{\dot{p}} & 0 & 0 \\ mz_G & 0 & -mx_G - M_{\dot{w}} & 0 & I_y - M_{\dot{q}} & 0 \\ -my_G & mx_G - N_{\dot{v}} & 0 & 0 & 0 & I_z - N_{\dot{r}} \end{bmatrix}^{-1} \begin{bmatrix} \Sigma X \\ \Sigma Y \\ \Sigma Z \\ \Sigma K \\ \Sigma M \\ \Sigma N \end{bmatrix} \quad (2)$$

With

$$\Sigma X = X_{res} + X_{|u|u}u|u| + (X_{wq} - m)wq + (X_{qq} + mx_G)q^2 + (X_{vr} + m)vr + (X_{rr} + mx_G)r^2 - my_Gpq - mz_Gpr + X_{prop}$$

$$\Sigma Y = Y_{res} + Y_{|v|v}v|v| + Y_{r|r|}r|r| + my_Gr^2 + (Y_{ur} - m)ur + (Y_{wp} + m)wp + (Y_{pq} - mx_G)pq + Y_{uv}uv - my_Gp^2 + mz_Gqr + Y_{uu\delta_r}u^2\delta_r$$

$$\Sigma Z = Z_{res} + Z_{|w|w}w|w| + Z_{q|q|}q|q| + (Z_{uq} + m)uq + (Z_{vp} - m)vp + (Z_{rp} - mx_G)rp + Z_{uw}uw + mz_G(p^2 + q^2) - my_Grq + Z_{uu\delta_s}u^2\delta_s$$

$$\Sigma K = K_{res} + K_{p|p|}p|p| - (I_z - I_y)qr + m(uq - vp) - mz_G(wp - ur) + K_{prop}$$

$$\Sigma M = M_{res} + M_{w|w|}w|w| + M_{q|q|}q|q| + (M_{uq} - mx_G)uq + (M_{vp} + mx_G)vp + [M_{rp} - (I_x - I_z)]rp + mz_G(vr - wp) + M_{uw}uw + M_{uu\delta_s}u^2\delta_s$$

$$\Sigma N = N_{res} + N_{v|v|}v|v| + N_{r|r|}r|r| + (N_{ur} - mx_G)ur + (N_{wp} + mx_G)wp + [N_{pq} - (I_y - I_x)]pq - my_G(vr - wp) + N_{uv}uv + N_{uu\delta_r}u^2\delta_r$$

In this paper the nonlinear system of AUV in equation (2) model can be linearied with Jacobian approach where the nonlinear AUV system in general as follows:[4]

$$\dot{x}(t) = f(x(t), u(t), t)$$
$$y(t) = g(x(t), u(t), t) \quad (3)$$

So Jacobian matrix form is given as follows:

$$J_{x1} = \begin{bmatrix} \frac{\partial \Sigma X}{\partial u} & \frac{\partial \Sigma X}{\partial v} & \frac{\partial \Sigma X}{\partial w} & \frac{\partial \Sigma X}{\partial p} & \frac{\partial \Sigma X}{\partial q} & \frac{\partial \Sigma X}{\partial r} \\ \frac{\partial \Sigma Y}{\partial u} & \frac{\partial \Sigma Y}{\partial v} & \frac{\partial \Sigma Y}{\partial w} & \frac{\partial \Sigma Y}{\partial p} & \frac{\partial \Sigma Y}{\partial q} & \frac{\partial \Sigma Y}{\partial r} \\ \frac{\partial \Sigma Z}{\partial u} & \frac{\partial \Sigma Z}{\partial v} & \frac{\partial \Sigma Z}{\partial w} & \frac{\partial \Sigma Z}{\partial p} & \frac{\partial \Sigma Z}{\partial q} & \frac{\partial \Sigma Z}{\partial r} \\ \frac{\partial \Sigma K}{\partial u} & \frac{\partial \Sigma K}{\partial v} & \frac{\partial \Sigma K}{\partial w} & \frac{\partial \Sigma K}{\partial p} & \frac{\partial \Sigma K}{\partial q} & \frac{\partial \Sigma K}{\partial r} \\ \frac{\partial \Sigma M}{\partial u} & \frac{\partial \Sigma M}{\partial v} & \frac{\partial \Sigma M}{\partial w} & \frac{\partial \Sigma M}{\partial p} & \frac{\partial \Sigma M}{\partial q} & \frac{\partial \Sigma M}{\partial r} \\ \frac{\partial \Sigma N}{\partial u} & \frac{\partial \Sigma N}{\partial v} & \frac{\partial \Sigma N}{\partial w} & \frac{\partial \Sigma N}{\partial p} & \frac{\partial \Sigma N}{\partial q} & \frac{\partial \Sigma N}{\partial r} \end{bmatrix}$$

From the results of the partial derivatives of the above in order to obtain Jacobian matrix as follows:

$$J_{x2} = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m - Y_{\dot{v}} & 0 & -mz_G & 0 & mx_G - Y_{\dot{r}} \\ 0 & 0 & m - Z_{\dot{w}} & my_G & -mx_G - Z_{\dot{q}} & 0 \\ 0 & -mz_G & my_G & I_x - K_{\dot{p}} & 0 & 0 \\ mz_G & 0 & -mx_G - M_{\dot{w}} & 0 & I_y - M_{\dot{q}} & 0 \\ -my_G & mx_G - N_{\dot{v}} & 0 & 0 & 0 & I_z - N_{\dot{r}} \end{bmatrix}^{-1} J_{x1} \quad (4)$$

For jacobian matrix control performed partial derivatives as follows:

$$J_{u1} = \begin{bmatrix} \frac{\partial \Sigma X}{\partial X_{prop}} & \frac{\partial \Sigma X}{\partial \delta_r} & \frac{\partial \Sigma X}{\partial \delta_s} & \frac{\partial \Sigma X}{\partial K_{prop}} & \frac{\partial \Sigma X}{\partial \delta_s} & \frac{\partial \Sigma X}{\partial \delta_r} \\ \frac{\partial \Sigma Y}{\partial X_{prop}} & \frac{\partial \Sigma Y}{\partial \delta_r} & \frac{\partial \Sigma Y}{\partial \delta_s} & \frac{\partial \Sigma Y}{\partial K_{prop}} & \frac{\partial \Sigma Y}{\partial \delta_s} & \frac{\partial \Sigma Y}{\partial \delta_r} \\ \frac{\partial \Sigma Z}{\partial X_{prop}} & \frac{\partial \Sigma Z}{\partial \delta_r} & \frac{\partial \Sigma Z}{\partial \delta_s} & \frac{\partial \Sigma Z}{\partial K_{prop}} & \frac{\partial \Sigma Z}{\partial \delta_s} & \frac{\partial \Sigma Z}{\partial \delta_r} \\ \frac{\partial \Sigma K}{\partial X_{prop}} & \frac{\partial \Sigma K}{\partial \delta_r} & \frac{\partial \Sigma K}{\partial \delta_s} & \frac{\partial \Sigma K}{\partial K_{prop}} & \frac{\partial \Sigma K}{\partial \delta_s} & \frac{\partial \Sigma K}{\partial \delta_r} \\ \frac{\partial \Sigma M}{\partial X_{prop}} & \frac{\partial \Sigma M}{\partial \delta_r} & \frac{\partial \Sigma M}{\partial \delta_s} & \frac{\partial \Sigma M}{\partial K_{prop}} & \frac{\partial \Sigma M}{\partial \delta_s} & \frac{\partial \Sigma M}{\partial \delta_r} \\ \frac{\partial \Sigma N}{\partial X_{prop}} & \frac{\partial \Sigma N}{\partial \delta_r} & \frac{\partial \Sigma N}{\partial \delta_s} & \frac{\partial \Sigma N}{\partial K_{prop}} & \frac{\partial \Sigma N}{\partial \delta_s} & \frac{\partial \Sigma N}{\partial \delta_r} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{uu\delta_r}u^2 & 0 & 0 & 0 & Y_{uu\delta_r}u^2 \\ 0 & 0 & Z_{uu\delta_s}u^2 & 0 & Z_{uu\delta_s}u^2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & M_{uu\delta_s}u^2 & 0 & M_{uu\delta_s}u^2 & 0 \\ 0 & N_{uu\delta_r}u^2 & 0 & 0 & 0 & N_{uu\delta_r}u^2 \end{bmatrix}$$

Control Jacobian matrix thus obtained are:

$$J_{u2} = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m - Y_{\dot{v}} & 0 & -mz_G & 0 & mx_G - Y_{\dot{r}} \\ 0 & 0 & m - Z_{\dot{w}} & my_G & -mx_G - Z_{\dot{q}} & 0 \\ 0 & -mz_G & my_G & I_x - K_{\dot{p}} & 0 & 0 \\ mz_G & 0 & -mx_G - M_{\dot{w}} & 0 & I_y - M_{\dot{q}} & 0 \\ -my_G & mx_G - N_{\dot{v}} & 0 & 0 & 0 & I_z - N_{\dot{r}} \end{bmatrix}^{-1} J_{u1} \tag{5}$$

So that the linearization of non linear plant AUV systems are:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = J_{x2} \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix} + J_{u2} \begin{bmatrix} X_{prop} \\ \delta_r \\ \delta_s \\ K_{prop} \\ \delta_s \\ \delta_r \end{bmatrix} \tag{6}$$

III. DESIGN SLIDING MODE CONTROL OF AUV

Sliding Mode Control is a control method based on robust means the system is working to address the problem of modeling uncertainties [5][6].

A. Switching Function

Let a dynamic system:

$$\dot{x}^{(n)}(t) = f(x,t) + b(x,t)u + d(t) \tag{7}$$

Where u control input, x a state vector, $f(x,t)$ and $b(x,t)$ the form of limited functionality, $d(t)$ external interference. Switching function that surface $S(x,t)$ in the state space $\mathbb{R}^n$, meet the general equation:

$$S(x,t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e(t) \tag{8}$$

Sliding conditions are defined as follows:

$$\dot{V} = S\dot{S} \leq -\eta|S| \tag{9}$$

Inequality (9) is called the condition of sliding. The sliding condition can be written in several forms, namely:

$$S\dot{S} < 0$$

Or

$$\dot{S}\, sgn(S) \leq -\eta \tag{10}$$

With a positive constant.

In the equation (6) can be expressed in the form:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} & B_{15} & B_{16} \\ B_{21} & B_{22} & B_{23} & B_{24} & B_{25} & B_{26} \\ B_{31} & B_{32} & B_{33} & B_{34} & B_{35} & B_{36} \\ B_{41} & B_{42} & B_{43} & B_{44} & B_{45} & B_{46} \\ B_{51} & B_{52} & B_{53} & B_{54} & B_{55} & B_{56} \\ B_{61} & B_{62} & B_{63} & B_{64} & B_{65} & B_{66} \end{bmatrix} \begin{bmatrix} X_{prop} \\ \delta_r \\ \delta_s \\ K_{prop} \\ \delta_s \\ \delta_r \end{bmatrix} \tag{11}$$

From equation (11) then carried out starting from the design of the SMC state u, v, w, p, q and r. The discussion is only written u state, to other states adjust.

$$\dot{u} = A_{11}u + A_{12}v + A_{13}w + A_{14}p + A_{15}q + A_{16}r + B_{11}X_{prop} + B_{12}\delta_r + B_{13}\delta_s + B_{14}K_{prop} + B_{15}\delta_s + B_{16}\delta_r \tag{12}$$

Tracking error for the surge is:

$$\tilde{u} = u - u_d$$

With $u_d = constant$. Because the system of the order of 1, then the switching function is formed as follows:

$$S(u,t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{u}$$

$$S(u,t) = \left(\frac{d}{dt} + \lambda\right)^{1-1} \tilde{u}$$

$$S(u,t) = \tilde{u}$$

$$S(u,t) = u - u_d \tag{13}$$

While derivative of is:

$$\dot{S}(u,t) = \dot{u} - \dot{u}_d \tag{14}$$

Because u_d constant so $\dot{u}_d = 0$. Substituting equation (12) to (14) becomes:

$$\dot{S}(u,t) = A_{11}u + A_{12}v + A_{13}w + A_{14}p + A_{15}q + A_{16}r + B_{11}X_{prop} + B_{12}\delta_r + B_{13}\delta_s + B_{14}K_{prop} + B_{15}\delta_s + B_{16}\delta_r \tag{15}$$

Further specified value $\tilde{X}_{prop}$ from equation (15) with value $\dot{S} = 0$.

$$A_{11}u + A_{12}v + A_{13}w + A_{14}p + A_{15}q + A_{16}r + B_{11}X_{prop} + B_{12}\delta_r + B_{13}\delta_s + B_{14}K_{prop} + B_{15}\delta_s + B_{16}\delta_r = 0 \tag{16}$$

Thus obtained $\tilde{X}_{prop}$ are:

$$\tilde{X}_{prop} = -\left(\frac{A_{11}u + A_{12}v + A_{13}w + A_{14}p + A_{15}q + A_{16}r}{B_{11}} + \frac{B_{12}\delta_r + B_{13}\delta_s + B_{14}K_{prop} + B_{15}\delta_s + B_{16}\delta_r}{B_{11}} \right) \tag{17}$$

Based control law which satisfy the conditions of sliding is:

$$X_{prop} = \tilde{X}_{prop} - KSgn(S) \tag{18}$$

From equation (17) and (18) obtained:

$$X_{prop} = -\left(\frac{A_{11}u + A_{12}v + A_{13}w + A_{14}p + A_{15}q + A_{16}r}{B_{11}} + \frac{B_{12}\delta_r + B_{13}\delta_s + B_{14}K_{prop} + B_{15}\delta_s + B_{16}\delta_r}{B_{11}} \right)$$

$$-KSgn(S) \tag{19}$$

Substituting equation (19) to equation (15) obtained:

$$\dot{S}(u,t) = A_{11}u + A_{12}v + A_{13}w + A_{14}p + A_{15}q + A_{16}r + B_{11}$$

$$\left[-\left(\begin{array}{c}\dfrac{A_{11}u + A_{12}v + A_{13}w + A_{14}p + A_{15}q + A_{16}r}{B_{11}} + \\ \dfrac{B_{12}\delta_r + B_{13}\delta_s + B_{14}K_{prop} + B_{15}\delta_s + B_{16}\delta_r}{B_{11}}\end{array}\right) - KSgn(S)\right]$$

$$+ B_{12}\delta_r + B_{13}\delta_s + B_{14}K_{prop} + B_{15}\delta_s + B_{16}\delta_r$$

$$\dot{S}(u,t) = -B_{11}KSgn(S) \tag{20}$$

In order to satisfy the condition of sliding namely:

$$S\dot{S} \leq -\eta|S| \tag{21}$$

Then the value K will be designed by substituting the equation (20) to equation (21) thus obtained:

$$-SB_{11}KSgn(S) \leq -\eta|S|$$

$$-B_{11}KSgn(S) \leq -\frac{\eta|S|}{S}$$

$$K \geq \frac{\eta}{B_{11}Sgn(S)} \tag{22}$$

If $S > 0$ so $K \geq \frac{\eta}{B_{11}}$ and if $S < 0$ so $K \geq -\frac{\eta}{B_{11}}$ or can be written $-K \leq \frac{\eta}{B_{11}}$.

From both these inequalities can be expressed in the form:

$$-K \leq \frac{\eta}{B_{11}} \leq K$$

$$\left|\frac{\eta}{B_{11}}\right| \leq K \tag{23}$$

From equation (23) shows that the value K is:

$$K = max\left|\frac{\eta}{B_{11}}\right| \tag{24}$$

Then to be used a boundary layer to minimize chattering by changing the signum function (sgn) in equation (18) becomes a function of saturation (sat) as follows:

$$X_{prop} = \tilde{X}_{prop} - Ksat\left(\frac{S}{\Phi}\right) \tag{25}$$

Thus SMC for state control surge on the AUV is obtained from substituting equation (17) and (24) to equation (25) is as follows:

$$X_{prop} = -\left(\begin{array}{c}\frac{A_{11}u+A_{12}v+A_{13}w+A_{14}p+A_{15}q+A_{16}r}{B_{11}} + \\ \frac{B_{12}\delta_r+B_{13}\delta_s+B_{14}K_{prop}+B_{15}\delta_s+B_{16}\delta_r}{B_{11}}\end{array}\right)$$

$$-max\left|\frac{\eta}{B_{11}}\right| sat\left(\frac{S}{\Phi}\right) \tag{26}$$

In the same way the steps of designing the SMC to sway, heave, roll, pitch and yaw.

IV. Computational Result

AUV control of the design is then performed simulations using the SMC as follows:

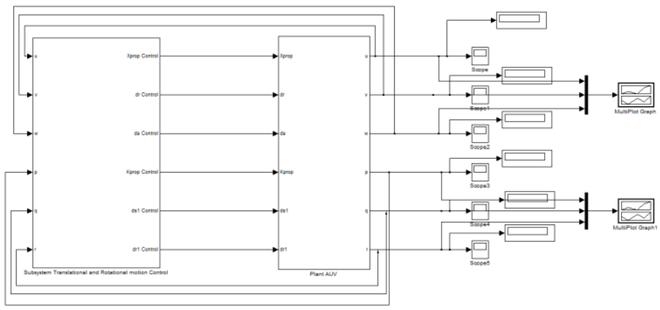


Fig. 2. Block Diagram AUV

Figure. 2 shows a block diagram of SMC AUV 6 DOF, while the simulation results shown in the next figure.

Figure 3 shows the translational motion of SMC on AUV control, steady state of motion surge occurs at a speed of 0.60349 m/s. steady state sway motion occurs at a speed of 0.973441 m/s. Both are headed to the desired setpoint value. In the tenth second surge of motion show that the system is already headed to the desired setpoint, while the sway motion showed faster trend stable. As for the heave motion with steady state at a speed -0.228119 m/s stable and there are errors from the desired setpoint.

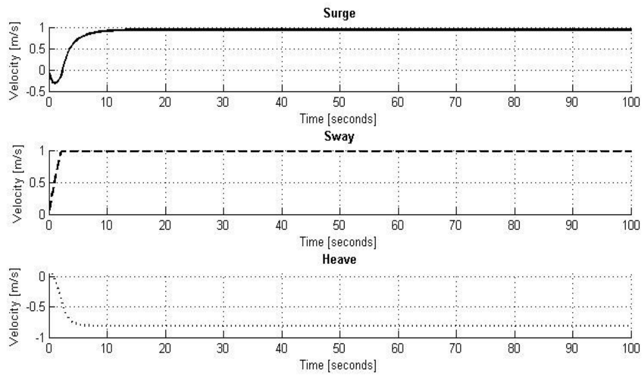


Fig. 3. AUV Translation Motion

Figure 4 shows a graph of rotational motion SMC controls on AUV which rotational motion is divided on the motion roll apparent that the steady state at a negative speed 0.398994 m/s, while the steady state for pitch motion at a speed -0.192733 m/s, and the steady yaw motion -1.146269 state at a speed of m/s.

