TESIS – SS14 2501

FORECASTING EXCHANGE RATE ACROSS COUNTRIES WITH GOLD PRICE AS EXOGENOUS VARIABLE USING TRANSFER FUNCTION AND VARI-X MODEL

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Peramalan Nilai Tukar antar Negara dengan Harga Emas sebagai Variabel Eksogen Menggunakan Fungsi Transfer dan VARI-X

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The thesis is structured to meet one of the requirements to obtain a Master of Science (M.Si) degree
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PERAMALAN NILAI TUKAR ANTAR NEGARA DENGAN HARGA EMAS SEBAGAI VARIABEL EKSOGEN MENGGUNAKAN FUNGSI TRANSFER DAN VARI-X

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ABSTRAK

Investor dan kolektor menyimpan emas sebagai tabungan dan kekayaan mereka dalam jumlah yang banyak. Emas tidak dikenai bunga layaknya obligasi harta atau tabungan, tetapi harga emas sering mengalami kenaikan dan penurunan. Penelitian ini bertujuan untuk membuat model statistik untuk memahami hubungan antara nilai tukar dan harga emas di beberapa negara yang merupakan anggota utama dalam pengekspor emas di dunia. Selain itu, penelitian ini juga bertujuan untuk mendapatkan model terbaik untuk peramalan nilai tukar dan harga emas sebagai variabel eksogen lintas negara dengan membandingkan akurasi prediksi antara Fungsi Transfer dan VARI-X. Nilai tukar tiga negara digunakan sebagai studi kasus terhadap harga emas yaitu Australia, Brasil, dan Afrika Selatan. Dalam penelitian ini, model ARIMA digunakan untuk meramalkan data harga emas sebagai input untuk Fungsi Transfer dan VARI-X. Model Fungsi Transfer hanya mempertimbangkan hubungan antara harga emas sebagai input dengan nilai tukar di masing-masing negara, sedangkan model VARI-X mempertimbangkan juga keterkaitan antara nilai tukar di ketiga negara tersebut. Data yang digunakan adalah data harian periode 1 Juni 2010 hingga 28 Februari 2018. The Root Mean Square Error (RMSE) dan Mean Absolute Percentage Error (MAPE) adalah kriteria yang dipakai untuk memilih model terbaik. Hasil kajian menunjukkan bahwa VARI-X adalah model terbaik untuk meramalkan nilai tukar di Australia, sedangkan Fungsi Transfer adalah model terbaik untuk meramalkan nilai tukar di Afrika Selatan dan Brasil.

Kata kunci: *Fungsi Transfer, VARI-X, Nilai Tukar, Harga Emas, RMSE, MAPE.*

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FORECASTING EXCHANGE RATE ACROSS COUNTRIES WITH GOLD PRICE AS EXOGENOUS VARIABLE USING TRANSFER FUNCTION AND VARI-X MODEL

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ABSTRACT

Investors and collectors hold gold as protection for their savings and wealth at large. Gold does not pay interest like treasure bonds or savings account but current gold prices often reflect increases and decreases. This research aims to provide a model for understanding the relationship between exchange rate and gold price across countries as key members in the export of gold in the world. Also, it finds the best method to exchange rate and exogenous gold price data across countries by comparing the forecast accuracy between transfer function and VARI-X. Three countries exchange rates are used as a case study against the gold price i.e. Australia, Brazil, and South Africa. In this research, the ARIMA model is used for forecasting gold price data as an input for Transfer Function and VARI-X models. Transfer function model only considers the relationship between the gold price as input with the exchange rate in each country, whereas VARI-X model also considers the interrelationship between exchange rates in these three countries. Daily data is used for the period 1st June 2010 to the 28th February 2018. The Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE) is used as criteria for selecting the best model. The results show that VARI-X is the best model for forecasting the Australian exchange rate, whereas Transfer Function is the best model for forecasting South African and Brazilian exchange rates.

Keywords: Transfer Function, VARI-X, Exchange Rate, Gold Price, RMSE, MAPE

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STATEMENT OF AUTHENTICITY

I, the undersigned,

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Declare that my thesis entitled:

"FORECASTING EXCHANGE RATE ACROSS COUNTRIES WITH GOLD PRICE AS AN EXOGENOUS VARIABLE USING TRANSFER FUNCTION AND VARI-X"

Is a completely independent work of mine, completed without using any illegal information, neither the work of others that I recognize as my own work.
All citations are listed in the references section of this thesis.

If it turns out that the statement is not true, I am willing to accept the consequences in accordance with the regulations.

Surabaya, 7 July 2018
Yours Sincerely

Alhassan Sesay
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Author

Alhassan

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDIX

CHAPTER 1
INTRODUCTION

1.1 Background

Gold is one of the most valuable metal in the world. Investors and collectors hold gold as protection for their savings and for their wealth at large. Gold is also seen as a hedge against inflation and currency devaluation. Currency values fluctuate but gold values in terms of ounces can stay more stable in the long term. Humans could use gold as a form of wealth and also as a form of currency.

According to the economist point of view the U.S dollar was chosen for the Bretton Woods system because the United States was easily the world strongest economy coming out of the Second World War. Unlike the other strong nations, the united states do not have to repair infrastructure as a result of this the price of gold was pegged to the U.S dollar.

Forecasting can be used in many ways but one of its most objectives is to predict its future values. Forecasting of gold price provides the valuable information to the government, management, and other stockholders in order to prevent risks (Mombeini and Chamzini, 2015). Gold behaves like a product whose long-term asset depends on the future of demand and supply (Ismail et al., 2009).

Several researchers have proposed different mathematical models that can be used to forecast gold price and exchange rates. Some of these models are Generalized Autoregressive Conditional hetroskedacity (GARCH) model, Threshold Autoregressive (TAR) model, Self-Exciting Threshold Autoregressive (SETAR) model (Basikhasteh, 2014), Artificial Neural network (ANN) (Mombeini and Chamzini, 2015).

Similarly, Pitigalaarachchi et al. (2016) investigated the determinants of gold price from some different macroeconomic factors in Sri Lanka by using ARIMA and VAR models. They found out that the percentage of the gold price for the previous months was highly affected by the percentage of gold in the current month

Ahmad et al. (2015) applied to time series regression and ARIMAX on their monthly currency inflow and outflow series data whose purpose was to get a model to predict inflow and outflow currency in East Java. Their results indicate that the more complex models may not necessarily produce a more accurate forecast.

Akal (2015) proposed a VAR model with non-stochastic exogenous variables (X) based on the direction of causalities to estimate for China, the United States, Japan, and EU by being the largest energy consumers and the most interactive leading economies among themselves in the world. The VAR aspect of their VARX model was able to show dynamic effects of domestic energy intensities of the countries. Also, the exogenous part of VARX model shows them the effect of exogenous variables on endogenous ones.

In another research, Ikoku (2014) developed a model on Currency in Circulation. They developed a forecast from AR(1) models to VECMs that incorporated a number of structural variables. They found out that, depending on the specification of the model, structural variables such as the exchange rate and interbank rate and dummy variables for elections and holidays were significant in explaining changes on currency in circulation.

Evans and Lyons (2016) found out that daily interdealer order flow explains a significant percent of daily exchange rate changes and thus argued that flows are a proximate cause of exchange rate movements. Bank (2001) also found out that weekly flows help explain exchange rate movements. Others such as Kim (1997) and Cai, Cheung, Lee, and Melvin (2001) established that the position of large traders explains currency volatility far better than do news announcements or fundamentals. Froot and Ramadorai (2002) asserted that investor flows are important for understanding deviations of exchange rates from fundamentals but not for understanding long-run currency values.

According to Siourounis et al. (2003), they used unrestricted VAR’s to investigate capital flows and exchange rates and found out that dynamic forecasts from an equity augmented-VAR provide support for exchange rate predictability and outperform a random walk and a standard VAR that includes only exchange rates and interest rates differentials. However, a specification that produces a superior forecast performance depends on the exchange rate and forecast horizon.

Koutsoyiannis (1983) examined that the price of gold which is completely coated in US dollars has become closely linked to the state of the economy and that silver and stocks are substitutes for gold providing alternative investment opportunities in pursuit of capital gains, they also noticed a negative relationship between the gold prices and US dollar compared to other researchers. Their findings suggest that gold price is not efficient in the very short run.

Harmston (1998) looked at the relationship of gold price and that of its purchasing power parity of the United States, Britain, France, Germany and Japan within the periods of 1861–1879, 1933 -1934 and 1968 –1997 and he noticed that one of the reasons for the fluctuation was due to its dual economic function, the second reason was that papers were not redeemable. Even though the price of gold and the price index rose during the war but the price of gold does not rise enough to maintain its purchasing power parity.

Smith (2001) examined the short-run correlation between the price of gold and stock price indices for the United States for the period January 1991 to October 2001. In his research he used three gold prices set in London and one set in New York together with six stock price indices of different coverage, he noticed that there is no cointegration involved in gold price and US stock price index.

In another study, Smith (2002) explores the relationship between gold prices and the prices of stocks in Europe and Japan. He tried to distinguish between short and long run by using cointegration and short-term linear relationship. He noticed that there is no cointegration involved in gold price and stock price index, which simply means there is no long-run equilibrium and the series do not share a common stochastic trend.

Ghosh et al. (2002) used monthly gold price data and cointegration regression techniques to analyze the general inflation in the USA and the world at large, US dollar exchange rate and another shock. They suggested that sizeable short-run movements in the price of gold are related with the gold price rising by means of the total rate of inflation. They also confirmed that movements in the nominal price of gold are controlled by short-run influences.

Sjaastad (2008) tested the relationship between the major exchange rates like the DM (euro), US dollar spot and forward exchange rates, yen spot and forward exchange rates and the price of gold using forecast error data. He found a relationship between the forward and spot prices. Also, he asserted that the major gold producers of the world South Africa, Australia and Russia seemed to have no significant influence over the world price of gold.

According to the U.S Geological Survey in the past fourteen years, China nearly doubled its gold mine production to 440 metric tonnes with a reserve of 2,200 and rose to the top slot, while South Africa cut its production in half and fell from first to seventh with a mine production of 145 metric tonnes. Australia held steady in and around second slots with a mine production of 300 in 2017 and 250 tonnes in 2016 and Brazil which has been below the first ten countries before increasing their mine production to 85 and reserves of 2,400 metric tonnes in 2017.

Australia has a number of additional advantages in terms of international trade with South Africa. The business culture, accounting practices, and the legal system are rather similar, and English is the official language in both countries. Recently, there has also been a strong business migration from South Africa to Australia. Australia ostensibly views South Africa as the gateway into the Southern African Development Community (SADC) region, as it is already the regional powerhouse (Jordaan, 2015)

South Africa and Brazil also enjoy strong bilateral relations as symbolized by high-level visits and various agreements signed across a number of sections since 1994. The relationship between the two countries is also underpinned by a common desire to influence the global agenda in the 21st century in a manner that reflects the aspirations of developing countries. The key sectoral areas of cooperation include trade and industry, health, arts and culture, tourism and environmental affairs. Brazil remains South Africa's largest trading partner in Latin America. Trade between South Africa and Brazil has seen steady growth in recent years.

There has been an economic cooperation and trade between these countries. Australia is the sixth largest export destination for South African goods. Majority of the product exported from South Africa to Australia are finished goods. However, gold mining has been in existence during the era of batter system was the trade of goods and services became the main medium of exchange. Also, these countries been the key players in the export of gold, the research consider them as a result of their factor endowment in the production and supply of gold in the world market which has a significant impact on the socio-economic activities of their countries.

Based on the background review, this research will focus on forecasting exchange rates across countries with gold price as an exogenous variable using transfer function and VARI-X compared to the models of the ARIMA model. The result of this model can be obtained by looking at the smallest RMSE value.

1.2 Problem Statement

Based on the existing exchange rate and price of gold problems, the statement of the problem to be examined in this research includes:

i. How to create a descriptive analysis on the Exchange rate data and gold price across these countries (Australia, Brazil and South Africa)?
ii. How to assess the effect of gold price in dollars to the exchange rate of these countries (Australia, Brazil and South Africa) using transfer function and VARI-X?
iii. What is the best model derived from the comparison after using ARIMA, transfer function, and VARI-X based on the smallest RMSE and MAPE value to forecast the exchange rates and gold price?

1.3 Objectives

The objectives of this study are:

i. To create a descriptive analysis on the Exchange rate and gold price data across these countries
ii. To assess the effect of gold price in dollars to the exchange rate of these countries using Transfer function and VARI-X
iii. To derive the best model from the comparison after using transfer function and VARI-X based on the smallest RMSE and MAPE value to forecast the exchange rates and gold price

1.4 The significance of the Research

The significance of this research is that it will serve as a source of data and academic documentary for future and prospective researchers on this subject matter and in different areas. It will provide scientific insights concerning models. The benefit also is that it will create a platform for future studies into forecasting exchange rate across countries with gold price as an exogenous variable using ARIMA, transfer function, and VARI-X.

1.5 Outline of the Research

This thesis is composed of five themed chapters. Chapter 1 introduces the research background, problem definition, objectives and questions to answer by the research. Chapter 2 is dedicated to a literature review of previous literature on exchange rates and gold price and then proposed a model. Chapter 3 presents a prediction model for ARIMA, transfer function and VARI-X. Chapter 4 discusses the results obtained using this methodology in a case study. Finally, chapter 5 gives the conclusion and recommendations of the research.

1.6 Limitation of the Study

The purpose of this study is to look into forecasting the exchange rates across countries with gold price as an exogenous variable using ARIMA, transfer function, and VARI-X model.

i. The data used is purely limited to the main players of gold that is Australia, Brazil, and South Africa that have the tendency to influence the Exchange rates around the world. Other countries were excluded as they do not have much impact to influence the exchange rate and also due to a lot of missing data. However, these limitations would not in any way affect the result of the findings
ii. Modeling relies on the human interpretation of the analysis which is made to detect trends and outliers thus there is a large amount of judgment involved in assessing the most appropriate model and parameters.

CHAPTER 2
LITERATURE REVIEW

The literature review support problem-solving in knowledge related to exchange rates and price of gold in the various countries. Some of the things discussed are related to the ARIMA model, transfer function, and VARI-X the way their exchange rate has been doing during the past years after those shocks they went through.

2.1 The Concept of Time Series Analysis

Time series analysis was introduced by George Box and Gwilym Jenkins in 1976. Time series is a series of observation data that occur based on the time index in sequence with a fixed time interval. Time series analysis is a statistical procedure applied to predict the probable structure of future circumstances for decision making. The research data used is intertwined by time, so there is a correlation between current events with data from one previous period. Although it is closely related to the time sequence, it does not rule out having a close relationship with other dimensions such as space. Time series is also applied in various fields, such as agriculture, business and economics, engineering, health, meteorology, quality control and social sciences. In business and economics, time series is applied in observing stock prices, interest rates, monthly price indices, quarterly sales, and annual revenues (De Gooijer and Hyndman, 2006). The development of time series analysis that has more than one variable is called multivariate time series analysis which is used to model and forecast the interaction and movement between the time series variables. The multivariate time series analysis model used in this research is the transfer function and VARI-X model.

2.1.1 Stationarity

Stationary data is a data that experiences growth or decline. Stationary data roughly have a horizontal pattern along the time axis. In other words, the fluctuation of the data is around a constant average value, independent of the time and variance of the fluctuation is substantially constant at all times. If the time series data is not stationary in mean then the data is made stationary by way of differentiating or differencing. Let's say differencing order 1 where W_t is the value after the differentiation or differencing when the stationary conditions in the variance are not met, Box and Cox in Wei (2006) introduces power transformation, that is

$$T(Z_t) = \frac{Z_t^{\lambda} - 1}{\lambda} \qquad (2.1)$$
$$T(Z_t) = Z_t^{\lambda}$$

The Box-Cox transformation form for some of the most commonly used (lambda) estimates is shown in Table 2.1 (Wei, 2006).

Table 2.1 Box Cox Transformation

Value from λ (lambda)	**Transformation**
-1.0	$\frac{1}{Z_t}$
-0.5	$\frac{1}{\sqrt{Z_t}}$
0.0	$\ln(Z_t)$
0.5	$\sqrt{Z_t}$
1.0	Z_t

Some use of λ value, as well as its relation to the transformation, is shown in Table 2.1 Some provisions to stabilize the transformation is as follows.

1. Transformation should only be done for the Z_t series that is positive
2. Transformation is done before doing differencing and modeling time series
3. Transformation not only stabilizes the variance but often also improve the distribution approach with a normal distribution

2.1.2 Autocorrelation Function (ACF)

One of the simplest identification of stationarity (Z) is based on the Autocorrelation Function (ACF) or the correlation between Z_1 and. ACF in the lag to-k is defined as follows (Wei, 2006)

$$\hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0} = \frac{\sum_{i=1}^{n-k}(Z_t - \bar{Z})(Z_{t+k} - \bar{Z})}{\sum_{t=1}^{n}(Z_t - \bar{Z})^2} \qquad k = 0, 1, 2, \ldots \tag{2.2}$$

Autocorrelation Function (ACF) can be used to identify the time series model and see the data structure in the mean. If ACF goes down slowly then it can be concluded that the data has not been stationary in the mean.

2.1.3 Partial Autocorrelation Function (PACF)

PACF is used as a tool to measure the level of closeness between Z_t and Z_{t+k} after eliminating dependency $Z_{t+1}, Z_{t+2}, \ldots, Z_{t+k-1}$. PACF is denoted by $\hat{\phi}_{k+1,j}$ with the calculation as in Wei (2006), namely:

$$\hat{\phi}_{k+1,K+1} = \frac{\hat{\rho}_{k+1} - \sum_{j=1}^{k}\hat{\phi}_{kj}\hat{\rho}_{k+1-j}}{1 - \sum_{j=1}^{k}\hat{\phi}_{kj}\hat{\rho}_j} \tag{2.3}$$

where $\hat{\phi}_{k+1,j} = \hat{\phi}_{kj} - \hat{\phi}_{k+1,k+1}\hat{\phi}_{k,k+1-j}$ with $j = 1, 2, \ldots, k$.

2.2 Identification of ARIMA Model

Selection of the ARIMA model is appropriate for one-time series data that can be done by using the Box-Jenkins procedure. The ARIMA modeling procedure includes several stages of identification, diagnostic check, and forecasting. The identification step is carried out by observing the ACF and PACF plots of data subsequently used to obtain the provisional estimation of the appropriate ARIMA model. The next stage estimates and tests the significance of the parameters of whether the estimated intermediate model has been sufficiently accurate with the time series data. The steps of establishing ARIMA model based on the Box-Jenkins method can be illustrated in Figure 2.1 and its explanation

2.2.1 Identification

ARIMA Identification model (*p*, *d*, *q*) is done after stationary data. If the data is not differenced, then *d* is 0 and if the data becomes stationary after differencing the 1 then *d* is 1 and so on. Box Jenkins (ARIMA) model is divided into 3 groups, namely: autoregressive model (AR), moving average (MA), and ARMA mixed model (autoregressive moving average) characteristic of the first two models. The ARIMA timetable model has other forms:

a. Autoregressive Model (AR)

A linear equation is said to be an autoregressive model if the model denotes *Z* as a linear function of a certain amount of *Z* over the previous period together with the current error. The shape of this model with the order *P* or AR (*p*) or ARIMA model (*p*, 0, 0) are:

$$Z_t = \emptyset_1 \dot{Z}_{t-1} + \emptyset_1 \dot{Z}_{t-2} + \cdots + \emptyset_p \dot{Z}_{t-p} + a_t \quad (2.4)$$

or,

$$\phi_p(B) z_t = a_t \quad (2.5)$$

where

$\phi_p(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$ and $\dot{Z}_t = Z_t - \mu$ with

z_t = Time series data as response variable at time *t*

$Z_{t-1}, \cdots, Z_{t-p}$ = data from time to $t-1, t-1, \cdots, t-p$

$\phi_1, \phi_2, \cdots, \phi_p$ = autoregressive parameters

a_t = error value at time *t*.

b. Moving Average Model (MA)

Unlike the autoregressive model which denotes Z_t as a linear function of Z_t based on a combination of past errors (lag). The shape of this model with the order *q* or model (0, 0, *q*) in general is:

$$Z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q} \quad (2.6)$$

or

$$Z_t = \theta_q(B) a_t \quad (2.7)$$

where

$\theta_q(B)$ $= (1 - \theta_1 B - \cdots - \theta_q B^q)$ with:

$\theta_1, \theta_2, \cdots, \theta_q$ = Moving average parameters

$a_t, a_{t-1}, \cdots, a_{t-q}$ = error value in the period of time $t, t-1, \ldots, t-q$

It can be seen from the model that Z_t is the weighted average error of q last period used for the moving average model. If a model used two past errors then it is called the moving average model order 2 or MA (2)

c. Autoregressive Moving Average (ARMA)

To get a model that accurately predicts time series data, it often sees a mixture of AR and MA processes in a single model. The combination of these two processes is commonly known as the autoregressive moving average (ARMA), the shape of this model with the order (*p*, *q*) or ARMA (*p*, *q*) or ARIMA model is:

$$\phi_p(B)Z_t = \theta_q(B)a_t \qquad (2.8)$$

d. Autoregressive Integrated Moving Average (ARIMA)

A time series model which is stationary through differencing process is referred to as the ARIMA model. Thus, if stationary data is processed differencing *d* times, with ARMA basis (*p*, *q*), then the model formed becomes ARIMA (*p, d, q*) where *p* is the autoregressive order (AR) or AR level, d is the level process of differencing and q is addressing the moving average or MA level. The general form of the ARIMA model is

$$\phi_p(B)(1-B)^d Z_t = \theta_q(B)a_t \qquad (2.9)$$

where:

$(1-B)^d$ = differencing in the order of *d*

B = backward shift operator, $BZ_t = Z_{t-1}$

2.2.2 Seasonal ARIMA Model

In addition to being used for non-seasonal models, ARIMA models can also be used for the seasonal model. The seasonal time series model is a time series that has certain "repetitive after-hours" properties, e.g. one yearly for monthly time series, or one weekly. Seasonal ARIMA models are expressed (Wei, 2006)

$$\Phi_p(B^s)\phi_P(B)(1-B)^d(1-B^s)^D Y_t = \theta_q(B)\Theta_Q(B^s)a_{t,} \tag{2.10}$$

where

(p,d,q) = AR (p) order, differencing order (d), MA (q) order for non-Seasonal Patterns

$(P,D,Q)^s$ = AR (*P*) order, differencing order (*D*), MA (*Q*) order for a Seasonal pattern

$\phi_p(B)$ = coefficients of non-seasonal AR components *p* degree, with the following description

$$\phi_p(B) = (1-\phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p),$$

$\phi_p(B^s)$ = coefficients of the seasonal AR component *s* with translation follows

$$\phi_p(B^S) = (1-\phi_1 B^S - \phi_2 B^{2s} - \cdots - \phi_p B^{ps}),$$

$\theta_q(B)$ = MA component coefficients are non-seasonal with *q* degree, with the translation as follows

$$\theta_p(B) = (1-\theta_1 B - \theta_2 B^2 - \cdots - \theta_p B^p),$$

$\Theta_Q(B^s)$ = Seasonal MA component coefficient *s* with translation as follows

$$\Theta_Q(B^S) = (1-\Theta_1 B^S - \Theta_2 B^{2s} - \cdots - \Theta_Q B^{Qs}),$$

$(1-B)^d$ = operator for differencing order d

$(1-B^s)^D$ = operator for seasonal differencing *s* order *D*,

a_t = the residual value at the time *t* already meet the white noise assumption

It is a summary of the characteristics of ACF and PACF to determine the order of *p* and *q*.

