SEACMA 2013

PROCEEDING

South East Asian Conference on Mathematics and Its Applications

"Mathematical Harmony in Science and Technology"

November, 14-15 2013

Mathematical Harmony in Science and Technology

SEACMA 2013

isbn 978-979-96152-8-2

PROCEEDING

SOUTH EAST ASIAN CONFERENCE ON MATHEMATICS AND ITS APPLICATIONS

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"Mathematical Harmony in Science and Technology"

Published by:

MATHEMATICS DEPARTMENT

INSTITUT TEKNOLOGI SEPULUH NOPEMBER SURABAYA, INDONESIA

PROCEEDING

SOUTH EAST ASIAN CONFERENCE ON MATHEMATICS AND ITS APPLICATIONS

SEACMA 2013

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ISBN 978-979-96152-8-2

This Conference is held by cooperation with
Mathematics Department, ITS

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Message

from the

Dean Faculty of Mathematics and Natural Sciences

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On behalf of the Faculty of Mathematics and Natural Sciences, Institut Teknologi Sepuluh Nopember, it is a great honor and sincere to welcome all participants to the second of South East Asian Conference on Mathematics and Its Application (SEACMA 2013).

This year, Department of Mathematics, Institut Teknologi Sepuluh Nopember, have honor to organize this meaningful conference. I believe that the purpose of this conference is not only sharing knowledge among the mathematician and scholars in related fields but also to hearten new generation of expertise in mathematics to realize the science and technology advancement.

It is undeniable that there is mathematical harmony in science and technologies. Many disciplines like engineering, computer science, information technology, operational research, logistics management, risk management and many others are all the products of mathematics. Thus, it is essential that we must hold this annual conference as a stage for all scholars in finding new ideas and applications on Mathematics.

Greatly thank to all supportive session including organizing committee, keynote speakers, invited speakers, paper reviewers, participants and sponsors. This event will not achieve without you all. Finally, I hope that the outcome of SEACMA 2013 will be pleasing and most useful to everybody.

Sincerely yours,

Prof. Dr. R.Y. Perry Burhan
Dean

Message *from the* **Chairman Organizing Committee**

On behalf of the organizing committee it is my pleasure to welcome you to the South East Asian Conference on Mathematics and Its Applications. The conference aims to provide a forum for academics, researchers, and practitioners to exchange ideas and recent developments on mathematics. The conference is expected to faster networking, collaboration and joint effort among the conference participants to advance the theory and practice as well as to identify major trends in mathematics.

We are also very pleased to welcome keynote speakers of the conference: Prof. Dr. Ir. Arnold W. Heemink, from Delft Institute of Applied Mathematics, Netherlands, Prof. Dr. Muhammad Isa Irawan, MT, from Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia, and Dr. Said Munzir, M.EngSc, from Syiah Kuala University, Indonesia, Dr. Nguyen Van Sanh, from Department of Mathematics, Faculty of Science, Mahidol University, Thailand.

We will spend about a day together for the conference. This conference is attended delegates and contributors from Indonesia, Malaysia, Thailand, and Netherlands Such a spread of participation from around the world confirms the appropriateness of the "National" label of this conference. There are 35 papers to be presented orally. Papers presented in the conference will be included in the conference proceeding to be published at the conference.

The organizing committee would like to express our deepest appreciation to ITS Rector, keynote speakers, head of departments of Mathematics of ITS, and sponsors for the support, without all mentioned this conference may not be happened. Furthermore, my appreciation goes to the members of the committee for their hard work and cooperative teamwork in the preparation of the conference. Finally, we wish all participants enjoy a fruitful scientific and human discussion.