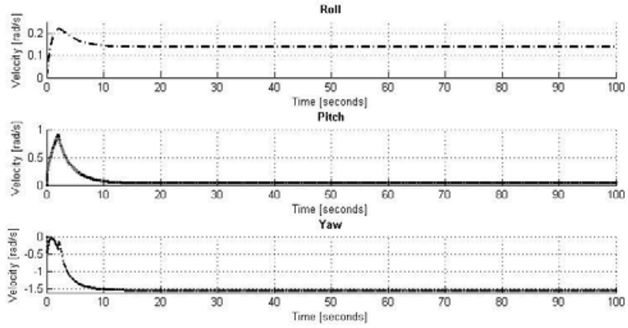


Fig. 4. AUV Rotation Motion

V. CONCLUDING REMARKS

In this paper, an Autonomous Underwater Vehicle (AUV) model with six degrees of freedom being linearized using jacobian matrix. Sliding Mode Control to be able to apply AUV. AUV control using SMC method generates performances: error steady state of surge motion (39,651%), sway (2,4099%), heave (77,1881%), pitch (119,2731%), roll (60,1008%) dan yaw (42,6865%).

VI. ACKNOWLEDGMENT

This research received support from Mechatronics Laboratory Engineering and Mathematical Modelling and Simulation Laboratory of ITS.

REFERENCES

[1] Oktafianto.K, Mardlijah, and Nurhadi. H, "Design of Autonomous Underwater Vehicle Control System Using Method Proportional Integral Derivative (PID)," SEMNASTIKA. Surabaya, pp. 209-218, 25 April 2015.

[2] Yang, C, "Modular Modelling and Control for Autonomous Vehicle (AUV)". Master Thesis of Engineering, 2007, Department of Mechanical Engineering National University of Singapore.

[3] Fossen, T. I, A Nonlinear Unified State-space Model for Ship Maneuvering and Control in A Seaway-Journal of Bifurcation and Chaos, 2005, Journal of Bifurcation and Chaos.

[4] Subiono, Mathematical System (second edition), 2010, Department of Mathematics Sepuluh Nopember Institut of Technology, Surabaya.

[5] Mardlijah, Submarine Motion Control Using SMC, 2010, South East Asian Conference Mathematic Applied (SEACMA), Surabaya.

[6] Nurhadi.H, "Sliding-Mode (SM) and Fuzzy-Sliding-Mode (FSM) Controllers for High-Precisely Linear Piezoelectric Ceramic Motor (LPCM)". The 2013 IEEE International Conference on Robotics, Biomeimetics, Intelligent Computational Systems (Robionetics 2013), Yogyakarta, 25-27 November 2013.

[7] Nurhadi.H, Kuo. W.M and Tarng Y.S "Study on Controller Designs for High-Precisely Linear Piezoelectric Ceramic Motor (LPCM)". The 5th IEEE Conference on Industrial Electronics and Applications (ICIEA 2010), Taichung, R.O.C., 15-17 June 2010.

Int'l Conf. on Advanced Mechatronics, Intelligent Manufacture, and Industrial Automation 2015 (ICAMIMIA 2015)
Surabaya, Indonesia, on October 15-17, 2015

Estimate and Control Position Autonomous Underwater Vehicle Based on Determined Trajectory using Fuzzy Kalman Filter Method

[1]Zunif Ermayanti
Mathematics Department of Institut Teknologi Sepuluh Nopember,
Surabaya, Indonesia
Zunif.e@gmail.com

[2]Erna Apriliani
Mathematics Department of Institut Teknologi Sepuluh Nopember,
Surabaya, Indonesia
april@matematika.its.ac.id

[3]Hendro Nurhadi*
Mechanical Engineering of Institut Teknologi Sepuluh Nopember,
Surabaya, Indonesia
hdnurhadi@me.its.ac.id
*Corresponding Author, IEEE Member

[4]Teguh Herlambang
Post Graduate Program in Marine Technologi of Institut Teknologi Sepuluh Nopember,
Surabaya, Indonesia
teguh@unusa.ac.id

***Abstract*— Unmanned Underwater Vehicle (UUV), known as underwater drones, are any vehicle that are able to operate underwater without human occupant. AUV (Autonomous Underwater Vehicle) are one of categories of these vehicles which operate independently of direct human input. This AUV is required to have a navigation system that can manoeuvred 6 Degree of Freedom (DOF) and able to estimate the exact position based on the determined trajectory. Fuzzy Kalman Filter (FKF) method is used to estimate the position of the AUV. This process is used to maintain the accuracy of the trajectory. The performance of FKF algorithm on some several trajectory cases show that this method has relatively small Root Means Square Error (RSME), which is less than 10%.**

Keywords*— *AUV, estimation, Fuzzy Kalman Filter

I. INTRODUCTION

Unmanned Underwater Vehicle (UUV) are any vehicle that are able to operate underwater without human occupant. These vehicles are divided into two, there are Remote Operational Vehicle (ROV), which is operated by remote control, and Autonomous Underwater Vehicle (AUV), which is a machine in the water that operate independently by direct human input [1]. AUV is now quite widely used for several purposes in many fields, i.e. science, environment, marine industry, military, national defense and security. In archipelago country, such as Indonesia, which has widely ocean area, this AUV can be used as a surveillance tool to see untouched underwater conditions and can supervise the defense or border areas in the territory of the Republic of Indonesia. In addition, the AUV can also be used to see and find out the state of the sea bottom, i.e. conditions and natural resources in the sea, geological sampling, inspection of underwater structures, and construction and maintenance of underwater structures.

A research which has been conducted on the AUV are investigate the estimation on the AUV by using Ensemble Kalman Filter [2]. That research estimate some translational motion, i.e. surging, swaying, and heaving. However, general forces, such as drag force and lift force, on AUV aren't taken into account in detail. The next research is conducted by [1] with the same topic and method, the difference is those research also calculate the drag and lift forces of the AUV. A research which use Fuzzy Kalman Filter method to solve the problem is the research which is conducted by Mahmuri, H (2011) about the estimation of the cancer cells development by using Fuzzy Kalman method [3].

Due to its importance and previous researches, this research will be further developed on the estimated position and control on AUV by using all existing motion on AUV, there are 6 DOF (Degree of Freedom) both translational and rotational motion, whereas the method used is Fuzzy Kalman Filter.

In this research, we use Fuzzy Kalman Filter as our method because FKF can be used for any parameter variations. In addition, the merger between Fuzzy and Kalman Filter is occurred because the Fuzzy system can be used for anything inappropriate and ambiguous.

The goal of this research is to get the estimated position of the AUV in accordance to the determined trajectory with a relatively small error.

II. AUV MODELS

Two important things to note for analyzing AUV are Earth Fixed Frame (EFF) and Body Fixed Frame (BFF) [4]. EFF is used to describe the position and orientation of the AUV with the position of the x-axis direct to the north, the y-axis to the east, and the z-axis toward the center of the Earth. While, BFF is used to describe the speed and acceleration of the AUV with the starting point at the center of gravity.

Motion of AUV have 6 DOF (Degree of Freedom) where 3 DOF for translation motion and 3 DOF for rotational motion in point x, y, and z. General equation of motion consists of 3 equations for translational motion and 3 motions for rotational motion. The general equation of motion translation and rotation are surge, sway, and heave as motion translation and roll, pitch, and yaw as rotation [4].

Position and Angle Euler

$\eta = [\eta_1^T, \eta_2^T]^T, \eta_1 = [x, y, z]^T, \eta_2 = [\phi, \theta, \psi]^T$

Linear and Angular Velocity

$v = [v_1^T, v_2^T]^T, v_1 = [u, v, w]^T, v_2 = [p, q, r]^T$

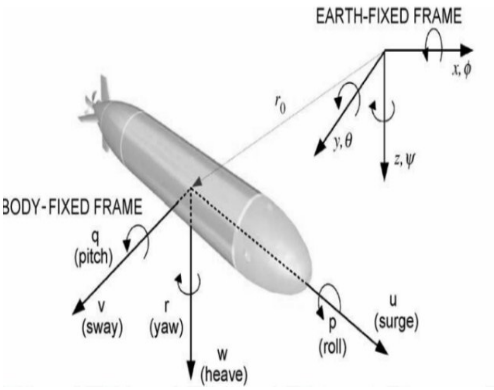
Force and Moment

$\tau = [\tau_1^T, \tau_2^T]^T, \tau_1 = [X, Y, Z]^T, \tau_2 = [K.M.N]^T$

TABLE 1. AUV COORDINATE

DOF	Note	Force/ Moment	Velocity	Position
1	*Surge*	X	u	x
2	*Sway*	Y	v	y
3	*Heave*	Z	w	z
4	*Roll*	K	p	ϕ
5	*Pitch*	M	q	θ
6	*Yaw*	N	r	ψ

Fig 1. Six Degree of Freedom AUV



There for the AUV models can be written as follows:

Surge

$$m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X_{res} + X_{|u|u}u|u| + X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop} \quad (1)$$

Sway

$$m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(pq + \dot{r})] = Y_{res} + Y_{|v|v}v|v| + Y_{r|r|}r|r| + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r \quad (2)$$

Heave

$$m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = Z_{res} + Z_{|w|w}w|w| + Z_{q|q|}q|q| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq}uq + Z_{vp}vp + Z_{rp}rp + Z_{uw}uw + Z_{uu\delta_s}u^2\delta_s \quad (3)$$

Roll

$$I_x\dot{p} + (I_z - I_y)qr + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = K_{res} + K_{p|p|}p|p| + K_{\dot{p}}\dot{p} + K_{prop} \quad (4)$$

Pitch

$$I_y\dot{q} + (I_x - I_z)rp + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] = M_{res} + M_{w|w|}w|w| + M_{q|q|}q|q| + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uq}uq + M_{vp}vp + M_{rp}rp + M_{uw}uw + M_{uu\delta_s}u^2\delta_s \quad (5)$$

Yaw

$$I_z\dot{r} + (I_y - I_z)p + m\,[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = N_{res} + N_{v|v|}v|v| + N_{r|r|}r|r| + N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv + N_{uu\delta_r}u^2\delta_r \quad (6)$$

III.RESULT AND DISCUSSION

This chapter describes the settlement of AUV models which are used to estimate the position and determine trajectory that may be taken by the AUV. Further, we will estimate position of the AUV motion due to the AUV's determined trajectory using Fuzzy Kalman Filter method. Furthermore, based on the results of the estimation, we conduct a control on the AUV system in order to keep AUV moves at the determined trajectory.

A. AUV Models Solution

Based on the AUV model on Equation 1-6, those equations can be built on matric as follows:

$$E \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} \quad (7)$$

Or can be written as:

$$E\,\dot{x} = F \quad (8)$$

where

$$E=\begin{bmatrix} \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} & \begin{matrix} 0 & \frac{mx_G}{m-X_{\dot u}} & \frac{-my_G}{m-X_{\dot u}} \\ -\frac{mz_G}{m-Y_{\dot v}}+ & 0 & \frac{mx_G-Y_{\dot r}}{m-Y_{\dot v}} \\ \frac{my_G}{m-Z_{\dot w}} & \frac{-mx_G-Z_{\dot q}}{m-Z_{\dot w}} & 0 \end{matrix} \\ \begin{matrix} 0 & \frac{m\,y_G}{I_x-K_{\dot p}} & -\frac{m\,z_G}{I_x-K_{\dot p}} \\ \frac{m\,z_G}{I_y-M_{\dot q}} & 0 & \frac{-m\,x_G-M_{\dot w}}{I_y-M_{\dot q}} \\ -\frac{my_G}{I_z-N_{\dot r}} & \frac{mx_G-N_{\dot v}}{I_z-N_{\dot r}} & 0 \end{matrix} & \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \end{bmatrix} \tag{9}$$

and

$$F_1=\frac{X_{res}+X_{|u|u}u|u|+X_{wq}wq+X_{qq}qq+X_{vr}vr+X_{rr}rr+X_{prop}+}{m-X_{\dot u}}$$
$$\frac{-m[-vr+wq-x_G(q^2+r^2)+pq\,y_G+pr\,z_G]}{m-X_{\dot u}} \tag{10}$$

$$F_2=\frac{Y_{res}+Y_{|v|v}v|v|+Y_{r|r|}r|r|+Y_{ur}ur+Y_{wp}wp+Y_{pq}pq+Y_{uv}uv+}{m-Y_{\dot v}}$$
$$\frac{Y_{uu\delta_r}u^2\delta_r-m[-wp+ur-y_G(r^2+p^2)+qr\,z_G+pq\,x_G]}{m-Y_{\dot v}} \tag{11}$$

$$F_3=\frac{Z_{uu\delta_s}u^2\delta_s-m[-uq+vp-z_G(p^2+q^2)+rp\,x_G+rq\,y_G]+}{m-Z_{\dot w}}$$

$$\frac{Z_{res}+Z_{|w|w}w|w|+Z_{q|q|}q|q|+Z_{uq}uq+Z_{vp}vp+Z_{rp}rp+Z_{uw}uw}{m-Z_{\dot w}} \tag{12}$$

$$F_4=\frac{K_{res}+K_{p|p|}p|p|+K_{prop}-\left((I_z-I_y)qr\right)+}{I_x-K_{\dot p}}$$
$$\frac{-m[y_G(-uq+vp)-z_G(-wp+ur)]}{I_x-K_{\dot p}} \tag{13}$$

$$F_5=\frac{M_{res}+M_{w|w|}w|w|+M_{q|q|}q|q|+M_{uq}uq+M_{vp}vp+M_{rp}rp+}{I_y-M_{\dot q}}$$

$$\frac{M_{uw}uw+M_{uu\delta_s}u^2\delta_s-((I_x-I_z)rp)+}{I_y-M_{\dot q}}$$
$$\frac{-m[z_G(-vr+wq)-x_G(-uq+vp)]}{I_y-M_{\dot q}} \tag{14}$$

$$F_6=\frac{N_{res}+N_{r|r|}r|r|+N_{ur}ur+N_{wp}wp+N_{pq}pq+N_{uv}uv+N_{uu\delta_r}u^2\delta_r+}{I_z-N_{\dot r}}$$
$$\frac{N_{v|v|}v|v|-\left((I_y-I_z)pq-m[x_G(-wp+ur)-y_G(-vr+wq)]\right)}{I_z-N_{\dot r}} \tag{15}$$

B. Linearization

Model of AUV is a non-linear model, therefore, this model will be converted into common forms as:

$$\dot x = Ax + Bc \tag{16}$$

Where c is control.

Equation 8 will be formed into the Equation 16 by means of a function F in Jacobi to the speed and control. So we get Equation 17 and 18 :

- Jacobi to the speed

$$J_x=\begin{bmatrix} \frac{\partial F_1}{\partial u} & \frac{\partial F_1}{\partial v} & \frac{\partial F_1}{\partial w} & \frac{\partial F_1}{\partial p} & \frac{\partial F_1}{\partial q} & \frac{\partial F_1}{\partial r} \\ \frac{\partial F_2}{\partial u} & \frac{\partial F_2}{\partial v} & \frac{\partial F_2}{\partial w} & \frac{\partial F_2}{\partial p} & \frac{\partial F_2}{\partial q} & \frac{\partial F_2}{\partial r} \\ \frac{\partial F_3}{\partial u} & \frac{\partial F}{\partial v} & \frac{\partial F_3}{\partial w} & \frac{\partial F_3}{\partial p} & \frac{\partial F_3}{\partial q} & \frac{\partial F_3}{\partial r} \\ \frac{\partial F_4}{\partial u} & \frac{\partial F_4}{\partial v} & \frac{\partial F_4}{\partial w} & \frac{\partial F_4}{\partial p} & \frac{\partial F_4}{\partial q} & \frac{\partial F_4}{\partial r} \\ \frac{\partial F_5}{\partial u} & \frac{\partial F_5}{\partial v} & \frac{\partial F_5}{\partial w} & \frac{\partial F_5}{\partial p} & \frac{\partial f_5}{\partial q} & \frac{\partial F_5}{\partial r} \\ \frac{\partial F_6}{\partial u} & \frac{\partial F_6}{\partial v} & \frac{\partial F_6}{\partial w} & \frac{\partial F_6}{\partial p} & \frac{\partial F_6}{\partial q} & \frac{\partial F_6}{\partial r} \end{bmatrix} \tag{17}$$

- Jacobi to control

$$J_C=\begin{bmatrix} \frac{\partial F_1}{\partial X_{prop}} & \frac{\partial F_1}{\partial \delta_r} & \frac{\partial F_1}{\partial \delta_s} & \frac{\partial F_1}{\partial K_{prop}} & \frac{\partial F_1}{\partial \delta_s} & \frac{\partial F_1}{\partial \delta_r} \\ \frac{\partial F_2}{\partial X_{prop}} & \frac{\partial F_2}{\partial \delta_r} & \frac{\partial F_2}{\partial \delta_s} & \frac{\partial F_2}{\partial K_{prop}} & \frac{\partial F_2}{\partial \delta_s} & \frac{\partial F_2}{\partial \delta_r} \\ \frac{\partial F_3}{\partial X_{prop}} & \frac{\partial F_3}{\partial \delta_r} & \frac{\partial F_3}{\partial \delta_s} & \frac{\partial F_3}{\partial K_{prop}} & \frac{\partial F_3}{\partial \delta_s} & \frac{\partial F_3}{\partial \delta_r} \\ \frac{\partial F}{\partial X_{prop}} & \frac{\partial F_4}{\partial \delta_r} & \frac{\partial F_4}{\partial \delta_s} & \frac{\partial F_4}{\partial K_{prop}} & \frac{\partial F_4}{\partial \delta_s} & \frac{\partial F_4}{\partial \delta_r} \\ \frac{\partial F_5}{\partial X_{prop}} & \frac{\partial F_5}{\partial \delta_r} & \frac{\partial F_5}{\partial \delta_s} & \frac{\partial F_5}{\partial K_{prop}} & \frac{\partial F_5}{\partial \delta_s} & \frac{\partial F_5}{\partial \delta_r} \\ \frac{\partial F_6}{\partial X_{prop}} & \frac{\partial F_6}{\partial \delta_r} & \frac{\partial F_6}{\partial \delta_s} & \frac{\partial F_6}{\partial K_{prop}} & \frac{\partial F_6}{\partial \delta_s} & \frac{\partial F_6}{\partial \delta_r} \end{bmatrix} \tag{18}$$

Therefore, we obtained matrices A and B as follows:

$$A = E * J_x \tag{19}$$

$$B = E * J_C \tag{20}$$

C. Discretization Model

AUV equation of motion should be changed into the form of discretization because FKF algorithm can only be implemented on discrete system. To be able to use different discretization forward, namely:

$$\dot x = \frac{dx}{dt} \approx \frac{x_{k+1}-x_k}{\Delta t} \tag{21}$$

Discretization Equation 9 is obtained generally as follows:

$$\frac{x_{k+1}-x_k}{\Delta t} = Ax + Bc \tag{22}$$

$$x_{k+1} = (Ax + Bc)\Delta t + x_k \tag{23}$$

$$x_{k+1} = (A\Delta t + 1)x + B\Delta t\, c \tag{24}$$

D. Fuzzy Kalman Filter Implementation

1. Fuzzification

Fuzzification is a process that converts input from crisp shape (firmly) into the form of fuzzy (linguistic variables) that are usually presented in the form of fuzzy set.