Table 2.2 Characteristics of theoretical ACF and PACF for stationary processes

Process	ACF	PACF
AR(p)	Dies down	Cuts off after lag *p*
MA(q)	Cuts off after lag *q*	Dies down
ARMA(p ,q)	Dies down after lag (*q* – *p*)	Dies down after lag (*p* –*q*)

2.2.3 Parameter Estimation

After obtaining the temporary model for time series data, the next complete step is to estimate the parameter model. There are several ways that can be used to get ARIMA model parameters (Wei, 2006), among others:

a. Moment Method

The moment method is performed by substituting a sample of moments such as a sample of mean, sample variance and an ACF sample to solve the resulting equation and to estimate unknown parameters. The simplest example of this moment method is to estimate the mean of a series μ to use the mean sample $\bar{Z}$

b. Maximum Likelihood Method

The Likelihood moment method has been widely used in estimation, the general model for stationary ARMA (*p*, *q*) is:

$$Z_t = \emptyset_1 Z_{t-1} + \cdots + \emptyset_p Z_{t-p} + a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q}$$
(2.11)

where $\dot{Z}_t = Z_t - \mu$ and $\{a_t\}$ is identically and normally distributed $(0, \sigma_a^2)$, Joint probability for $a = (a_1, a_2, \cdots, a_n)$ is:

$$p(a / \phi, \mu, \theta, \sigma_a^2) = (2\pi\sigma_a^2)^{-n/2} \exp[-\frac{1}{2\sigma_a^2} \sum_{t=1}^{n} a_t^2] \qquad (2.12)$$

so the equation (2.11) becomes

$$a_t = \theta_1 a_{t-1} + \cdots + \theta_q a_{t-q} + Z_1 - \emptyset_1 Z_{t-1} - \cdots - \emptyset_p Z_{t-p} \qquad (2.13)$$

then the likelihood function for the parameter $((\phi,\mu,\theta,\sigma_a^2)$ can search if $Z=(Z_1,Z_2,...Z_n)'$ and insured that $Z_*=(Z_{1-p},\mathrm{K}\ ,Z_{-1,}Z_0)'$ and $a_*=(a_{1-q},\mathrm{K}\ ,a_{-1},a_0)'$ is known. The conditional function is as follows:

$$\ln L(\phi,\mu,\theta)=-\frac{n}{2}\ln 2\pi\sigma_a^2-\frac{S_*(\phi,\mu,\theta)}{2\sigma_a^2} \tag{2.14}$$

where $S_*(\phi,\mu,\theta)=\sum_{t=p+1}^{n}a_t^2(\phi,\mu,\theta\setminus Z_*,a_*,Z)$ is a conditional sum of squares function. The sum of $\hat{\phi},\hat{\mu}$ and $\hat{\theta}$ by maximizing the function is called the conditional maximum likelihood estimator. $\ln L(\phi,\mu,\theta,\sigma_a^2)$, including data only through (ϕ,μ,θ), this estimator is equal to the conditional least square estimator obtained by minimizing the conditional sum of the square from function s (ϕ,μ,θ) without parameter σ_a^2

c. Ordinary Least Square (OLS) Method

As an illustration of the application of the least square method for estimation of AR(1) parameter model, i.e.

$$Z_t-\mu=\phi_1(Z_{t-1}-\mu)+a_t \tag{2.15}$$

The least squares estimation is done by finding the parameter value that minimizes the sum of squares error, i.e.

$$S_*(\phi_1,\mu)=\sum_{t=2}^{n}[(Z_t-\mu)-\phi_1(Z_{t-1}-\mu)]^2 \tag{2.16}$$

then lowered to X and then equated to 0, we get the estimated parameters of model AR (1) as follows

$$\hat{\mu}=\frac{\sum_{t=2}^{n}Z_t-\phi_1\sum_{t=2}^{n}Z_{t-1}}{(n-1)(1-\phi_1)}. \tag{2.17}$$

The above equation can be reduced to

$$\mu\approx\frac{\bar{Z}-\phi_1\bar{Z}}{(1-\phi_1)}=\bar{Z} \tag{2.18}$$

in the same way, lower ϕ_1 will be obtained

$$\phi_1 = \frac{\sum_{t=2}^{n}(Z-\overline{Z})(Z_{t-1}-\overline{Z})}{\sum_{t=2}^{n}(Z_{t-1}-\overline{Z})^2} \quad . \tag{2.19}$$

In general, let δ be a parameter in the ARIMA model (including ϕ, θ *and* μ) and $\hat{\delta}$ is the estimation value of the parameter and s.e ($\hat{\delta}$) is the standard error of the estimated value $\hat{\delta}$ then the parameter significance test can be done as follows:

Hypothesis:

$H_0 : \delta = 0$ (Parameter is not significant)

$H_1 : \delta \neq 0$ (Parameter is Significant)

Test statistic:

$$t = \frac{\hat{\delta}}{s.e(\hat{\delta})}$$

Rejection region: Reject H_0 if | $|t| > t_{\alpha/2:n-m}$ with m = total parameter

2.2.4 Diagnostic Check

One of the most important stages in building a time series model is the residual diagnosis of the model, i.e. residual white noise (residual indicating no serial or residual independent correlation) and identical (homoscedastic). In addition to residual white noise, also normal distribution. White noise assumption test using the Ljung-Box test with the hypothesis is as follows

$H_0 : \rho_1 = \rho_2 = \text{L} = \rho_K = 0$ (Residual is either correlated or white noise)

$H_1 :$ atleast one of $\rho_k \neq 0$ For K=1, 2, …, K (Residual not correlated or white noise)

The test statistic used in the Ljung-Box test is as follows

$$Q^* = n(n+2)\sum_{k-1}^{k}(n-k)^{-1}\hat{\rho}_k^2 \tag{2.20}$$

where *n* is the number of observational data, $\hat{\rho}_k$ is the ACF residual lag *k* residual Q^* is a chi-square distributed parameter with free degree *K – p – q* where *p* is the order of AR and *q* is the order of the MA. If $Q^* > \chi^2(\alpha, df = k - p - q)$ or p-value smaller than α then H_0 is rejected

2.2.5 Normal Distribution Test

In addition to the assumption of white noise, residuals also must meet the assumption of normal distribution. Testing of normal distribution is conducted using Kolmogorov-Smirnov test with the following hypothesis.

$H_0 : F_n(a_t) = F_0(a_t)$ (Residual is normally distributed)

$H_1 : F_n(a_t) \neq F_0(a_t)$ (Residual is not normally distributed)

The test statistic used in the normal distribution test is as follows

$$D = Sup \setminus F_n(a_t) - F_0(a_t))$$

where $F_n(a_t)$ is the cumulative opportunity function of the data Sample, $F_0(a_t)$ is the cumulative opportunity value of the normal distribution, Sup is the maximum value of the absolute price then *D* is the farthest vertical distance between $F_n(a_t)$ and $F_0(a_t)$. However, if the condition is not met a nonparametric test should be done

2.3 Transfer Function Model

Transfer function model is based on the relationship between time series response variable data (output series) with one or more predictor variables (input series). In other words, the transfer function is a model that describes the predicted value of time ahead of a time series variable based on past values and/or based on one or more variables of another time series that has a relationship with the time series. The general form of a transfer function for a single input (X_t) and the single output (Y_t) is as follows

$$y_t = v_0 x_t + v_1 x_{t-1} + v_2 x_{t-2} + \cdots + \eta_t = v(B) x_t + \eta_t \quad (2.21)$$

With y_t is a stationary output array x_t is a series whose input is stationary, and n_t is an error component to follow an ARMA model.

with

$$v(B) = \frac{\omega_s(B) B^b}{\delta_r(B)} \text{ and } n_t = \frac{\theta_q(B)}{\phi_p(B)} a_t \quad (2.22)$$

where

$\omega_s(B) = \omega_0 - \omega_1 B - \cdots - \omega_s B^s$

: Moving average operator of order *s*,

$\delta_r(B) = 1 - \delta_1 B - \cdots - \delta_r B^r$

: The autoregressive operator of order *r*

$a_t =$ Residual value at time *t*.

2.3.1 Identification of Transfer Function Model

According to Wei (2006), the stages in doing the process of identifying the form of transfer function model are as follows

1. Prepare the input and output series

   The set of inputs and output should be a series that is already stationary. If the series is not stationary in mean or variance, the differencing process should be done in order to make the series stationary in mean and the transformation process to be stationary series invariance.

2. Prewhitening the input series

   In the process of prewhitening or bleaching aims for making the input series more manageable and eliminating the whole pattern in there so that the series becomes white noise. The identification of the input model follows the ARMA model which can be expressed in the following equation

$$\phi_x(B) x_t = \theta_x(B) \alpha_t \quad (2.23)$$

where $\phi_x(B)$ is an autoregressive operator, $\theta_x(B)$ moving average operators and α_t is an already white noise input series with mean 0 and variance σ_a^2 which is the result of prewhitening. Input series x_t then converted into a series α_t so that it becomes:

$$\alpha_t = \frac{\phi_x(B)}{\theta_x(B)} X_t \qquad (2.24)$$

3. Prewhitening the output series

   The prewhitening process is also performed on the y_t output series the transfer function can map x_t into y_t. In prewhitening the output series, the resulting series is not necessarily the white noise series. This is due to that the series output is modeled forcibly by using the input series model. Here is the equation for prewhitening output series.

$$\beta_t = \frac{\phi_x(B)}{\theta_x(B)} Y_t \qquad (2.25)$$

   β_t is the output series that has experienced prewhitening while y_t is the corresponding output sequence

4. Calculation of cross-correlation function or CCF (cross-correlation function) and autocorrelation for input and output series that prewhitening has been done.

   CCF is used to measure the level of relationship between values x at time t with y at the time $t+k$. The CCF coefficient of input x_t and output y_t for the k-lag is defined as follows.

$$\hat{\rho}_{xy}(k) = \frac{\gamma_{xy}(k)}{\sigma_x \sigma_y} \quad k = 0, \pm 1, \pm 2, \ldots, \qquad (2.26)$$

   is estimated by the sample cross-correlation function

$$\hat{\rho}_{xy}(k) = \frac{\hat{\gamma}_{xy}(k)}{S_x S_y} \quad k = 0, \pm 1, \pm 2, \ldots,$$

   where

$$\hat{\gamma}_{xy}(k)=\begin{cases}\frac{1}{n}\sum_{t=1}^{n-k}(x_t-\bar{x})(y_{t+k}-\bar{y}), & k\geq 0,\\ \frac{1}{n}\sum_{t=1-k}^{n}(x_t-\bar{x})(y_{t+k}-\bar{y}), & k<0,\end{cases}$$

$S_x=\sqrt{\hat{\gamma}_{xx}(0)},\quad S_x=\sqrt{\hat{\gamma}_{yy}(0)},$

$\bar{x}$ and $\bar{y}$ are the same means as the x_t and y_t series respectively (Wei, 2006)

5. Determination of (*b*, *r*, *s*) for the transfer function model connecting the input and output series

   The three key parameters in the transfer function model are (*b, r, s*) with *r* denoting the degree of function $\delta(B)$, S shows the degree of function $\omega(B)$, and b indicates the delay recorded in x_{t-b} in the equation:

$$y_t=\frac{\omega_s(B)}{\delta_r(B)}x_{t-b}+\frac{\theta_q(B)}{\phi(B)}a_t \tag{2.27}$$

   Here are the rules that can be used to estimate the value of (*b*, *r*, *s*) in a transfer function (Wei, 2006)

a. The value *b* states that y_t is not influenced by x_t period $t+b$. The value of b is obtained by looking at the CCF plot, i.e. starting the first significant b-lag

b. The value of *s* denotes how long a series y_t is influenced by $x_{t-b-1},x_{t-b-2},\ldots,x_{t-b-s}$ so that the value of *s* is the number in the CCF lag before the pattern declines.

c. The value *r* denotes that y_t is influenced by the past time which is $y_{t-1},y_{t-2},\ldots,y_{t-r}$

6. Initial assessment of noise series

   The weight of v is measured directly and indicated to the calculation of preliminary estimate value of a series of disturbances n_t

$$y_t=v(B)x_t+n_t \tag{2.28}$$

   then

$$n_t = y_t - v_0 x_t - v_1 x_{t-1} - v_2 x_{t-2} - \cdots - v_g x_{t-g} \tag{2.29}$$

where g is the practical value chosen

7. Determination (p_n, q_n) for the ARIMA model $(p_n, 0, q_n)$ of the series disturbance n_t .The value of n_t is analyzed by means of ordinary ARIMA for determining the exact ARIMA model to be obtained by the value (p_n, q_n). In this way the functions $\phi_n(B)$ and $\theta_n(B)$ for a series of n_t disturbances can be obtained to get the equation:

$$\phi_n(B) n_t = \theta_n(B) a_t \tag{2.30}$$

2.3.2 Estimation of Transfer Function Parameter Model

After the identification of the transfer function model, the transfer function parameter is obtained as follows (Wei, 2006):

$$y_t = \frac{\omega_s(B)}{\delta_r(B)} x_{t-b} + \frac{\theta(B)}{\phi(B)} a_t \tag{2.31}$$

and then estimate the parameters, $\delta = (\delta_1, \text{K}\ \delta_r)'$, $\omega = (\omega_0, \omega_1, \text{K}\ , \omega_s)'$, $\phi = (\phi_1, \text{K}\ , \phi_p)'$, $\delta = \left(\delta_1, \cdots, \delta_p\right)'$, $\theta = \left(\theta_1, \cdots, \theta_p\right)'$ and σ_a^2 . Equation (2.31) could be written as

$$\delta_r(B)\phi(B) y_t = \phi(B)\omega_s(B) x_{t-b} + \delta_r(B)\theta(B) a_t, \tag{2.32}$$

or, the same with

$$c(B) y_t = d(B) x_{t-b} + e(B) a_t \tag{2.33}$$

where

$$c(B) = \delta(B)\phi(B) = (1-\delta_1 B - \cdots - \theta_r B^r)(1-\phi_1 B - \cdots - \phi_p B^p)$$
$$= (1 - c_1\mathbf{B} - c_2\mathbf{B}^2 - \cdots - c_{p+r}\mathbf{B}^{p+r}),$$
$$d(B) = \phi(B)\omega(B) = (1-\phi_1 B - \cdots - \phi_p B^p)(\omega_0 - \cdots - \omega_s B^s)$$
$$= (d_0 - d_1\mathbf{B} - d_2\mathbf{B}^2 - \cdots - d_{p+s}\mathbf{B}^{p+s}),$$
$$e(B) = \delta(B)\phi(B) = (1-\delta_1 B - \cdots - \theta_r B^r)(1-\theta_1 B - \cdots - \theta_q B^q)$$
$$= (1 - e_1\mathbf{B} - e_2\mathbf{B}^2 - \cdots - e_{r+q}\mathbf{B}^{r+q})$$

then

$$a_t = y_t - c_1 y_{t-1} - \text{K} - c_{p+r} y_{t-p-r} - d_0 x_{t-b} - d_1 x_{t-b-1} - \text{K} - d_{p+s} x_{t-b-p-s} + e_1 a_{t-1} + \text{K} + e_{r+q} a_{t-r-q} \quad (2.34)$$

where c_i, d_j and e_k are functions of $\delta_i, \omega_j, \phi_k,$ and θ_1 . Under the assumption that the a_t are $N(0, \sigma_a^2)$ white noise series, we have the following conditional likelihood function:

$$L(\delta, \omega, \phi, \theta, \sigma_a^2 \mid b, x, y, x_0, y_0, a_0) = (2\pi\sigma_a^2)^{-n/2} \exp\left[-\frac{1}{2\pi\sigma_a^2}\sum_{t=1}^{n} a_i^2\right], \quad (2.35)$$

where x_0, y_0, a_0 are some proper starting values for computing a_t from … Similar to the starting values needed in the estimation of univariate ARIMA models. The method of estimation can also be used to estimate the parameters $\omega, \delta, \phi, \theta,$ and σ_a^2 . For instance, by setting the unknown a's equal to their conditional expected values of zero, the nonlinear least squares estimate of these parameters is obtained by minimizing

$$S(\delta, \omega, \theta \mid b) = \sum_{t=t_0}^{n} a_t^2 \quad (2.36)$$

where

$$t_0 = \max\{p + r + 1, b + p + s + 1\}$$

2.3.3 Diagnostic Testing of Transfer Function Model

According to Wei, (2006), after the model has been identified and its parameters estimated, it is necessary to check the model adequacy before we can use it for forecasting, control and other purposes. In the transfer function model, we assume that the a_t is white noise and is independent of the input series x_t and hence are also independent of the pre-whitened input series α_t. Those in the diagnostic checking of a transfer function model, we have to estimate the residuals $\hat{a}_t$ from the noise model as well as the residuals α_t from the prewhitened input model to see whether the assumption hold.

1. **Cross-correlation (CCF) test between residual model series noise (a_t) with a prewhitening input selection (α_t)**

   This test is performed to determine whether the noise series and input series that have been prewhitening are independent that is by calculating CCF between residuals a_t and α_t . For an adequate model the sample $(\hat{\rho}_{\alpha a}(k))$ between $\hat{a}_t$ and α_t should show no patterns and lie within their two standard errors $2(n-k)^{-1/2}$. The following portmanteau test can also be used:

$$Q_0 = m(m+2)\sum_{j=0}^{k}(m-j)^{-1}\hat{\rho}^2_{\alpha\hat{a}}(j), \qquad (2.37)$$

   Which approximately follows a χ^2 distribution with ($(k+1)-M$ degrees of freedom, where $m = n - t_0 + 1$, M is the number of parameters δ_i and ω_j estimated in the transfer function $v(B) = \omega(B)/\delta(B)$

2. **The autocorrelation test for the residual model of the noise series (a_t) which connects the input and output series**

   This test is done to determine the suitability of the model that is already capable of producing white noise. The idea is to see if the ACF and PACF of the residual do not show a certain pattern. In addition, a portmanteau test can also be used which is similar to the previous one. It can be calculated as

$$Q_1 = m(m+2)\sum_{j=1}^{k}(m-j)^{-1}\hat{\rho}_a^2(j) \tag{2.38}$$

The Q_1 statistic approximately follows a χ^2 distribution with $(k-p-q)$ degrees of freedom depending on the number of parameters in the noise model.

2.3.4 The Use of Transfer Function Model for Forecasting

Modeling of the transfer function is done by a way of simultaneously modeling all the variables already identified earlier on so that the model becomes:

$$y_t = \frac{\omega_s(B)}{\delta_r(B)}B^b x_t + \frac{\theta_q(B)}{\phi_p(B)}a_t \tag{2.39}$$

Once the model of the corresponding transfer function is obtained it can then be used to predict the value of the series output y_t based on the past value of the output series itself and the input series x_t that affect it

2.4 Autoregressive Integrated Moving Averages with Exogenous Variable

Autoregressive Integrated Moving Average (ARIMA) Model is a combination of Autoregressive (AR) and Moving Average (MA) as well as a differencing process to time series data with an additional variable known as an exogenous variable. In general, the ARIMA model is written as ARIMA (*p*, *d*, *q*) with a mathematically model as follows:

$$\phi_p(B)(1-B)^d Z_t = \theta_0 + \theta_q(B)a_t \tag{2.40}$$

where,

(p,d,q) : Order AR (*p*), differencing order (*d*), order MA (*q*) for non-seasonal patterns.

$\phi_p(B)$: Coefficient of AR with *p* degree.

$\phi_p(B) = (1-\phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p)$,

$\theta_q(B)$: Coefficient of MA with q degree.

$\theta_p(B) = (1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_p B^p)$,

The ARIMAX model is an ARIMA model with an additional variable. There are several additional types of variables, e.g. dummy variables for the effects of daily variations and a deterministic trend. Daily variations are patterns with varying length of the period. Depending on the day or season you choose as the reference category (Suhartono et al., 2015). Here is an ARIMAX Model with a deterministic trend.

$$Z_t = \beta_0 + \beta_1 V_{1,t} + \beta_2 V_{2,t} + \cdots + \beta_k V_{k,t} + N_t \quad (2.41)$$

$$N_t = \frac{\theta_q(B)}{\phi_p(B)} a_t, \quad (2.42)$$

where,

$V_{k,t}$ = dummy variable for day of the week to k

$\phi_p(B) = (1 - \phi_1 B - \cdots - \phi_p B^p)$,

$\phi_p(B) = (1 - \phi_1 B - \cdots - \phi_p B^p)$,

N_t = residual of the time series regression process

a_t = residual of the ARIMAX process

2.5 Outlier Identification

Time series data is often influenced by external events. If the cause of the occurrence is known, then the event is known as an intervention, but if it's not known then it is called an outlier. In univariate time series there are 4 types of outliers, namely: Additive Outlier (AO), Innovational Outlier (IO), Level shift (LS), and Temporary or Transient change (TC), but in multivariate time series there are only two that is Additive outlier (AO) and Innovational Outlier (IO). The outlier model is written as follows (Ahmad & Masun, 2015):

$$Z_t = \sum_{j=1}^{k} w_j v_j(B) I_j^{T_j} + X_t \tag{2.43}$$

where

X_t **:** Time series model without outlier, with $X_t = \frac{\theta(B)}{\phi(B)} a_t$

$I_j^{(T_j)}$ **:** Indicator variable for outlier at the time – t period i.e. $I_j^{(T)} = \begin{cases} 1, & t = T \\ 0, & t \neq T \end{cases}$

$V_j(B)$ =1 for AO,

$V_j(B) = \frac{\theta(B)}{\phi(B)}$ for IO,

$V_j(B) = \frac{1}{(1-B)}$ for LS and

$V_j(B) = \frac{1}{(1-\delta B)}; 0 < \delta < 1$ for TC

2.6 VARMA (Vector Autoregressive Moving Averages)

Series data in some empirical studies often consists of observations of some variables, otherwise known as multivariate time series (Box Jenkis &Reinsel, 1994). When the study is about economic variables etc. The variable that may be seen is the Exchange rate variable which is especially useful when comparing one country to another because it shows the relative performance of those countries. The univariate VARMA model is expressed in the following equation:

$$\Phi_p(B) Z_t = \Theta_q(B) a_t \tag{2.44}$$

where

$$\Phi_p(B) = I_k - \Phi_1 B - \ldots - \Phi_p B^p$$

and

$$\Theta_q(B) = I_k - \Theta_1 B - \ldots - \Theta_q B^q$$

where $Z(t)$ is a multivariate time series, multivariate correction of its real value Φ_p and $\Theta_q(B)$ respectively is an autoregressive matrix p and q average polynomial moving average with a nonsingular matrix size of $m \times$ m

2.7 VARX (Vector Autoregressive with Exogenous variables)

Vector Autoregressive with Exogenous Variables (VARX) is an expansion of the VAR model by adding exogenous *X* variables at the right equation. If the data is not stationary in means, then it should be differenced such that the model becomes Vector Autoregressive Integrated with Exogenous Input (Apriliadara et., 2016). The VARI-X (p,d,s*) model can be written as:

$$\Phi_p(B)(1-B)^d Z_t = \Theta^*_{s*}(B)X_t + a_t \tag{2.45}$$

where,

$$\Phi_p(B) = (I - \Phi_1 B - \Phi_2 B^2 - \text{K} - \Phi_p B^p), \Phi^*_{s*}(B) = (\Theta^*_0 - \Theta^*_1 B - \Theta^*_2 B^2 - \text{K} - \Theta^*_{s*} B^{s*})$$

and d is the order of differencing. This research employs differencing with order one denoted as VAR-X (p, 1, s*):

$$\Phi_p(B)(1-B)^d Z_t = \Theta^*_{s*}(B)X_t + a_t \tag{2.46}$$

$$Z_t = \Phi_1 Z_{t-1} + \Phi_2 Z_{t-2} + \cdots + \Phi_p \dot{Z}_{t-p} - \Phi_0 \dot{Z}_{t-1} - \Phi_1 Z_{t-2} - \Phi_2 Z_{t-3} - \cdots - \Phi_p \dot{Z}_{t-(p+1)}$$
$$+ \Phi_0 \dot{Z}_t + \Phi_1 \dot{Z}_{t-1} + \cdots + \Phi^*_{s^*} X_{t-s*} + a_t$$

$$\dot{Z}_t = \sum_{t=1}^{p} \Phi_t Z_{t-i} - \sum_{t=1}^{p+1} \Phi_{t-i} Z_{t-i} + \sum_{t=1}^{s*} \Theta^*_t X_{t-i} + a_t$$

where

$\dot{Z}_t$: is the $m \times 1$ series vector, $\Phi_0 = -1$ is the $m \times m$ identity matrix, and m is the
number of series.