Subchan, PhD
Conference Chairman

CONTENTS

Full Conditional Distributions of Bayesian Poisson-Lognormal 2-Level Spatio-Temporal Extension for DHF Risk Analysis

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Abstract. Bayesian Poisson-lognormal 2-level (BP2L) spatio-temporal arranged to analyze the DHF relative risk. The parameters of model are estimated by its full conditional distributions. Analytical procedures to obtain the full conditional distributions for BP2L spatio-temporal have been described in proceedings of 3rd Annual Basic Science International Conference 2013. Random effects of the BP2L spatio-temporal have considered as spatial. In the real phenomena, the random effects show generally space-time varying. This paper is to extend the BP2L which is considering the random effects as space-time varying. The extension is called the extended of BP2L (EoBP2L) spatio-temporal. The purpose of this study is to determine the expression of the full conditional distributions of EoBP2L spatio-temporal. The investigation result showed that the full conditional distributions of the EoBP2L spatio-temporal are closed form; hence, the Gibbs sampler can be used for estimating the parameters.

Keywords: Bayesian Poisson-Lognormal, closed form, DHF, full conditional distribution

1 Background

DHF case is influenced by heterogeneity and uncertainty factors [1,2]. Spatial convolution or Poisson-Lognormal model introduced to accommodate these factors [3,4,5]. By [6,7] has applied the model that implemented in 31 districts of Surabaya DHF data on 2010. The result has shown a realistic model.

In another hand, it has been shown that DHF case varies temporally also [8,9] introduced the development of convolution model into a spatio-temporal term by adding temporal trend using Bayesian approach. DHF case is also hierarchically structured data, as shown in [6]. It has been demonstrated that DHF case is nested to district as level 1 and each district is nested to Surabaya city as level 2 (or 2-level hierarchy). By [10] has considered the development of

convolution model in 2-level hierarchy, called Bayesian Poisson-Lognormal 2-level (BP2L) spatio-temporal. The uncertainty factors in the BP2L spatio-temporal have considered as spatial term.

The aim of this paper is to develop the BP2L spatio-temporal which is considering the uncertainty factors in space-time term, called the extended of BP2L (EoBP2L) spatio-temporal. The EoBP2L spatio-temporal would have a complex joint posterior of parameters model, so that parameter estimation needs the computational intensive approach. One way to solve the estimation is by constructing full conditional distribution and employing Gibbs sampler or Metropolis-Hasting algorithm. These algorithms requires the appropriate method for generating the parameters of model [11,12,13]. When the full conditional distribution has closed form, the Gibbs sampler is simpler be used. Otherwise, Metropolis-Hasting is more appropriate [14,15,16]. The simplest way to identify the closed-form of full conditional distribution is by characterizing the pattern of functional form of Poisson distribution, normal distribution, gamma distribution, logarithmic functions, exponential functions, or Taylor series. When Gibbs sampler is appropriate, WinBUGS is used to estimate these parameters [17,18].

2 Bayesian Poisson-Lognormal 2-level Spatio-Temporal Extension

The most commonly encountered model based on district count is Poisson model. It is appropriate when there is a relatively rare event of DHF in relatively large population each district. Suppose the DHF count y_{st} is identically distributed Poisson with parameter λ_{st}. Poisson variability is influenced by λ_{st} that depends on district s and time t. The relationship with this issue is that the extended of Bayesian Poisson-Lognormal 2-level (EoBP2L) spatio-temporal. The EoBP2L spatio-temporal is the extension of the model as described by [10], where it is expressed as,

$$\begin{aligned} & y_{st} \mid \lambda_{st} \sim Poisson(\lambda_{st}) \\ & \lambda_{st} = e_{st} \exp\left[\left(\beta_0 + \sum_{p=1}^{P} x_{pst}^{\mathrm{T}} \beta_p + u_{st} + v_{st} + (\alpha + \delta_s) t_z\right)\right], s = 1,\ldots,S,\ t = 1,\ldots,T,\ p = 1,\ldots,P, \end{aligned} \tag{1}$$

where S is the number of locations, T is length of time observation, P is the number of covariates, e_{st} is an expected count in district s^{th} at time t^{th}, x_{pst} is p^{th} covariate in district s^{th} at time t^{th}, u_{st} is uncorrelated random effect at district s^{th} at time t^{th}, v_{st} is correlated random effect (CAR model) at district s^{th} at time t^{th}, and