TABLE 2. INITIALIZATION

Symbol	Note	Initialization
u^-	Minimum surge speed	0
u^+	Maximum surge speed	1
⋮	⋮	⋮
r^-	Minimum yaw speed	0
r^+	Maximum yaw speed	1

- ❑ Minimum Surge

$$\mu_{u1} = \frac{u-u^-}{u^+-u^-} \tag{25}$$

- ❑ Maximum Surge

$$\mu_{u1} = \frac{u^+-u}{u^+-u^-} \tag{26}$$

- ❑ For more motion performed in the same way

2. Determining the Basic Rules

Basic rules are determined from a combination of the maximum and minimum as many as $2n$ where n is the number of models or variables. So that the equation of the AUV motion with 6 DOF had 6 models or variables. The possibilities that may occur are $2^6 = 64$ (see Table 3). By using the basic rules in general, namely:

Rule: if a is A_i then $x_{k+1}^i = \mu_A^i(a)a\, x_k$

Thus fuzzy basic rules which are obtained are as follows: the ground rules i are numbering 64 fuzzy logic rules where the value $\mu_A^i(a)a\, x_k = A_i$, which will be estimated by using the Kalman Filter method [5].

- System and measurement model

$$x_{k+1} = A_i\, x + Bu + w_k \tag{27}$$

$$z_k = Hx + v_k. \tag{28}$$

$$w_k \sim N(0, Q_k), v_k \sim N(0, R_k), x_o \sim N(\overline{x_0}, P_{xo})$$

- Initial condition

$$\hat{x}(0) = \hat{x}_0 \ dan\ P(0) = P_0$$

- Time Update

$$\hat{x}_{k+1}^- = A_k^i \hat{x}_k + B_k u_k \tag{29}$$

$$P_{k+1}^- = A_k P_k A_k^{i\,T} + Q \tag{30}$$

- Measurement Update

Kalman Gain :

$$K_{k+1} = P_{k+1}^- H_{K+1}^T (H_{k+1} P_{k+1}^- H_{K+1}^T + R_{k+1})^{-1} \tag{31}$$

Update estimation :

$$\hat{x}_{k+1} = \hat{x}_{k+1}^- + K_{k+1}(z_{k+1} - H\hat{x}_{k+1}^-) \tag{32}$$

Update Covarian Error :

$$P_{k+1} = (1 - K_{k+1} H_{k+1}) P_{k+1}^- \tag{33}$$

TABLE 3. DETERMINING THE BASIC RULES

u	v	w	p	Q	R	output
1	1	1	1	1	1	A_1
1	1	1	1	1	0	A_2
1	1	1	1	0	1	A_3
1	1	1	1	0	0	A_4
1	1	1	0	1	1	A_5
⋮	⋮	⋮	⋮	⋮	⋮	⋮
0	0	0	0	0	0	A_{64}

Defuzzification

After the basic rules that are applied to the Kalman Filter have been obtained, we gain 64 estimations at each step as below:

$$\hat{x}_{k+1}^i = \begin{bmatrix} \hat{x}_{k+1}^1 \\ \hat{x}_{k+1}^2 \\ \hat{x}_{k+1}^3 \\ \vdots \\ \hat{x}_{k+1}^{64} \end{bmatrix} \tag{34}$$

Fuzzification process is done by:

$$\hat{x}_{k+1}^i = \frac{w_{b1}\hat{x}_{k+1}^1 + w_{b2}\hat{x}_{k+1}^2 + \cdots + w_{b64}\hat{x}_{k+1}^{64}}{w_{b1} + w_{b2} + \cdots + w_{b64}} \tag{35}$$

Where:

$$w_{b1} = \mu_{u2}\mu_{v2}\mu_{w2}\mu_{p2}\mu_{q2}\mu_{r2} \tag{36}$$

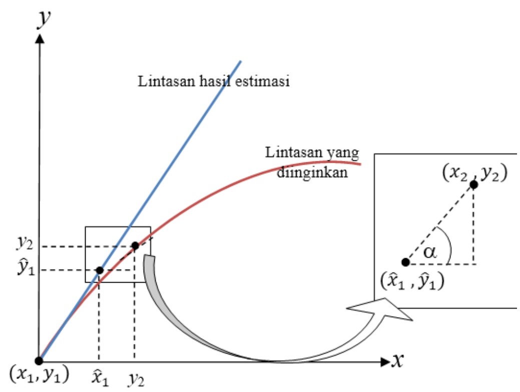
⋮

$$w_{b64} = \mu_{u1}\mu_{v1}\mu_{w1}\mu_{p1}\mu_{q1}\mu_{r1} \tag{37}$$

E. Control position

In correction step, we get the result of position estimation from the AUV motion. Further, based on the estimated value, we will conduct the system control by changing the steering angle of the motion. In general overview, to determine the steering angle based on the results of the estimation is given as follows [3]:

Fig.2 Control Position



In the initial position, we assume the position of AUV is at the point $(x_1,\ y_1)$, further the AUV move in the position $(\hat{x}_1,\ \hat{y}_1)$ when the AUV should be at determined trajectory which at the point $(x_1,\ y_1)$ so that the value of α which is the steering angle obtained by:

$$\tan \alpha_k = \frac{y_{k+1} - \hat{y}_k}{x_{k+1} - \hat{x}_k} \tag{38}$$

$$\alpha_k = arctan\ \frac{y_{k+1} - \hat{y}_k}{x_{k+1} - \hat{x}_k} \tag{39}$$

F. Result and Simulation

Simulations carried out by applying the Fuzzy Kalman Filter algorithm in motion dynamics model of AUV. The simulation will be presented in two-dimensional graph that describes the position on AUV. The simulation results of this research will be compared between the trajectories determined by the results of the estimation using the Fuzzy Kalman Filter Method. In each case to the estimated position, the changes that exist in every movement AUV in the form into the coordinates x,y, and z. by way of [6]:

$$\dot{x} = u\cos\psi - v\sin\psi \tag{40}$$

$$\dot{y} = u\sin\psi + v\cos\psi \tag{41}$$

$$\dot{z} = w \tag{42}$$

$$\dot{\psi} = r \tag{43}$$

Part of this simulation will show the performance of Fuzzy Kalman Filter. In this study used a model error is 10 % of the initial conditions. For each case there is provided a measurement system at 4 motions i.e. surge, sway, heave, and yaw. At this simulation given its initial velocity i.e.
$u_0 = 1.5\,\frac{m}{s}$, $v_0 = 1.5\,\frac{m}{s}$, $w_0 = 1.5\,\frac{m}{s}$, $p_o = 0\,\frac{rad}{s}$, $q_o = 0\,\frac{rad}{s}$, $r_o = 0\,\frac{rad}{s}$.
The initial angle $\delta_r = 5°$and $\delta_s = 5°$ so that the point corresponding to the trajectory and the value of the time change $\Delta t = 0.1$. The number of experimental results of running as many as 30 times.

Fig.3 Case 1

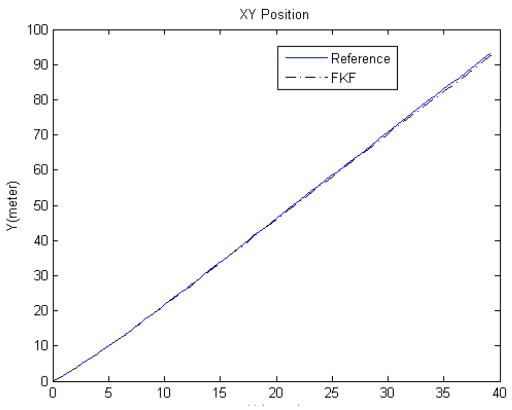


Fig.4 Case 2

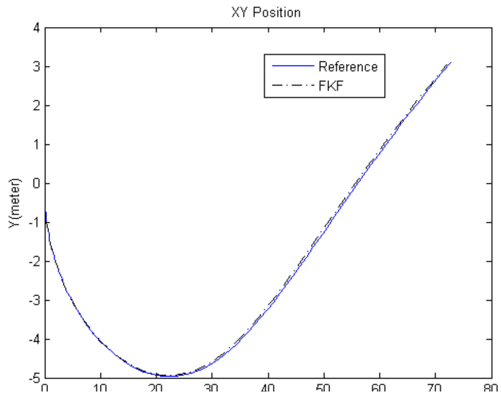


Fig.5 Case 3

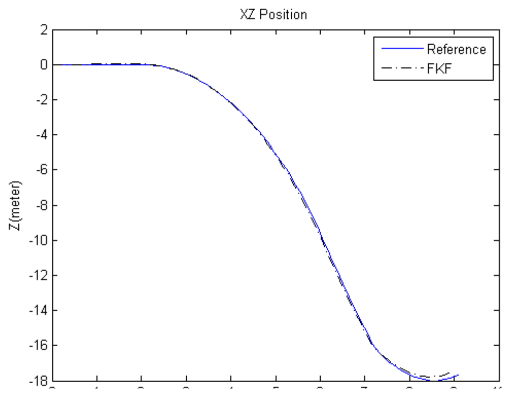


Fig.6 Case 4

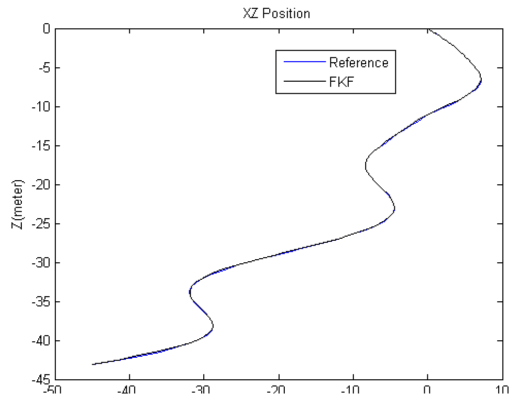


TABLE 4. RMSE

POSITION	CASE 1	CASE 2	CASE 3	CASE 4
X	0.02192	0.06361	0.015666	0.011044
Y	0.0368	0.000805	-	-
Z	-	-	0.004019	0.022705
Angle	0.00096	0.00115	0.001165	0.005212
Time	0.782187	0.8982	0.80199	0.9228

From the case above, we can conclude that FKF can work better to estimate the position of the determined trajectory. However, the time required by every case depends on the complexity of the trajectory. This condition occurs because the speed of six DOF on AUV is varying depend on its trajectory. RSME on the X axis is greater if the trajectory is made on the XY dimension than on the XZ dimension. As for the Y axis, the straight trajectory has greater error than the curved trajectory because the distance of the straight trajectory farther than the curved. However, on the Z axis or diving trajectory, RMSE on the trajectory which has more curve is greater than the RMSE on the trajectory with less curve. Therefore, the angle RMSE on the Case 4 is greater than others.

IV. CONCLUSION

Fuzzy Kalman Filter and Kalman Filter methods can be used to estimate the position of AUV with the desired trajectory. Due to the parameter measurements, i.e. in motion surge, sway, heave, and yaw, each position has relatively small RMSE. In other words, this estimation method can be applied to translational and rotational motion on AUV.

V. CONTRIBUTION OF THIS WORK

This research is one of our contribution as a tool to support for the next research on AUV field in science, environment, marine industry, military, and national defense purposes.

REFERENCES

[1] Hendro Nurhadi, Subchan, Gustiyadi FR, Design of Position Estimation Algorithm of Navigation and Trajectory System for Unmanned Underwater Vehicle ITS AUV-01 using Ensemble Kalman Filter (ENKF) Method, 13th Seminar on Intelligent Technology and Its Applications (SITIA 2012), Surabaya, 23 May 2012.
[2] Fitria, Risa.2011.Implementation Ensemble Kalman Filter on estimates Speed of the submarine. Institute Teknologi Sepuluh Nopember,. Surabaya
[3] Mahmuri, H. (2011), Estimates spread of cancer cells by using fuzzy Kalman Filter, Department Mathematics,Intitute Sepuluh Nopember, Surabaya.
[4] Yang, C.(2007), Modular Modeling and Control for Autonomous Underwater Vehicle (AUV), Department of Mechanical Engineering National University of Singapore, Singapore.
[5] [Chen, G.(1997), *"Fuzzy Kalman Filtering ",* Department of Electrical and Computer Engineering University of Houston, Houston.
[6] Ataei,M.Koma,A.Y(2014), "*Three-dimensional optimal Path planning For waypoint guidance of an autonomous Underwater vehicle* ", Center of Advanced System and Technologies, Faculty of Mechanical Engineering, College of Engineering, University of Tehran, Iran.
[7] Nurhadi,H. (2011)."Rule Based Positioning Optimization for High – Precision LPAT", IEEE Transactions on Instrumental and Measurement, Volume 60, Issue 10, on Page: 3411-3443.

Navigation and Guidance Control System of AUV with Trajectory Estimation of Linear Modelling

Teguh Herlambang
Post Graduate Program in Marine Technology, ITS, Indonesia
teguh@unusa.ac.id

Eko Budi Djatmiko
Department of Ocean Engineering, ITS, Indonesia
ebdjatmiko@oe.its.ac.id

Hendro Nurhadi*
Department of Mechanical Engineering, ITS, Indonesia
hdnurhadi@me.its.ac.id
(*Coresponding Auhor, IEEE Member)

***Abstract*—This paper put forwards a study on the development of navigation and guidance systems for AUV. The restriction in AUV model and estimation on the degree of freedom are recognized as the common problem in AUV's navigation and guidance systems. In this respect a linear model, derived from the linearization using the Jacobian matrix, will be utilized. The so obtained linear model is then estimated by the Ensemble Kalman Filter (EnKF). The implementation of EnKF algorithm on the linear model is carried out by establishing two simulations, namely by generating 300 and 400 ensembles, respectively. The simulations exhibit that the generation of 400 ensembles will give more accurate results in comparison to the generation of 300 ensembles. Furthermore, the best simulation yields the tracking accuracy between the real and simulated trajectories, in translational modes, is in the order of 99.88%, and in rotational modes is in the order of 99.99%.**

Keywords*— *AUV, EnKF, Navigation

I. INTRODUCTION

Over 70% of Indonesia is cover by ocean, so this very potential requires attention and good technology to fully secure the potential of the Indonesian oceans. Underwater robotics technology is very necessary in this case to assist human to do exploration in Indonesian oceans [1]. AUV is very useful for ocean observation since it does not require a tethered cable and so swims freely without restriction [2]. AUV can be used for underwater exploration, mapping and underwater defense system equipment. AUV must clarify its observability and controllability based on a mathematical model [1]. The mathematical model contains various hydrodynamic force and moments expressed collectively in terms of hydrodynamic coefficients [3].

This paper is a study on the development of navigation and guidance systems for AUV. The navigation and guidance is initially modelled as a linear system, derived from the linearization of the non-linear system using the Jacobian matrix, to determine the trajectory in controlling the AUV movement. One of basic Navigation system is trajectory estimation is the Kalman Filter, it is a good candidate method for positioning [4], and we need accurate position estimation [5]. The resulting linear system model is further implemented in the Ensemble Kalman filter (EnKF). In the EnKF method, the algorithm is executed by generating a number of specific ensemble to calculate the mean and covariance error state variables [6].

This paper present trajectory estimation of linear AUV SEGOROGENI ITS system, which is obtain by linearizing nonlinear 6 DOF AUV model with jacobian approach using Ensemble Kalman Filter (EnKF). We present the result of model estimation based on numeric simulation. This paper proposes to validate trajectory estimation of AUV numerically, then it is compared to trajectory reference to get a small root mean square error (RMSE). The implementation of EnKF algorithm on the linear model is carried out by establishing two simulations, namely by generating 300 and 400 ensembles, respectively.

II. AUTONOMOUS UNDERWATER VEHICLE

Two important things required to analyze the Autonomous Underwater Vehicle (AUV) that is Earth Fixed Frame (EFF) and Body Fixed Frame (BFF) in figure 1 [1,5]. EFF is used to describe the position and orientation of the AUV with the position of the x axis to the north, the y-axis to the east and the z-axis toward the center of the earth while BFF is used to describe the speed and acceleration of the AUV with the starting point at the center of gravity [6]. Motion of AUV have 6 DOF where 3 DOF for translational motion and 3 DOF for rotational motion in point x, y and z. Profile and Specification of AUV SEGOROGENI ITS are listed in figure 2 and Table 1.