Φ_p : is an $m \times m$ -matrix sized, while m is a matrix sized $m \times m$

Output Series with $Z = 4$ and AR order 2 will be generated with VAR(2) model equation that is:

$$Z_t = \Phi_1 Z_{t-1} + \Phi_2 Z_{t-2} + a_t \tag{2.47}$$

$$\begin{bmatrix} Z_{1t} \\ Z_{2t} \\ Z_{3t} \\ Z_{4t} \end{bmatrix} = \begin{bmatrix} \phi_{1,11} & \phi_{1,12} & \phi_{1,13} & \phi_{1,14} \\ \phi_{1,21} & \phi_{1,22} & \phi_{1,23} & \phi_{1,24} \\ \phi_{1,31} & \phi_{1,32} & \phi_{1,33} & \phi_{1,34} \\ \phi_{1,41} & \phi_{1,42} & \phi_{1,43} & \phi_{1,44} \end{bmatrix} \begin{bmatrix} Z_{1,t-1} \\ Z_{2,t-1} \\ Z_{3,t-1} \\ Z_{4,t-1} \end{bmatrix} + \begin{bmatrix} \phi_{2,11} & \phi_{2,12} & \phi_{2,13} & \phi_{2,14} \\ \phi_{2,21} & \phi_{2,22} & \phi_{2,23} & \phi_{2,24} \\ \phi_{2,31} & \phi_{2,32} & \phi_{2,33} & \phi_{2,34} \\ \phi_{2,41} & \phi_{2,42} & \phi_{2,43} & \phi_{2,44} \end{bmatrix} \begin{bmatrix} Z_{1,t-2} \\ Z_{2,t-2} \\ Z_{3,t-2} \\ Z_{4,t-2} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \\ a_{3t} \\ a_{4t} \end{bmatrix}$$

So we get the equation for each series of output as follows:

$$Z_{1,t} = \phi_{1,11}Z_{1,t-1} + \phi_{1,12}Z_{2,t-1} + \phi_{1,13}Z_{3,t-1} + \phi_{1,14}Z_{4,t-1} + \phi_{2,11}Z_{1,t-2} + \phi_{2,12}Z_{2,t-2} + \phi_{2,13}Z_{3,t-2} + \phi_{2,14}Z_{4,t-2} + a_{1t},$$
$$Z_{2,t} = \phi_{1,21}Z_{1,t-1} + \phi_{1,22}Z_{2,t-1} + \phi_{1,23}Z_{3,t-1} + \phi_{1,24}Z_{4,t-1} + \phi_{2,21}Z_{1,t-2} + \phi_{2,22}Z_{2,t-2} + \phi_{2,23}Z_{3,t-2} + \phi_{2,24}Z_{4,t-2} + a_{2t},$$
$$Z_{3,t} = \phi_{1,31}Z_{1,t-1} + \phi_{1,32}Z_{2,t-1} + \phi_{1,33}Z_{3,t-1} + \phi_{1,34}Z_{4,t-1} + \phi_{2,31}Z_{1,t-2} + \phi_{2,32}Z_{2,t-2} + \phi_{2,33}Z_{3,t-2} + \phi_{2,34}Z_{4,t-2} + a_{3t},$$
$$Z_{4,t} = \phi_{1,41}Z_{1,t-1} + \phi_{1,42}Z_{2,t-1} + \phi_{1,43}Z_{3,t-1} + \phi_{1,44}Z_{4,t-1} + \phi_{2,41}Z_{1,t-2} + \phi_{2,42}Z_{2,t-2} + \phi_{2,43}Z_{3,t-2} + \phi_{2,44}Z_{4,t-2} + a_{1t},$$

Vector Autoregressive with Exogenous Variable (VARX) model are models with the addition of exogenous variables. The model for VARX (p,s*) can be written as:

$$\dot{Z} = \sum_{j=1}^{p} \Phi_i \dot{Z}_{t-1} + \sum_{i=0}^{s*} \Theta X_{i-1} + a_t \qquad (2.47)$$
$$\Phi(B)\hat{Z}_t = \Theta*(B)X_t + a_t$$

where

$$\Phi(B) = I_k - \Phi_1 B - \cdots \Phi_p B^p$$
$$\dot{Z}_t = I_k - \Theta_1 B - \cdots - \Theta_{s*} B^{s*}$$
$$\dot{Z}_t = ((Z_{1t} - \mu), \cdots, (Z_{kt} - \mu))'$$
$$a_t = (a_{1t}, \cdots, a_{kt})'$$
$$X_t = (x_{1t}, \cdots, a_{kt})'$$

Φ_i is a $k \times k$ matrix, whereas $\Theta_i *$ is $k \times r$ sized matrix

2.7.1 VARI-X Parameter Estimation Model

The parameter estimation on the VARI-X model (*p*, *q*, *s*) is the same as the VARMA model that is done by minimizing the sum of squares by using the LS (Least Square) method. Given the following model

$$Z = X\beta + B*\delta + U \qquad (2.48)$$

where

$$Z = (Z_{1,}Z_2, \cdots, Z_n)$$
$$B^* = [\Phi : \Theta]$$
$$U = (u_1, u_2, \cdots u_n)$$
$$Z = Vec(Z)$$
$$\delta = vec(\delta)$$
$$\beta = vec(\beta)$$
$$u = vec(U)$$

or

$$vec(Y) = vec(B^* X) + vec(Z\delta) + vec(U)$$
$$= (X \otimes I_k)\beta + (Z' \otimes I_k)vec(\delta) + vec(u)$$

Can be written as:

$$y = (X \otimes I_k)\beta + Z' \otimes I_k)\delta + u$$
$$u = y - (X \oplus I_k)\beta - (Z' \otimes I_k)\delta \qquad (2.49)$$

where u covariance matrix is as follows:

$$\Sigma_u = I_n \otimes \Sigma_n$$

The meaning of multivariate OLS estimation of p and q, means selecting the estimator by minimizing

$$S(\beta, \delta) = u'(I_n \otimes \Sigma_u^{-1})u = u'(I_n \otimes \Sigma_u^{-1})u$$
$$= [y - (X \otimes I_k)\beta - (Z \otimes I_k)\delta]'(I_n \otimes \Sigma_u^{-1}[X' \otimes I_k)\beta] - (Z \otimes I_k)\delta)$$
$$= Vec(Y - BX - Z\delta)'(I_n \otimes \Sigma_u^{-1})Vec(Y - BX - Z\delta)$$

To find the minimum value of the equation (2.40), note that

$$S(\beta, \delta) = [y(I_n \otimes \Sigma_u^{-1})y' - y'(I_n \otimes \Sigma_u^{-1})(X' \otimes I_k)]\beta - y'(I_n \otimes \Sigma_u^{-1})(Z' \otimes I_k)\delta$$
$$- (X \otimes I_k)\beta'(I_n \otimes \Sigma_u^{-1})y + \beta'(X \otimes I_k)(I_n \otimes \Sigma_u^{-1})(X' \otimes I_k)\beta$$
$$+ \beta(X \otimes I_k)(I_n \otimes \Sigma_u^{-1})(Z' \otimes I_k)\delta - (Z \otimes I_k)\delta'(I_n \otimes \Sigma_u^{-1})y$$
$$+ (Z \otimes I_k)\delta'(I_n \otimes \Sigma_u^{-1})(X \otimes I_k)\beta + \delta'(Z \otimes I_k)(I_n \otimes \Sigma_u^{-1})((Z' \otimes I_k)\delta$$
$$= y(I_n \otimes \Sigma_u^{-1})y' - 2\beta'(X \otimes I_k)y + \beta'(X'X) \otimes \Sigma_u^{-1})\beta - 2\delta'(Z \otimes \Sigma_u^{-1})y$$
$$+ 2\beta'(ZX' \otimes \Sigma_u^{-1})\beta + \delta'(Z'Z) \otimes \Sigma_u^{-1})\delta$$

Then there are two parameters to be derived ie β and δ, where

-parameter β for the VARMAX model, then the function S (β, δ) becomes

$$\frac{\partial \mathrm{S}(\beta, \delta)}{\partial \beta} = 0, \ \textit{where}$$

$$\frac{\partial(\mathrm{S}(\beta,\delta))}{\partial\beta} = -2(X\otimes\Sigma_u{}^{-1})y + 2((X\,'X)\otimes\Sigma_u{}^{-1})\beta\,' + 2\delta\,'(ZX\,'\otimes\Sigma_u{}^{-1})$$

Minimum Function $\mathrm{S}(\beta,\delta)$ when $\frac{\partial(\mathrm{S}(\beta,\delta))}{\partial\beta} = 0$

$$2((X\,'X\otimes\Sigma_u{}^{-1})\hat{\beta} = 2(X\otimes\Sigma_u{}^{-1})y - 2(ZX\,'\otimes\Sigma_u{}^{-1})\delta\,'$$

$$\hat{\beta} = (XX)^{-1}Xy - (XX)^{-1}ZX\,'\delta\,'$$

$$\hat{\beta} = (XX)^{-1}X\,'X(y - Z\delta\,'$$

$$\frac{\partial(\mathrm{S}(\beta,\delta))}{\partial\beta} = -2(X\otimes\Sigma_u{}^{-1})y + 2((X\,'X)\otimes\Sigma_u{}^{-1})\beta\,' + 2\delta\,'(ZX\,'\otimes$$

The parameter δ for exogenous variable, the function $\mathrm{S}(\beta,\delta)$ becomes

$\frac{\partial(\mathrm{S}(\beta,\delta))}{\partial\beta} = 0$ with

$$\frac{\partial(\mathrm{S}(\beta,\delta))}{\partial\beta} = 2(Z\otimes\Sigma_u{}^{-1})y + 2(ZX\,'\otimes\Sigma_u{}^{-1})\beta + 2((Z\,'Z)\otimes\Sigma_u{}^{-1})\delta\,'$$

Minimum Function $\mathrm{S}(\beta,\delta)$ when $\frac{\partial(\mathrm{S}(\beta,\delta))}{\partial\beta} = 0$

$$2((Z\,'Z)\otimes\Sigma_u{}^{-1})\hat{\delta} = 2(Z\otimes\Sigma_u{}^{-1})y - 2(ZX\,'\otimes\Sigma_u{}^{-1})\beta$$

$$\hat{\delta} = (Z\,'Z)^{-1}Zy - (Z\,'Z)^{-1}ZX\,'\beta$$

$$\hat{\delta} = (Z\,'Z)^{-1}ZZ\,'(y - X\beta)$$

2.7.2 VARI-X Identification Model

The identification of the time series vector model is similar to that of the univariate time series model. If given time series vectors $Z_{1,}Z_2,\ldots,Z_n$. Identification can be done by looking at the pattern of the sample correlation matrix Function and Matrices Partial Autoregression after the stationary data (Wei, 2006)

2.7.3 Sample Correlation Matrix Function

If there is a time series vector with observations of n, i.e. $Z_1,Z_2,\ldots,Z_n$ then the sample correlation matrix equation is as given

$$\hat{\rho}(k) = [\hat{\rho}_y(K)] \qquad (2.50)$$

where $\hat{\rho}_y(k)$ is the cross-sample correlation for the i-th and j-series components expressed in the following equation

$$\hat{l}_{ij}(K)=\frac{\sum_{t=1}^{n-k}(Z_{i,t}-\bar{Z}_i)(Z_{j,t+k}-\bar{Z}_j)}{\left[\sum_{t=1}^{n}(Z_{i,t}-\bar{Z}_i)^2\sum_{t=1}^{n}(Z_{j,t}-\bar{Z}_i)^2\right]^{1/2}} \qquad (2.51)$$

$\bar{Z}_i$ and $\bar{Z}_j$ is the sample average of the corresponding series components. Bartlett (1996) in Wei (2006) has decreased the variance and covariance of the cross-correlated cross-sectional quantities of the samples. Based on the hypothesis that two-time series data Z_i and Z_j are not correlated, so Bartlett shows the following equation:

$$\sigma[\hat{l}_{ij}(k)]\cong\frac{1}{n-k}\left[1+2\sum_{s=1}^{\infty}\rho_y(s)\rho_y(s)\right],\quad |k|>q \qquad (2.52)$$

when Z_i and Z_j are white noise series then we get the following equation:

$$Cov[\hat{l}_{ij}(k),\hat{l}_{ij}(k+s)\cong\frac{1}{n-k},$$

(2.53)

$$\sigma[\hat{l}_{ij}(k)]\cong\frac{1}{n-k} \qquad (2.54)$$

For large sample sizes, n-k in the above equation is often substituted in front of n. The sample correlation matrix equation is used to determine the order in the moving average (MA) model. However, the matrix and graph form will be more complex as the vector dimension increases. To overcome this, Tiao and Box (1981) in Wei (2006) introduce an appropriate method to summarize the sample correlation explanation, using symbols (+), (-), and in the sample correlation matrix position (i, j). The symbol (+) denotes a value less than 2 times the standard error and implies a positive correlation relationship. The symbol (-) denotes the value less than (-2) times the standard error or the existence of a negative correlation. The symbol (.) Denotes the value between ± 2 times the standard error which means there is no correlation relationship (Wei, 2006)

2.7.4 The Matrix of Partial Cross-Correlation Function (MPCCF)

In univariate time series, the partial autocorrelation function (PACF) equation is very important for determining the order in AR model. The generalization of the PACF concept into the time series vector form is done by Tiao & Box (1981) in Wei (2006), defines the partial autoregression matrix form in s with p (s) notation, as the last matrix coefficient when the data is applied into an autoregressive vector process of the order s, the Partial autoregression matrix function is defined as follows:

$$p(s) = \begin{cases} \Gamma(1)[\Gamma(0)]^{-1}, & s = 1, \\ \{\Gamma(s) - c'(s)[A(s)^{1} b(s)\}\{\Gamma(0) - b'(s)[A(s)]^{-1} b(s)\}^{-1}, & s > 1, \end{cases} \tag{2.55}$$

For $s \geq 2$ then the values of A (s), b (s), and c (s) are as follows

$$A(s) = \begin{bmatrix} \Gamma(0) & \Gamma(1) & \cdots & \Gamma(s-2) \\ \Gamma(1) & \Gamma(0) & \cdots & \Gamma(s-3) \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma(s-2) & \Gamma(s-3) & \cdots & \Gamma(0) \end{bmatrix}, \tag{2.56}$$

$$b(s) = \begin{bmatrix} \Gamma(s-1) \\ \Gamma(s-2) \\ \vdots \\ \Gamma(1) \end{bmatrix}, \quad c(s) = \begin{bmatrix} \Gamma(1) \\ \Gamma(2) \\ \vdots \\ \Gamma(s-1) \end{bmatrix} \tag{2.57}$$

2.7.5 Diagnostic Test for VARI-X

There are two types of the test from a diagnostic test of VARI-X model. They are a significant parameter and suitability test (test multivariate white noise assumption or multivariate normal distribution from residual).

i. Significant Parameter Test:

A good VARI-X model is a model whose parameter is significant and the parameters differ from zero. $\phi_{i,jq}$ Start estimate from the parameter SE ($\phi_{i,jq}$)

which has a standard error from $\phi_{i,jq}$, the test significant parameter can be written as:

$$H_0 : \phi_{i,jq} = 0$$

$$H_0 : \phi_{i,jq} \neq 0$$

Test statistic is

$$\text{t-statistic} = \frac{\hat{\phi}_{i,jq}}{SE(\hat{\phi}_{i,jq})}$$

Reject H_0 when |t-statistic|> $t_{(\alpha/2;df=n-np)}$ or the value of p which is the largest parameter or from P-value. It means that to reject H_0 if p-value < α which α is a level of significance.

ii. Test the suitability of the model

This test aims to determine whether the model has presented the data well. This test is a test of whether the residual obtained using the parameter matrix is of white noise and the normal multivariate residual meet the assumption

a. Residual test from multivariate white noise

Multivariate white noise test is used for finding residual from a model that is not already correlated to another with portmanteau test. The Multivariate white noise test from residual is as shown below:

$H_0 :$ $E(\hat{e}_t\hat{e}'_{t-1}) = 0$ or white noise model

$H_1 :$ at least has $E(\hat{e}_t\hat{e}'_{t-1}) \neq 0$, $i = 1, 2, \ldots, h > p$ or not white noise model with test statistic. The test statistic used is as follows:

$$Q_h = n\sum_{i=1}^{h} tr(\hat{C}_i{}'\hat{C}_0{}^{-1}\hat{C}_i C_0{}^{-1}) \qquad (2.58)$$

where

$$\hat{C}_i = n^{-1}\sum_{t=i+1}^{n} \hat{a}_t\hat{a}'_{t-1}$$

$H_0 :$ rejected if Q $Q_h > \chi^2$ or with p-value < α (Lutkepohl, 2006)

b. Residual assumption test with Multivariate normal

The Assumption which is mostly known from VAR model is residual that is used in multivariate normal distribution. Inspection of multivariate normal distribution that can be used with the creation of *q-q* plot from d_T^2 (Johnson and Wichern, 2002)

$$d_T^2 = \left(\left(\hat{e}_T - \bar{\hat{e}}\right)' \hat{\Omega}^{-1}(\hat{e}_T - \bar{\hat{e}})\right), \ T = 1, 2, \ldots n \qquad (2.59)$$

where:

$\hat{e}_T$: Residual of each observation from vector column

$\bar{\hat{e}}_T$: Average vector residual of each column

The hypothesis used is:

H_0 : Residual from multivariate normal distribution

H_1 : Residual is not from multivariate normal distribution

Accept H_0 if $d_T^2 \le \chi^2_{K,0.5}$ a percentage greater than 50% of data has.

2.8 Model Usage for Forecasting

Once the data meet the assumptions on diagnostic testing, the next step is to forecast or calculate the values of the forward forecast. There are two estimates of forecasting, they are forecast points and forecast intervals. The equation for calculating the forecast of the point at a time $n + l$ is as follows

$$\hat{Z}_n(l) = E(Z_{n+l} \mid Z_n, Z_{n-1}, \ldots, Z_1) \qquad (2.60)$$

where $\hat{Z}_n(l)$ is the forecast for l the future period from Z_{n+l} (Wei, 2006)

2.9 Selection of the Best Model

The selection of the best models in this study is done by using Root Mean Square Error forecasting criteria (RMSE) for out-sample. Hyndman (2006) stated that to implement and compare different method on the data with the same scale then MSE is the most appropriate. But often RMSE is preferred to MSE because

RMSE can be equated to the scale of the data used (R. J. Hyndman & Koehler, 2005). The RMSE formula is as shown below (Wei, 2006)

$$RMSE = \sqrt{\frac{\sum_{l=1}^{L} (Y_{n+l} - \hat{Y}_n(l))^2}{L}} \tag{2.61}$$

where

Y_{n+l} = data out- sample

$\hat{Y}_n(l)$ = Forecast value

L = total number of out-sample data.

2.10 Previous Researches

In this study the other previous researches used are shown in Table 2.3:

Table 2.3 Previous Research

Authors	Method	Data	Results
Abken (1979)	Gold prices were taken as endogenous variables and lagged values of gold prices and interest rates are taken as exogenous variables in a regression analysis	Monthly data between January 1973 and December 1979 were used.	There result shows that the explanatory ratio of the regression equation is low when the similar relationship is used between future prices and future spot prices.
James et al., (1985)	A VARMA model was employed to look at the relationship between the variables.	Yearly transformed series data was used from 1962 to 1981	In this article, they investigated simultaneously the relations among stock returns, real activity, inflation, and money supply changes using Vector autoregressive moving average (VARMA). Their empirical results strongly support the causality model.
Liu (1987)	This study demonstrates how multivariate forecasting models can be effectively used to generate high-performance forecasts for business applications. Secondly, the study compares the forecasts generated by a simultaneous transfer function model and a white noise regression.	In this study monthly data were used for two years	It is found out that ignoring the residual serial correlation can greatly degrade the forecasting performance of a multivariable model, and in some situations cause a multivariable model to perform inferior to a univariate ARIMA model.

Table 2.3 Previous Research (Continue)

Authors	Method	Data	Results
Casals et al., (2002)	They proposed a simple and structured procedure for decomposing a vector of time series into trend, cycle, seasonal and irregular components. They also applied a wide range of data representations including ARIMA, VARMAX, and univariate transfer functions.	They generated 200 random draws from the data generating process and compute the smoothed estimate of the components.	The method is independent of particular model specifications, as it is based on the innovation model which encompasses standard processes such as ARIMA, VARMAX or transfer function. The flexibility of the model and the state space methods easily accommodates nonstandard situations defined by missing values and multivariate time series.
Montgomery & Weatherby (2007)	They used a linear transfer function to model the interrelationships between input and output series. Also, a survey of intervention analysis was applied in which the input series is an indicator of a variable corresponding to an isolated events	Monthly electricity consumption data was used in megawatt Hours from January 1951 to April 1977.	The result of the methodology is that there is a significant improvement in the quality of forecast obtained from the stochastic models. The identification-estimation diagnostic checking cycle employed in building univariate model is paralleled in the transfer function model building.

Table 2.3 Previous Research (Continue)

Authors	Method	Data	Results
Sjaastad (2008)	Ordinary Least Square Estimate on restricted and unrestricted regressions was used	Monthly data from January 1991 to May 1994	He found out that since the dissolution of the Bretton Woods international monetary system, floating exchange rates among the major currencies have been a major source of price instability in the world gold market and appreciation and depreciation of the dollar would have strong effects on the price of gold in other currencies
Otok and Suhartono (2009)	In this research they found the best method to most rainfall index data by comparing the forecast accuracy among ARIMA, ASTAR, Single input Transfer function and multi-input Transfer function models.	The study used monthly data from January 1989 to December 2008.	The comparison of forecast accuracy at out-sample data showed that single input transfer function model yields better forecast at the various locations.
Sujit and Kumar (2011)	Using techniques of time series such as Vector autoregressive and cointegration.	The study takes daily data from 2nd January 1998 to 5th June 2011 constituting 3485 observations.	The result shows that the exchange rate is highly affected by changes in other variables and one of the models suggest that there is weak long-term relationship among variables.

Table 2.3 Previous Research (Continue)

Authors	Method	Data	Results
Temmuz and Oma (2012)	The study uses regression to look at the effect of gold trading and gold prices on the flow of funds in the financial system.	The study uses yearly data from January 2002 to December 2011.	The result suggests that there is a positive relationship between national gold prices, Istanbul Stock Exchange 100 Index and the exchange rate between Turkish Lira and the Dollar
Contuk et al., (2013).	They tried to determine whether or not the use of a GARCH model would be appropriate for using heteroscedasticity test in gold prices on ISE 100 index	They used daily prices and index data from 1st January 2009 to 31st December 2012.	There test results shows that there was an ARCH effect in both variables and GARCH modeling could be used.
Tufail and Batool (2013)	They aimed to capture the potential effects of gold and stock prices on inflation in Pakistan by using time series econometric techniques such as cointegration and vector error correction models.	Yearly data was used from 1960 to 2009. The exchange rate is given in Rupees per USD(annual average) and the weighted average rate of return on precious metals is used as a proxy for the rate of return on gold and stock exchange in percent per annum.	They find out that gold is a potential determinant of inflation in Pakistan. On the other hand, it also provides a complete hedge against expected inflation, although stock exchange securities outperform gold and real estate as a hedge against unexpected inflation

Table 2.3 Previous Research (Continue)

Authors	**Method**	**Data**	**Results**
Contuk et al., (2013).	They tried to determine whether or not the use of a GARCH model would be appropriate for using heteroscedasticity test in gold prices on ISE 100 index	They used daily prices and index data from 1st January 2009 to 31st December 2012	There test results shows that there was an ARCH effect in both variables and GARCH modeling could be used.
Arumugam and Anithakumari (2013)	In this research, transfer function was used when there is an output series and the fitted transfer function model was used for forecasting.	Monthly data was used for the period from January 1991 to December 2012. The production were collected for 264 months.	The time series forecast based on transfer function method was compared to production and sales in the forecast horizon. It was concluded that the quality of predictions using the proposed technique is considerably good compared with other standard time series models when output series are influenced by input series.
Basikhasteh et al., (2014)	The main objective of this study was to determine factors affecting the gold prices by using MGARCH model	The study uses monthly data from June, 1992 to March 2010	According to their empirical findings, the highest correlation is found between gold prices and the USA exchange rate negatively and also there was a positive correlation between gold prices and oil prices

Table 2.3 Previous Research (Continue)

Authors	Method	Data	Results
Ibrahim et al. (2014)	The research analysis factors affecting the prices of gold by using multiple linear Regression Model to determine the significant relationship between dependent and independent variables.	They used data covering 10 years period from 2003 to 2012.	This research uses three independent variables that affect the prices of gold and their empirical results found out that there is a negative relationship between inflation rates and exchange rates on gold prices while crude oil is positively significant.
Tsoku and Metsileng (2015)	The study deals with Box-Jenkins Methodology (Seasonal ARIMA) was used to forecast South African gold sales	Box Jenkins time series technique was used to perform time series analysis on monthly gold sales for the period January 2000 to June 2013	The forecast value shows that there will be a decrease in the overall gold sales for the first six months of 2014. It is hoped that the study will demonstrate the significance of the Box-Jenkins technique for this area of research
Mombeini and Yazdani-Chamzini (2015)	They developed an artificial neural network (ANN) model for gold price and compared it with the traditional ARIMA model.	They used an information which includes 220 monthly observations of the gold price per ounce against its affecting parameters from April 1990 to July 2008.	In this research, they used seven input parameters for the gold price forecasting and three performance measures which include: The coefficient of determination (R2), RMSE and MAE were utilized to evaluate the performance of different models. The results show that the ANN model outperforms the ARIMA model in terms of different performance criteria

Table 2.3 Previous Research (Continue)

Authors	Method	Data	Results
Apriliadara et al., (2016)	They employed the use of VARI model to forecast the currency inflow and outflow in Indonesia. The work also considers the effect of Eid Fitir as exogenous variables	This research used monthly data currency inflow and outflow from 2003 until 2014. The data was divided into in-sample and out-sample	The results showed that the dynamic of currency inflow was influenced by the amount of outflow. Moreover, the empirical evidence also proved that Eid Fitr has a significant influence on both inflow and outflow.
Zakaria et al., (2015)	The study was conducted to determine the factors influencing gold prices by using Pooled ordinary Least Squares (POL) methodology	This research employed the use of monthly data for 14 years spanning from 2000 until 2013	The result revealed that the rates of inflation, exchange and interest rate were significantly related with gold prices in Malaysia in different magnitude and direction.
S.Subhashini and Dr. S. Poornima (2018)	The research focuses on cointegration and causality test	The relationship between gold price, exchange rate, and crude oil uses the period from the 1st January 2009 to 31st December 2013	Based on his previous research and the Indian sentiment towards investment in gold as the gold investment it is looked as a safe place for those who hold ideal money.