$(\alpha+\delta_s)t_z$ is trend temporal. Likelihood and joint prior distribution of (1) are defined respectively as,

$$p(y_{1t},\ldots,y_{St}|\boldsymbol{\lambda})=\prod_{t=1}^{T}\left(\prod_{i=1}^{S}\frac{\left[e_{st}\exp\left(\beta_0+\sum_{p=1}^{P}\beta_p x_{pst}^{\mathrm{T}}+u_{st}+v_{st}+(\alpha+\delta_i)t_z\right)\right]^{y_{it}}}{y_{st}!}\right)\times$$

$$\exp\left[-\sum_{t=1}^{T}\sum_{s=1}^{S}e_{st}\exp\left(\beta_0+\sum_{p=1}^{P}\beta_p x_{pst}^{\mathrm{T}}+u_{st}+v_{st}+(\alpha+\delta_s)t_z\right)\right], \tag{2}$$

$\boldsymbol{\lambda}=\{\beta_0,\beta_p,\alpha,u_i,v_s,\delta_s,\tau_\beta,\tau_u,\tau_v,\tau_\alpha\}$

and

$$p(\boldsymbol{\lambda})=p(\beta_0)\,p(\beta_p|\tau_\beta)\,p(\alpha|\tau_\alpha)\,p(u_{st}|\tau_u)\,p(v_{st}|\tau_v)\,p(\delta_s|\tau_\delta)\,p(\tau_\alpha)\,p(\tau_u)\,p(\tau_v)\,p(\tau_\delta)\,p(\tau_\beta). \tag{3}$$

Based on equation (2) and (3), joint posterior would be

$$p(\boldsymbol{\lambda}|y_{1t},\ldots,y_{St})\propto A\times B\times p(\beta_0)\,p(\beta_p|\tau_\beta)\,p(\alpha|\tau_\alpha)\,p(u_{st}|\tau_u)\,p(v_{st}|\tau_v)\,p(\delta_s|\tau_\delta)\,p(\tau_\alpha)\,p(\tau_u)\,p(\tau_v)\,p(\tau_\delta)\,p(\tau_\beta), \tag{4}$$

where

$$A=\prod_{t=1}^{T}\left(\prod_{s=1}^{S}\frac{\left[e_{st}\exp\left(\beta_0+\sum_{p=1}^{P}\beta_p x_{pst}^{\mathrm{T}}+u_{st}+v_{st}+(\alpha+\delta_s)t_z\right)\right]^{y_{st}}}{y_{st}!}\right),$$

$$B=\exp\left[-\sum_{t=1}^{T}\sum_{s=1}^{S}e_{st}\exp\left(\beta_0+\sum_{p=1}^{P}\beta_p x_{pst}^{\mathrm{T}}+u_{st}+v_{st}+(\alpha+\delta_s)t_z\right)\right].$$

Definition 1. (Full conditional distribution) [16]. *Suppose joint posterior (4),*

$$p(\boldsymbol{\lambda}|y_{1t},\ldots,y_{St})=\frac{p(y_{1t},\ldots,y_{St}|\boldsymbol{\lambda})\,p(\boldsymbol{\lambda})}{\int_{\Omega_\lambda}p(y_{1t},\ldots,y_{St}|\boldsymbol{\lambda})\,p(\boldsymbol{\lambda})\,d\boldsymbol{\lambda}},$$

$$\propto p(y_{1t},\ldots,y_{St}|\boldsymbol{\lambda})\,p(\boldsymbol{\lambda}),\ \boldsymbol{\lambda}=\{\beta_0,\beta_p,\alpha,u_{st},v_{st},\delta_s,\tau_\beta,\tau_u,\tau_v,\tau_\alpha\}$$