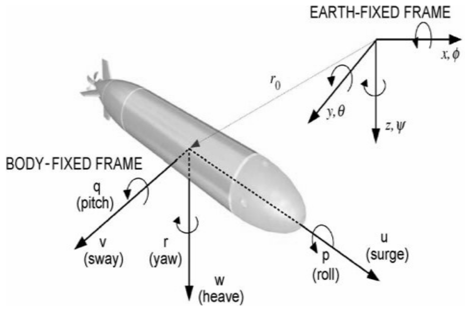


Fig. 1 6 DOF in AUV

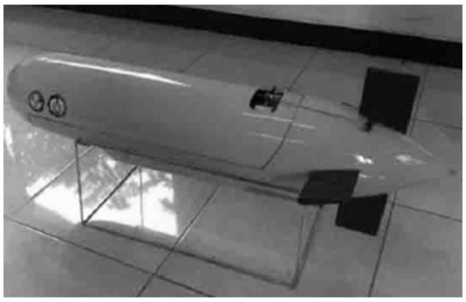


Fig. 2 Profile of AUV SEGOROGENI ITS

Table 2. specification of AUV SEGOROGENI ITS

Weight	15 Kg
Overall Length	980 mm
Beam	180 mm
Controller	Ardupilot Mega 2.0
Communication	Wireless Xbee 2.4 GHz
Camera	TTL Camera
Battery	Li-Pro 11,8 V
Propulsion	12V motor DC
Propeller	3 Blades OD : 40 mm
Speed	1,94 knots (1m/s)

General equation of motion in 6-DOF AUV consists of translational and rotational as follows [9]:

Surge:

$$m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X_{|u|u}u|u| + X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop} \quad (1)$$

Sway :

$$m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(pq + \dot{r})] = Y_{res} + Y_{|v|v}v|v| + Y_{r|r|}r|r| + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r \quad (2)$$

Heave :

$$m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = Z_{res} + Z_{|w|w}w|w| + Z_{q|q|}q|q| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq}uq + Z_{vp}vp + Z_{rp}rp + Z_{uw}uw + Z_{uu\delta_s}u^2\delta_s \quad (3)$$

Roll:

$$I_x\dot{p} + (I_z - I_y)qr + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = K_{res} + K_{p|p|}p|p| + K_{\dot{p}}\dot{p} + K_{prop} \quad (4)$$

Pitch :

$$I_y\dot{q} + (I_x - I_z)rp + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] = M_{res} + M_{w|w|}w|w| + M_{q|q|}q|q| + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uq}uq + M_{vp}vp + M_{rp}rp + M_{uw}uw + M_{uu\delta_s}u^2\delta_s \quad (5)$$

Yaw :

$$I_z\dot{r} + (I_y - I_z)pq + m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = N_{res} + N_{v|v|}v|v| + N_{r|r|}r|r| + N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv + N_{uu\delta_r}u^2\delta_r \quad (6)$$

Translational motion u, v and w are representation of surge, sway and heave. Rotational motion p, q and r are representation of roll, pitch and yaw. The nonlinear system of AUV model can be linearized with Jacobian matrix where the nonlinear AUV system in general as follows :

$$\dot{x}(t) = f(x(t), u(t), t)$$
$$y(t) = g(x(t), u(t), t) \quad (7)$$

So the Jacobian matrix is formed as follows [10] :

$$\frac{\partial f(\bar{x},\bar{u},t)}{\partial x} = \begin{bmatrix} \frac{\partial f_1(\bar{x},\bar{u},t)}{\partial x_1} & \frac{\partial f_1(\bar{x},\bar{u},t)}{\partial x_2} & \cdots & \frac{\partial f_1(\bar{x},\bar{u},t)}{\partial x_n} \\ \frac{\partial f_2(\bar{x},\bar{u},t)}{\partial x_1} & \frac{\partial f_2(\bar{x},\bar{u},t)}{\partial x_2} & \cdots & \frac{\partial f_2(\bar{x},\bar{u},t)}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n(\bar{x},\bar{u},t)}{\partial x_1} & \frac{\partial f_n(\bar{x},\bar{u},t)}{\partial x_2} & \cdots & \frac{\partial f_n(\bar{x},\bar{u},t)}{\partial x_n} \end{bmatrix} \quad (8)$$

So equation 1 - 6 can be expressed as follows :

$$\begin{bmatrix} \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} & \begin{matrix} 0 & \frac{mz_G}{m-X_{\dot{u}}} & \frac{-my_G}{m-X_{\dot{u}}} \\ -\frac{mz_G}{m-Y_{\dot{v}}}+ & 0 & \frac{(mx_G-Y_{\dot{r}})}{m-Y_{\dot{v}}} \\ \frac{my_G}{m-Z_{\dot{w}}} & -\frac{(mx_G+Z_{\dot{q}})}{m-Z_{\dot{w}}} & 0 \end{matrix} \\ \begin{matrix} 0 & -\frac{m\,z_G}{I_x-K_{\dot{p}}} & \frac{m\,y_G}{I_x-K_{\dot{p}}} \\ \frac{m\,z_G}{I_y-M_{\dot{q}}} & 0 & -\frac{(m\,x_G+M_{\dot{w}})}{I_y-M_{\dot{q}}} \\ -\frac{my_G}{I_z-N_{\dot{r}}} & \frac{(mx_G-N_{\dot{v}})}{I_z-N_{\dot{r}}} & 0 \end{matrix} & \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix} \quad (9)$$

Where $f_1, f_2, f_3, f_4, f_5, f_6$ expressed as follows :

$$f_1 = \frac{X_{res}+X_{|u|u}u|u|+X_{wq}wq+X_{qq}qq+X_{vr}vr+X_{rr}rr+X_{prop}-m[-vr+wq-x_G(q^2+r^2)+pq\,y_G+pr\,z_G]}{m-X_{\dot{u}}} \quad (10)$$

$$f_2 = \frac{Y_{res}+Y_{|v|v}v|v|+Y_{r|r|}r|r|+Y_{ur}ur+Y_{wp}wp+Y_{pq}pq+Y_{uv}uv+Y_{uu\delta_r}u^2\delta_r-m[-wp+ur-y_G(r^2+p^2)+qr\,z_G+pq\,x_G]}{m-Y_{\dot{v}}} \quad (11)$$

$$f_3 = \frac{Z_{res}+Z_{|w|w}w|w|+Z_{q|q|}q|q|+Z_{uq}uq+Z_{vp}vp+Z_{rp}rp+Z_{uw}uw+Z_{uu\delta_s}u^2\delta_s-m[-uq+vp-z_G(p^2+q^2)+rp\,x_G+rq\,y_G]}{m-Z_{\dot{w}}} \quad (12)$$

$$f_4 = \frac{K_{res}+K_{p|p|}p|p|+K_{prop}-\left((I_z-I_y)qr+m\begin{bmatrix} y_G(-uq+vp)- \\ z_G(-wp+ur) \end{bmatrix}\right)}{I_x-K_{\dot{p}}} \quad (13)$$

$$f_5 = \frac{M_{res}+M_{w|w|}w|w|+M_{q|q|}q|q|+M_{uq}uq+M_{vp}vp+M_{rp}rp+M_{uw}uw+M_{uu\delta_s}u^2\delta_s-((I_x-I_z)rp+m[z_G(-vr+wq)-x_G(-uq+vp)])}{I_y-M_{\dot{q}}} \quad (14)$$

$$f_6 = \frac{N_{res}+N_{v|v|}v|v|+N_{r|r|}r|r|+N_{ur}ur+N_{wp}wp+N_{pq}pq+N_{uv}uv+N_{uu\delta_r}u^2\delta_r-((I_y-I_z)pq+m[x_G(-wp+ur)-y_G(-vr+wq)])}{I_z-N_{\dot{r}}} \quad (15)$$

Furthermore linear system is obtained as follows [7]:

$$\dot{x}(t) = A\,x(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \quad (16)$$

with

$$A = J_x = \begin{bmatrix} \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} & \begin{matrix} 0 & \frac{mz_G}{m-X_{\dot{u}}} & \frac{-my_G}{m-X_{\dot{u}}} \\ -\frac{mz_G}{m-Y_{\dot{v}}}+ & 0 & \frac{(mx_G-Y_{\dot{r}})}{m-Y_{\dot{v}}} \\ \frac{my_G}{m-Z_{\dot{w}}} & -\frac{(mx_G+Z_{\dot{q}})}{m-Z_{\dot{w}}} & 0 \end{matrix} \\ \begin{matrix} 0 & -\frac{m\,z_G}{I_x-K_{\dot{p}}} & \frac{m\,y_G}{I_x-K_{\dot{p}}} \\ \frac{m\,z_G}{I_y-M_{\dot{q}}} & 0 & -\frac{(m\,x_G+M_{\dot{w}})}{I_y-M_{\dot{q}}} \\ -\frac{my_G}{I_z-N_{\dot{r}}} & \frac{(mx_G-N_{\dot{v}})}{I_z-N_{\dot{r}}} & 0 \end{matrix} & \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \end{bmatrix}^{-1} \begin{bmatrix} a_1 & b_1 & c_1 & d_1 & e_1 & g_1 \\ a_2 & b_2 & c_2 & d_2 & e_2 & g_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 & g_3 \\ a_4 & b_4 & c_4 & d_4 & e_4 & g_4 \\ a_5 & b_5 & c_5 & d_5 & e_5 & g_5 \\ a_6 & b_6 & c_6 & d_6 & e_6 & g_6 \end{bmatrix} \quad (17)$$

$$B = J_u = \begin{bmatrix} \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} & \begin{matrix} 0 & \frac{mz_G}{m-X_{\dot{u}}} & \frac{-my_G}{m-X_{\dot{u}}} \\ -\frac{mz_G}{m-Y_{\dot{v}}}+ & 0 & \frac{(mx_G-Y_{\dot{r}})}{m-Y_{\dot{v}}} \\ \frac{my_G}{m-Z_{\dot{w}}} & -\frac{(mx_G+Z_{\dot{q}})}{m-Z_{\dot{w}}} & 0 \end{matrix} \\ \begin{matrix} 0 & -\frac{m\,z_G}{I_x-K_{\dot{p}}} & \frac{m\,y_G}{I_x-K_{\dot{p}}} \\ \frac{m\,z_G}{I_y-M_{\dot{q}}} & 0 & -\frac{(m\,x_G+M_{\dot{w}})}{I_y-M_{\dot{q}}} \\ -\frac{my_G}{I_z-N_{\dot{r}}} & \frac{(mx_G-N_{\dot{v}})}{I_z-N_{\dot{r}}} & 0 \end{matrix} & \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \end{bmatrix}^{-1} \begin{bmatrix} A_1 & B_1 & C_1 & D_1 & E_1 & G_1 \\ A_2 & B_2 & C_2 & D_2 & E_2 & G_2 \\ A_3 & B_3 & C_3 & D_3 & E_3 & G_3 \\ A_4 & B_4 & C_4 & D_4 & E_4 & G_4 \\ A_5 & B_5 & C_5 & D_5 & E_5 & G_5 \\ A_6 & B_6 & C_6 & D_6 & E_6 & G_6 \end{bmatrix} \quad (18)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } D = 0$$

So
$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = A \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix} + B \begin{bmatrix} X_{prop} \\ \delta_r \\ \delta_s \\ K_{prop} \\ \delta_s \\ \delta_r \end{bmatrix} \qquad (19)$$

Where $a_1, a_2, \ldots, g_6$ and $A_1, A_2, \ldots, G_6$ Component of Matrix A and B (Result of linearization using Jacobian Matrix) in Table 3 and 4.

Table 3. Component of Matrix A

$a_1 = \frac{\partial f_1}{\partial u}$	$b_1 = \frac{\partial f_1}{\partial v}$	$c_1 = \frac{\partial f_1}{\partial w}$
$a_2 = \frac{\partial f_2}{\partial u}$	$b_2 = \frac{\partial f_2}{\partial v}$	$c_2 = \frac{\partial f_2}{\partial w}$
$a_3 = \frac{\partial f_3}{\partial u}$	$b_3 = \frac{\partial f_3}{\partial v}$	$c_3 = \frac{\partial f_3}{\partial w}$
$a_4 = \frac{\partial f_4}{\partial u}$	$b_4 = \frac{\partial f_4}{\partial v}$	$c_4 = \frac{\partial f_4}{\partial w}$
$a_5 = \frac{\partial f_5}{\partial u}$	$b_5 = \frac{\partial f_5}{\partial v}$	$c_5 = \frac{\partial f_5}{\partial w}$
$a_6 = \frac{\partial f_6}{\partial u}$	$b_6 = \frac{\partial f_6}{\partial v}$	$c_6 = \frac{\partial f_6}{\partial w}$
$d_1 = \frac{\partial f_1}{\partial p}$	$e_1 = \frac{\partial f_1}{\partial q}$	$g_1 = \frac{\partial f_1}{\partial r}$
$d_2 = \frac{\partial f_2}{\partial p}$	$e_2 = \frac{\partial f_2}{\partial q}$	$g_2 = \frac{\partial f_2}{\partial r}$
$d_3 = \frac{\partial f_3}{\partial p}$	$e_3 = \frac{\partial f_3}{\partial q}$	$g_3 = \frac{\partial f_3}{\partial r}$
$d_4 = \frac{\partial f_4}{\partial p}$	$e_4 = \frac{\partial f_4}{\partial q}$	$g_4 = \frac{\partial f_4}{\partial r}$
$d_5 = \frac{\partial f_5}{\partial p}$	$e_5 = \frac{\partial f_5}{\partial q}$	$g_5 = \frac{\partial f_5}{\partial r}$
$d_6 = \frac{\partial f_6}{\partial p}$	$e_6 = \frac{\partial f_6}{\partial q}$	$g_6 = \frac{\partial f_6}{\partial r}$

Table 4. Component of Matrix B

$A_1 = \frac{\partial f_1}{\partial X_{prop}}$	$B_1 = \frac{\partial f_1}{\partial \delta_r}$	$C_1 = \frac{\partial f_1}{\partial \delta_s}$
$A_2 = \frac{\partial f_2}{\partial X_{prop}}$	$B_2 = \frac{\partial f_2}{\partial \delta_r}$	$C_2 = \frac{\partial f_2}{\partial \delta_s}$
$A_3 = \frac{\partial f_3}{\partial X_{prop}}$	$B_3 = \frac{\partial f_3}{\partial \delta_r}$	$C_3 = \frac{\partial f_3}{\partial \delta_s}$
$A_4 = \frac{\partial f_4}{\partial X_{prop}}$	$B_4 = \frac{\partial f_4}{\partial \delta_r}$	$C_4 = \frac{\partial f_4}{\partial \delta_s}$
$A_5 = \frac{\partial f_5}{\partial X_{prop}}$	$B_5 = \frac{\partial f_5}{\partial \delta_r}$	$C_5 = \frac{\partial f_5}{\partial \delta_s}$
$A_6 = \frac{\partial f_6}{\partial X_{prop}}$	$B_6 = \frac{\partial f_6}{\partial \delta_r}$	$C_6 = \frac{\partial f_6}{\partial \delta_s}$
$D_1 = \frac{\partial f_1}{\partial K_{prop}}$	$E_1 = \frac{\partial f_1}{\partial \delta_s}$	$G_1 = \frac{\partial f_1}{\partial \delta_r}$
$D_2 = \frac{\partial f_2}{\partial K_{prop}}$	$E_2 = \frac{\partial f_2}{\partial \delta_s}$	$G_2 = \frac{\partial f_2}{\partial \delta_r}$
$D_3 = \frac{\partial f_3}{\partial K_{prop}}$	$E_3 = \frac{\partial f_3}{\partial \delta_s}$	$G_3 = \frac{\partial f_3}{\partial \delta_r}$
$D_4 = \frac{\partial f_4}{\partial K_{prop}}$	$E_4 = \frac{\partial f_4}{\partial \delta_s}$	$G_4 = \frac{\partial f_4}{\partial \delta_r}$
$D_5 = \frac{\partial f_5}{\partial K_{prop}}$	$E_5 = \frac{\partial f_5}{\partial \delta_s}$	$G_5 = \frac{\partial f_5}{\partial \delta_r}$
$D_6 = \frac{\partial f_6}{\partial K_{prop}}$	$E_6 = \frac{\partial f_6}{\partial \delta_s}$	$G_6 = \frac{\partial f_6}{\partial \delta_r}$

III. Ensemble Kalman Filter

The algorithm *Ensemble Kalman Filter* (EnKF) can be seen [11]:

Model system and measurement model

$$x_{k+1} = f(x_k, u_k) + w_k \qquad (20)$$

$$z_k = Hx_k + v_k \qquad (21)$$

$$w_k \sim N(0, Q_k), \; v_k \sim N(0, R_k) \qquad (22)$$

1. Inisialitation

   Generate N *ensemble* as the first guess $\bar{x}_0$

   $$x_{0,i} = [x_{0,1} \quad x_{0,2} \quad \cdots \quad x_{0,N}] \qquad (23)$$

   The first value: $\hat{x}_0 = \frac{1}{N}\sum_{i=1}^{N} x_{0,i}$ (24)

2. Time Update

   $\hat{x}_{k,i}^- = f(\hat{x}_{k,-1,i}, u_{k-1,i}) + w_{k,i}$ where $w_{k,i} = N(0, Q_k)$ (25)

   Estimation : $\hat{x}_k^- = \frac{1}{N}\sum_{i=1}^{N} \hat{x}_{k,i}^-$ (26)

   Error *covariance*:

   $$P_k^- = \frac{1}{N-1}\sum_{i=1}^{N}(\hat{x}_{k,i}^- - \hat{x}_k^-)(\hat{x}_{k,i}^- - \hat{x}_k^-)^T \qquad (27)$$

3. Measurement Update

   $z_{k,i} = Hx_{k,i} + v_{k,i}$ where $v_{k,i} \sim N(0, R_k)$ (28)

   Kalman gain : $K_k = P_k^- H^T (HP_k^- H^T + R_k)^{-1}$ (29)

   Estimation : $\hat{x}_{k,i} = \hat{x}_{k,i}^- + K_k(z_{k,i} - H\hat{x}_{k,i}^-)$ (30)

   $\hat{x}_k = \frac{1}{N}\sum_{i=1}^{N} \hat{x}_{k,i}$ (31)

   Error covariance : $P_k = [I - K_k H]P_k^-$ (32)

IV. Computational Result

This simulation was carried out by implementing an algorithm Ensemble Kalman Filter (EnKF) in the AUV model. The result of the simulation was evaluated and compared with real condition, estimaton result by EnKF. This simulation consist of two types of simulations. That is the first simulation by generating 300 ensemble and the second simulation by generating 400 ensembles. The simulations were conducted by assuming surge (u), sway (v), heave (w), roll (p), pitch $(q)$and yaw (r). The value of Δt has been used was $\Delta t = 0{,}1$. The initial condition used were $u_0 = 0\,m$, $v_0 = 0\,m$, $w_0 = 0\,m$,$p_0 = 0\,rad$, $q_0 = 0\,rad$ and $r_0 = 0\,rad$.