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CHAPTER 3
RESEARCH METHODOLOGY

3.1 Data Source

This study uses daily data of the International Monetary Fund (IMF) and World Gold Council data from 1 June 2010 to 28th February 2018. The data contains the US dollar exchange rate and of other countries. The data is divided into training and testing sets i.e. training data from 1st June 2010 to 28th February 2017 and testing data from 1st March 2017 to 28th February 2018

3.2 Research Variables

The variables used in this study are exogenous and response variables

Table 3.1 Research Variables

Variable	Variable Name
x_t	GP for the price of 1 ounce of Gold in US dollar at time t
$Z_{1,t}$	SAER/ USD for South Africa exchange rate at time t
$Z_{2,t}$	BER/ USD for Brazil exchange rate at time t
$Z_{3,t}$	AER/ USD for Australia exchange rate at time t

3.3 Data Structure

In this study, the data structure used is data with response variable and exogenous variable as shown in Table 3.2

Table 3.2 Data Structure

Time (t)	Date	Variables			
		GP/USD (x_t)	SAER/USD($Z_{1,t}$)	BER/USD($Z_{2,t}$)	AER/USD($Z_{3,t}$)
1		x_1	$Z_{1,1}$	$Z_{2,1}$	$Z_{3,1}$
2		x_2	$Z_{1,2}$	$Z_{2,2}$	$Z_{3,2}$
⋮	⋮	⋮	⋮	⋮	⋮
1936		x_{1936}	$Z_{1,1936}$	$Z_{2,1936}$	$Z_{3,1936}$

3.3.1 Step by Step Analysis

An important aspect of this thesis is model building. It is essential that we ensure the selected model fits the data well. For that reason, it is crucial that we begin with an exploratory data analysis in order to get a better picture of how the variables interact with each other. The purpose of this exploratory analysis is to select a distribution for the response variables, a link function for the Transfer function and VARI-X method for Exchange rate

a. To answer the first objective is to create a descriptive analysis on the exchange rate and price of gold data by conducting a descriptive analysis on Exchange rate data and Gold price by using time series plot, mean, standard deviation, minimum and maximum, correlation matrix and bar chart.

3.3.2 Forecasting with ARIMA Method of Gold Price

The steps that need to be done in the forecasting stage with ARIMA method are as follows:

1. plot gold price data
2. If the data has not been stationary on the variance then it is necessary to do the Box-Cox transformation and if the data has not been stationary on average then differencing needs to be done
3. If the data has been stationary, look at the ACF and PACF pattern for identification of the ARIMA model order
4. Estimation of parameters and testing of significance parameter based on selected models
5. Conduct diagnostic test for white noise and residual normality through Ljung-Box and Kolmogorov-Sminov test
6. Forecasting gold price with ARIMA model
7. Calculate the RMSE value of testing data

3.3.3 Forecasting with Transfer Function Method

To answer the second objective is to model the average exchange rate and gold price data using Transfer function and VARI-X as follows

1. Preparing the input series for single input (Gold price) and output series Exchange rates
2. Identify time series plots, ACF plots, and PACF. If it is not stationary invariance then transformation is done, while if not stationary in mean then Differencing is done.

3. Test the suitability of the model by fulfilling the white noise assumption and normality
4. Do Prewhitening on the input series to obtain α_t .
5. Do Prewhitening on each output series to obtain β_t
6. Perform cross-correlation calculations (Cross-correlation) and autocorrelation for the series of inputs and outputs that have been in prewhitening
7. Set the value (*b*, *r*, *s*) that connects the input series and output to guess the model of the transfer function
8. Estimate the initial noise series (n_t) and calculate autocorrelation, partial autocorrelation and line spectrum for this series
9. Assign (p_n, q_n) to model ARIMA $(p_n, 0, q_n)$ from noise series (n_t)
10. Estimate the transfer function model parameters. Estimate parameters of the transfer function model using the method of Conditional Least Square.
11. Perform diagnostic test of the transfer function model by calculating the autocorrelation for the residual value of the model (*b*, *r*, *s*) connecting the output series and the input series and then calculate cross-correlation between the residual value $(\hat{a}_t)$ with the residual (α_t) that has been prewhitening
12. Forecast the values that will come with the transfer function model
13. Calculate the RMSE value of data testing for the Transfer function

3.3.4 Forecasting with ARIMA method of Gold Price

The steps that need to be done in the forecasting stage with the VARI-X method are as follow

i. Check the stationary of the in-sample data using Box-Cox transformation and Dickey-Fuller test
ii. Identify the order of VAR model based on the lowest value of AICc and MPCCF plot

iii. Modeled the exchange rate and exogenous price of gold in-sampled data using the VARI-X method, and parameter significance checking at a maximum alpha of 10% and white noise assumption
iv. Calculate the value of in-sample estimates and forecast out-sample or intervals using the resulting model at points (iii)
v. Calculate the estimation of the in-sample covariance matrix and the matrix forecast out-sample covariance based on point (iv)
vi. Modeling VARI-X model based on the result of the lowest AICc and MPCCF plot followed by calculating the performance of the proposed model using RMSE
vii. Forecast the values of the exchange rate and exogenous price of gold

3.3.5 Analysis of Forecasting Performance

Analysis of forecasting performance to answer the third objective is done by comparing the RMSE and MAPE value of the modeling method of the ARIMA method gold price, Single-input transfer function, and VARI-X, and then draw a conclusion of the best method used for modeling exchange rate and exogenous price of gold across these countries.

3.4 Flow Chart of Research Methods

The data analysis methods described in the previous sections can generally be summarized in Figure 3.1 for the ARIMA method, figure 3.2 ARIMA gold output price method, single input transfer function and VARI-X, Figure 3.3 for the transfer function followed by Figure 3.4 for VARI-X.

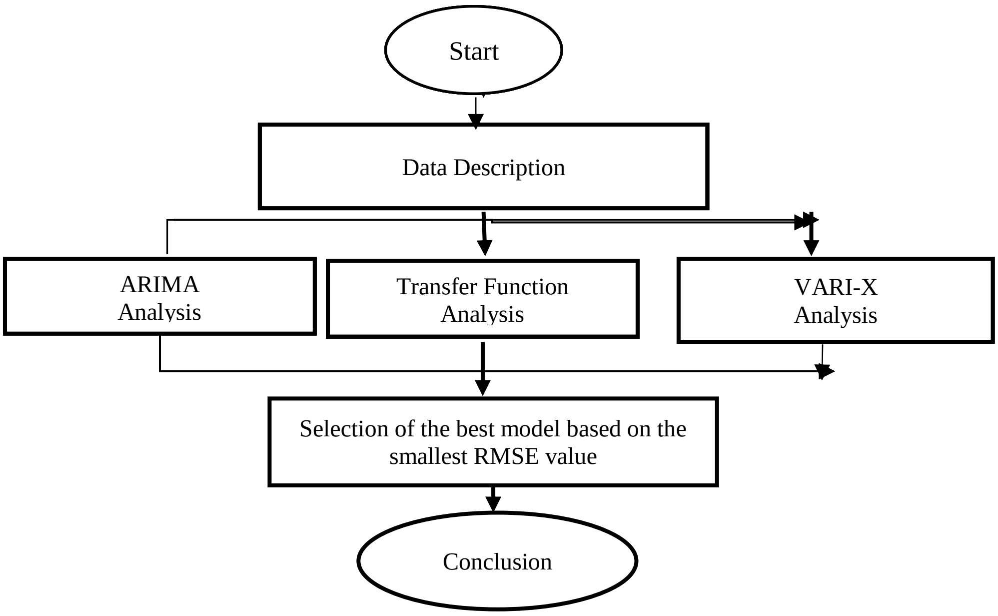

Figure 3.1 Research diagram flowchart

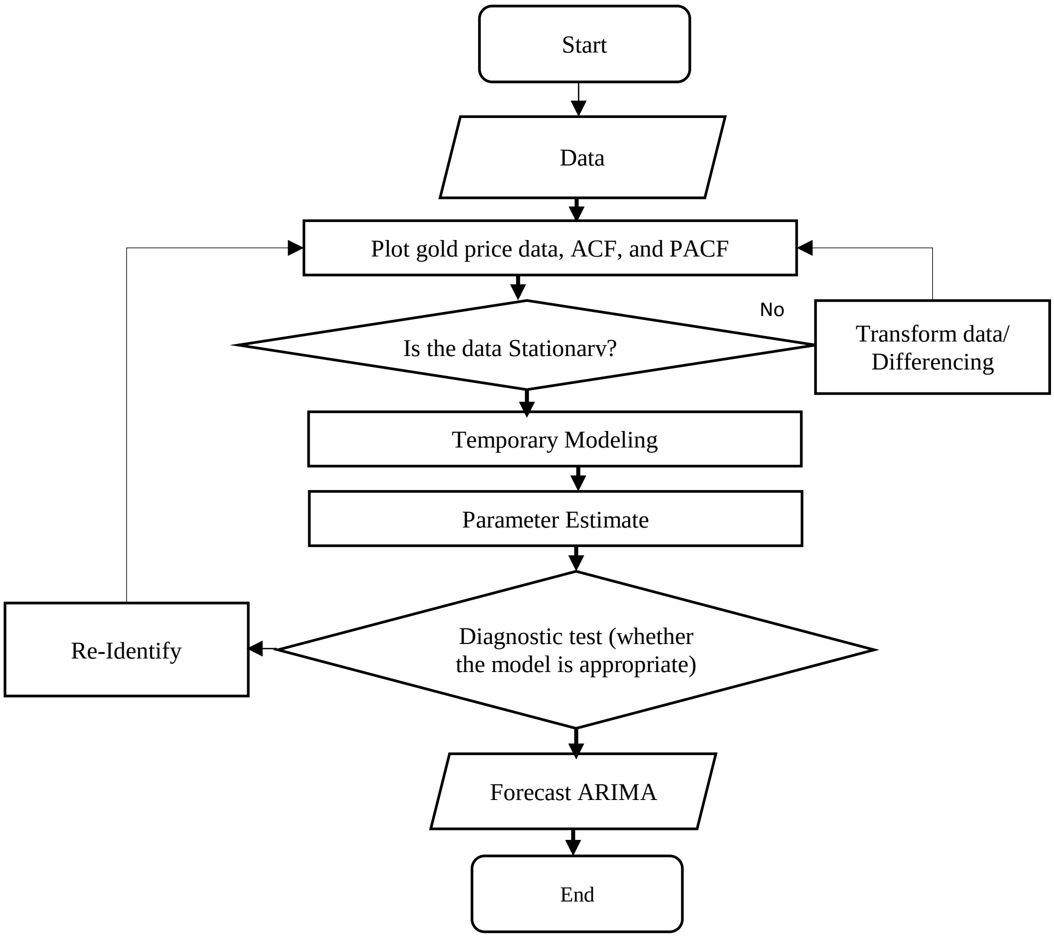

Figure 3.2 ARIMA Model of Gold Price

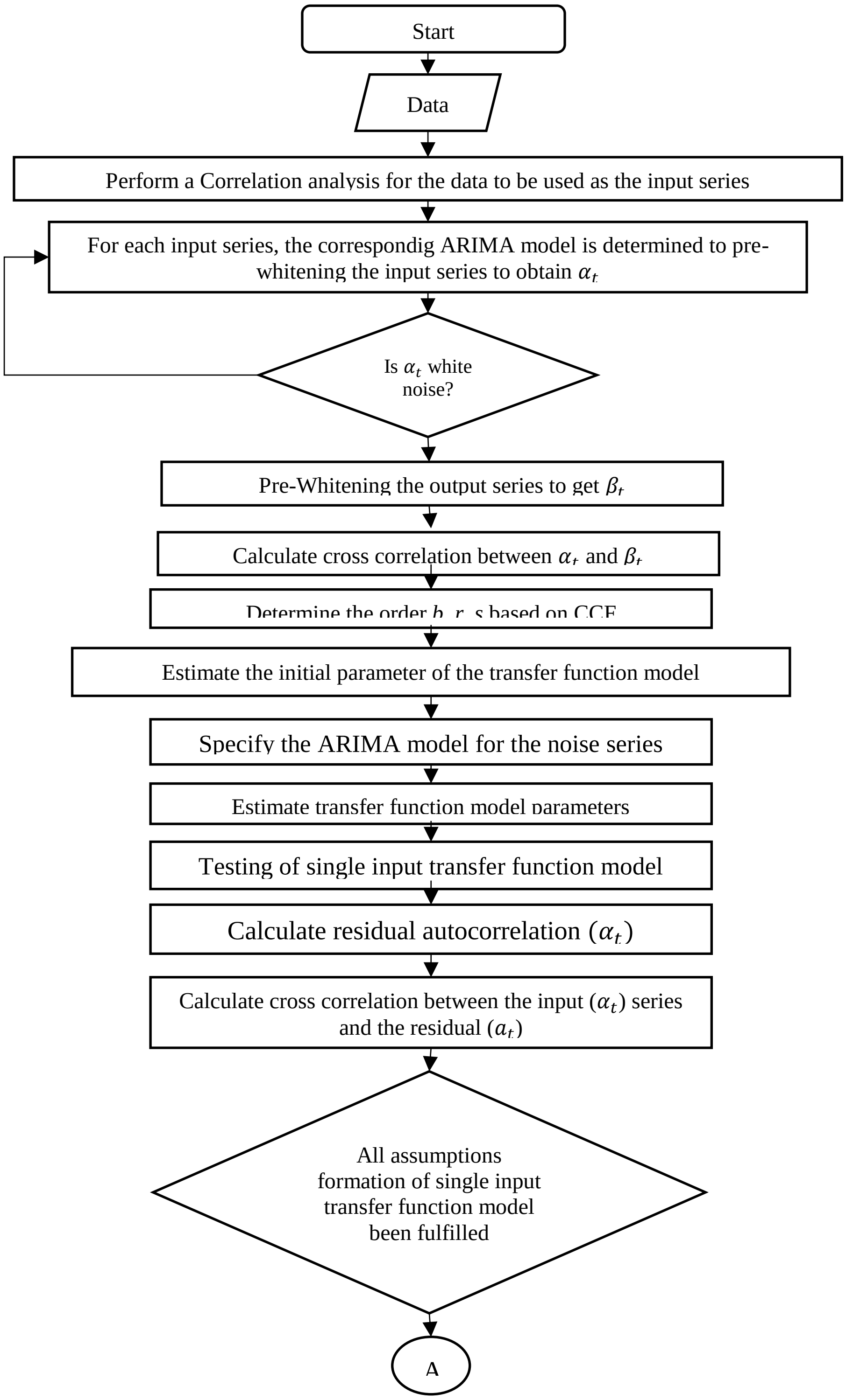

Figure 3.3 Transfer Function Single Input Flow Chart

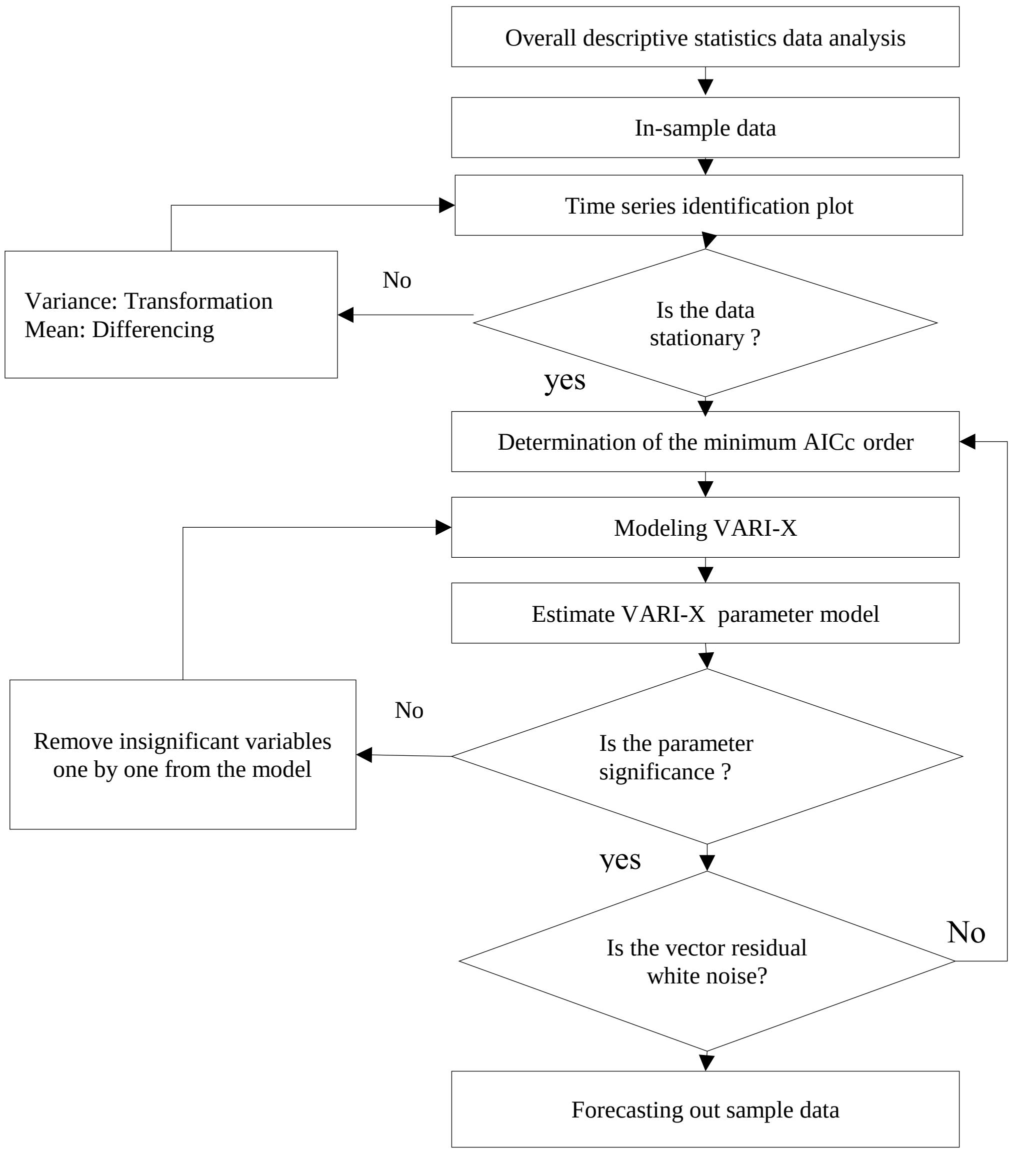

Figure 3.4 VARI-X Flow Chart Representation

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CHAPTER 4

ANALYSIS AND DISCUSSION

This chapter discusses the exchange rate across countries with gold price as an exogenous variable using ARIMA, transfer function and VARI-X. In this research ARIMA model, the appropriate type for gold price data, transfer function which is one of the most popular technique in time series modeling is used for forecasting whose major objective is to identify the role of gold price which is an input series in determining each of the output series and VARI-X which is a model that do not only predict more than one variable but can also see the interaction between countries exchange rate variables with each other. The best method is obtained by comparing the RMSE, then forecasting is done using VARI-X. Before forecasting, descriptive statistics of the data is first done.

4.1 Characteristics of Exchange Rate across Countries with Gold Price

In this descriptive analysis, patterns formed from exchange rates in South Africa, Brazil, Australia and gold price data is as shown using the time series plot in Figure 4.1.

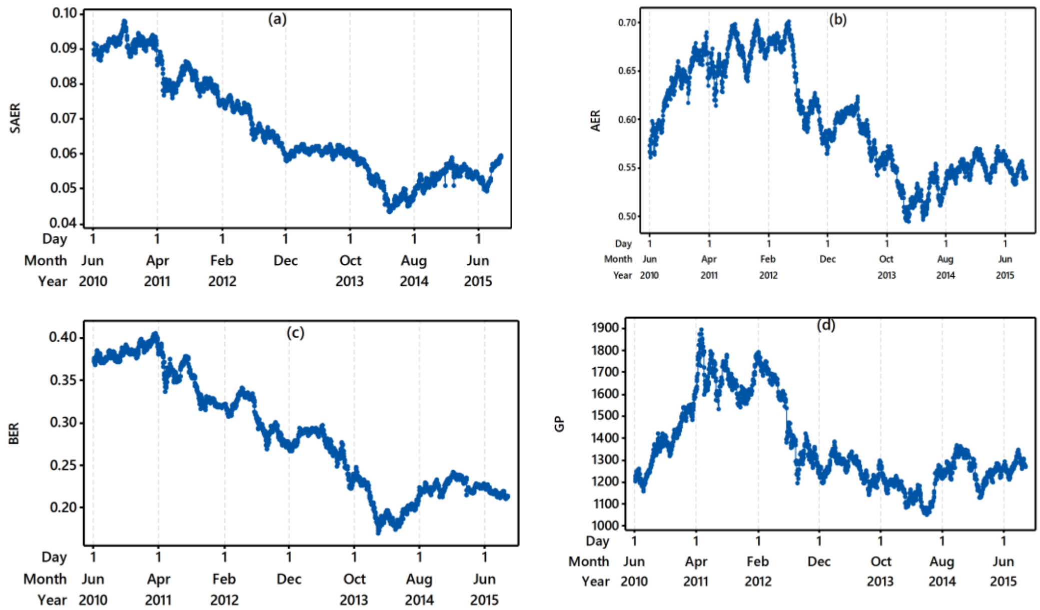


Figure 4. 1 Time Series Plot for Exchange rate of SAER/ USD (a), AER / USD (b), BER /USD, (c) and Gold Price

Figure 4.1 is a time series plot for exchange rate and gold price from 1st June 2010 to 28th February 2018 indicating that for South Africa the exchange rate increases at an initial stage and then start decreasing until it reaches a particular point again where it started increasing but at a slower rate, the Brazilian exchange rate started falling at an early stage and then continue increasing fluctuating, the Australian exchange rate also started increasing slowly but decreased at a rapid rate and while the gold price initially started increasing until its get to a particular pick when it started falling steadily this shows that there are some outliers. The occurrence of outliers purportedly might be due to an increase and decrease in exchange rate around the world or as a result of the financial crisis.

In addition to using the time series plot to know the characteristics of the exchange rate in Australia, Brazil, South Africa and gold price data we also use descriptive statistics. The descriptive statistics of the exchange rate and gold price data are shown in Table 4.1.

Table 4.1 Descriptive Statistics of Exchange Rate and Gold Price

Variable	No.obs	Mean	Median	St.Dev	Min	Max
Gold Price	1936	1364	1296	193.9439	1049	1895
Australia	1936	0.5982	0.5920	0.0559	0.4947	0.7020
Brazil	1936	0.2854	0.2851	0.0665	0.1701	0.4052
South Africa	1936	0.0670	0.0614	0.0148	0.0431	0.0980

Based on Table 4.1, it was found out that the total observations in this study are 1936, mean of gold price from 1st June 2010 to 28th February 2018 is 1341 and median of 1296, the standard deviation of 193.9439, 0.0559 for Australia, 0.0665 dollars for Brazil and 0.0431 dollars for South Africa. The highest exchange rate is of Australia with 0.7020 and the lowest exchange rate is 0.0980 of South Africa with Brazil 0.4052.