then full conditional distribution for example u_{st} *is defined as*

$$p(u_{st}|y_{1t},\ldots,y_{St},\beta_0,\beta_p,\alpha,v_{st},\delta_s,\tau_\beta,\tau_u,\tau_v,\tau_\alpha)\equiv[u_{st}|.]\propto p(u_{st}|\tau_u)\,p(y_{1t},\ldots,y_{St}|\boldsymbol{\lambda}),$$

and treats the other parameters except u_{st} *are constant.*

Definition 2. (Closed form) [15]. *Suppose* $p(u_{st}|\tau_u)$ *follows standard distribution as in Figure 1, the full conditional distribution for example* $[u_{st}|.] \propto p(u_{st}|\tau_u)p(y_{1t},...,y_{St}|\boldsymbol{\lambda})$, *is closed form if it gives a standard distribution.*

$p(\beta_0) \sim flat()$	$p(\tau_\delta) \sim G(e_1, e_2)$	$p(u_{st}) \sim N(0, \tau_u)$
$p(\alpha) \sim N(0, \tau_\alpha)$	$p(\tau_\alpha) \sim G(b_1, b_2)$	
$p(\tau_u) \sim G(c_1, c_2)$		
$p(\tau_v) \sim G(d_1, d_2)$	$p(\delta_s) \sim N\left(\rho \frac{\sum_{j\in\varepsilon(s)}^{S} \delta_j}{D_s}, \frac{1}{\tau_\delta D_s}\right)$	$p(v_{st}) \sim N\left(\rho \frac{\sum_{j\in\varepsilon(s)}^{S} v_j}{D_s}, \frac{1}{\tau_v D_s}\right)$
$p(\tau_\beta) \sim G(a_1, a_2)$		
$p(\beta_p) \sim N(0, \tau_\beta)$		

Figure 1. Prior distribution of EoBP2L spatio-temporal

Based on Definition 1, prior distribution in Figure 1, and joint posterior (4) are used to create a full conditional distribution for each parameter of model (1). Analytical procedure the full conditional distributions of the model are only illustrated for example u_{st} and v_{st}.

<u>Full conditional distribution for u_{st}</u>

$$\left[u_{st}|\ \right] \propto \left[\frac{1}{\sqrt{2\pi\tau_u}}\right]^{ST} \exp\left[\sum_{t=1}^{T}\sum_{s=1}^{S} u_{st}y_{st} - \sum_{t=1}^{T}\sum_{s=1}^{S} e_{st}\exp(u_{st}) - \frac{\sum_{t=1}^{T}\sum_{s=1}^{S} u_{st}^2}{2\tau_u}\right]. \tag{5}$$

Taylor series around $v_{st}^* = 0$ in (5) is used to approximate the $\exp(u_{st})$ [19], then obtained

$$\left[u_{st}|.\right] \propto \left[\frac{1}{\sqrt{2\pi\tau_u}}\right]^{ST} \exp\left[\sum_{t=1}^{T}\sum_{s=1}^{S}\left((y_{st} - e_{st})u_{st} - \frac{1}{2\tau_u}(1+\tau_u e_{st})u_{st}^2\right)\right]. \tag{6}$$

Suppose

$$B_{21} = \sum_{t=1}^{T}\sum_{s=1}^{S}(y_{st} - e_{st}),\ B_{22} = \sum_{t=1}^{T}\sum_{s=1}^{S}(1+\tau_u e_{st}),$$

then (6) can be written

$$\left[u_{st}|.\right] \propto \exp\left(-\frac{2\tau_u B_{21}}{B_{22}}\right)^2 \left[\frac{1}{\sqrt{2\pi\tau_u}}\right]^{ST} \exp\left[-\frac{1}{2\tau_u}\left(\frac{-2\tau_u B_{21}}{B_{22}} + u_{st}\right)^2\right]. \tag{7}$$

Based on the Definition 2, full conditional distribution for u_{st} is closed form,

$$u_{st} \sim N\left(\frac{2\tau_u^{(0)}B_{21}}{B_{22}}, \left(\tau_u^{(0)}\right)^{ST} \exp\left(-\frac{2\tau_u^{(0)}B_{21}}{B_{22}}\right)^2\right), \ \tau_u^{(0)} \text{ is initial value.}$$