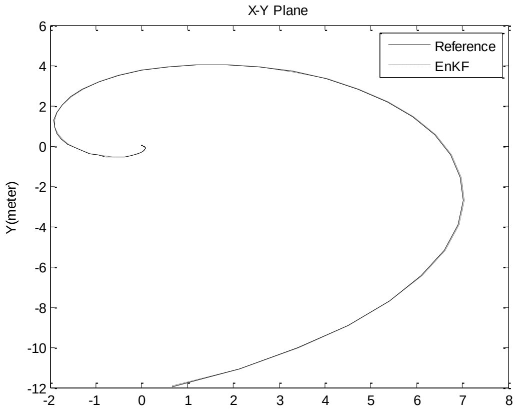


Figure 2. Trajectory Estimation of 6 DOF for XY Plane with 400 ensemb

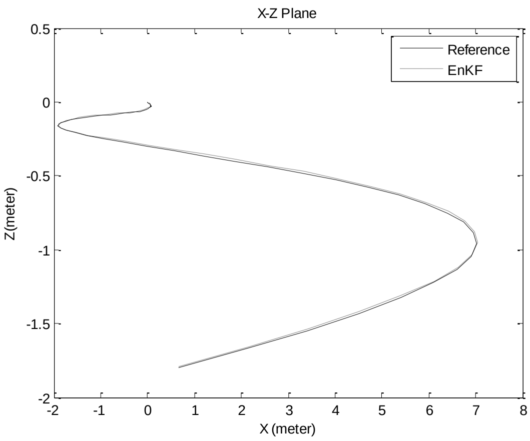


Figure 3. Trajectory Estimation of 6 DOF for XZ Plane with 400 ensemble

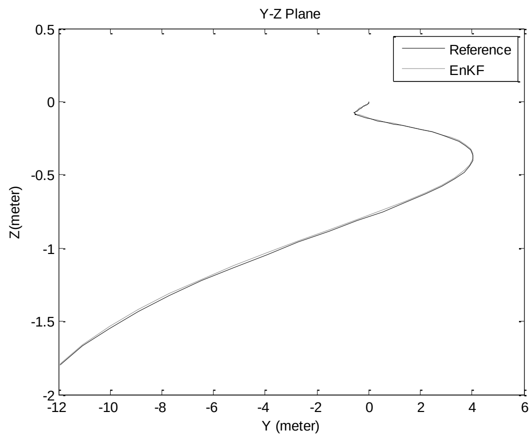


Figure 4. Trajectory Estimation of 6 DOF for YZ Plane with 400 ensemble

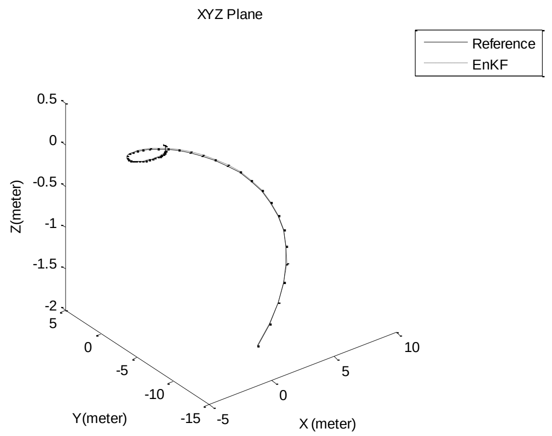


Figure 5. Trajectory Estimation of 6 DOF for XYZ Plane with 400 ensemble

Figure 2, 3, 4 and 5 shows AUV moves following the desire trajectory both on XY, XZ, YZ and XYZ plane with high accuracy. In figure 2, AUV goes to the left and then turn arround clock wise. In fiu 3, AUV dives by turning right and left until depth of 1,9 meter. In general as seen in the Table 5, the results of the two simulations were highly accurate. The first simulation by generate 300 ensemble with tracking error of translational motion 0,0082 m/s or accuracy of 99,82% and rotational motion 0,00099 rad/s or accuracy of 99,99%. the second simulation by generate 400 ensemble with tracking error of translational motion 0,007 m/s or accuracy of 99,88% and rotational motion 0,00091 rad/s or accuracy of 99,99%. Time simulation of the two simulation results with 300 ensemble faster than 400 because more ensemble generated the longer time simulation.

Table 5. RMSE value from Computational Result

	300 Ensemble		400 Ensemble	
	RMSE	Accuracy	RMSE	Accuracy
Surge	0.0071 m/s	99,98%	0.0077m/s	99,96%
Sway	0.0094m/s	99,8%	0.0071m/s	99,93%
Heave	0.0081m/s	99,7%	0.0063 m/s	99,75%
Roll	0.00099 rad/s	99,99%	0.00098 rad/s	99,99%
Pitch	0.00094 rad/s	99,99%	0.00088 rad/s	99,99%
Yaw	0.00105rad/s	99,99%	0.00088 rad/s	99,99%
Time	3.8125 s		5.0469 s	

V. CONCLUSION

Based on analysis of the two simulation results, EnKF method could be applied to estimate of linear system trajectory of AUV SEGOROGENI ITS with considerably high accuracy. Of the two simulation by generating both 300 and 400 ensembles, the estimation results were all accurate.

REFERENCES

[1] Herlambang, T., Nurhadi H and Subchan, Preliminary Numerical Study on Designing Navigation and Stability Control Systems for ITS AUV, Applied Mechanics and Materials Vol. 493 (2014) pp 420-425 Trans Tech Publications, Switzerland

[2] Kondo, H., and Ura, T., Navigation of an AUV for investigation of underwater structures, Control Engineering Practice, 12 (2004) 1551–1559 Elsevier.

[3] Kim, K., and Choi, H.S., 2007, Analysis on the controlled nonlinear motion of a test bed AUV-SNUUV 1, Ocean Engineering 34 (2007) 1138–1150

[4] Kalman, R.E., and Bucy, R.S., 1961. New Result in Linear Filtering and Prediction Theory. ASME Journal of Basic Engineering.

[5] Nurhadi. H, 2011 "Multistage rule-based positioning Optimization for High-Precision LPAT", IEEE Transaction on instrumentation and Measurement, Vol 60, No 10, October 2011.

[6] Evensen, G, Data Asimilation The Ensemble Kalman Filter (second edition),2009 Springer-Verlag Berlin Hiedelberg London and New York.

[7] Herlambang, T., Djatmiko E.B and Nurhadi H, Optimization with Jacobian Approach for ITS AUV System, 2014, international conference on Marine Technology

[8] Fossen, T. I, A Nonlinear Unified State-space Model for Ship Maneuvering and Control in A Seaway-Journal of Bifurcation and Chaos, 2005, Journal of Bifurcation and Chaos.

[9] Yang, C, Modular Modelling and Control for Autonomous Vehicle (AUV). Master Thesis of Engineering, 2007, Department of Mechanical Engineering National University of Singapore.

[10] Subiono, Mathematical System (second edition), 2010, Department of Mathematics Sepuluh Nopember Institut of Technology, Surabaya.

[11] Herlambang, T, Square Root Ensemble Kalman Filter (SR-EnKF) for Estimation of Missile Position", Magister Thesis, 2012, Department of Mathematics Sepuluh Nopember Institut of Technology, Surabaya

Praise Worthy Prize

International Review of Mechanical Engineering (I.RE.M.E.), Vol. 9, N. 6
ISSN 1970 - 8734 *November 2015*

Ensemble Kalman Filter with a Square Root Scheme (EnKF-SR) for Trajectory Estimation of AUV SEGOROGENI ITS

Teguh Herlambang, Eko Budi Djatmiko, Hendro Nurhadi

Abstract – *Results of a study on the development of navigation system and guidance for AUV are presented in this paper. The study was carried to evaluate the behavior of AUV SEGOROGENI ITS, designed with a characteristic length of 980 mm, cross-section diameter of 180 mm, for operation in a 3.0 m water depth, at a maximum forward speed of 1.94 knots. The most common problem in the development of AUVs is the limitation in the mathematical model and the restriction on the degree of freedom in simulation. In this study a model of linear system was implemented, derived from a non-linear system that is linearized utilizing the Jacobian matrix. The linear system is then implemented as a platform to estimate the trajectory. In this respect the estimation is carried out by adopting the method of Ensemble Kalman Filter Square Root (EnKF-SR). The EnKF-SR method basically is developed from EnKF at the stage of correction algorithm. The implementation of EnKF-SR on the linear model comprises of three simulations, each of which generates 100, 200 and 300 ensembles. The best simulation exhibited the error between the real tracking and the simulation in translation mode was in the order of 0.009 m/s, whereas in the rotation mode was some 0.001 rad/s. These fact indicates the accuracy of higher than 95% has been achieved.* ***Copyright © 2015 Praise Worthy Prize S.r.l. - All rights reserved.***

***Keywords**: AUV, EnKF-SR, Linear System, Trajectory Estimation*

Nomenclature

AUV	Autonomous Underwater Vehicle
DOF	Degree of Freedom
SVD	Singular Value Decomposition
EnKF-SR	Ensemble Kalman Filter Square Root
SNAME	The Society of Naval Architects and Marine Engineers
$\eta = [\eta_1^T, \eta_2^T]^T$	The position and orientation vector in the earth-fixed coordinates
$\eta_1 = [x, y, z]^T$	The linear position vector in the earth-fixed coordinates
$\eta_2 = [\emptyset, \theta, \Psi]^T$	The angular position vector in the earth-fixed coordinates
$v = [v_1^T, v_2^T]^T$	The linear and angular velocity vector in the body-fixed coordinates
$v_1 = [u, v, w]^T$	The linear velocity vector in the body-fixed coordinates
$v_2 = [p, q, r]^T$	The angular velocity vector in the body-fixed coordinates
$\tau = [\tau_1^T, \tau_2^T]^T$	The forces and moments acting on the vehicle in the body-fixed coordinates
$\tau_1 = [X, Y, Z]^T$	The forces acting on the vehicle in the body-fixed coordinates
$\tau_2 = [K, M, N]^T$	The moments acting on the vehicle in the body-fixed coordinates
$[x_G, y_G, z_G]$	The AUV's center of gravity in body fixed coordinates
$[I_x, I_y, I_z]$	The moments of inertia about the X, Y, Z axes respectively
$f_1, f_2, f_3, f_4, f_5, f_6$	Surge, Sway, Heave, Roll, Pitch and Yaw for Function in Jacobian Matrix
$a_1, a_2, \ldots, g_6$	Component of Matrix *A* (Result of linearization using Jacobian Matrix)
$A_1, A_2, \ldots, G_6$	Component of Matrix *B* (Result of linearization using Jacobian Matrix)

I. Introduction

Geographical area of Indonesia consists of islands and waters. Approximately two-thirds of the total area of Indonesia is water. Its strategic position, tropical climate and abundant natural resources offer high economic potential as well as national defensive potential, thus sophisticated underwater robotics technology is required to keep both national security and sea treasure of Indonesia. Underwater robotics technology currently being developed is an Autonomous Underwater Vehicle (AUV). AUV is capably underwater vehicle in moving automatically without direct control by humans according to the trajectory.

AUV can be used for underwater exploration, mapping, underwater defense system equipment, sensor off board submarines, inspection of underwater structures, natural resources and the condition of the Earth's surface plates etc [1]. Two important things required to analyze the Autonomous Underwater Vehicle (AUV) that is Earth Fixed Frame (EFF) and Body Fixed Frame (BFF) [2].

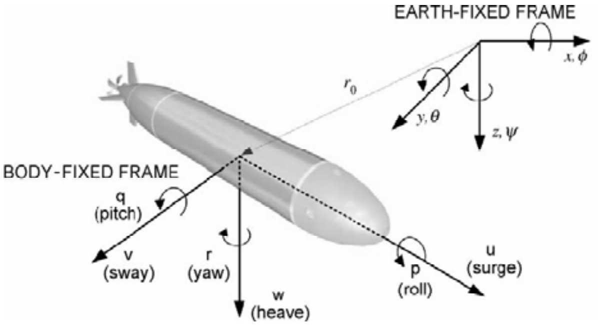


Fig. 1. 6 DOF in AUV

In a great number of envioned mission scenarios, AUV will be need to follow inertial reference trajectory accurately [3]. To achieve that purpose, the navigation system must be designed and implemented on AUV.

One of basic Navigation system is trajectory estimation was was introduced in the 1961s and the most popular of estimation methods is the Kalman Filter.

Kalman filter is method of a state variable estimation from linear discrete dynamic system which minimizes the estimation error covariance [4], [12]-[15]. Another approximation is an extension of the Kalman filter called Ensemble Kalman Filter (EnKF) and Ensemble Kalman Filter Square Root (EnKF-SR).

In the EnKF method, the algorithm is executed by generating a number of specific ensemble to calculate the mean and covariance error state variables [5]. Ensemble Kalman Filter Square Root (EnKF-SR) is development of EnKF algorithm which Square root scheme is one scheme can be implemented in correction step [6].

The main contribution of this paper is trajectory estimation of linear AUV SEGOROGENI ITS system with Ensemble Kalman Filter Square Root (EnKF-SR). Linear model is obtained by linearizing nonlinear 6 DOF AUV model with Jacobian matrix. Linear system is platform to make trajectory estimation. The result of this paper is numeric simulation by comparing real trajectory and the result of trajectory estimation to get a small root mean square error (RMSE).

This paper consists of three types of simulations which the first, second and third simulation by generate 100, 200 and 300 ensemble. Profile of SEGOROGENI ITS depicted in Fig. 2.

Fig. 2. Profile of AUV SEGOROGENI ITS

II. Mathematical Model

Two important things required to analyze the Autonomous Underwater Vehicle (AUV) that is Earth Fixed Frame (EFF) and Body Fixed Frame (BFF) (Yang, 2007) [2]. EFF is used to describe the position and orientation of the AUV with the position of the x axis to the north, the y-axis to the east and the z-axis toward the center of the earth while BFF is used to describe the speed and acceleration of the AUV with the starting point at the center of gravity. x-axis to the ship's bow, the positive y axis direct to the right side of the ship and the positive z-axis direct [7], [8]. Motion of AUV have 6 DOF where 3 DOF for translational motion and 3 DOF for rotational motion in point *x*, *y* and *z* are listed in Table I.

TABLE I
NOTATION OF AUV MOTION AXIS [9]

DOF	Translational And Rotational	Force/ Moment	Linear and Anguler Velocity	Potition/ Angle Euler
1	Surge	X	U	x
2	Sway	Y	V	y
3	Heave	Z	W	z
4	Roll	K	P	ϕ
5	Pitch	M	Q	θ
6	Yaw	N	R	

The dynamics of the AUV there are external forces influencing the movement follows as [9]:

$$\begin{aligned} \tau = \tau_{hidrostatis} + \tau_{addedmass} + \\ +\tau_{drag} + \tau_{lift} + \tau_{control} \end{aligned} \tag{1}$$

$$\begin{aligned} \eta = [\eta_1^T, \eta_2^T]^T, \eta_1 = [x, y, z]^T, \eta_2 = [\emptyset, \theta, \Psi]^T \\ v = [v_1^T, v_2^T]^T, v_1 = [u, v, w]^T, v_2 = [p, q, r]^T \\ \tau = [\tau_1^T, \tau_2^T]^T, \tau_1 = [X, Y, Z]^T, \tau_2 = [K, M, N]^T \end{aligned} \tag{2}$$

where η vector position the position and orientation of the EFF, v vector velocity of linear and anguler of the BFF, the position and orientation of the BFF, and τ description of force and moment in AUV of the BFF.

By combining equations hydrostatic, lift, added mass, drag, thrust and assuming a diagonal tensor of inertia (I_o) is zero then the total forces and moments of models obtained from the following [2].

General equation of motion in 6-DOF AUV consists of translational and rotational as follows [2]:

Surge:

$$\begin{aligned} m[\dot{u} - vr + wq - x_G(q^2 + r^2) + \\ +y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = \\ X_{|u|u}u|u| + X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}qq + \\ +X_{vr}vr + X_{rr}rr + X_{prop} \end{aligned} \tag{3}$$

Sway:

$$\begin{aligned} m[\dot{v} - wp + ur - y_G(r^2 + p^2) + \\ +z_G(qr - \dot{p}) + x_G(pq + \dot{r})] = \\ Y_{res} + Y_{|v|v}v|v| + Y_{r|r|}r|r| + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur}ur + \\ +Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r \end{aligned} \tag{4}$$

Heave:

$$\begin{gathered} m[\dot{w} - uq + vp - z_G(p^2 + q^2) + \\ + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = \\ Z_{res} + Z_{|w|w} w|w| + Z_{q|q|} q|q| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + \\ + Z_{uq} uq + Z_{vp} vp + Z_{rp} rp + Z_{uw} uw + Z_{uu\delta_s} u^2 \delta_s \end{gathered} \quad (5)$$

Roll:

$$\begin{gathered} I_x \dot{p} + \left(I_z - I_y\right) qr + m[y_G(\dot{w} - uq + vp) + \\ - z_G(\dot{v} - wp + ur)] = \\ K_{res} + K_{p|p|} p|p| + K_{\dot{p}}\dot{p} + K_{prop} \end{gathered} \quad (6)$$

Pitch:

$$\begin{gathered} I_y \dot{q} + (I_x - I_z) rp + m[z_G(\dot{u} - vr + wq) + \\ - x_G(\dot{w} - uq + vp)] = \\ M_{res} + M_{w|w|} w|w| + M_{q|q|} q|q| + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + \\ + M_{uq} uq + M_{vp} vp + M_{rp} rp + M_{uw} uw + M_{uu\delta_s} u^2 \delta_s \end{gathered} \quad (7)$$

Yaw:

$$\begin{gathered} I_z \dot{r} + \left(I_y - I_z\right) pq + m[x_G(\dot{v} - wp + ur) + \\ - y_G(\dot{u} - vr + wq)] = \\ N_{res} + N_{v|v|} v|v| + N_{r|r|} r|r| + N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + \\ + N_{ur} ur + N_{wp} wp + N_{pq} pq + N_{uv} uv + N_{uu\delta_r} u^2 \delta_r \end{gathered} \quad (8)$$

Translational motion u, v and w are representation of surge, sway and heave. Rotational motion p, q and r are representation of roll, pitch and yaw.

This type of AUV SEGOROGENI ITS using only one propeller on the tail AUV which will produce x_{prop} and additional moments K_{prop}. External forces and moments acting on the AUV are the hydrostatic force, thrust and hydrodynamic force and where every object in the water will have a hydrostatic force consisting of gravity and buoyancy forces.

While hydrodynamic component consists of added mass, drag and lift. Specification of AUV SEGOROGENI ITS in Table II.