The mean for Australia Exchange rate is 0.5982 with a median of 0.5920 while that of Brazil is 0.2854 dollars with a median of 0.2851 dollars of maximum range. In addition, it can also be stated that the exchange rate variation is very high in Australia than the other countries while the gold price was at its maximum price of 1895 dollars during this particular pick. This can be seen in the standard deviation value and the minimum exchange rate which is still greater than the other countries.

The correlation movement between exchange rate and gold price is done by using Table 4.2

Table 4.2 Correlation Matrix Showing Correlation Coefficients of a Combination of 4 Variables

Variable	Australia	Brazil	South Africa	Gold Price
Australia	1.00	0.86	0.81	0.87
Brazil	0.86	1.00	0.97	0.68
South Africa	0.81	0.97	1.00	0.63
Gold Price	0.87	0.68	0.63	1.00

The correlation matrix table shows the correlation coefficients between exchange rate and gold price variables. Each random variable in table 4.2 is correlated with each of the other values. This allows us to see which pair have the highest correlation. Australian exchange rate and Brazilian Exchange rate has a strong positive linear relationship of 0.87 and 0.86 with gold price while South Africa's exchange rate also has but a little bit less than Australia and Brazil. Comparatively, it is clearly seen that Australia has the strongest linear relationship with gold price than the other countries while also South Africa and Brazil exchange rate has a very strong relationship than Australia. It can now be seen that there is an interrelationship between these countries exchange rate and gold price. However, the correlation plot can be used to show the relationship in Figure 4.2.

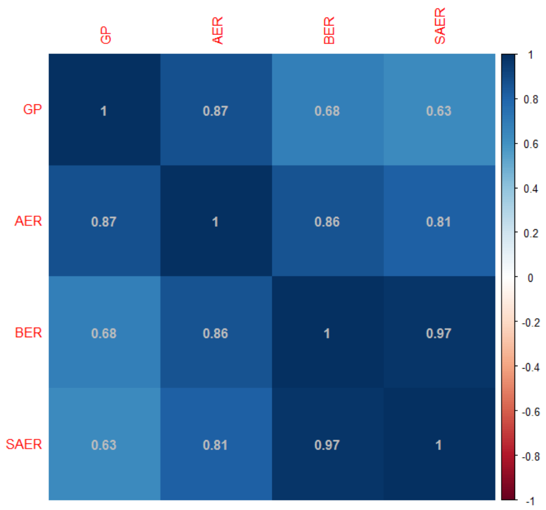


Figure 4. 2 A Correlation Matrix Correlogram

Positive correlations are displayed in blue and negative correlations in red. Color intensity and the size of the circle are proportional to the correlation coefficients. On the right side of the correlogram, the legend color shows the correlation coefficients and the corresponding color.

Furthermore, the values from the exchange rate variables can be used to summarize the series. The names of each variable will be used as categories in the chart.

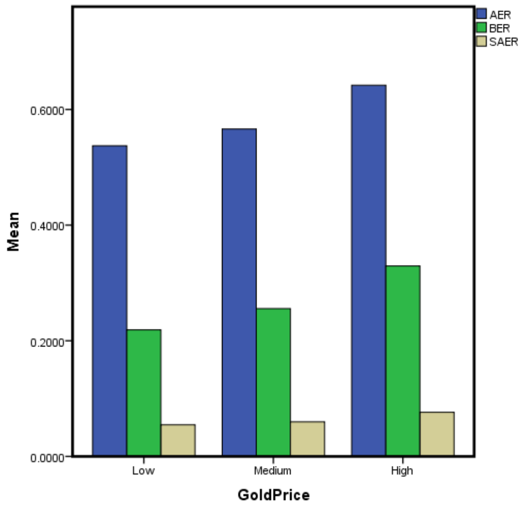


Figure 4. 3 Mean Exchange Rate and Gold Price

Based on figure 4.3 Australia experiences a high exchange rates than those in the major economies within the first half of the 2000s when the major economies experienced a downturn and monetary policy was eased in those countries another reason is that a change in the size of the risk premium influences the relative demand for Australia dollar which has effect on their exchange rate. Brazilian exchange rate has been doing well comparatively when compared with Australia but much higher than the South African rate with respect to the mean. The little strong upward pressure on the exchange rate is as a result of the monetary policies in the western world and high commodity prices but been managed by imposing a lot of controls to minimize capital inflows. Exporters and importers in South Africa and elsewhere that trade with the country also constitute a critical source of demand and supply. These low exchange rates are also as a result of certain people who are normally referred to as speculators who take a position in the market depending on their speculation of the currency that might work in their favor. South Africa like any other that relies on trade for it economic wellbeing and therefore depend on it commodities.

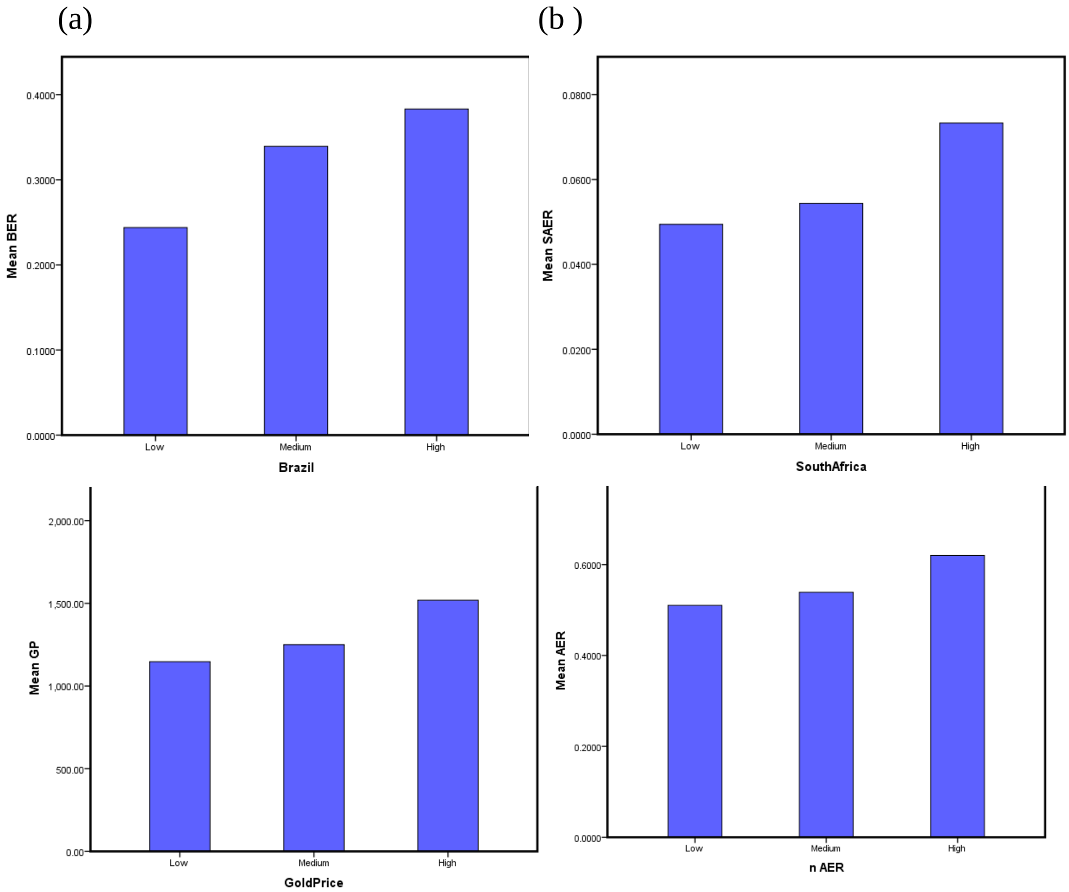


Figure 4. 4 Exchange Rate and Gold Price

Figure 4.4 shows the mean exchange rate of South Africa, Australia, Brazil and Gold price in the world in which each and every one of them indicating a high exchange rate within 2017 and 2018 periods but with Australia the highest.

4.2 Modeling of the Exchange rate and Gold Price Using ARIMA

The following is the ARIMA modeling of exchange rate and gold price data. In ARIMA modeling, the exchange rate and gold price data are divided into two, i.e. in-sample data consisting of 1st June 2010 through 30 December 2016 and out-sample data consisting of 1st January 2017 to 28th February 2018. ARIMA modeling for exchange rate and Gold Price is done based on Box-Jenkins that is parameter identification, parameter estimation, and diagnostic check and forecasting.

In time series before doing data modeling of the exchange rate and gold price across countries, the data must be stationary invariance and mean as well. To detect stationary data in a univariate variance using the Box-Cox plot by looking at the value of lambda (λ) or lower-class limit (LCL) and upper-class limit (UCL) boundary that contains the value of 1. The result of the Box-Cox is shown in table 4.3.

Table 4.3 Box Cox Plot Result

Variable	**Lambda** (λ)	**LCL**	**UCL**
Gold Price	-0.50	-0.84	-0.22
Australia	1.00	0.27	1.25
Brazil	1.00	0.84	1.22
South Africa	0.50	0.34	0.73

Based on table 4.3, the exchange rate variables that is the Australian exchange rate 1.00, Brazilian Exchange rate 1.00, South African Exchange rate 0.05 and Gold price variable is found to be -0.50. It is clearly seen that Gold Price and South African Exchange rate requires transformation as they are less than 1 while the Australian and Brazilian exchange rate does not need to be transformed. To be stationary invariance, the gold price should be expressed as one divided by the square root of the variable, the South African exchange rate as the Square root of the variable $\sqrt{Z_t}$ based on Table 4.3. The result of the transformation for each exchange rate and gold price data is shown in Table 4.4.

Table 4.4 Box-Cox Plot after Transformation

Variable	Lambda (λ)	LCL	UCL
Gold Price	1.00	0.39	1.68
Australia	1.00	0.27	1.25
Brazil	1.00	0.84	1.22
South Africa	1.00	0.72	1.49

From Table 4.4 the Box-Cox plot after transformation shows that the exchange rate and the gold price are now stationary in variance as all of them exhibit the same value of lambda which is one (1). The mean can further be identified unilaterally by looking at the ACF and PACF plots as supported by the Augmented Dickey-Fuller (ADF) test as it can be seen in figure 4.5.

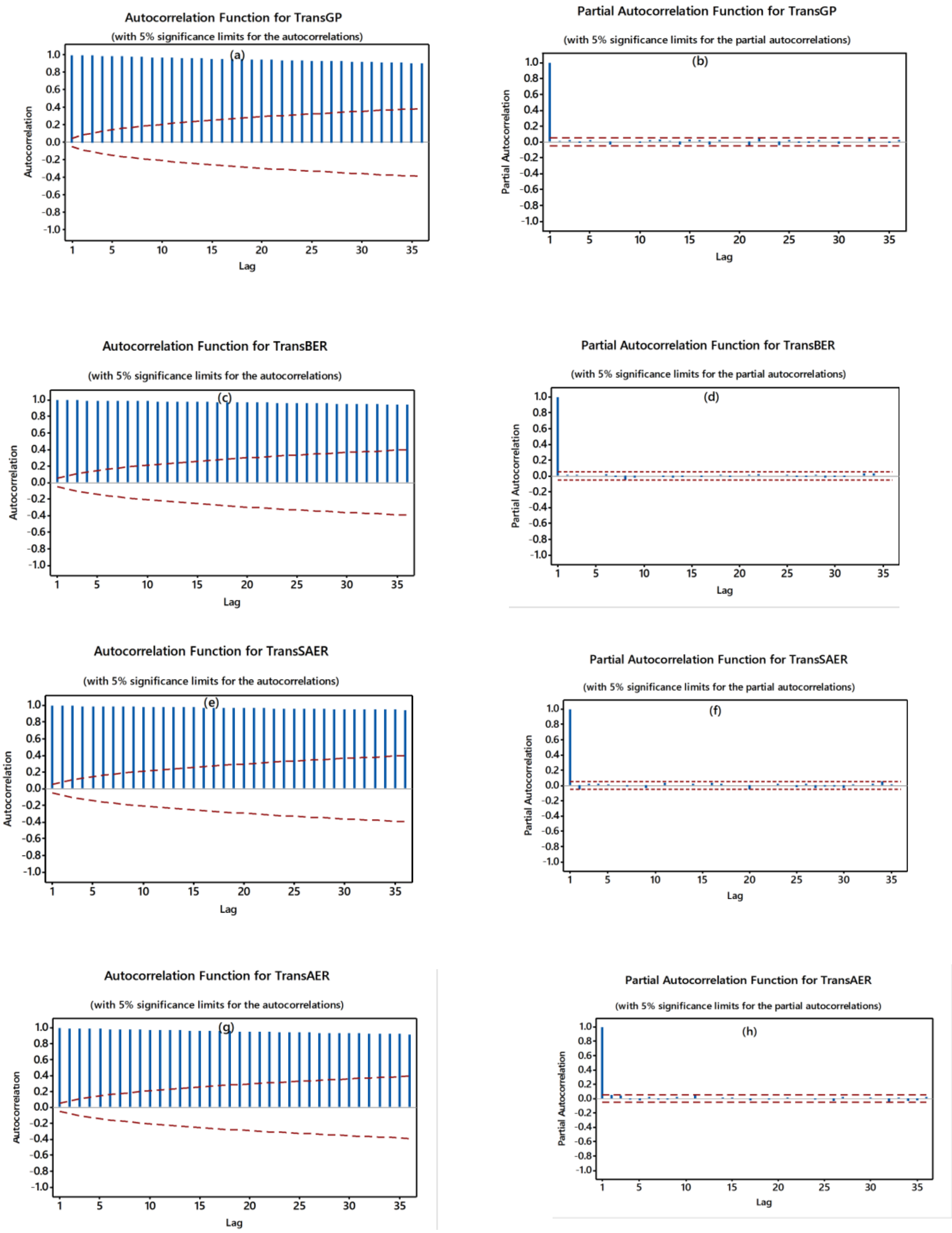


Figure 4. 5 ACF and PACF of Gold Price, Brazil, South Africa and Australia Exchange Rate after Transformation

Based on figure 4.5, the ACF dies down slowly which shows that it is not stationary in mean and hence needs differencing.

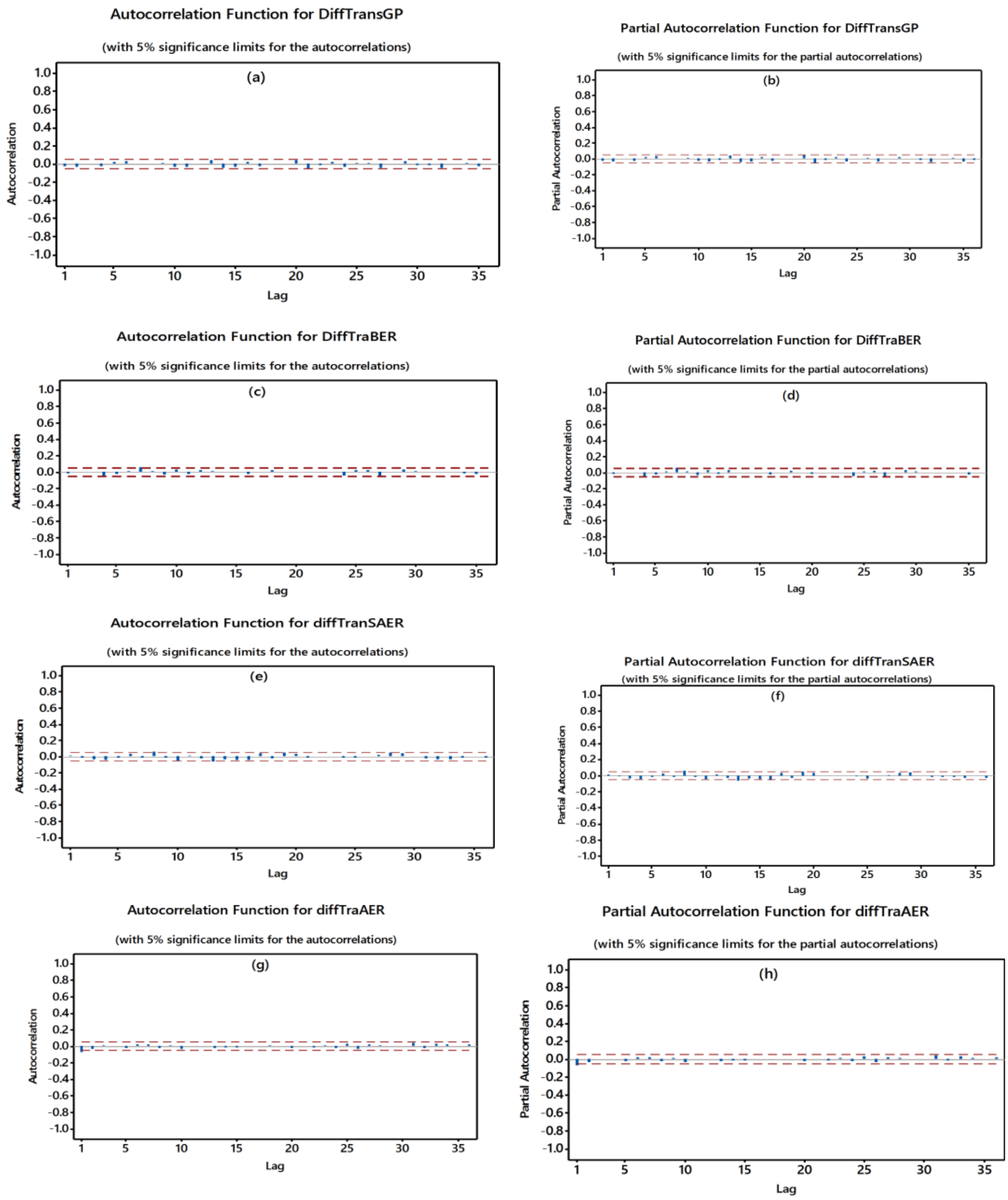


Figure 4. 6 Gold Price, Brazil, Australia and South Africa Exchange Rate after Differencing

Figure 4.6 indicates that the data is stationary in mean as it can be seen that the ACF plot and PACF significant parameter. statistically, one lag is seen in both ACF and PACF of Brazilian Exchange rate and more than two partial autocorrelations are significant. From the overall pattern, however, $\hat{\rho}k$ cuts off after lag 1 and $\hat{\phi}_{kk}$ tails off while Gold price does not meet the white noise assumption since both ACF and PACF are significant at lag 21 which means that in order for the white noise residual assumption to be met one has to perform what is commonly known as subset ARIMA. So the tentative models are ([21], 1, 0) and (0, 1, [21]) for gold price as shown in Table 4.5.

Table 4.5 Significance Test for Gold Price Parameter ARIMA Result

ARIMA Model	Parameter	Estimation	S.E	t-value	P-Value
([21], 1, 0)	ϕ_{21}	-0.054297	0.024613	-2.2061	0.027
(0, 1, [21])	θ_{21}	-0.056635	0.025111	-2.2554	0.024

Based on Table 4.5, all the parameter estimation values have been significant at a p-value less than 0.05. Next step is the residual assumption test which includes the white noise test and normally distributed test shown in Table 4.6.

Table 4.6 Residual Assumption Test for Gold Price

	White Noise Test				Normality test
ARIMA Model	Lags	χ^2	df	P-value	P-value
([21], 1, 0)	6	5.48	5	0.3604	D=0.081 P-value<0.01
	12	9.99	11	0.5310	
	18	20.39	17	0.2548	
	24	26.84	23	0.2828	
(0,1,[21])	6	5.44	5	0.3633	D=0.040 P-value<0.01
	12	10.04	11	0.5266	
	18	20.49	17	0.2500	
	24	26.94	23	0.2585	

Based on Table 4.6 the P-value from ARIMA ([21], 1, 0) model and ARIMA (0, 1, [21]) still have greater values than 0.05 so the assumption of white noise has been met. Also, by testing the p-value assumption of normality which is less than 0.05, it means that the data has been modeled by a normal distribution and the normal distributed residual assumptions have not been met. The model chosen as the best model is seen based on the smallest AIC value. Table 4.7 is the AIC value of each ARIMA model of Gold Price.

Table 4.7 Criteria of the best fit for Input Model Series

ARIMA Model	AIC
ARIMA ([21], 1, 0)	13716.01
ARIMA (0,1[21])	13715.81

The model with the smallest AIC value in Table 4.7 is the ARIMA model (0, 1, [21]). So the model is suitable for predicting Gold price. Therefore, the alleged Gold Price ARIMA model is presented with its significant parameters in Table 4.8.

Table 4.8 Proposed ARIMA Model for Significance Exchange Rate

Series	ARIMA Model	Parameter	Estimation	S.E	t-value	P-value
South Africa	([8,13,16],1,0)	ϕ_8	0.0547	0.0249	2.1974	0.028
		ϕ_{13}	-0.0503	0.0249	-2.0195	0.043
		ϕ_{16}	-0.0484	0.0249	-1.9410	0.052
Australia	(1,1,0)	ϕ_1	-0.0699	0.0246	-2.8449	0.004
	(0,1,1)	θ_1	-0.07483	0.0253	-2.9564	0.003
Brazil	([7],1,0)	ϕ_7	0.0663	0.0246	2.6932	0.007
	(0,1,[7])	θ_7	0.0663	0.0245	2.6984	0.007

Based on Table 4.8 it is now clearly seen that all the data is already significant as indicated by a p-value that is less than 0.05 which simply means that the terms should be kept in the model. The next step is the residual assumption test which is done by the use of Ljung-Box chi-square statistics and the autocorrelation function of the residuals which determines whether the model meets the assumption that the residuals are independent for the gold price.

Table 4.9 Residual Assumption test for Exchange ate ARIMA Models

			White Noise Test			Normality test
Series	ARIMA Model	Lags	χ^2	Df	P-value	P-value
South Africa	([8,13,16],1,0)	6	7.40342	4	0.06009	D=0.05 P-value<0.01
		12	14.91814	10	0.09321	
		18	23.98385	16	0.06536	
		24	31.97573	22	0.05888	
Australia	(0,1,1)	6	4.22733	4	0.23794	D=0.04 P-value<0.01
		12	7.93644	10	0.54056	
		18	9.45876	16	0.85233	
		24	11.61359	22	0.94946	
Brazil	(0,1,[7])	6	4.80777	4	0.18643	D=0.05 P-value<0.01
		12	10.35982	10	0.32215	
		18	12.08134	16	0.67286	
		24	15.03462	22	0.82121	

After knowing the significant parameters then the next stage is to test the residual assumption which includes the white noise test and normal distribution. This test aims to know whether the residuals of the data are independent and normally distributed. Based on Table 4.9 all the models have met the white noise assumption. This noise is indicated by a p-value greater than α (0.05). Based on normality testing, there are several models whose p-values are less than α (0.05) this means the normal distribution is not fulfilled. This abnormality may be as a result of the outliers on the exchange rate and gold price allegedly due to the financial crisis or increase and a decrease in the exchange rate but has already been transformed.

The next step is to write the ARIMA model equation. The ARIMA model equations for each data can be written as follows:

a. The equation of the exogenous gold price subset ARIMA model of (0,1,[21]) could be written as

$$
\begin{aligned}
X_{1,t}^{*} &= a_t - \theta_{21} a_{t-21} \\
X_{1,t} - X_{1,t-1} &= a_t - \theta_{21} a_{t-21} \\
X_{1,t} &= X_{1,t-1} + \theta_{21} a_{t-21} + \theta_{21} a_{t-22} + a_t \\
X_{1,t} &= X_{1,t-1} - 0.0566 a_{t-21} + a_t
\end{aligned} \tag{4.1}
$$

b. The equation for the South African Exchange Rate subset ARIMA model of ([8,13,16],1,0) could be written as

$$
\begin{aligned}
Z_{1,t}^{*} &= a_t + \phi_8 Z*_{1,t-8} + \phi_{13} Z*_{1,t-13} + \phi_{16} Z*_{1,t-16} \\
Z_{1,t} - Z_{1,t-1} &= a_t + \phi_8 Z_{1,t-8} + \phi_8 Z_{1,t-9} + \phi_{13} Z_{1,t-13} + \phi_8 Z_{1,t-14} + \phi_{16} Z_{1,t-16} + \phi_8 Z_{1,t-17} \\
Z_{1,t} &= Z_{1,t-1} + 0.0547 Z_{t-8} - 0.0503 Z_{t-13} - 0.0484 Z_{t-16} + a_t
\end{aligned} \tag{4.2}
$$

c. The equation for the Australian Exchange Rate subset ARIMA model of (0,1,1) is as follow

$$
\begin{aligned}
Z_{2,t}^{*} &= a_t - \theta a_{t-1} \\
Z_{2,t} - Z_{2,t-1} &= a_t + \theta_1 a_{t-1} \\
Z_{2,t} &= Z_{2,t-1} + a_t - \theta_1 a_{t-1} \\
Z_{2,t} &= Z_{2,t-1} + 0.07483 a_{t-1} + a_t
\end{aligned} \tag{4.3}
$$

d. The equation for the Brazilian Exchange Rate subset ARIMA Model of (0,1,[7]) could be written as

$$
\begin{aligned}
Z_{3,t}^{*} &= a_t - \theta_7 a_{t-7} \\
Z_{3,t} - Z_{3,t-1} &= a_t - \theta_7 a_{t-7} \\
Z_{3,t} &= Z_{3,t-1} + a_t - \theta_7 a_{t-7} \\
Z_{3,t} &= Z_{3,t-1} - 0.06632 a_{t-7} + a_t
\end{aligned} \tag{4.4}
$$

By using the above models to forecast in-sample and out-sample data to predict the exchange rate and gold price directly, a comparison of RMSE, MAE, and MAPE resulted from forecasting these out sample data. The results obtained are as follows.