<u>Full conditional distribution for</u> v_{st}

$$\left[v_{st}|.\right] \propto \exp\left[\sum_{t=1}^{T}\sum_{s=1}^{S} v_{st}y_{st} - \sum_{t=1}^{T}\sum_{s=1}^{S} e_{st}\exp(v_{st}) - \frac{\tau_v}{2}\sum_{t=1}^{T}\sum_{s=1}^{S} D_s v_{st}^2 + \rho\tau_v\sum_{t=1}^{T}\left(\sum_{s=1}^{S} v_{st}\sum_{j\in\varepsilon(s)}^{S} v_j\right) - \frac{\rho^2\tau_v\left(\sum_{j\in\varepsilon(s)}^{S} v_j\right)^2}{2D_s}\right]\left[\frac{\tau_v D_s}{2\pi}\right]^{\frac{ST}{2}}. \quad (8)$$

Taylor series around $v_{st}^{*} = 0$ in (8) is used to approximate the $\exp(v_{st})$,

$$\left[v_{st}|.\right] \propto \exp\left[\sum_{t=1}^{T}\left\{\sum_{s=1}^{S}\left(y_{st} - e_{st} + \rho\tau_v\sum_{j\in\varepsilon(s)}^{S} v_j\right)v_{st} - \frac{1}{2\tau_v}\left(\tau_v^2 D_s + \tau_v\right)v_{st}^2\right\}\right]\left[\frac{\tau_v D_s}{2\pi}\right]^{\frac{ST}{2}}. \quad (9)$$

Suppose

$$C_{21} = \sum_{t=1}^{T}\sum_{s=1}^{S}\left(y_{st} - e_{st} + \rho\tau_v\sum_{j\in\varepsilon(s)}^{S} v_j\right), \ C_{22} = \sum_{t=1}^{T}\sum_{s=1}^{S}\left(\tau_v^2 D_s + \tau_v\right),$$

then (9) can be expressed

$$\left[v_{st}|.\right] \propto \left\{\exp\left[\sum_{t=1}^{T}\sum_{s=1}^{S} e_{st} - \left(\frac{2\tau_v C_{21}}{C_{22}}\right)^2\right]\right\}\left[\frac{\tau_v D_s}{2\pi}\right]^{\frac{ST}{2}} \exp\left[-\frac{1}{2\tau_v}\left(-\frac{2\tau_v C_{21}}{C_{22}} + v_{st}\right)^2\right].$$

Based on the Definition 2, full conditional distribution for v_{st} is closed form,

$$v_{st} \sim N\left(\frac{2\tau_v^{(0)}C_{21}}{C_{22}}, \left[\tau_v^{(0)}D_s\right]^{-\frac{ST}{2}} \exp\left[\sum_{t=1}^{T}\sum_{s=1}^{S} e_{st} - \left(\frac{2\tau_v^{(0)}C_{21}}{C_{22}}\right)^2\right]\right), \ \tau_v^{(0)} \text{ is initial value.}$$

Full conditional distributions for the other parameters are also closed form which created similar to u_{st} and v_{st}, given in Figure 2.

Corollary. *Given EoBP2L spatio-temporal* (*1*)*, if set of likelihood* (*2*) *and prior* (*3*) *are following the assumptions in Figure 1, then its full conditional distributions are closed form.*

3 Conclusions and Future Research

Full conditional distributions of EoBP2L spatio-temporal are closed form (Figure 2). Gibbs sampler is, therefore, used for estimating the parameters. Further research is to apply EoBP2L spatio-temporal using Surabaya DHF data.