TABLE II
SPECIFICATION OF AUV SEGOROGENI ITS

Weight	15 kg
Overall Length	980 mm
Beam	180 mm
Controller	Ardupilot Mega 2.0
Communication	Wireless Xbee 2.4 GHz
Camera	TTL Camera
Battery	Li-Pro 11,8 V
Propulsion	12V motor DC
Propeller	3 Blades OD : 40 mm
Speed	1,94 knots (1m/s)

III. Linearization

In this paper the nonlinear system of AUV model can be linearized with Jacobian matrix where the nonlinear AUV system in general as follows:

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), t) \\ y(t) &= g(x(t), u(t), t) \end{aligned} \quad (9)$$

So the Jacobian matrix is formed as follows [10]:

$$\frac{\partial f(\bar{x}, \bar{u}, t)}{\partial x} = \begin{bmatrix} \frac{\partial f_1(\bar{x}, \bar{u}, t)}{\partial x_1} & \frac{\partial f_1(\bar{x}, \bar{u}, t)}{\partial x_2} & \cdots & \frac{\partial f_1(\bar{x}, \bar{u}, t)}{\partial x_n} \\ \frac{\partial f_2(\bar{x}, \bar{u}, t)}{\partial x_1} & \frac{\partial f_2(\bar{x}, \bar{u}, t)}{\partial x_2} & \cdots & \frac{\partial f_2(\bar{x}, \bar{u}, t)}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n(\bar{x}, \bar{u}, t)}{\partial x_1} & \frac{\partial f_n(\bar{x}, \bar{u}, t)}{\partial x_2} & \cdots & \frac{\partial f_n(\bar{x}, \bar{u}, t)}{\partial x_n} \end{bmatrix} \quad (10)$$

So Eq. (3) - (8) can be expressed as follows:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \frac{m z_G}{m - X_{\dot{u}}} & \frac{-m y_G}{m - X_{\dot{u}}} \\ 0 & 1 & 0 & -\frac{m z_G}{m - Y_{\dot{v}}} + & 0 & \frac{(m x_G - Y_{\dot{r}})}{m - Y_{\dot{v}}} \\ 0 & 0 & 1 & \frac{m y_G}{m - Z_{\dot{w}}} & -\frac{(m x_G + Z_{\dot{q}})}{m - Z_{\dot{w}}} & 0 \\ 0 & -\frac{m\, z_G}{I_x - K_{\dot{p}}} & \frac{m\, y_G}{I_x - K_{\dot{p}}} & 1 & 0 & 0 \\ \frac{m\, z_G}{I_y - M_{\dot{q}}} & 0 & -\frac{(m\, x_G + M_{\dot{w}})}{I_y - M_{\dot{q}}} & 0 & 1 & 0 \\ -\frac{m y_G}{I_z - N_{\dot{r}}} & \frac{(m x_G - N_{\dot{v}})}{I_z - N_{\dot{r}}} & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix} \quad (11)$$

where $f_1, f_2, f_3, f_4, f_5, f_6$ expressed as follows:

$$f_1 = \frac{\begin{array}{c} X_{res} + X_{|u|u} u|u| + X_{wq} wq + X_{qq} qq + X_{vr} vr + X_{rr} rr + X_{prop} + \\ -m[-vr + wq - x_G(q^2 + r^2) + pq\, y_G + pr\, z_G] \end{array}}{m - X_{\dot{u}}} \tag{12}$$

$$f_2 = \frac{\begin{array}{c} Y_{res} + Y_{|v|v} v|v| + Y_{r|r|} r|r| + Y_{ur} ur + Y_{wp} wp + Y_{pq} pq + Y_{uv} uv + \\ +Y_{uu\delta_r} u^2 \delta_r - m[-wp + ur - y_G(r^2 + p^2) + qr\, z_G + pq\, x_G] \end{array}}{m - Y_{\dot{v}}} \tag{13}$$

$$f_3 = \frac{\begin{array}{c} Z_{res} + Z_{|w|w} w|w| + Z_{q|q|} q|q| + Z_{uq} uq + Z_{vp} vp + Z_{rp} rp + \\ +Z_{uw} uw + Z_{uu\delta_s} u^2 \delta_s - m[-uq + vp - z_G(p^2 + q^2) + rp\, x_G + rq\, y_G] \end{array}}{m - Z_{\dot{w}}} \tag{14}$$

$$f_4 = \frac{\begin{array}{c} K_{res} + K_{p|p|} p|p| + K_{prop} - \left(\left(I_z - I_y\right) qr + \right. \\ \left. m[y_G(-uq + vp) - z_G(-wp + ur)] \right) \end{array}}{I_x - K_{\dot{p}}} \tag{15}$$

$$f_5 = \frac{\begin{array}{c} M_{res} + M_{w|w|} w|w| + M_{q|q|} q|q| + M_{uq} uq + M_{vp} vp + M_{rp} rp + \\ +M_{uw} uw + M_{uu\delta_s} u^2 \delta_s - \left((I_x - I_z) rp + m[z_G(-vr + wq) - x_G(-uq + vp)] \right) \end{array}}{I_y - M_{\dot{q}}} \tag{16}$$

$$f_6 = \frac{\begin{array}{c} N_{res} + N_{v|v|} v|v| + N_{r|r|} r|r| + N_{ur} ur + N_{wp} wp + N_{pq} pq + \\ +N_{uv} uv + N_{uu\delta_r} u^2 \delta_r - \left(\left(I_y - I_z\right) pq + m[x_G(-wp + ur) - y_G(-vr + wq)] \right) \end{array}}{I_z - N_{\dot{r}}} \tag{17}$$

Furthermore linear system is obtained as follows [11]:

$$\begin{aligned} \dot{x}(t) &= A\, x(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{18}$$

with:

$$A = J_x = \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{mz_G}{m - X_{\dot{u}}} & \frac{-my_G}{m - X_{\dot{u}}} \\ 0 & 1 & 0 & -\frac{mz_G}{m - Y_{\dot{v}}} + & 0 & \frac{(mx_G - Y_{\dot{r}})}{m - Y_{\dot{v}}} \\ 0 & 0 & 1 & \frac{my_G}{m - Z_{\dot{w}}} & -\frac{(mx_G + Z_{\dot{q}})}{m - Z_{\dot{w}}} & 0 \\ 0 & -\frac{m\, z_G}{I_x - K_{\dot{p}}} & \frac{m\, y_G}{I_x - K_{\dot{p}}} & 1 & 0 & 0 \\ \frac{m\, z_G}{I_y - M_{\dot{q}}} & 0 & -\frac{(m\, x_G + M_{\dot{w}})}{I_y - M_{\dot{q}}} & 0 & 1 & 0 \\ -\frac{my_G}{I_z - N_{\dot{r}}} & \frac{(mx_G - N_{\dot{v}})}{I_z - N_{\dot{r}}} & 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} a_1 & b_1 & c_1 & d_1 & e_1 & g_1 \\ a_2 & b_2 & c_2 & d_2 & e_2 & g_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 & g_3 \\ a_4 & b_4 & c_4 & d_4 & e_4 & g_4 \\ a_5 & b_5 & c_5 & d_5 & e_5 & g_5 \\ a_6 & b_6 & c_6 & d_6 & e_6 & g_6 \end{bmatrix} \tag{19}$$

$$B = J_u = \begin{bmatrix} \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} & \begin{matrix} 0 & \frac{mz_G}{m - X_{\dot{u}}} & \frac{-my_G}{m - X_{\dot{u}}} \\ -\frac{mz_G}{m - Y_{\dot{v}}} + & 0 & \frac{(mx_G - Y_{\dot{r}})}{m - Y_{\dot{v}}} \\ \frac{my_G}{m - Z_{\dot{w}}} & -\frac{(mx_G + Z_{\dot{q}})}{m - Z_{\dot{w}}} & 0 \end{matrix} \\ \begin{matrix} 0 & -\frac{m\,z_G}{I_x - K_{\dot{p}}} & \frac{m\,y_G}{I_x - K_{\dot{p}}} \\ \frac{m\,z_G}{I_y - M_{\dot{q}}} & 0 & -\frac{(m\,x_G + M_{\dot{w}})}{I_y - M_{\dot{q}}} \\ -\frac{my_G}{I_z - N_{\dot{r}}} & \frac{(mx_G - N_{\dot{v}})}{I_z - N_{\dot{r}}} & 0 \end{matrix} & \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \end{bmatrix}^{-1} \tag{20}$$

$$\begin{bmatrix} A_1 & B_1 & C_1 & D_1 & E_1 & G_1 \\ A_2 & B_2 & C_2 & D_2 & E_2 & G_2 \\ A_3 & B_3 & C_3 & D_3 & E_3 & G_3 \\ A_4 & B_4 & C_4 & D_4 & E_4 & G_4 \\ A_5 & B_5 & C_5 & D_5 & E_5 & G_5 \\ A_6 & B_6 & C_6 & D_6 & E_6 & G_6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } D = 0 \tag{21}$$

So:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = A \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix} + B \begin{bmatrix} X_{prop} \\ \delta_r \\ \delta_s \\ K_{prop} \\ \delta_s \\ \delta_r \end{bmatrix} \tag{22}$$

and then discretized can be show:

$$\begin{bmatrix} u_{k+1} \\ v_{k+1} \\ w_{k+1} \\ p_{k+1} \\ q_{k+1} \\ r_{k+1} \end{bmatrix} = A \begin{bmatrix} u_k \\ v_k \\ w_k \\ p_k \\ q_k \\ r_k \end{bmatrix} + B \begin{bmatrix} X_{prop} \\ \delta_r \\ \delta_s \\ K_{prop} \\ \delta_s \\ \delta_r \end{bmatrix} \tag{23}$$

If we write completely, so the discrete models in Eq. (23) can be written generally in a linear function bellow:

$$x_{k+1} = f(x_k, u_k) \tag{24}$$

Due to some assumptions, the stochastic factor in noise must be added to each equations. Thus Eq. (23) can be formulated as follows [4]:

$$x_{k+1} = f(x_k, u_k) + w_k \tag{25}$$

$$z_k = Hx_k + v_k \tag{26}$$

whereas $f(x_k, u_k)$ is nonlinear or linear function will be applied Ensemble Kalman Filter Square Root (EnKF-SR) Algorithm.

The system noise w_k and measurement noise $v_k$are generated by computer and ussualy normally distributed, so the mean zero.

Generally, Q_k states the system noise varians and R_k states the measurement noise varians. Both are depend on time [4].

IV. Ensemble Kalman Filter Square Root

This section present EnKF-SR algorithm to estimated nonlinear or linear dynamic system and measurement model, the algorithm *Ensemble Kalman Filter Square Root* (EnKF-SR) can be seen [6]:

Model system and measurement model

$$x_{k+1} = f(x_k, u_k) + w_k \tag{27}$$

$$z_k = Hx_k + v_k \tag{28}$$

$$w_k \sim N(0, Q_k), \qquad v_k \sim N(0, R_k) \tag{29}$$

1. Initialization

Generate N *ensemble* as the first guess $\bar{x}_0$:

$$x_{0,i} = [x_{0,1} \quad x_{0,2} \quad \cdots \quad x_{0,N}] \tag{30}$$

The first Mean *Ensemble*:

$$\bar{x}_{0,i} = x_{0,i}\, 1_N \tag{31}$$

The first *Ensemble* error:

$$\tilde{x}_{0,i} = x_{0,i} - \bar{x}_{0,i} = x_{k,i}(I - 1_N) \tag{32}$$

2. Time Update:

$$\hat{x}^-_{k,i} = f(\hat{x}_{k,-1,i}, u_{k-1,i}) + w_{k,i} \tag{33}$$

where:

$$w_{k,i} = N(0, Q_k)$$

Mean *Ensemble*:

$$\bar{x}^-_{k,i} = \hat{x}^-_{k,i}\, 1_N \tag{34}$$

Error *Ensemble*:

$$\tilde{x}^-_{k,i} = \hat{x}^-_{k,i} - \bar{x}^-_{k,i} = \hat{x}^-_{k,i}\,(I - 1_N) \tag{35}$$

3. Measurement Update:

$$z_{k,i} = Hx_{k,i} + v_{k,i} \tag{36}$$

where:

$$v_{k,i} \sim N(0, R_k)$$

$$\begin{gathered} S_k = H\tilde{x}^-_{k,i}, E_k = (v_1, v_2, \ldots, V_N) \\ C_k = S_k\, S_k^T + E_k\, E_k^T \end{gathered} \tag{37}$$

Mean Ensemble:

$$\bar{x}_{k,i} = \bar{x}^-_{k,i} + \tilde{x}^-_{k,i} S_k^T C_k^- (\bar{z}_{k,i} - H\bar{x}^-_{k,i}) \tag{38}$$

Square Root Scheme:

- eigenvalue decompotition from:

$$C_k = U_k\, \Lambda_k U_k^T \tag{39}$$

- determine matrix:

$$M_k = \Lambda_k^{\frac{1}{2}}\, U_k^T\, S_k^- \tag{40}$$

- determine SVD from:

$$M_k = Y_k\, L_k\, V_k^T \tag{41}$$

Ensemble Error:

$$\tilde{x}_{k,i} = \tilde{x}^-_{k,i}\, V_k\, (I - \mathrm{L}_k\, L_k^T)^{\frac{1}{2}} \tag{42}$$

Ensemble Estimation:

$$\hat{x}_{k,i} = \tilde{x}_{k,i} + \bar{x}_{k,i} \tag{43}$$

To evaluate of estimation result accuracy from EnKF algorithm, can be show with calculate Root Mean Square Error (RMSE) [6]:

$$RMSE = \sqrt{\frac{\sum_i^n \left(X_{obs,i}(k) - X_{model,i}(k)\right)^2}{n}} \tag{44}$$

with:
$X_{obs,i}(k)$ = observation data;
$X_{model,i}(k)$ = model data;
n = iteration.

V. Computational Result

This simulation has been carried out by implementing an algorithm Ensemble Kalman Filter (EnKF) in the AUV model.

The result of simulation will be evaluated and compared with real condition, estimaton result with EnKF. This simulation consist of three types of simulations. in which the first, second, third simulation by generate 100,200 and 300 ensemble. Simulations have been done by assuming surge (u), sway (v), heave (w), roll (p), pitch $(q)$and yaw (r).

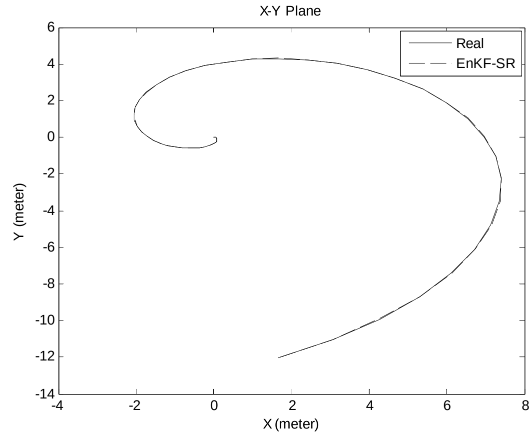
The value of Δt has been used was $\Delta t = 0{,}1$. Initial condition has been used were $u_0 = 0\ m, v_0 = 0\ m$, $w_0 = 0\ m$, $p_0 = 0$ rad, $q_0 = 0$ rad and $r_0 = 0$ rad.

Figs. 3 show result of trajectory estimation AUV by generate 200 ensemble. Fig. 3(a) shows the result of trajectory estimation in XY plane, Fig. 3(b) in XZ plane, Fig. 3(c) in YZ plane and Fig. 3(d) in XYZ plane.

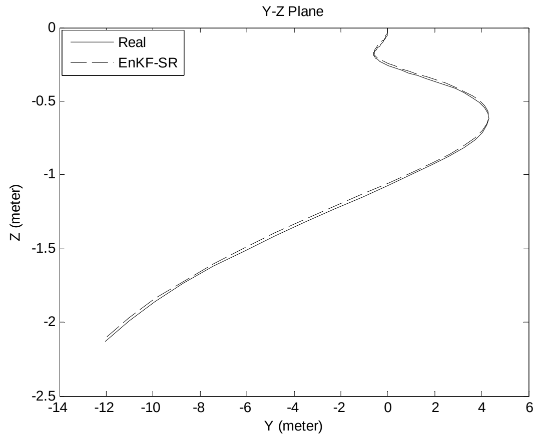
Fig. 3(a) shows result of trajectory estimation highly accurate with tracking error of surge 0,0094 m/s and accuracy of 99,22%. Tracking error of sway 0,011 m/s and accuracy of 97,8%. AUV moves forward and turns around within XY plane to the right direction reaching more or less 270 degrees. Surge motion is influenced by the propeller X_{prop}. The sway motion is influenced by vertical fin or rudder δ_r. The angle of rudder position will affect sway motion of AUV so we need control system for control the angle of rudder position.

Fig. 3(b) shows result of trajectory estimation highly accurate with tracking error of surge 0,0094 m/s and accuracy of 99,22%. Tracking error of heave 0,0081 m/s and accuracy of 96,8%. AUV dives moving right and left to the depth of 2,2 meters. The heave motion is influenced by horizontal fin or stern δ_s. The angle of stern position will affect heave motion of AUV, so we need control system for control The angle of stern position. Fig. 3(c) shows the result of trajectory estimation highly accurate for YZ plane. Fig. 4(d) shows result of trajectory estimation highly accurate for XYZ Plane with tracking error of translational motion 0,009 m/s and accuracy of 97,39%. Tracking error of rotational motion 0,001 rad/s and accuracy of 99,97%.

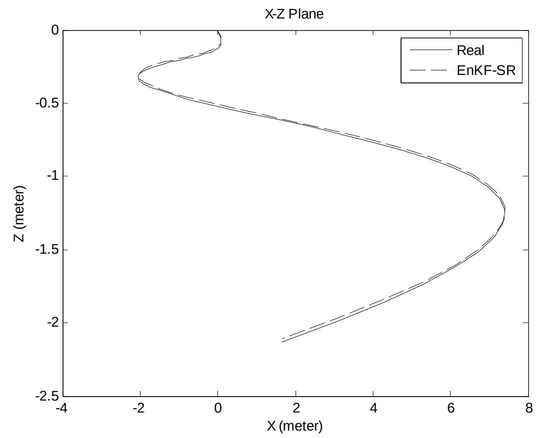
In general as seen in the Table III, the results of the three simulations were highly accurate. The first simulation by generate 100 ensemble with tracking error of translational motion 0,0093 m/s or accuracy of 97,39% and rotational motion 0,0012 rad/s or accuracy of 99,978%.