Table 4.10 Comparison of the Best Forecast Approach on the Exchange Rate and Gold Price

Data	Training			Testing		
	MAPE	**RMSE**	**MAE**	**MAPE**	**RMSE**	**MAE**
Gold Price	15.58855	10.58967	0.75698	91.99921	80.1317	6.57042
South Africa	0.00029	0.0004	0.64226	1248.3389	1247.4725	99.99563
Australia	0.01037	0.0030	0.4997	1247.855	1246.9888	99.9568
Brazil	0.00245	0.0017	0.6247	1248.16477	1247.2982	99.98164

From Table 4.10 looking at the in-sample and out sample forecasting data. Exchange rate and Gold price ARIMA forecasting method show a better accuracy rate. It is also clearly seen that gold price training data has the highest RMSE and MAPE value when compared with that of the exchange rate forecasting method. The difference between the actual data and the result of the forecasted exchange rate and gold price in-sample and out sample data is shown in Figure 4.7

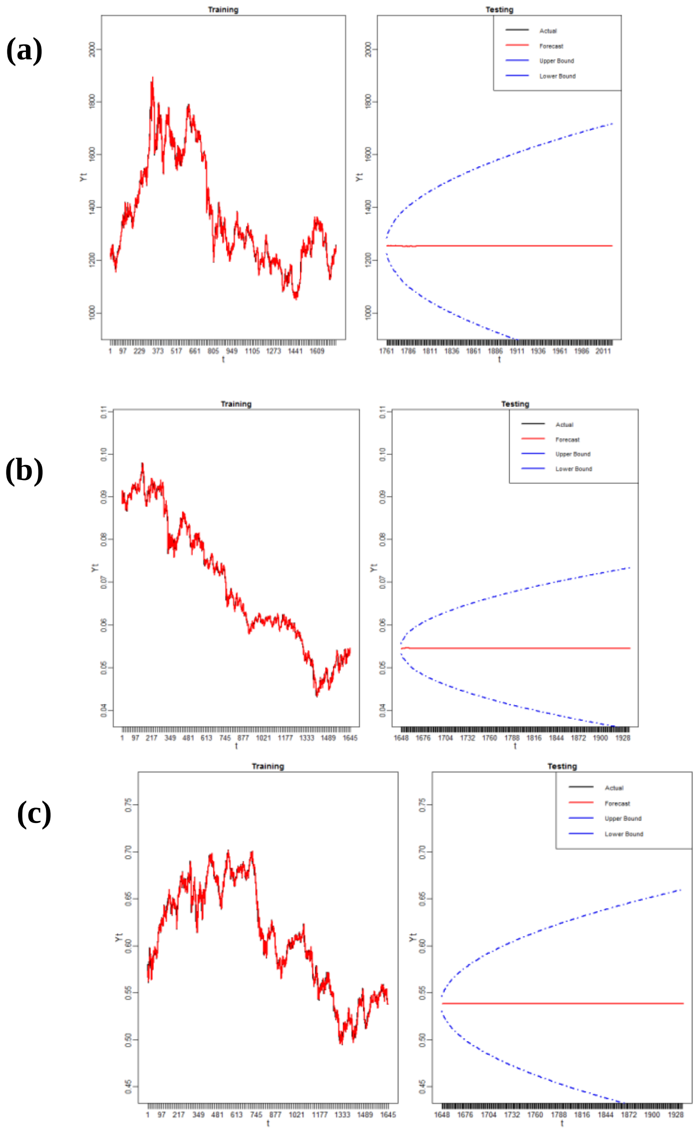
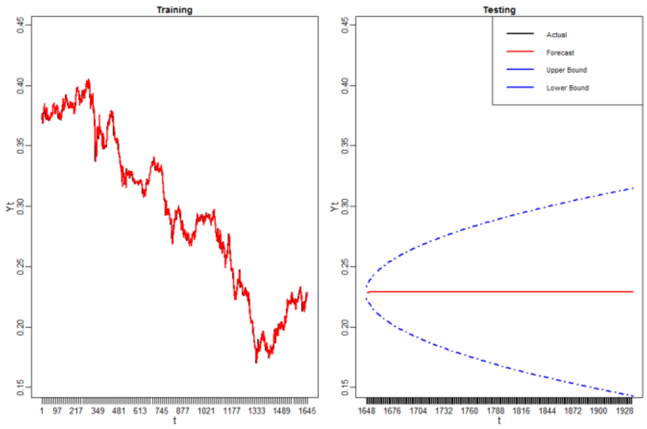


Figure 4. 7 A Comparison of ARIMA Models and Forecasted Results (a) Plots of Gold Price training and testing (b) Plots of South Africa training and testing (c) Plots of Australia training and testing (d) Plots of Brazil training and testing

(d)



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Based on Figure 4.7 the actual data gap with the forecasted data result on the ARIMA model for exchange rate and gold price on in-sample data is quite good even though there are little fluctuations on the out sample of the South African exchange rate but overall its tend to be good. This shows that the ARIMA model captures the pattern of the data. The best value based on ARIMA model RMSE and MAPE criteria for exchange rate and price of gold is presented in Table 4.11.

Table 4.11 The Values of the Best ARIMA Model

Series	Out Sample	
	MAPE	RMSE
Gold Price (0,1,[21])	0.3460	80.1317
South Africa ([8,13,16],1,0)	1.1285	0.1791
Australia (0,1,1)	2.6243	0.0171
Brazil ([7],1,0)	1.8158	0.0090

From Table 4.11 looking at the forecasting out-sample data, exchange rate and gold price ARIMA forecasting method show a better accuracy rate. It is also clearly seen that gold price testing data has the highest RMSE value when compared with that of the exchange rate forecasting method and South Africa has a high RMSE of 0.1791 comparatively to Australia and Brazil while the gold price has a very small MAPE to that of the exchange rate countries

4.3 Modeling of Exchange Rate and Gold Price Using Transfer Function

In transfer function, modeling is divided into in-sample data from 1st June 2010 to 30th December 2016 and out-sample data for the period from 3rd January

2017 to 28th February 2018. These are the steps in the transfer function model formation of the exchange rate at which gold price is used as an input series.

4.3.1 Data Modification in Mean and Variance

The preliminary stage of transfer function model formation is the identification of the input transfer function model. Prior to model identification, the assumptions that must be met is that the data must be stationary invariance and mean. If the data is not stationary invariance then the transformation is done whereas if the data is not stationary in mean differencing is done. Here is the time series plot of the exogenous gold price variable from 1st June 2010 to 27 February to 2017.

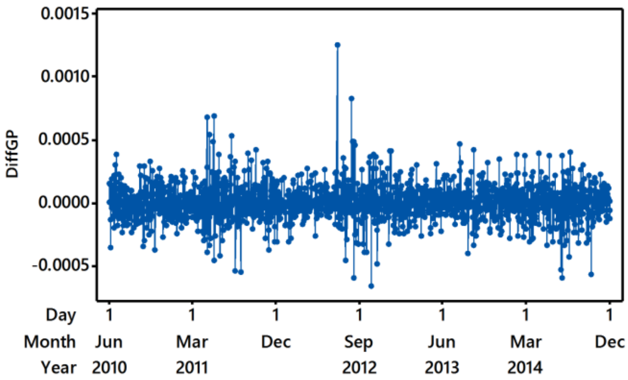

Figure 4. 8 Gold Price in the World

According to the time series plot in Figure 4.8, it appears that the gold price data is already stationary invariance and mean. The stationary variance is indicated by data fluctuations that are not too far away. The stationary mean can be done by looking at the time series plot which indicates that the gold price data is in the mean area and does not have a seasonal pattern. Stationary in the mean can be further investigated by looking at the ACF and PACF plots.

4.3.2 ARIMA Model and Prewhitening Input and Output Series

The ARIMA model of gold price data as input series is also divided into two, the i.e. in-sample period from 1st June 2010 to 30th December 2016 and out-sample data for the period from 1st January 2017 to 28th February 2018. The following is an ACF and PACF plot of gold price in-sample data after differencing

Checking the diagnosis residual used to determine the feasibility of the ARIMA model formed. The input series used in transfer function modeling must meet the white noise assumptions intended to find out if the residuals of the data are independent.

Table 4.12 Best Criteria for input ARIMA

ARIMA Model	**AIC**
ARIMA ([21], 1, 0)	13716.01
ARIMA (0,1[21])	13715.81

The model with the smallest AIC value in Table 4.12 is the ARIMA (0, 1, [21]) model. So the transfer function input gold price model is ARIMA (0, 1, [21]) and can be written as follows

$$X_t = a_t + 0.5664a_{t-21} \quad (4.5)$$

The next step is whitening the input and output series. Here is a prewhitening of input gold price series

$$\alpha_t = a_t + 0.5664a_{t-21} \quad (4.6)$$

So the whitening for the output series is as follows

$$\beta_t = a_t + 0.5664a_{t-21} \quad (4.7)$$

4.3.3 Modeling of the Exchange rate and Gold Price

The formation of the initial model of the transfer function in a way to get the CCF between the input series and the output series has experience whitening process. CCF shows how far the input series is able to affect the output array. CCF plot is used as a basis for determining the *b*, *r*, and *s* order to be used as the bases of the temporary transfer function. Then do an initial assessment of noise series and ARIMA modeling of noise series. After obtaining the estimated transfer function parameters, test diagnostic and the last stage is to forecast exchange rate and gold price.

4.3.4 Outline of an Exchange rate transfer function

The approximate (*b*, *r*, *s*) value for the transfer function model is determined based on CCF plot results between α_t and β_t . The Parameter *b* is the delay period before the input series affects the output series on the first *x* lag affect significantly of y. While the determination of s is by estimating how long the series y continues to be influenced by the series x, while the value of r shows that the value of y is influenced by its past value so it will form a pattern. Here is the CCF between α_t and β_t

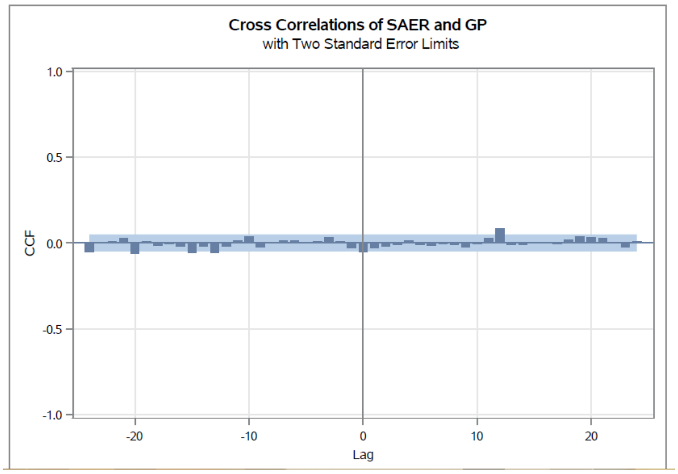

Figure 4. 9 CCF Plot of South African Exchange rate based on Prewhitening Series

Figure 4.9 shows that there is a significant lag in both the lower and upper limits of the lags. This means that the exchange rate has an influence on gold price. The value of b = 12, s = 0, and r = 0 since the plot shows a certain pattern even though it is also stationary in mean, however, it implies that it will be similar to $v(B)x_t = v(B)x_t = \omega_0 x_{t-2}$. Next, we look at the Cross-correlation of Gold Price and Australia

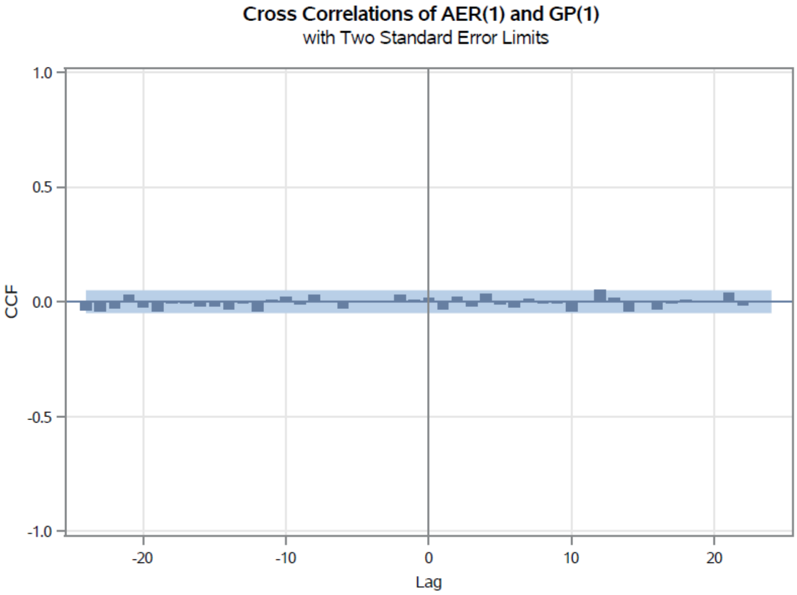

Figure 4. 10 CCF Plot of Gold Price and Australian Exchange Rate

Based on Figure 4.10, there is a significant lag in the upper limit of the cross-correlation function which means that gold price has effect on Australian exchange rate. The value of b = 12, s = 0 and r = 0 as indicated by the plot above and hence could be similar to the model $v(B)x_t = v(B)x_t = \omega_0 x_{t-2}$

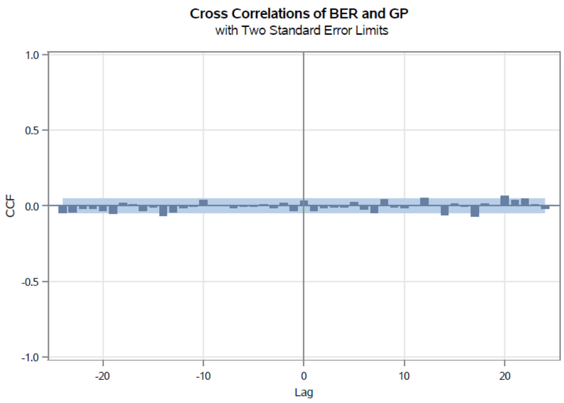

Figure 4. 11 CCF of Gold Price and Brazilian Exchange Rate

According to Figure 4.11 shows that there are significant lags at lag 12, lag 15 and lag 20 of both lower and upper limits of the lags. This means that gold price has an influence on the exchange rate. The value of b = 12, s = [3], and r = 0 since the plot does show a certain pattern along the stationary mean however it implies that it will be similar to $v(B)x_t = \dfrac{\omega_0}{(1-\delta_1 B - \delta B^2)} x_{t-2}$

The Next part of the estimate, check for autocorrelations for white noise. This output has the same form as the autocorrelation check for residuals. The autocorrelation check for white noise is shown in Table 4.13.

Table 4.13 Autocorrelation Check for White noise Component in Transfer Function Model between Exchange Rate and Gold Price

ARIMA Model	Lags	χ^2	df	P-value
South Africa	6	7.18	6	0.3048
	12	18.75	12	0.0949
	18	36.25	18	0.0066
	24	43.85	24	0.0079
Australia	6	12.04	6	0.1823
	12	16.20	12	18.23
	18	17.62	18	0.4810
	24	20.11	24	0.6904
Brazil	6	4.82	6	0.5676
	12	17.30	12	0.1387
	18	19.21	18	0.3789
	24	22.11	24	0.5726

Table 4.13 shows that Australia and Brazil met the white noise check as their p-values are all greater than α (0.05) which simply means their models are quite adequate enough for the series but South Africa still has more information that can be used by a more complex. Checking further for Autocorrelation by using white noise residual shown in Table 4.14.

Table 4.14 Autocorrelation Check of Residuals

ARIMA Model	Lags	χ^2	df	P-value
South Africa	12	0.2371	7	0.2371
	18	17.15	13	0.1927
	24	18.56	19	0.4855
	30	28.90	25	0.2681
Australia	12	7.24	11	0.7789
	18	8.12	17	0.9640
	24	10.61	23	0.9868
	30	15.25	29	0.9830
Brazil	12	7.27	10	0.7002
	18	10.37	16	0.8467
	24	14.20	22	0.8941
	30	24.34	28	0.6635

Based on Table 4.14, the test statistics for the residuals series indicate whether the residuals are uncorrelated (white noise) or contain additional information that might be used by a more complex model. In this case the test statistics do not reject the autocorrelation hypothesis as all the p-values are greater than 0.05 for the exchange rate in each country.

Figure 4.12 shows the graphical check of the residuals from the model. The residual correlation and white noise test plots show that one cannot reject the hypothesis that the residuals are uncorrelated for the gold price and exchange rate

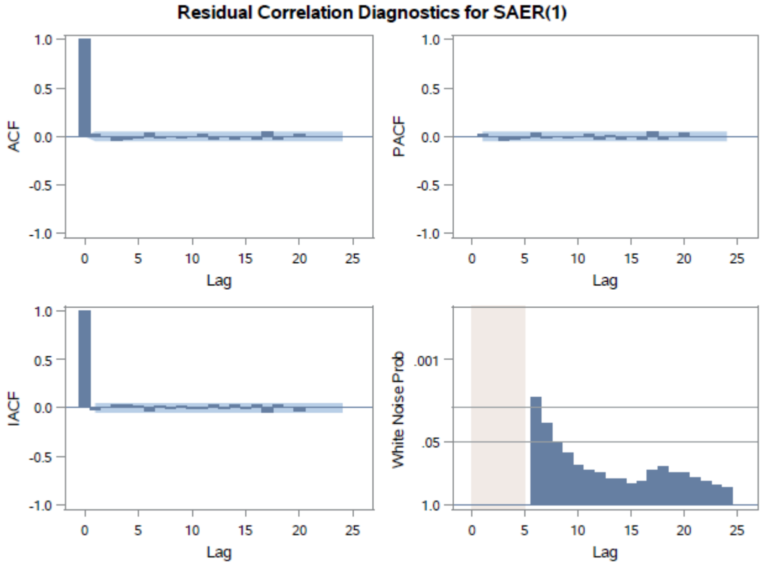

Figure 4. 12 White Noise Check of Residuals for South African Exchange Rate

Another graphical check for residuals is the normality test which is shown in figure 4.12, it shows departure from normality as it can be established that the model for gold price and the South African Exchange rate is adequate.

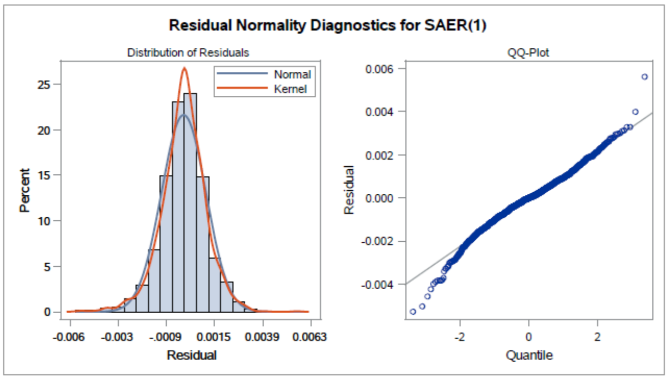

Figure 4. 13 Normality Check of Residuals for South African Exchange Rate and Gold Price

Based on Figure 4.13 below, it shows that the white noise test for gold price and Australian exchange rate could be used to make better predictions and thus conclude that no complex series should be used.

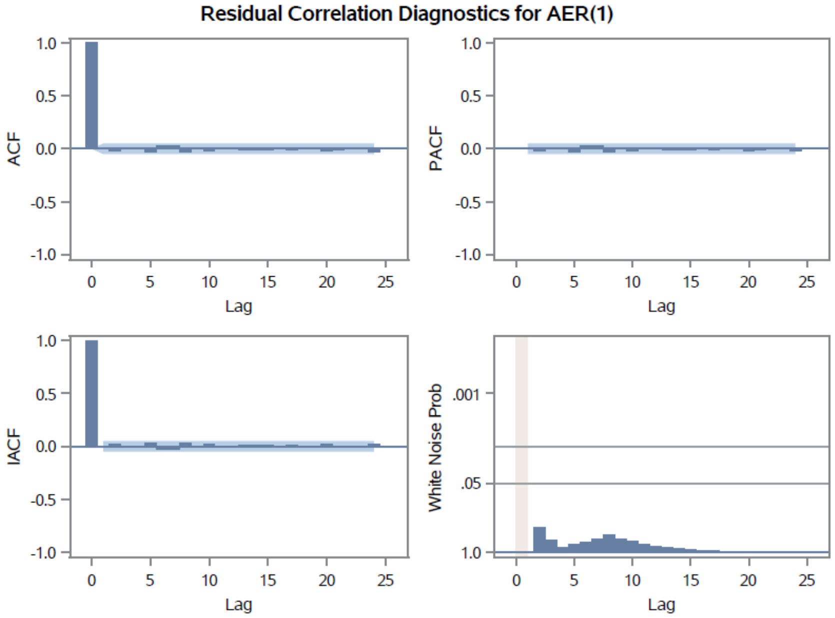

Figure 4. 14 White Noise Check of Residuals for Australian Exchange Rate and Gold Price

Figure 4.15 shows similarly that gold price and Australian exchange rate can be established and the model is acceptable for prediction.

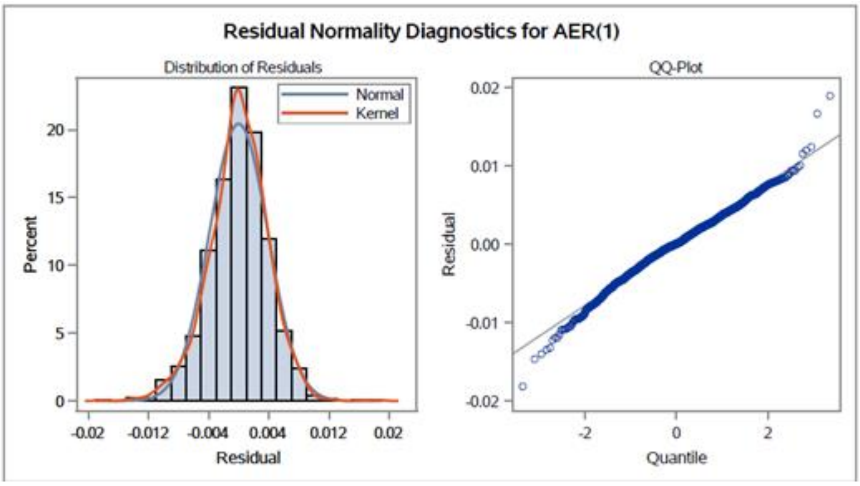

Figure 4. 15 Normality Check of Residuals for Australian Exchange Rate and Gold Price

In addition to the South African and Australian exchange rate is the Brazilian graphical representation of the residual correlation and normality diagnostic check that also shows that it model is adequate to make a prediction.