4 Acknowledgments

This article is a p art of Laboratory's research grant and doctoral research at Statistics Department of Institut Teknologi Sepuluh Nopember (ITS), Surabaya, Indonesia, granted by LPPM Institut Teknologi Sepuluh Nopember (ITS). We thank Head of BPS and BMKG Surabaya city.

$$\beta_0 \sim \exp\left[\sum_{t=1}^{T}\sum_{s=1}^{S}\left(y_{st}\beta_0^{(0)} - e_{st}\exp(\beta_0^{(0)})\right)\right].$$

$$\beta_p \sim N\left(\frac{2\tau_\beta^{(0)}A_{11}}{A_{12}}, \tau_\beta^{(0)}\exp\left(-\left(\frac{2\tau_\beta^{(0)}A_{11}}{A_{12}}\right)^2\right)\right), A_{11} = \sum_{t=1}^{T}\sum_{s=1}^{S}\left(y_{st} - e_{st}\sum_{p=1}^{P}x_{pst}^{\mathrm{T}}\right), A_{12} = \sum_{t=1}^{T}\sum_{s=1}^{S}\left(1 + \tau_\beta^{(0)}e_{st}\sum_{p=1}^{P}\left(x_{pst}^{\mathrm{T}}\right)^2\right).$$

$$u_{st} \sim N\left(\frac{2\tau_u^{(0)}B_{11}}{B_{12}}, \left(\tau_u^{(0)}\right)^S\exp\left(-\left(\frac{2\tau_u^{(0)}B_{11}}{B_{12}}\right)^2\right)\right), B_{11} = \sum_{t=1}^{T}\sum_{s=1}^{S}\left(y_{st} - e_{st}\right), B_{12} = \sum_{t=1}^{T}\sum_{s=1}^{S}\left(\tau_u^{(0)}e_{st} + 1\right).$$

$$v_{st} \sim N\left(\frac{2\tau_v^{(0)}C_{11}}{C_{12}}, \left(\tau_v^{(0)}D_s\right)^{-\frac{S}{2}}\exp\left[-\left(\frac{2\tau_v^{(0)}C_{11}}{C_{12}}\right)^2\right]\right), C_{11} = \sum_{t=1}^{T}\sum_{s=1}^{S}\left(y_{st} - e_{st} + \rho\tau_v^{(0)}\sum_{j\in\varepsilon(s)}^{S}v_j\right), C_{12} = \sum_{t=1}^{T}\sum_{s=1}^{S}\left(\tau_v^{(0)}e_{st} - \left(\tau_v^{(0)}\right)^2 D_s\right).$$

$$\alpha \sim N\left(\frac{2\tau_\alpha^{(0)}F_{11}}{F_{12}}, \tau_\alpha^{(0)}\exp\left[-\left(\frac{2\tau_\alpha^{(0)}F_{11}}{F_{12}}\right)^2\right]\right), F_{11} = \sum_{t=1}^{T}\sum_{s=1}^{S}\left(y_{st} - e_{st}\right), F_{12} = \left(\sum_{t=1}^{T}\sum_{s=1}^{S}\left(e_{st}\tau_\alpha^{(0)} + 1\right)\right).$$

$$\delta_s \sim N\left(\frac{2\tau_\delta^{(0)}G_{11}}{G_{12}}, \left[\frac{1}{\tau_\delta^{(0)}D_s}\right]^{-\frac{S}{2}}\exp\left[\left(\frac{\sqrt{\tau_\delta^{(0)}}\rho\sum_{j\in\varepsilon(s)}^{S}\delta_j}{\sqrt{2D_s}}\right)^2 - \left(\frac{2\tau_\delta^{(0)}G_{11}}{G_{12}}\right)^2\right]\right), G_{11} = \sum_{t=1}^{T}\sum_{s=1}^{S}\left(y_{st} - e_{st} + \tau_\delta^{(0)}\rho\sum_{j\in\varepsilon(s)}^{S}\delta_j\right), G_{12} = \sum_{t=1}^{T}\sum_{s=1}^{S}\left(\tau_\delta^{(0)}e_{st} - \left(\tau_\delta^{(0)}\right)^2 D_s\right).$$

Figure 2. Full conditional distributions of EoBP2L spatio-temporal

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