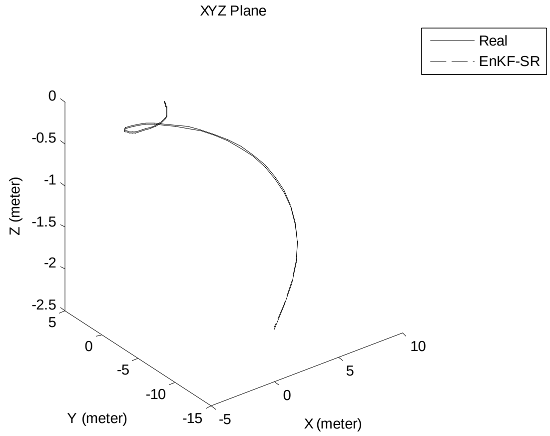
(a) Trajectory Estimation of 6 DOF for XY Plane with 200 ensemble



(c) Trajectory Estimation of 6 DOF for YZ Plane with 200 ensemble



(b) Trajectory Estimation of 6 DOF for XZ Plane with 200 ensemble



(d) Trajectory Estimation of 6 DOF for XYZ Plane with 200 ensemble

Figs. 3. Trajectory estimation AUV by generate 200 ensemble

The second simulation by generates 200 ensemble with tracking error of translational motion 0,0096 m/s or accuracy of 97,94% and rotational motion 0,0015 rad/s or accuracy of 99,96%. The third simulation by generate 300 ensemble with tracking error of translational motion 0,0095 m/s or accuracy of 97,31% and rotational motion 0,0019 rad/s or accuracy of 99,94%. Time simulation of the three simulation results with 100 ensemble faster than 200 and 300 ensemble because more ensemble generated the longer time simulation

TABLE III
SIMULATION RESULTS

	100 ens RMSE	Accuracy	200ens RMSE	Accuracy	300ens RMSE	Accuracy
Surge	0.0082 m/s	99,41%	0.0094 m/s	99,22%	0.0083 m/s	99,3%
Sway	0.01 m/s	97,77%	0.0112 m/s	97,8%	0.0101 m/s	97,76%
Heave	0.0096 m/s	94,98%	0.0081 m/s	96,8%	0.01085 m/s	94,86%
Roll	0.0012 rad/s	99,99%	0.0012 rad/s	99,95%	0.0019 rad/s	99,93%
Pitch	0.0012 rad/s	99,97%	0.0017 rad/s	99,96%	0.0018 rad/s	99,95%
Yaw	0.001 rad/s	99,97%	0.0015 rad/s	99,96%	0.0020 rad/s	99,95%
Time	0.6875 s		1.0469 s		1.4531 s	

VI. Conclusion

Based on analysis of the three simulation results, EnKF-SR method could be applied to estimate of linear system trajectory of AUV SEGOROGENI ITS with considerably high accuracy. Of the three simulation by generating both 100, 200 and 300 ensembles, the estimation results were all accurate. Square root scheme is one scheme can be implemented in the EnKF.

This scheme can affect the estimation results because it implemented in correction step in EnKF Algorithm.

References

[1] Herlambang, T., Nurhadi H and Subchan., 2014a "Preliminary Numerical Study on Designing Navigation and Stability Control Systems for ITS AUV", *Applied Mechanics and Materials* Vol. 493 (2014) pp 420-425 Trans Tech Publications, Switzerland.

[2] Yang, C. 2007. *Modular Modelling and Control for Autonomous Vehicle (AUV).* Department of Mechanical Engineering National University of Singapore

[3] Pascoal, A. (1994).The AUV MARIUS: Mission Scenarios, Vehicle Design, Construction and Testing. *Proceedings of the 2nd Workshop on Mobile Robots for Subsea Environments.* Monterey Bay Aquarium, Monterey, California, USA.

[4] Lewis, L Frank. (1986), *"Optimal Estimation, With An Introduction To Stochastic Control Theory"*, John Wiley and Sons, New York.

[5] Evensen, G (2009), "*Data Asimilation The Ensemble Kalman Filter (second edition)"*, Springer-Verlag Berlin Hiedelberg London and New York

[6] Herlambang, T. (2012), "*Square Root Ensemble Kalman Filter (SR-EnKF) for Estimation of Missile Position*", Magister Thesis, Department of Mathematics Sepuluh Nopember Institut of Technology, Surabaya.

[7] Fosen, T. I.2005. A Nonlinear Unified State-space Model for Ship Maneuvering and Control in A Seaway-Journal of Bifurcation and Chaos, *Int. J. Bifurcation Chaos*, 15, 2717 (2005)..

[8] Perez, T. O. N. Smogeli, T.I Fossen and A.J Sorensen. 2005. An Overview of marine Systems Simulator (MSS): A Simulink Toolbox for Narine Control System . *SIMS2005-Scandanavian Conference on Simulation and Modelling*.

[9] SNAME, The Society of Naval Architects and Marine Engineers, "Nomenclature for Treating the Motion of A Submerged Body Through A Fluid", *Technical and Research Bulletin*, no. 1-5, 1950.

[10] Subiono. 2010. "Mathematical system". Department of Mathematics Sepuluh Nopember Institut of Technology, Surabaya.

[11] Herlambang, T., Djatmiko E.B and Nurhadi H., 2014b. "Optimization with Jacobian Approach for ITS AUV System", international conference on Marine Technology.

[12] Bennassar, A., Abbou, A., Akherraz, M., Barara, M., A new sensorless control design of induction motor based on backstepping sliding mode approach, (2014) *International Review on Modelling and Simulations (IREMOS)*, 7 (1), pp. 35-42.

[13] Angrisani, L., Liccardo, A., Pasquino, N., Lo Moriello, R.S., Bifulco, P., Laracca, M., Lanzolla, A.M., On the suitability of DEKF for improving GPS location in car accidents, (2013) *International Review on Modelling and Simulations (IREMOS)*, 6 (5), pp. 1600-1606.

[14] Ferdowsi, M.H., Passive range estimation using two and three optical cameras, (2013) *International Review on Modelling and Simulations (IREMOS)*, 6 (2), pp. 613-618.

[15] Moujahed, M., Ben Azza, H., Jemli, M., Boussak, M., Speed estimation by using EKF techniques for sensor-less DTC of PMSM with Load Torque Observer, (2014) *International Review of Electrical Engineering (IREE)*, 9 (2), pp. 270-279.

Authors' information

Teguh Herlambang was born in 1987, He is currently a Ph.D student at the Department of Ocean EngIneering at Institute of Technology Sepuluh Nopember (ITS) and received his B.Sc and M.Sc. degree from Department of Mathematics at Institute of Technology Sepuluh Nopember (ITS). His area of interest is modelling, navigation, guidance and control of dynamics system.

Eko Budi Djatmiko is currently Professor of Naval Architecture and Ocean Engineering, Institut Teknologi Sepuluh Nopember (ITS), Surabaya, Indonesia. He received the Ph. D (1992) and Master Degree (1988) from the University of Glasgow, UK. Prof. Djatmiko is the dean of the Faculty of Marine Tecnology at ITS. He has published numerous technical papers in his field and successfully supervised more than 10 doctoral students in the subjects of ship and offshore structure hydrodynamics, operability, reliability, as well as surface and underwater vehicle control and guidance.

Hendro Nurhadi Received the Dipl.Ing.(FH) degree from the University of Applied Science Georg-Simon-Ohm Fachhochshule Nuremberg, Nuremberg, Germany, in 2001 and the Ph.D degree from the National Taiwan University of Science and Technology (NTUST), Taipei, Taiwan, in 2009. He is currently with the Department of Mechanical Engineering at Institute of Technology Sepuluh Nopember (ITS), Surabaya, Indonesia, in charged as a Head of Mechatronics Laboratory, also assigned as a coordinator of national consortium for mechatronics for defense, unmanned systems and industrial machineries. He has authored numerous international journal papers and International conferences, as well as reviewer and editor for various international journal papers and international proceedings. His research interests and consulting activities are in the areas of control system, robotics and automation, advanced mechatronics, automated optical inspection (AOI), machine tools, dynamic systems, automation of manufacturing processes, computer-aided design and manufacturing, optimization applications, digital signal processing, artificial intelligence and related fields.

Optimization with Jacobian Approach for ITS AUV System

Teguh HERLAMBANG[1,2)], Eko B.DJATMIKO[3)], Hendro NURHADI[4)]
[1)]Department of Electrical Engineering, Faculty of Engineering, UNUSA, Indonesia
[2)]Post Graduate Program in Marine Technology, ITS, Indonesia
[3)]Department of Ocean Engineering, ITS, Indonesia
[4)]Department of Mechanical Engineering, ITS, Indonesia
teguh@unusa.ac.id[1,2]), ebdjatmiko@oe.its.ac.id[3)],hdnurhadi@me.its.ac.id[4)]

Abstract

Inthis paper theoptimization of AUV system with Jacobian approach for AUV systemis studied. With jacobian approach, linearization of nonlinear AUV system to analyze controllabity and observability without control system could be accomplished. Linear system of AUV has a 6 DOF model, which are surge, sway, heave, roll,pitch and yaw. Results of the optimization with Jacobian approach show that the AUV system is controllable and observable.

Keywords:AUV, optimization, Jacobian,linear and nonlinear system, controllable, observable

1. Introduction

More than 70% of Indonesian teritory comprises of seas, so it has a great potency which need to be looked after. Advanced technology is required to aid in managing the potential resources at sea. Autonomous Underwatwer Vehicle (AUV) is one of the advanced technology necessary in this case,in particular to assist various activities of underwater exploration in the deepsea. AUV is very useful for ocean observation since it does not require a tethered cable, and it can swim freely without restriction [1]. AUVcanbe usedforunderwater exploration, mapping, underwaterdefensesystemequipment, sensoroffboardsubmarines, inspection ofunderwaterstructuresand natural resources, observing condition ofthe earth surfaceplates, and so on.

One important aspect that should be established in the design of AUV is the clarification on its observability and controllability, based on a mathematical model [2]. The mathematical model contains various hydrodynamic force and moment expressed collectively in terms of hydrodynamic coefficients [3]. AUV nonlinear system causes many uncertainties in the modeling, so requires linearization to obtain more viable results.

This paper present a study to solve problemsin optimization utilizing theJacobian approach for ITS AUV system. Optimization of AUV systemis considered as the foundation with regards to navigation, control and guidance system in ITS AUV.This study emphasized on basic development control, navigation and guidance of AUV.

2. Autonomous Underwater Vehicle (AUV) Model

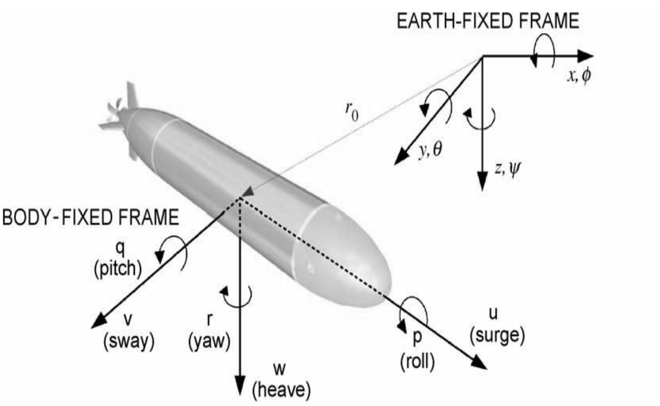


Figure 1. 6 DOF in AUV motions

Two important things need to be first recognized on the Autonomous Underwater Vehicle (AUV), that is the Earth Fixed Frame (EFF) and the Body Fixed Frame (BFF) [4]. EFFis used to describethe positionandorientationof theAUVwiththe position ofthe x-axisdirectto the north, the y-axis to the east and the z-axis toward the center of the earth.While BFF is used to describethe speedandaccelerationof theAUV with thestarting pointatthe center of gravity, x-axis direct to the ship bow, positive y-axis direct to the right hand side of the ship and positive z-axis direct downward [5,6].

As shown in Figure 1 and Table 1, an AUV has 6 DOF mode of motions, where 3 DOF for translational motion and 3 DOF for rotational motion in with regards to$x$, y and z axis. In the dynamics problem, motion oftheAUVis influenced byexternal forcesas follows [7]:

$$\tau = \tau_{hidrostatis} + \tau_{addedmass} + \tau_{drag} + \tau_{lift} + \tau_{control} \quad (1)$$

Table 1. Notation of AUV Motion Axis [4,8]

DOF	Translational And Rotational	Force / Moment	Linear and Anguler Velocity	Potition /Angle Euler
1	Surge	*X*	*U*	*x*
2	Sway	*Y*	*V*	*y*
3	Heave	*Z*	*W*	*z*
4	Roll	*K*	*P*	ϕ
5	Pitch	*M*	*Q*	*θ*
6	Yaw	*N*	*R*	Ψ

General equation of AUV motions in 6 DOF consists of 3 first equation for translational motion and 3 second equation for rotational motions, as described in the following.

$$\eta = [\eta_1^T, \eta_2^T]^T, \quad \eta_1 = [x, y, z]^T, \quad \eta_2 = [\emptyset, \theta, \Psi]^T;$$
$$v = [v_1^T, v_2^T]^T, \quad v_1 = [u, v, w]^T, \quad v_2 = [p, q, r]^T; \quad (2)$$
$$\tau = [\tau_1^T, \tau_2^T]^T, \quad \tau_1 = [X, Y, Z]^T, \quad \tau_2 = [K, M, N]^T;$$

Where $\boldsymbol{\eta}$ vector is the position and orientation of the EFF, $\boldsymbol{v}$ vector velocity of linear and angular of the BFF, the position and orientation of the BFF, and τ description of force and moment in AUV of the BFF.

By combining equations hydrostatic force, lift added mass, drag, thrust and assuming a diagonal tensor of inertia (I_o) is zero then the total forces and moments of models obtained from the following [4,8].

Surge:

$$m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] =$$
$$X_{res} + X_{|u|u}u|u| + X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop} \quad (3)$$

Sway :

$$m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(pq + \dot{r})] =$$
$$Y_{res} + Y_{|v|v}v|v| + Y_{r|r|}r|r| + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r \quad (4)$$

Heave :

$$m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] =$$
$$Z_{res} + Z_{|w|w}w|w| + Z_{q|q|}q|q| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq}uq + Z_{vp}vp + Z_{rp}rp + Z_{uw}uw +$$
$$Z_{uu\delta_s}u^2\delta_s \quad (5)$$

Roll:

$$I_x\dot{p} + (I_z - I_y)qr + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] =$$
$$K_{res} + K_{p|p|}p|p| + K_{\dot{p}}\dot{p} + K_{prop} \quad (6)$$

Pitch :

$$I_y\dot{q} + (I_x - I_z)rp + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] =$$

$$M_{res} + M_{w|w|}w|w| + M_{q|q|}q|q| + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uq}uq + M_{vp}vp + M_{rp}rp + M_{uw}uw + M_{uu\delta_s}u^2\delta_s \quad (7)$$

Yaw :

$$I_z\dot{r} + (I_y - I_z)pq + m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] =$$

$$N_{res} + N_{v|v|}v|v| + N_{r|r|}r|r| + N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv + N_{uu\delta_r}u^2\delta_r \quad (8)$$

Translational x, y and z are representation of surge, sway and heave. Withthe position ofthe x-axisdirectto the north, the y-axisto the eastandthe z-axistowardthe center ofthe earth. Rotational p, q and r are representation of roll, pitch and yaw. Thistype ofAUV, shown in Table 2,using only singlepropelleron the tailAUVwhichwill produces x_{prop} andadditionalmomentsK_{prop} . External forcesandmomentsacting on theAUVare thehydrostaticforce, thrustand hydrodynamicforceandwhereeveryobjectin the waterwillhave ahydrostaticforceconsisting ofgravity andbuoyancy forces. Whilehydrodynamiccomponentconsistsofadded mass, dragandlift. Thrustuse fin to control the balance of the ship which require a constant rate.

Table 2. Principal dimension of ITS AUV

Symbol	Value	Symbol	Value
M	19,8 kg	x_G	0.062 m
L	1.5 m	y_G	0.0013 m
V	0.00219444 m^3	z_G	0.05 m
D	0.2 m	x_B	0.062 m
I_x	0.08583 kg m^2	y_B	0 m
I_y	1.11575 kg m^2	z_B	0 m
I_z	1.11575 kg m^2		

In this paper the nonlinear system of AUV model can be linearized with Jacobian approach where the nonlinear AUV system in general as follows :

$$\dot{x}(t) = f(x(t), u(t), t)$$
$$y(t) = g(x(t), u(t), t) \quad (9)$$

Sothe Jacobian matrix is formed as follows [2]:

$$\frac{\partial f(\bar{x},\bar{u},t)}{\partial x} = \begin{bmatrix} \frac{\partial f_1(\bar{x},\bar{u},t)}{\partial x_1} & \frac{\partial f_1(\bar{x},\bar{u},t)}{\partial x_2} & \cdots & \frac{\partial f_1(\bar{x},\bar{u},t)}{\partial x_n} \\ \frac{\partial f_2(\bar{x},\bar{u},t)}{\partial x_1} & \frac{\partial f_2(\bar{x},\bar{u},t)}{\partial x_2} & \cdots & \frac{\partial f_2(\bar{x},\bar{u},t)}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n(\bar{x},\bar{u},t)}{\partial x_1} & \frac{\partial f_n(\bar{x},\bar{u},t)}{\partial x_2} & \cdots & \frac{\partial f_n(\bar{x},\bar{u},t)}{\partial x_n} \end{bmatrix} \quad (10)$$