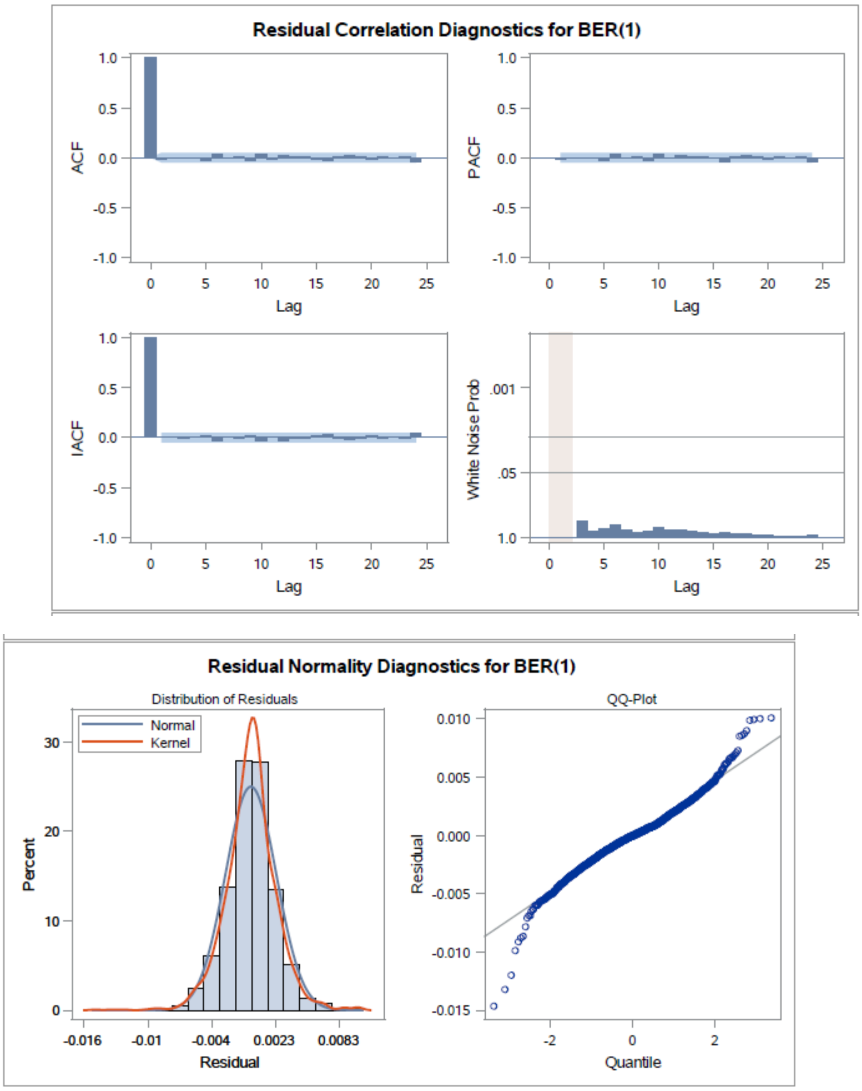

Figure 4. 16 Normality and White Noise Residual Check for Brazilian Exchange Rate

Table 4.15 is a parameter estimate which shows the parameters in the model, each parameter table shows the estimated value and the standard error and t-value for the estimate. The table also indicates the lag and the number of the impulse response at which the parameters appear

Table 4.15 Parameter Estimation for South African Exchange Rate

Order (b,r,s)	Parameter	Estimate	SE	t-value	p-value
	ω_0	0.64249	0.18770	3.42	0.0006
	ϕ_8	0.04893	0.02467	1.98	0.0475
	ϕ_{10}	-0.04807	0.02468	-1.95	0.0516
(b=12,s=0,r=0)	ϕ_{13}	-0.06538	0.02472	-2.64	0.008
	ϕ_{15}	-0.04763	0.024272	-1.93	0.0542
	ϕ_{19}	0.05748	0.02476	2.32	0.0204

Table 4.15 indicates that there are six parameters in the model with no mean term, it estimated values are given with their corresponding autoregressive parameters as they are also referred to as the lagged value of South African Exchange rate with 12 shifts. The values provide a significance test for the parameter estimates and indicate whether some terms in the model might not be necessary. As it is clearly seen that the p-values are also statistically significant. The transfer function equation model and the noise time series for the South African Exchange rate could be written as:

$$Z_{1,t} = \mu + \omega_0 x_{t-12} + \eta_t$$

$$Z_{1,t} = 0.6424 x_{t-12} + \frac{1}{1 - 0.04893B^8 + 0.04807B^{10} + 0.0653B^{13} + 0.04763B^{15} - 0.05748B^{19}} a_t$$

This is preceded by looking at the parameter estimate of Australian exchange rate also in Table 4.16.

Table 4.16 Parameter Estimation for Australian Exchange Rate

Order (*b*,*r*,*s*)	Parameter	Estimate	SE	t-value	p-value
(*b*=12,*r*=0,*s*=0)	$\emptyset_1$	-0.06860	0.02470	-2.78	0.0055
	ω_0	1.45006	0.66371	2.18	0.0290

Table 4.16 shows that there are two parameters in the model with no mean term. It estimated values are given with one autoregressive parameter and coefficient of the input term. The p-values are seen to be significant. The noise time series and transfer function model for the Australian exchange rate could be written mathematically as:

$$Z_{2,t} = \mu + \omega_0 x_t + \eta_t$$

$$Z_{2,t} = 1.45006 x_t + \frac{1}{1 + 0.0686B} a_t$$

Table 4.17 shows the parameters in the model for the Brazilian exchange rate and their respective standard error, t-values, and p-values

Table 4.17 Parameter Estimation for Brazilian Exchange Rate

Order (*b*,*r*,*s*)	Parameter	Estimate	SE	t-value	p-value
Brazilian	ϕ_1	0.06716	0.02486	2.70	0.0070
	ω_0	0.74915	0.40949	1.83	0.0675
(*b*=12,*r*=0,*s*=[2,5,8])	ω_2	0.98323	0.40990	2.40	0.0166
	ω_5	1.18950	0.40821	2.91	0.0036
	ω_8	-1.19711	0.40801	-2.93	0.0034

Table 4.17 shows that there are five parameters with no mean term, it estimated values are given with one autoregressive parameter and coefficient of the input terms. The p-values are seen to be significant with the order of parameters been (*b*=12, *r*=0, *s*=[2,5,8]). The noise time series and transfer function could be written as:

$$Z_{3,t} = \mu + (\omega_0 - \omega_2 B^2 - \omega_5 B^5 - \omega_8 B^8)x_t + \eta_t$$

$$Z_{3,t} = 0.74915x_t - 0.98323x_{t-2} - 1.18950x_{t-5} - 1.19711x_{t-8} + \frac{1}{1-0.06716B}a_t$$

Table 4.18 Test for Normality between Exchange Rate and Gold Price in Transfer

Series	D-Value (Kolmogorov-Smirnov)	p-value
South Africa	0.0612	<0.0100
Australia	0.0154	<0.0100
Brazil	0.0683	<0.0100

Table 4.18 shows the normality test results against the residual model where the model for South Africa, Australia, and Brazil did not fulfil the normal distribution assumption. These residual abnormalities are suspected due to outlier data that cannot be captured by the transfer function model.

Table 4.19 Forecasting Exchange Rate Result and Gold Price Using Transfer Function

Obs	South Africa	Australia	Brazil
1648	0.2332	0.5381	0.2285
1649	0.2334	0.5379	0.2285
1650	0.2334	0.5380	0.2286
1651	0.2335	0.5381	0.2293
1652	0.2337	0.5382	0.2294
1653	0.2337	0.5383	0.2294
1654	0.2338	0.5383	0.2294
1655	0.2338	0.5384	0.2294
1656	0.2336	0.5385	0.2291
1657	0.2336	0.5384	0.2287
1658	0.2335	0.5384	0.2287
1659	0.2333	0.5382	0.2289

The forecasted out-sample for Exchange rate and gold price is shown on Table 4.19. Based on this table the exchange rate tends to fluctuate for all the countries been studied

Table 4.20 Transfer function best Criteria Model

Transfer function Models	RMSE	Information
South African Rate	0.17909	Not Normally distributed
Australian Rate	0.01032	Not Normally Distributed
Brazilian Rate	0.00218	Not Normally Distributed

Based on Table 4.20 it can be inferred from the transfer function criteria model that the best exchange rate for gold price is that of Brazilian with an RMSE of 0.00218 which is the smallest followed by the Australian exchange rate of 0.01032 and South Africa with a very high exchange rate of 0.17909

4.3.5 Comparison of ARIMA and Transfer Function

ARIMA which is a univariate model is used as an input for the transfer function. In econometrics, in particular, autoregressive integrated moving average (ARIMA) model is a generalization of an autoregressive moving average model. The purpose of this features is to make the model fit the data as well as possible. Thus comparing this two models helps to identify which one has a dominant effect on the exchange rate and gold price. The models are looked at using RMSE criteria that have the smallest out-sample data and the best model is then predicted for the next 12 periods. The following RMSE values comparison is presented of the two models.

Table 4.21 Comparison of ARIMA and Transfer Function

	ARIMA		Transfer Function	
Series	RMSE	MAPE	RMSE	MAPE
South Africa	**0.00029**	**0.0106**	0.00030	0.0167
Australia	**0.01037**	**0.01586**	0.01032	0.0159
Brazil	0.00245	0.07713	**0.00234**	**0.07385**

Based on Table 4.21 comparison values, the results show that ARIMA model is the best model for forecasting South African and Australian exchange rate with a Mean Absolute Percentage error of 0.0106 and 0.01586 where as Transfer function is the best model for forecasting Brazilian Exchange they are all indicated with a very low RMSE and MAPE. Here is the best model for each data.

(a) South Africa Exchange rate ARIMA model could be written as:

$$Z_{1,t} = \mu + \omega_0 x_{t-12} + \eta_t$$

$$Z_{1,t} = Z_{1,t-1} + 0.6424x_{t-12} + 0.6424x_{t-13}$$
$$+ \frac{1}{1 - 0.0489B^{8} + 0.0481B^{10} + 0.654B^{13} - 0.0575B^{19}} a_t$$

(b) Australian Exchange rate Transfer Function Mathematical is:

$$Z_{2,t} = 1.45006x_{t-12} + \frac{1}{1 + 0.0686B} a_t$$

(c) Brazilian Exchange rate Transfer function could be written as

$$Z_{3,t} = 0.74915x_{t-12} - 0.98323x_{t-14} - 1.19711x_{t-17} - 1.19711x_{t-20} + \frac{1}{1 - 06716B} a_t$$

4.4 Modeling of the Exchange rate and gold price using VARI-X

In VARI-X modeling, we only use in-sample data from 1st June 2010 to 28th February 2017 which is 1647 observations for each exchange rate and gold price data. In modeling of the exchange rate and gold price across countries, the data must be stationary invariance and mean as well. To detect stationary data in a univariate variance using the Box-Cox plot by looking at the value of lambda (λ) or lower-class limit (LCL) and upper-class limit (UCL) boundary that contains the value of 1. The result of the Box-Cox is shown in table 4.3.

The next step determines the significant lag using the PACF plot of the first model see Appendix, where it is found that some lags are significant. Therefore in non-seasonal ADF testing use the second model. The result of the ADF test using the second model supports definitely the identification of the initial visual, which is stated that the exchange rate data and gold price is not stationary in the mean. This is indicated by the value of *p* which is greater than 10% alpha, so first order differencing needs to be done on each data. The conditions in this non-seasonal ADF test apply to exchange rate and gold price. After doing first order differencing, to find out if the overall data is stationary in the mean, we visually use the ACF and PACF plot.

The results of each data plot on the exchange rate and gold price can be seen in details in Appendix 8 and 9. The plot results show that there are some parameters that are out of bounds (significant), but on the next lags the others are not out of bounds and can also be identified that no need to do seasonal differencing. It is also reinforced by the results of the seasonal ADF test in the Table 4.22, resulting in each data of the exchange rate and gold price by using first order differencing and as a result of this has met the condition for stationary in the mean.

Table 4.22 Dickey-Fuller Unit Root Test of Exchange Rate

Dickey-Fuller Unit Root Tests					
Variable	Zero Mean	Rho	Pr < Rho	Tau	Pr < Tau
SAER	Zero Mean	-1628.6	0.0001	-28.56	<.0001
	Single Mean	-1634.8	0.0001	-28.61	<.0001
	Trend	-1635.2	0.0001	-28.60	<.0001
BER	Zero Mean	-1685.5	0.0001	-29.03	<.0001
	Single Mean	-1692.4	0.0001	-29.08	<.0001
	Trend	-1692.5	0.0001	-29.07	<.0001
AER	Zero Mean	-1886.6	0.0001	-30.76	<.0001
	Single Mean	-1886.8	0.0001	-30.75	<.0001
	Trend	-1891.3	0.0001	-30.78	<.0001

The next step is using the minimum AICc values to choose the order for the VARI-X model to use. Based on Table 4.23, the Exchange rate indicates that the smallest AICc value in the AR order is lag4 which is -37.08432.

Table 4.23 AICc of Tentative Exchange Rate of VARI-X Model

Lag	MA 0
Lag 0	-36.82068
Lag 1	-37.04876
Lag 2	-37.07657
Lag 3	-37.07511
Lag 4	-37.08432
Lag 5	-37.08393
Lag 6	-37.07605
Lag 7	-37.07467
Lag 8	-37.07537
Lag 9	-37.06712
Lag 10	-37.06421
Lag 11	-37.05773
Lag 12	-37.04900

To further progress we look at the schematic representation of cross-correlation that shows MCCF.

Schematic Representation of Cross Correlations																									
Variable/Lag	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
SAER	+++	.++	.++	...	..-	.-.	...	...	+..	...	-.-	...	...	-..	...	...	...	...	...	+..	...	...	...	...	...
BER	+++	...	...	...	...	+..	...	.+.	...	...	...	...	...	...	...	...	...	...	+..	...	...	...	...	...	...
AER	+++	.+-	+..	...	...	...	...	.+.	...	...	...	...	...	-+.	...	-..	...	...	...	...	...	...	...	...	...
+ is > 2*std error, - is < -2*std error, . is between																									

Figure 4. 17 Matrix Autocorrelation Function

From Figure 4.17 it can be seen that the point of having a value is between 2 times the standard the error. The positive sign is less than the limit of 0.2 which means that there is a positive correlation relationship. The number of positive signs that occur simultaneously in the Matrix Correlation Function is significant at lag 0.

Schematic Representation of Partial Autoregression																						
Variable/Lag	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
SAER	.+-	...	...	...	.+.	...	...	+..	...	...	...	...	-..	...	...	...	...	.+.	...	...	...	...
BER	+-+	+..	+..	...	-..	...	.+.	..+	...	...	...	...	...	..+	...	...	...	...	...	...	...	...
AER	+.-	+.-	...	-..	...	...	...	...	...	-..	...	...	-..	...	.+.	...	...	...	...	...	-..	...
+ is > 2*std error, - is < -2*std error, . is between																						

Figure 4. 18 Matrix of Partial Autocorrelation Function (MPACF)

From Figure 4.18 it can be seen that there are cut off on lag 1, this is indicated with a positive (+) and negative (-) sign on the lag. Then The VARX order is selected in order 1 but this does not meet the white noise residual assumption having looked at the minimum information criteria which are of order 4. This research uses gold price as an exogenous variable. Therefore, the model obtains is VAR (4,1,20).

Wald Test which is used to see whether there is any relationship between the variables. Aall of the exchange rate variables such as Brazilian Exchange rate, South African and Australian exchange rate indicate that there is a relationship between each of these countries. Table 4.24 shows the Granger Causality Wald Test results.

Table 4.24 Granger-Causality Wald Test

Test	Group1 Variable	Group2 Variables	DF	Chi-Square	Pr > ChiSq
1	God Price	South Africa, Brazil, Australia	12	9.07	0.6967
2	Brazil	Australia, South Africa	8	325.91	<.0001
3	South Africa	Australia, South Africa	8	20.97	0.0072
4	Australia	South Africa, Brazil	22	113.29	<.0001

Based on Table 4.24 Granger-Causality test, it shows that in the exchange rate between countries all of them are statistically significant and has a strong relationship with each other and also they can influence gold price but gold price doesn't have a strong influence over them. It can be seen that gold price is the exogenous variable and fit VARX (4, 20) model, it also shows that you cannot reject Granger noncausality from the exchange rate variables for the gold price using a 0.05 significance level.

In accordance with the minimum information criteria detection, for instance, the Exchange rate level can be obtained from the alleged VARI-X (4, 1, 20) model of the exchange rate and gold price by using equation (2.13). So the model of the exchange rate across countries assumption is shown in equation (4.11) and in equation (4.12).

$$\begin{bmatrix} Z^*_{1,t} \\ Z^*_{2,t} \\ Z^*_{3,t} \end{bmatrix} = \begin{bmatrix} \phi_{1,11} & \phi_{1,12} & \phi_{1,13} \\ \phi_{1,21} & \phi_{1,22} & \phi_{1,23} \\ \phi_{1,31} & \phi_{1,32} & \phi_{1,33} \end{bmatrix} \begin{bmatrix} Z^*_{1,t-1} \\ Z^*_{2,t-1} \\ Z^*_{3,t-1} \end{bmatrix} + \begin{bmatrix} \phi_{2,11} & \phi_{2,12} & \phi_{2,13} \\ \phi_{2,21} & \phi_{2,22} & \phi_{2,23} \\ \phi_{2,31} & \phi_{2,32} & \phi_{2,33} \end{bmatrix} \begin{bmatrix} Z_{1,t-2} \\ Z_{2,t-2} \\ Z_{3,t-2} \end{bmatrix} + m + \begin{bmatrix} \phi_{11,11} & \phi_{11,12} & \phi_{11,13} \\ \phi_{11,21} & \phi_{1,22} & \phi_{11,23} \\ \phi_{11,31} & \phi_{11,32} & \phi_{11,33} \end{bmatrix} \begin{bmatrix} Z_{1,t-2} \\ Z_{2,t-2} \\ Z_{3,t-2} \end{bmatrix} + \begin{bmatrix} a_{1,t} \\ a_{2,t} \\ a_{3,t} \end{bmatrix}$$

Then to elaborate more by substituting first order differencing gives

$$\begin{bmatrix} (1-B)^1 & 0 & 0 \\ 0 & (1-B)^1 & 0 \\ 0 & 0 & (1-B)^1 \end{bmatrix} \begin{bmatrix} Z_{1,t} \\ Z_{2,t} \\ Z_{3,t} \end{bmatrix} = \begin{bmatrix} \phi_{1,11} & \phi_{1,12} & \phi_{1,13} \\ \phi_{1,21} & \phi_{1,22} & \phi_{1,23} \\ \phi_{1,31} & \phi_{1,32} & \phi_{1,33} \end{bmatrix} \begin{bmatrix} (1-B)^1 & 0 & 0 \\ 0 & (1-B)^1 & 0 \\ 0 & 0 & (1-B)^1 \end{bmatrix} \begin{bmatrix} Z^*_{1,t-1} \\ Z^*_{2,t-1} \\ Z^*_{3,t-1} \end{bmatrix} + \begin{bmatrix} \phi_{2,11} & \phi_{2,12} & \phi_{2,13} \\ \phi_{2,21} & \phi_{2,22} & \phi_{2,23} \\ \phi_{2,31} & \phi_{2,32} & \phi_{2,33} \end{bmatrix} \begin{bmatrix} Z^*_{1,t-2} \\ Z^*_{2,t-2} \\ Z^*_{3,t-2} \end{bmatrix} + m$$

$$\begin{bmatrix} \phi_{11,11} & \phi_{11,12} & \phi_{11,13} \\ \phi_{11,21} & \phi_{11,22} & \phi_{11,23} \\ \phi_{11,31} & \phi_{11,32} & \phi_{11,33} \end{bmatrix} \begin{bmatrix} (1-B)^1 & 0 & 0 \\ 0 & (1-B)^1 & 0 \\ 0 & 0 & (1-B)^1 \end{bmatrix} \begin{bmatrix} Z^*_{1,t-11} \\ Z^*_{2,t-11} \\ Z^*_{3,t-11} \end{bmatrix} + \begin{bmatrix} a_{1,t} \\ a_{2,t} \\ a_{3,t} \end{bmatrix}$$

Estimation of parameters in equations (4.11), (4.12) and (4.13) found that there are some insignificant parameters to the model at alpha 1%, 5%, or 10%. So some variables need to be removed one after another from the model, based on the lowest *t* value and the highest p-value. The results from parameter estimation suggest that the best exchange rate parameters of VARI-X models are shown in Table 4.25.

Table 4.25 Parameter Estimate for the Best VARI-X Model

South Africa $(Z_{1,t})$					Brazil $(Z_{2,t})$				
Parameter	Estimate	Std Error	t-value	p-value	Parameter	Estimate	Std.Error	t-value	p-value
$\phi_{0,11}$	-0.4283	0.1906	-2.25	0.023	$\phi_{12,21}$	0.667	0.3771	1.77	0.078
$\phi_{12,11}$	-0.6457	0.1897	3.40	0.001	$\phi_{14,21}$	-0.996	0.3790	-2.63	0.009
$\phi_{1,12}$	0.0209	0.0111	1.87	0.061	$\phi_{17,21}$	-1.119	0.3759	-2.98	0.004
$\phi_{2,13}$	0.0189	0.0073	2.56	0.011	$\phi_{20,21}$	0.951	0.3765	2.53	0.012
$\phi_{3,12}$	-0.0726	0.0252	-2.87	0.004	$\phi_{1,21}$	0.755	0.0514	14.69	0.00
$\phi_{3,13}$	0.0167	0.0072	2.30	0.021	$\phi_{1,22}$	-0.0888	0.0244	-3.63	0.00
$\phi_{4,11}$	-0.0473	0.0250	-1.88	0.060	$\phi_{1,23}$	0.0676	0.0151	4.46	0.00
$\phi_{4,12}$	-0.0189	0.0108	-1.74	0.083	$\phi_{2,21}$	0.1539	0.0535	2.87	0.00
					$\phi_{3,21}$	0.0967	0.0525	1.84	0.06
					$\phi_{4,22}$	-0.0449	0.0228	-1.97	0.04

Table 4.26 Parameter Estimate for the best VARI-X Model Continues

Australian Exchange Rate $(Z_{3,t})$				
Parameter	Estimate	Std.Error	t-value	p-value
$\phi_{12,31}$	1.15908	0.64638	1.79	0.0731
$\phi_{1,31}$	0.77205	0.08391	9.20	0.0001
$\phi_{1,33}$	-0.14287	0.02466	-5.79	0.0001
$\phi_{2,31}$	0.37298	0.08635	4.32	0.0001
$\phi_{2,33}$	-0.05362	0.02500	-2.15	0.0321
$\phi_{4,31}$	-0.17952	0.08279	-2.17	0.0303

Substituting the parameter estimation results in Table 4.26 to equations (4.11) , (4.12) and (4.13), then obtaining the model for Exchange rate on equations (4.11) and (4.12) and substituting the results of parameter estimates in Table 4.26 to equations (4.11) and (4.12), the best model of VARI-X (4,1,20) are obtained for the level of Exchange rate in equation (4.15) and (4.16). Here are the equations for the various VARI-X models

a. The Mathematical South African exchange rate model is:

$$Z^*_{1,t} = -0.4283X_t - 0.6457X^*_{t-12} + 0.0208Z^*_{2,t-1} - 0.0208Z_{2,t-2} - 0.0188Z^*_{2,t-3} + 0.0188Z^*_{3,t-2} - 0.07261Z^*_{2,t-3} + 0.07261Z^*_{2,t-4} + 0.0166Z^*_{3,t-3} - 0.0166Z^*_{3,t-4} - 0.0473Z^*_{2,t-4} + 0.0473Z^*_{2,t-5} - 0.0473Z^*_{2,t-4} + 0.0188Z^*_{2,t-5} + a_t$$

b. The Mathematical model for Brazilian exchange rate is:

$$Z^*_{2,t} = 0.6667X^*_{t-12} - 0.992X^*_{t-14} - 1.1191X^*_{t-17} + 0.9512X^*_{t-20} - 0.7553Z^*_{1,t-2} - 0.0887Z^*_{2,t-2} + 0.0887Z^*_{2,t-2} + 0.0675Z^*_{3,t-1} + 0.1538Z^*_{1,t-2} + 0.0967Z^*_{1,t-3} - 0.0967Z^*_{1,t-4} - 0.0449Z^*_{2,t-4} + a_t$$

c. The Mathematical model for Australian exchange rate is:

$$Z^*_{3,t} = 1.1590X^*_{t-12} + 0.7720Z^*_{1,t-1} - 0.7720Z^*_{1,t-2} - 0.1428Z_{3,t-1} + 0.1428Z^*_{3,t-2} + 0.3729Z^*_{1,t-2} - 0.3729Z^*_{1,t-3} - 0.0536Z^*_{3,t-2} + 0.0536Z^*_{3,t-3} - 0.1795Z^*_{1,t-4} + 0.1795Z^*_{1,t-5} + a_t$$

Based on the best model that has been obtained for each data, the forecasting for exchange rate and gold price from 1st June 2010 to 28th February 2017 is presented in Table 4.27.