So equation 3 - 8 can be expressed as follows:

$$f_1 = \frac{X_{res} + X_{|u|u}u|u| + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop} - m[-vr + wq - x_G(q^2 + r^2) + pq\,y_G + pr\,z_G]}{m - X_{\dot{u}}} \quad (11)$$

$$f_2 = \frac{Y_{res} + Y_{|v|v}v|v| + Y_{r|r|}r|r| + Y_{\dot{r}}\dot{r} + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r - m[-wp + ur - y_G(r^2 + p^2) + qr\,z_G + pq\,x_G]}{m - Y_{\dot{v}}} \quad (12)$$

$$f_3 = \frac{\begin{array}{c} Z_{res} + Z_{|w|w} w|w| + Z_{q|q|} q|q| + Z_{\dot{q}}\dot{q} + Z_{uq}\, uq + Z_{vp}\, vp + Z_{rp}\, rp + \\ Z_{uw}\, uw + Z_{uu\,\delta_s} u^2 \delta_s - m\left[-uq + vp - z_G\left(p^2 + q^2\right) + rp\; x_G + rq\; y_G\right] \end{array}}{m - Z_{\dot{w}}} \tag{13}$$

$$f_4 = \frac{K_{res} + K_{p|p|} p|p| + K_{prop} - \left(\left(I_z - I_y\right) qr + m\begin{bmatrix} y_G(-uq + vp) - \\ z_G(-wp + ur) \end{bmatrix}\right)}{I_x - K_{\dot{p}}} \tag{14}$$

$$f_5 = \frac{\begin{array}{c} M_{res} + M_{w|w|} w|w| + M_{q|q|} q|q| + M_{\dot{w}}\dot{w} + M_{uq}\, uq + M_{vp}\, vp + M_{rp}\, rp + \\ M_{uw}\, uw + M_{uu\,\delta_s} u^2 \delta_s - \left(\left(I_x - I_z\right) rp + m\left[z_G(-vr + wq) - x_G(-uq + vp)\right]\right) \end{array}}{I_y - M_{\dot{q}}} \tag{15}$$

$$f_6 = \frac{\begin{array}{c} N_{res} + N_{v|v|} v|v| + N_{r|r|} r|r| + N_{\dot{v}}\dot{v} + N_{ur}\, ur + N_{wp}\, wp + N_{pq}\, pq + \\ N_{uv}\, uv + N_{uu\,\delta_r} u^2 \delta_r - \left(\left(I_y - I_z\right) pq + m\left[x_G(-wp + ur) - y_G(-vr + wq)\right]\right) \end{array}}{I_z - N_{\dot{r}}} \tag{16}$$

Furthermore linear system is obtained as follows :

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{17}$$

with

$$A = J_x = \begin{bmatrix} \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} & \begin{matrix} 0 & \frac{m}{m - X_{\dot{u}}} & \frac{-m}{m - X_{\dot{u}}} \\ -\frac{m}{m - Y_{\dot{v}}} + & 0 & \frac{m}{m - Y_{\dot{v}}} \\ \frac{m}{m - Z_{\dot{w}}} & -\frac{m}{m - Z_{\dot{w}}} & 0 \end{matrix} \\ \begin{matrix} 0 & \frac{m\, y_G}{I_x - K_{\dot{p}}} & -\frac{m\, z_G}{I_x - K_{\dot{p}}} \\ \frac{m\, z_G}{I_y - M_{\dot{q}}} & 0 & -\frac{m\, x_G}{I_y - M_{\dot{q}}} \\ -\frac{m y_G}{I_z - N_{\dot{r}}} & \frac{m x_G}{I_z - N_{\dot{r}}} & 0 \end{matrix} & \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \end{bmatrix}^{-1} \begin{bmatrix} a_1 & b_1 & c_1 & d_1 & e_1 & g_1 \\ a_2 & b_2 & c_2 & d_2 & e_2 & g_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 & g_3 \\ a_4 & b_4 & c_4 & d_4 & e_4 & g_4 \\ a_5 & b_5 & c_5 & d_5 & e_5 & g_5 \\ a_6 & b_6 & c_6 & d_6 & e_6 & g_6 \end{bmatrix} \tag{18}$$

$$B = J_u = \begin{bmatrix} \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} & \begin{matrix} 0 & \frac{m}{m - X_{\dot{u}}} & \frac{-m}{m - X_{\dot{u}}} \\ -\frac{m}{m - Y_{\dot{v}}} + & 0 & \frac{m}{m - Y_{\dot{v}}} \\ \frac{m}{m - Z_{\dot{w}}} & -\frac{m}{m - Z_{\dot{w}}} & 0 \end{matrix} \\ \begin{matrix} 0 & \frac{m\, y_G}{I_x - K_{\dot{p}}} & -\frac{m\, z_G}{I_x - K_{\dot{p}}} \\ \frac{m\, z_G}{I_y - M_{\dot{q}}} & 0 & -\frac{m\, x_G}{I_y - M_{\dot{q}}} \\ -\frac{m y_G}{I_z - N_{\dot{r}}} & \frac{m x_G}{I_z - N_{\dot{r}}} & 0 \end{matrix} & \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \end{bmatrix}^{-1} \begin{bmatrix} A_1 & B_1 & C_1 & D_1 & E_1 & G_1 \\ A_2 & B_2 & C_2 & D_2 & E_2 & G_2 \\ A_3 & B_3 & C_3 & D_3 & E_3 & G_3 \\ A_4 & B_4 & C_4 & D_4 & E_4 & G_4 \\ A_5 & B_5 & C_5 & D_5 & E_5 & G_5 \\ A_6 & B_6 & C_6 & D_6 & E_6 & G_6 \end{bmatrix} \tag{19}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } D = 0 \tag{20}$$

Where value of $a_1, a_2, \ldots, g_6, A_1, A_2, \ldots, G_6$ are listed in Tables 3 and 4.

Table 3. Value of Jacobian component (matrix A)

$a_1 = \frac{\partial f_1}{\partial u} = \frac{2\bar{u}X_{\lvert u\rvert u}}{m - X_{\dot{u}}}$	$b_1 = \frac{\partial f_1}{\partial v} = \frac{\bar{r}(X_{vr} + m)}{m - X_{\dot{u}}}$	$c_1 = \frac{\partial f_1}{\partial w} = \frac{\bar{q}(X_{wq} - m)}{m - X_{\dot{u}}}$
$a_2 = \frac{\partial f_2}{\partial u} = \frac{r\, Y_{ur} + v\, Y_{uv} + 2u\, Y_{uu\delta_r}\delta_r - m\, r}{m - Y_{\dot{v}}}$	$b_2 = \frac{\partial f_2}{\partial v} = \frac{2\bar{v}Y_{\lvert v\rvert v} + \bar{u}Y_{uv}}{m - Y_{\dot{v}}}$	$c_2 = \frac{\partial f_2}{\partial w} = \frac{\bar{p}(Y_{wp} + m)}{m - Y_{\dot{v}}}$
$a_3 = \frac{\partial f_3}{\partial u} = \frac{\bar{u}Z_{uq} + \bar{w}Z_{uw}}{m - Z_{\dot{w}}}$	$b_3 = \frac{\partial f_3}{\partial v} = \frac{\bar{p}(Z_{vp} - m)}{m - Z_{\dot{w}}}$	$c_3 = \frac{\partial f_3}{\partial w} = \frac{2\bar{w}Z_{\lvert w\rvert w} + \bar{u}Z_{uw}}{m - Z_{\dot{w}}}$
$a_4 = \frac{\partial f_4}{\partial u} = \frac{m(\bar{q}y_G + \bar{r}z_G)}{I_x - K_{\dot{p}}}$	$b_4 = \frac{\partial f_4}{\partial v} = \frac{-m\,\bar{p}y_G}{I_x - K_{\dot{p}}}$	$c_4 = \frac{\partial f_4}{\partial w} = \frac{-m\,\bar{p}z_G}{I_x - K_{\dot{p}}}$
$a_5 = \frac{\partial f_5}{\partial u} = \frac{\bar{q}M_{uq} + \bar{w}M_{uw} + 2\,\bar{u}M_{uu\delta_s}\delta_s - m\,(\bar{q}x_G)}{I_y - M_{\dot{q}}}$	$b_5 = \frac{\partial f_5}{\partial v} = \frac{\bar{p}M_{vp} + m\,(\bar{r}z_G + \bar{p}x_G)}{I_y - M_{\dot{q}}}$	$c_5 = \frac{\partial f_5}{\partial w} = \frac{2\,\bar{w}\,M_{ww} + \bar{u}M_{uw} - m\,(\bar{q}z_G)}{I_y - M_{\dot{q}}}$
$a_6 = \frac{\partial f_6}{\partial u} = \frac{\bar{v}N_{uv} + 2\bar{u}N_{uu\delta_r}\delta_r + (N_{ur} - m\,x_G)\bar{r}}{I_z - N_{\dot{r}}}$	$b_6 = \frac{\partial f_6}{\partial v} = \frac{2\bar{v}N_{vv} + \bar{u}N_{uv} - m\,(\bar{r}y_G)}{I_z - N_{\dot{r}}}$	$c_6 = \frac{\partial f_6}{\partial w} = \frac{\bar{w}N_{wp} + \bar{q}N_{pq} + m\,(\bar{p}\,x_G + \bar{q}y_G)}{I_z - N_{\dot{r}}}$
$d_1 = \frac{\partial f_1}{\partial p} = \frac{-m(\bar{q}y_G + \bar{r}z_G)}{m - X_{\dot{u}}}$	$e_1 = \frac{\partial f_1}{\partial q} = \frac{2\bar{q}X_{qq} - m\begin{pmatrix}\bar{w} - 2\bar{q}x_G \\ +\bar{p}y_G + \bar{r}z_G\end{pmatrix}}{m - X_{\dot{u}}}$	$g_1 = \frac{\partial f_1}{\partial r} = \frac{\bar{v}X_{vr} + 2r\,X_{rr} + m(\bar{v} - 2\bar{r}x_G + \bar{p}z_G)}{m - X_{\dot{u}}}$
$d_2 = \frac{\partial f_2}{\partial p} = \frac{\bar{w}Y_{wp} + \bar{q}Y_{pq} + m(\bar{w} + 2\bar{p}y_G - \bar{q}x_G)}{m - Y_{\dot{v}}}$	$e_2 = \frac{\partial f_2}{\partial q} = \frac{\bar{p}Y_{pq} - m(\bar{r}z_G + \bar{p}x_G)}{m - Y_{\dot{v}}}$	$g_2 = \frac{\partial f_2}{\partial r} = \frac{2\bar{r}Y_{rr} + \bar{u}\,Yur - m(\bar{u} - 2\bar{r}y_G + \bar{q}z_G)}{m - Y_{\dot{v}}}$
$d_3 = \frac{\partial f_3}{\partial p} = \frac{\bar{v}Z_{vp} + \bar{r}Z_{rp} - m(-\bar{u} - 2\bar{q}z_G + \bar{r}y_G)}{m - Z_{\dot{w}}}$	$e_3 = \frac{\partial f_3}{\partial q} = \frac{2\bar{q}Z_{q\lvert q\rvert} + u\,Z_{uq} - m\begin{pmatrix}-\bar{u} - 2\bar{q}z_G \\ +\bar{r}y_G\end{pmatrix}}{m - Z_{\dot{w}}}$	$g_3 = \frac{\partial f_3}{\partial r} = \frac{\bar{p}Z_{rp} - m(\bar{p}x_G + \bar{q}y_G)}{m - Z_{\dot{w}}}$
$d_4 = \frac{\partial f_4}{\partial p} = \frac{2\bar{p}K_{p\lvert p\rvert} - m(\bar{v}y_G + \bar{w}z_G)}{I_x - K_{\dot{p}}}$	$e_4 = \frac{\partial f_4}{\partial q} = \frac{-[(I_z - I_y)\bar{q} - m\,\bar{u}z_G]}{I_x - K_{\dot{p}}}$	$g_4 = \frac{\partial f_4}{\partial r} = \frac{-[(I_z - I_y)r - m\,\bar{u}y_G]}{I_x - K_{\dot{p}}}$
$d_5 = \frac{\partial f_5}{\partial p} = \frac{\bar{v}M_{vp} + \bar{r}M_{rp} - [(I_x - I_z)\bar{r} + m(-\bar{v}x_G)]}{I_y - M_{\dot{q}}}$	$e_5 = \frac{\partial f_5}{\partial q} = \frac{2\bar{q}M_{qq} + \bar{u}\,M_{uq} - m(\bar{w}z_G + \bar{u}x_G)}{I_y - M_{\dot{q}}}$	$g_5 = \frac{\partial f_5}{\partial r} = \frac{\bar{p}M_{rp} - \begin{bmatrix}(I_y - I_z)\bar{p} \\ +m(-\bar{v}z_G)\end{bmatrix}}{I_y - M_{\dot{q}}}$
$d_6 = \frac{\partial f_6}{\partial p} = \frac{\bar{p}N_{wp} + \bar{q}N_{pq} - [(I_y - I_z)\bar{q} + m(-\bar{w}y_G)]}{I_z - N_{\dot{r}}}$	$e_6 = \frac{\partial f_6}{\partial q} = \frac{\bar{p}N_{pq} - \begin{bmatrix}(I_y - I_z)\bar{p} + \\ m(-\bar{w}y_G)\end{bmatrix}}{I_z - N_{\dot{r}}}$	$g_6 = \frac{\partial f_6}{\partial r} = \frac{2\bar{r}N_{rr} + \bar{u}N_{ur} - m\,(\bar{u}x_G + \bar{v}y_G)}{I_z - N_{\dot{r}}}$

Table 4. value of jacobian component (matrix B)

$A_1 = \frac{\partial f_1}{\partial X_{prop}} = \frac{1}{m - X_{\dot{u}}}$	$B_1 = \frac{\partial f_1}{\partial \delta_r} = 0$	$C_1 = \frac{\partial f_1}{\partial \delta_s} = 0$
$A_2 = \frac{\partial f_2}{\partial X_{prop}} = 0$	$B_2 = \frac{\partial f_2}{\partial \delta_r} = \frac{Y_{uu\,\delta_r} \bar{u}^2}{m - Y_{\dot{v}}}$	$C_2 = \frac{\partial f_2}{\partial \delta_s} = 0$
$A_3 = \frac{\partial f_3}{\partial X_{prop}} = 0$	$B_3 = \frac{\partial f_3}{\partial \delta_r} = 0$	$C_3 = \frac{\partial f_3}{\partial \delta_s} = \frac{Z_{uu\,\delta_s} \bar{u}^2}{m - Z_{\dot{w}}}$
$A_4 = \frac{\partial f_4}{\partial X_{prop}} = 0$	$B_4 = \frac{\partial f_4}{\partial \delta_r} = 0$	$C_4 = \frac{\partial f_4}{\partial \delta_s} = 0$
$A_5 = \frac{\partial f_5}{\partial X_{prop}} = 0$	$B_5 = \frac{\partial f_5}{\partial \delta_r} = 0$	$C_5 = \frac{\partial f_5}{\partial \delta_s} = \frac{\bar{u}^2 M_{uu\,\delta_s}}{I_y - M_{\dot{q}}}$
$A_6 = \frac{\partial f_6}{\partial X_{prop}} = 0$	$B_6 = \frac{\partial f_6}{\partial \delta_r} = \frac{\bar{u}^2 N_{uu\,\delta_s}}{I_z - N_{\dot{r}}}$	$C_6 = \frac{\partial f_6}{\partial \delta_s} = 0$
$D_1 = \frac{\partial f_1}{\partial K_{prop}} = 0$	$E_1 = \frac{\partial f_1}{\partial \delta_s} = 0$	$G_1 = \frac{\partial f_1}{\partial \delta_r} = 0$
$D_2 = \frac{\partial f_2}{\partial K_{prop}} = 0$	$E_2 = \frac{\partial f_2}{\partial \delta_s} = 0$	$G_2 = \frac{\partial f_2}{\partial \delta_r} = \frac{Y_{uu\,\delta_r} \bar{u}^2}{m - Y_{\dot{v}}}$
$D_3 = \frac{\partial f_3}{\partial K_{prop}} = 0$	$E_3 = \frac{\partial f_3}{\partial \delta_s} = \frac{Z_{uu\,\delta_s} \bar{u}^2}{m - Z_{\dot{w}}}$	$G_3 = \frac{\partial f_3}{\partial \delta_r} = 0$
$D_4 = \frac{\partial f_4}{\partial K_{prop}} = \frac{1}{I_x - K_{\dot{p}}}$	$E_4 = \frac{\partial f_4}{\partial \delta_s} = 0$	$G_4 = \frac{\partial f_4}{\partial \delta_r} = 0$
$D_5 = \frac{\partial f_5}{\partial K_{prop}} = 0$	$E_5 = \frac{\partial f_5}{\partial \delta_s} = \frac{\bar{u}^2 M_{uu\,\delta_s}}{I_y - M_{\dot{q}}}$	$G_5 = \frac{\partial f_5}{\partial \delta_r} = 0$
$D_6 = \frac{\partial f_6}{\partial K_{prop}} = 0$	$E_6 = \frac{\partial f_6}{\partial \delta_s} = 0$	$G_6 = \frac{\partial f_6}{\partial \delta_r} = \frac{\bar{u}^2 N_{uu\,\delta_s}}{I_z - N_{\dot{r}}}$

3.Controllability and Observability

Linear system in Equation 17 is said controllable if Matriks : $Ctr = (B|AB|A^2B|\ldots|A^{n-1}B)$ have the *n* rank. Observable if matriks

$$Obsv = \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{(n-1)} \end{pmatrix}$$ have the *n* rank [5].

In equation 18 and 19 obtained controllabity and observability matrix as follows $Ctr = (B|AB|A^2B|A^3B|A^4B|A^5B) = 6$ and

$$Obsv = \begin{pmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ CA^4 \\ CA^5 \end{pmatrix} = 6$$

So linier system of AUV with Jacobian approach is found to be controllable and observable.

4. Conclusion

Based on the analysis of Jacobian, controllability and observability AUV system is confirmed. It is also found that linearization of nonlinear AUV system can produce controllable and observable linear AUV system.

5. References

[1] Kondo, H. and Ura, T. (2003), Navigation of an AUV for Investigation of Underwater Structures, . *Elsevier Journal of Control Enginerring Practice*

[2] Yoerger, D.R. and Slotine, J.J.E. (1985), Robust Trajectory Control of Underwater Vehicle,*IEEE Journal of Ocean Engineering*, 10 (4)

[3] Kim, K.H. and Choi, H.S. (2006), Analysis on the Controlled Nonlinear Motion of Test Bed AUV-SNUUV I, *Elsevier Journal of Ocean Enginerring*

[4] Nurhadi, H., Subchan, and Gustiyadi, F.R. (2012), Design of Position Estimation Algorithm of Navigation and Trajectory System for Unmanned Underwater Vehicle ITS AUV-01 using Ensemble Kalman Filter (EnKF) Method, *Proc. of the 13th Seminar on Intelligent Technology and Its Applications*, SITIA 2012, Surabaya, May

[5] Perez, T., Smogeli, O.N., Fossen, T.I and Sorensen, A.J. (2005), An Overview of Marine Systems Simulator (MSS): A Simulink Toolbox for Marine Control System,*Proc. of Scandanavian Conference on Simulation and Modelling*, SIMS2005

[6] Fossen, T.I.(2005), A Nonlinear Unified State-Space Model for Ship Maneuvering and Control in A Seaway, *Journal of Bifurcation and Chaos*

[8] Yang, C. (2007), Modular Modelling and Control for Autonomous Vehicle (AUV), Department of Mechanical Engineering, National University of Singapore