Table 4.27 Forecasting Exchange Rate and Gold Price Using VARI-X

Obs	South Africa	Australia	Brazil
1648	0.23295	0.22820	0.53843
1649	0.23271	0.22800	0.53768
1650	0.23273	0.22787	0.53740
1651	0.23283	0.22818	0.53782
1652	0.23292	0.22837	0.53791
1653	0.23294	0.22842	0.53794
1654	0.23292	0.22839	0.53800
1655	0.23296	0.22836	0.53806
1656	0.23294	0.22815	0.53806
1657	0.23285	0.22781	0.53787
1658	0.23286	0.22778	0.53779
1659	0.23281	0.22787	0.53766

4.5 Comparison of the Transfer function and VARI-X

The Model that has been obtained is the transfer function model, ARIMA, and VARI-X but for the purpose of this study will compare only transfer function and VARI-X as they can be used for multivariate modeling since ARIMA is univariate. The models merit value would be looked at using RMSE criteria. Models that have the smallest RMSE values out-sample data would be the best model and then used to predict the next 12 periods. The following RMSE values comparison is presented of all models.

Table 4.28 Comparison Values

Series	Transfer Function		VARI-X	
	RMSE	MAPE	RMSE	MAPE
South Africa	**0.00029**	**0.16778**	0.00034	9.41001
Australia	0.01032	1.59100	**0.01029**	**1.55572**
Brazil	**0.00233**	**0.73852**	0.00290	1.02690

Based on Table 4.28 comparison values, the results show that VARI-X is the best model for forecasting the Australian exchange rate, whereas Transfer Function is the best model for forecasting South African and Brazilian exchange rates.

The lowest exchange rate is predicted for South Africa followed by Brazil and then Australia with the least. This is supposed as a result of demand and supply and certain speculators who take a position in the South African market depending on their speculation of the currency that might work in their favor. The Australian Exchange rate like any other relies on trade for it economic wellbeing and therefore depend on it commodities. A fall in prices of the commodity will contribute greatly to its sharp decline while Brazilian exchange rate is expected to drop drastically, this is as result of the alleged monetary policies been used by the government to minimize capital inflows. This is further looked at by comparing ARIMA, Transfer Function and VARI-X model.

Table 4.29 Comparison of ARIMA, Transfer Function and VARI-X

Series	**ARIMA**		**Transfer Function**		**VARI-X**	
	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
S/Africa	**0.00029**	**0.0106**	0.00029	0.01677	0.00034	0.01603
Australia	0.01037	0.01586	0.01032	0.01591	**0.01029**	**0.01555**
Brazil	0.00245	0.07713	**0.00234**	**0.07385**	0.00290	1.0269

From Table 4.29, it can be inferred that ARIMA is the best model for South African exchange rate with RMSE of 0.00029 and MAPE of 0.0106 while VARI-X has the best model for Australian Exchange rate with RMSE of 0.01029 and MAPE of 0.015557, Brazilian exchange rate which has RMSE of 0.00234 and Means Absolute Percentage error of 0.07385 for transfer function. This clearly means that each of this countries can be modeled by Autoregressive Integrated

Moving Averages, Transfer Function and Vector Autoregressive Integrated with Exogenous Variable.

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CHAPTER 5
CONCLUSION AND RECOMMENDATION

5.1 Introduction

This chapter covers the summary of all the findings of this thesis, the various conclusions are drawn from the findings and suggested alternatives and recommendations to forecasting the exchange rate across countries with gold price as an exogenous variable.

5.2 Conclusion

Based on the analysis and discussion that has been done, the following conclusions can be drawn.

1. The characteristics of the exchange rate of South Africa, Brazil, and Australia have a similar pattern from the 1st June 2010 to 27th February 2018 as it shows a fluctuation in the dollar. Based on the cross-correlation it also shows some speculations of strong positive correlation (inter-relationship) between the exchange rates and gold price variable.
2. In this research, the ARIMA model is used for forecasting gold price data univariately as an input for Transfer Function and VARI-X models.
3. Based on the Transfer function modeling, we found the in-sample and out sample forecast for exchange rate with gold price as an input variable that the transfer function model has met the requirement and that it is the best model for forecasting South African and Brazilian exchange rate when the RMSE criteria are used.
4. The best VARI-X model produced at each level based on the minimum information criteria is at lag 4 and at VARI-X (4, 1, 20) model. We also found out that it is the best model for forecasting the Australian exchange rate which has the smallest RMSE of 0.0168. It clearly shows that it does not only predict exchange rate across three countries but can also see the interaction between countries exchange rate with gold price as an exogenous variable. The result of these findings will be helpful to policymakers and

stakeholders to formulate effective policies for controlling the exchange rate and gold prices.

5.3 Recommendation

From the above Analysis, recommendations can be given for further research in which other models can be used to forecast exchange rate and gold prices such as Neural Network, GARCH, ARCH, VARMA, VAR, etc. Also, monthly prices can be forecasted using ARIMA, transfer function, and VARI-X. Few factors that are related to the exchange rate and gold prices were used in this research. This study can be expanded by considering other countries that have a tendency to influence the price of gold.

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APPENDIX

Appendix 1 Data Variable for Exchange Rate and Gold Price

Date	AER	BER	SAER	GP
1-Jun-10	0.573	0.376	0.088	1227.800
2-Jun-10	0.567	0.374	0.092	1215.000
3-Jun-10	0.580	0.372	0.090	1215.000
4-Jun-10	0.577	0.373	0.088	1203.500
7-Jun-10	0.561	0.374	0.089	1215.000
8-Jun-10	0.564	0.370	0.089	1246.000
9-Jun-10	0.565	0.369	0.089	1233.500
10-Jun-10	0.575	0.372	0.089	1217.500
⋮	⋮	⋮	⋮	⋮
21-Feb-18	0.542	0.212	0.059	1276.500
22-Feb-18	0.539	0.212	0.059	1275.000
23-Feb-18	0.539	0.213	0.060	1273.800
26-Feb-18	0.541	0.213	0.059	1266.500
27-Feb-18	0.541	0.213	0.059	1272.000

Appendix 2 Correlation Syntax Program for Exchange Rate in R

```
library(readr)
Data5 <- read_csv("D:/THESIS RESULTS 2018/Data5.csv")
attach(Data5)
head(Data5)
summary(Data5)
# Load libraries
library(MASS)
library(tseries)
library(forecast)
library(Hmisc)
library(ggplot2)
library(TSA)
library(lmtest)
# Descriptive statistics
summary(Data5)
sd(GP)
sd(AER)
sd(BER)
sd(SAER)
# Correlation matrix
Data2018_cor=cor(Data5[,c(2,3,4,5)])
Data2018_cor
Data2018_cor=round(Data2018_cor,2)
Data2018_cor
# input should be matrix always

library(corrplot)
corrplot(Data2018_cor)
corrplot(Data2018_cor,method = "ellipse")
corrplot(Data2018_cor,order = "AOE",method = "color",addCoef.col = "gray")
corrplot.mixed(Data2018_cor,order = "AOE")
```

Appendix 3 Arima Syntax for Exchange Rate and Gold Price

```
library(readr)
Data5 <- read.csv("D:/THESIS RESULTS 2018/Data5.csv")
DataTrans <- read.csv("D:/THESIS RESULTS 2018/DataTrans.csv")
attach(Data5)
attach(DataTrans)
head(Data5)
summary(Data5)
#names(DataTrans)

# Dickyfuller Test
diffTranSAER1<-na.remove(diffTranSAER)
adf.test(diffTranSAER1, alternative = c("stationary", "explosive"))

# Load libraries
library(MASS)
library(tseries)
library(forecast)
library(Hmisc)
library(ggplot2)
library(TSA)
library(lmtest)
# Descriptive statistics
summary(Data5)
sd(GP)
sd(AER)
sd(BER)
sd(SAER)

# Correlation matrix
Data2018_cor=cor(Data5[,c(2,3,4,5)])
Data2018_cor
Data2018_cor=round(Data2018_cor,2)
Data2018_cor

# input should be matrix always

library(corrplot)
corrplot(Data2018_cor)
corrplot(Data2018_cor,method = "ellipse")
corrplot(Data2018_cor,order = "AOE",method = "color",addCoef.col = "gray")
corrplot.mixed(Data2018_cor,order = "AOE")

# plot all the series
plot(AER,type = "l",xlab = "time",ylab = "Australia")
plot(BER,type = "l",xlab = "time",ylab = "Brazil")
plot(SAER,type = "l",xlab = "time",ylab = "South Africa")
plot(GP,type = "l",xlab = "time",ylab = "Gold Price")
```

Appendix 3 Arima Syntax for Exchange Rate and Gold Price (Continue)

```
# plot the  ACF, PACF of the series
acf(AER,lag.max = 36)
pacf(AER,lag.max = 36)
acf(BER,lag.max = 36)
pacf(BER,lag.max = 36)
acf(SAER,lag.max = 36)
pacf(SAER,lag.max = 36)
acf(GP,lag.max = 36)
pacf(GP,lag.max = 30)

# Take the training data, transform and then difference all of them
GoldPrice <- GP[1:1647]
GoldPrice
trans_GoldPrice <- 1/sqrt(GoldPrice)
diff_GoldPrice <- diff(trans_GoldPrice,1)
diff_GoldPrice
Australia <- AER[1:1647]
Australia
diff_Australia <- diff(Australia,1)
diff_Australia
Brazil <- BER[1:1647]
Brazil
diff_Brazil <- diff(Brazil,1)
diff_Brazil
SouthAfrica <- SAER[1:1647]
SouthAfrica
trans_SouthAfrica <- sqrt(SouthAfrica)
diff_SouthAfrica <- diff(trans_SouthAfrica,1)
diff_SouthAfrica

# Take the testing data
YtestGoldPrice <- as.ts(Data5[1648:1936,5])          #define testing data

# Plot The differenced acf and pacf series
acf(diff_GoldPrice,lag.max = 30)
pacf(diff_GoldPrice,lag.max = 30)
acf(diff_Australia,lag.max = 30)
pacf(diff_Australia,lag.max = 30)
acf(diff_Brazil,lag.max = 30)
pacf(diff_Brazil,lag.max = 30)

# Plot The differenced series
plot((diff_GoldPrice),type = "l",lwd =2,col="blue",xlab = "time",ylab ="Gold
Price",main = "Differenced Gold Price" )
plot((diff_Australia),type = "l",lwd =2,col="blue",xlab = "time",ylab = "Australia",main
= "Differenced Australian Exchange rate")
plot((diff_Brazil),type = "l",lwd =2,col="blue",xlab = "time",ylab = "Brazil",main =
"Differenced Brazilian Exchange Rate")
```

Appendix 3 Arima Syntax for Exchange Rate and Gold Price (Continue)

```
plot((diff_SouthAfrica),type = "l",lwd =2,col="blue",xlab = "time",ylab = "South
Africa",main = "Differenced South African Exchange Rate")
plot((diff_Chinese),type = "l",lwd =2,col="black",xlab = "time",ylab = "Chinese",main =
"Differenced Chinese Exchange Rate")

# Forecasting GoldPrice
# step1: Model identification
# Stationarity check - Dickey Fuller test)
adf.test(GoldPrice)
adf.test(diff_GoldPrice)

#CHECKING FOR STATIONARITY USING ACF PLOT#
tick=c(1,12,24,36)
par(mfrow=c(2,1),mar=c(2.8,3,1.2,0.4))    #the number of picture and its margin
par(mgp=c(1.7,0.5,0))                     #the distance between labels and axis
#ACF
acf(GoldPrice,lag.max=36,axes=F)
box()
axis(side=1,at=tick,label=tick,lwd=0.5,las=0,cex.axis=0.8)
abline(v=tick,lty="dotted", lwd=2, col="grey")
axis(side=2,lwd=0.5,las=2,cex=0.5,cex.axis=0.8)
#PACF

#ORDER IDENTIFICATION USING ACF AND PACF FROM STATIONARY DATA
tick=c(1,13,26,39)
par(mfrow=c(2,1),mar=c(2.8,3,1.2,0.4))    #the number of picture and its margin
par(mgp=c(1.7,0.5,0))                     #the distance between labels and axis
 #ACF
 acf(diff_GoldPrice,lag.max=36,axes=F)
 box()
 axis(side=1,at=tick,label=tick,lwd=0.5,las=0,cex.axis=0.8)
 abline(v=tick,lty="dotted", lwd=2, col="grey")
 axis(side=2,lwd=0.5,las=2,cex=0.5,cex.axis=0.8)
 #PACF
 pacf(diff_GoldPrice,lag.max=36,axes=F)
 box()
 axis(side=1,at=tick,label=tick,lwd=0.5,las=0,cex.axis=0.8)
 abline(v=tick,lty="dotted", lwd=2, col="grey")
 axis(side=2,lwd=0.5,las=2,cex=0.5,cex.axis=0.8)

#ARIMA MODELLING#
#non-seasonal ARIMA model (not meet white noise assumption)
modelARIMA=arima(GoldPrice, order = c(21,1,0),
        include.mean=FALSE)
summary(modelARIMA)                       #ARIMA (13,1,0)
coeftest(modelARIMA)                      #significance test for parameter
resi.ARIMA=as.ts(modelARIMA$residuals)           #define the residual value
fits.ARIMA=as.ts(fitted(modelARIMA))             #define forecast value for training
data
```

Appendix 3 Arima Syntax for Exchange Rate and Gold Price (Continue)

```
# Choose the one that has the least AIC from subset arima
# step2: model estimation
arima(GoldPrice,order = c([13,21],1,0))
arima(GoldPrice,order = c(0,1,[13,21]))


#subset ARIMA model (not meet white noise assumption)
modelARIMA=arima(GoldPrice, order = c(21,1,0),       #ARIMA ([13,14,21],1,0)
             transform.pars = FALSE,
             fixed=c(rep(0,20),NA),
             include.mean=FALSE)
summary(modelARIMA)                              #ARIMA ([1,2,11,12,13],0,0)
coeftest(modelARIMA)                          #significance test for parameter
resi.ARIMA.GP=as.ts(modelARIMA$residuals)                #define the residual value
fits.ARIMA=as.ts(fitted(modelARIMA))                  #define forecast value for training
data
tsdiag(modelARIMA)
write.csv(resi.ARIMA,"resiGP1.csv")

#DIAGNOSTIC CHECKING FOR ARIMA MODEL
#Independency test by using Ljung-Box test
lags <- c(6,12,18,24)                          #lag we used
p=3                                     #the number of ar parameter
q=0                                     #the number of ma parameter
LB.result<-matrix(0,length(lags),2)
for(i in seq_along(lags))
{
  LB.test=Box.test (resi.ARIMA.GP, lag = lags[i],type = c("Ljung-Box"),fitdf=p+q)
  LB.result[i,1]=LB.test$statistic
  LB.result[i,2]=LB.test$p.value
}
rownames(LB.result)<-lags
colnames(LB.result)<-c("statistics","p.value")
LB.result

#ACF and PACF for RESIDUAL ARIMA MODEL
tick=c(1,12,24,36,48)
par(mfrow=c(2,1),mar=c(2.8,3,1.2,0.4))    #the number of picture and its margin
par(mgp=c(1.7,0.5,0))                 #the distance between labels and axis
#ACF
acf(resi.ARIMA.GP,lag.max=24,axes=F)
box()
axis(side=1,at=tick,label=tick,lwd=0.5,las=0,cex.axis=0.8)
abline(v=tick,lty="dotted", lwd=2, col="grey")
axis(side=2,lwd=0.5,las=2,cex=0.5,cex.axis=0.8)
#PACF
pacf(resi.ARIMA.GP,lag.max=19,axes=F)
box()
```

Appendix 3 Arima Syntax for Exchange Rate and Gold Price (Continue)

```
axis(side=1,at=tick,label=tick,lwd=0.5,las=0,cex.axis=0.8)
abline(v=tick,lty="dotted", lwd=2, col="grey")
axis(side=2,lwd=0.5,las=2,cex=0.5,cex.axis=0.8)


#Normality test using Kolmogorov Smirnov
ks.test(resi.ARIMA.GP,"pnorm",mean=mean(resi.ARIMA),sd=sd(resi.ARIMA))

#FORECAST FOR TESTING DATA
fore.ARIMA=predict(modelARIMA, 290)$pred       #define forecast value for testing
data
se.fore.ARIMA=predict(modelARIMA, 290)$se      #define standard error for
forecasting result

#CALCULATE RMSE, MAE, AND MAPE CRITERIA
accuracies=matrix(0,3,2)
colnames(accuracies)=c("Training","Testing")
rownames(accuracies)=c("RMSE","MAE","MAPE")

accuracies[1,1]=accuracy(fits.ARIMA,GoldPrice)[1,2]
accuracies[2,1]=accuracy(fits.ARIMA,GoldPrice)[1,3]
accuracies[3,1]=accuracy(fits.ARIMA,GoldPrice)[1,5]
accuracies[1,2]=accuracy(as.vector(fore.ARIMA),YtestGoldPrice)[1,2]
accuracies[2,2]=accuracy(as.vector(fore.ARIMA),YtestGoldPrice)[1,3]
accuracies[3,2]=accuracy(as.vector(fore.ARIMA),YtestGoldPrice)[1,5]
accuracies

#CONSTRUCT INTERVAL PREDICTION
lower=fore.ARIMA-1.96*se.fore.ARIMA
upper=fore.ARIMA+1.96*se.fore.ARIMA

#COMPARISON BETWEEN ACTUAL AND FORECAST VALUE
a=min(min(fits.ARIMA),min(GoldPrice))            #lower bound for training data
b=max(max(fits.ARIMA),max(GoldPrice))            #upper bound for training data
c=min(min(fore.ARIMA),min(lower),min(YtestGoldPrice))    #lower bound for testing
data
d=max(max(fore.ARIMA),max(upper),max(YtestGoldPrice))    #upper bound for testing
data

par(mfrow=c(1,2),mar=c(2.3,2.7,1.2,0.4))  #the number of picture and its margin
par(mgp=c(1.3,0.5,0))                     #the distance between labels and axis

#PLOTTING FOR TRAINING DATA#
plot(as.ts(GoldPrice),ylab="Yt",xlab="t",lwd=2,axes=F,ylim=c(a*0.9,b*1.1))
box()
title("Training",line=0.3,cex.main=0.9)
axis(side=2,lwd=0.5,cex.axis=0.8,las=0)
axis(side=1,lwd=0.5,cex.axis=0.8,las=0,at=seq(1,1647,12))
lines(as.ts(fits.ARIMA),col="red",lwd=2)
```

Appendix 3 Arima Syntax for Exchange Rate and Gold Price (Continue)

```
#PLOTTING FOR TESTING DATA#
plot(as.ts(YtestGoldPrice),ylab="Yt",xlab="t",lwd=2,ylim=c(a*0.9,b*1.1),cex.lab=0.8,ax
es=F)
box()
title("Testing",line=0.3,cex.main=0.9)
axis(side=2,lwd=0.5,cex.axis=0.8,las=0)
axis(side=1,lwd=0.5,cex.axis=0.8,las=0,at=c(1:289),labels=c(1648:1936))
lines(as.vector(fore.ARIMA),col="red",lwd=2)
lines(as.vector(lower),col="blue2",lty="dotdash",lwd=2)
lines(as.vector(upper),col="blue2",lty="dotdash",lwd=2)

#DEFINE THE LEGEND#
legend("topright",c("Actual","Forecast","Upper Bound","Lower Bound"),
      col=c("black","red","blue2","blue2"),lwd=2,cex=0.7)
```

Appendix 4 SAS Studio Syntax Program on Transfer Function for Gold Price and Australian Exchange Rate.

```
proc arima data=hassan;
    /*--- Look at the input process ----------------------------*/
    identify AER=GP(1) nlag=24;
    run;
    /*--- Fit a model for the input ----------------------------*/
    estimate p=0 q=(21) noint method=cls;
    run;
    /*--- Crosscorrelation of prewhitened series ---------------*/
    identify var=AER(1) crosscorr=GP(1) nlag=24;
    run;
    /*--- Fit the 1st Transfer function model - look at residuals ---*/
    estimate p=1 q=0 input=( 12 $ (0) / (0) x ) method=cls noint plot;
    run;
    /*--- Forecast 30-ahead-forecast ---*/
    forecast lead=30;
    run;
    /*--- Fit the 2nd Transfer function model - look at residuals ---*/
    estimate p=0 q=1 input=( 12 $ (0) / (0) x ) method=cls noint plot;
    run;
    /*--- Forecast 30-ahead-forecast ---*/
    forecast lead=30 printall;
    run;
```

Appendix 5. SAS Studio Syntax Transfer Function Program on Gold Price and Brazilian Exchange Rate.

```
proc arima data=hassan;

   /*--- Look at the input process ----------------------------*/
   identify BER=GP(1) nlag=24;
   run;

   /*--- Fit a model for the input ----------------------------*/
   estimate p=0 q=(21) noint method=cls;
   run;

   /*--- Crosscorrelation of prewhitened series ---------------*/
   identify var=BER(1) crosscorr=GP(1) nlag=24;
   run;

    /*--- Fit the 1st Transfer function model - look at residuals ---*/
   estimate p=(4,7) q=0 input=( 12 $ (2,5,8) / (0) x ) method=cls noint plot;
   run;

   /*--- Forecast 30-ahead-forecast ---*/
   forecast lead=30 printall;
   run;
```

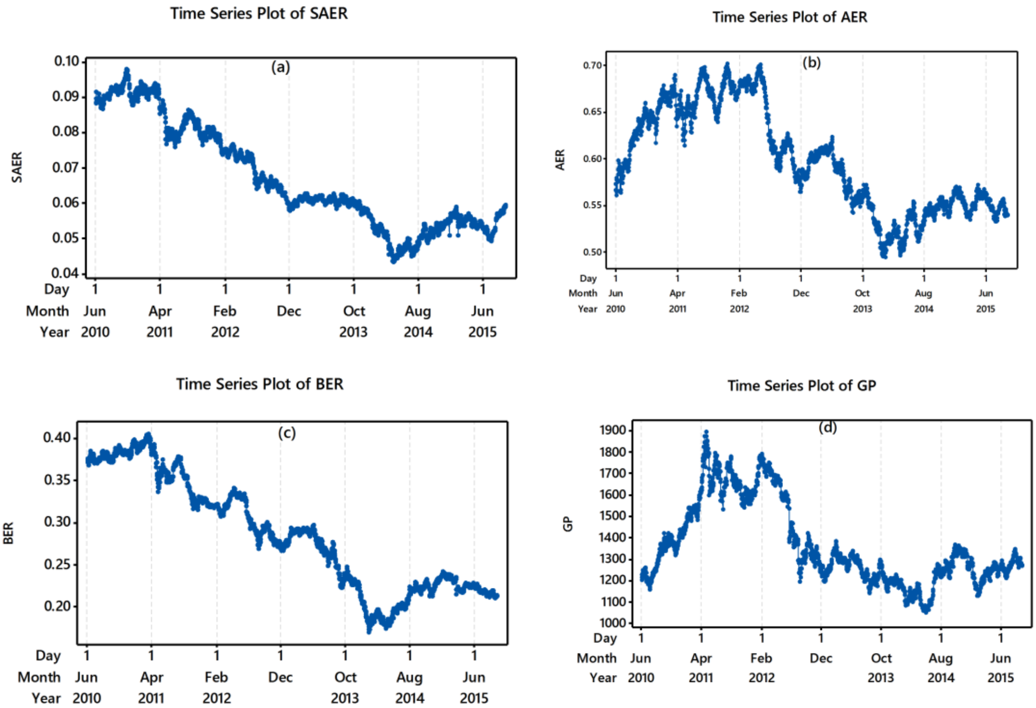
Appendix 6. SAS Studio Syntax Transfer Function Program on Gold Price and South African Exchange

```
proc arima data=hassan;

  /*--- Look at the input process ----------------------------*/
  identify var=GP(1) nlag=24;
  run;

  /*--- Fit a model for the input ----------------------------*/
  estimate p=0 q=(21) noint method=cls;
  run;

  /*--- Crosscorrelation of prewhitened series ---------------*/
  identify var=SAER(1) crosscorr=GP(1) nlag=24;
  run;

  /*--- Fit the 1st Transfer function model - look at residuals ---*/
  estimate p=(8,10,13,15,19) q=0 input=( 12 $ (0) / (0) x ) method=cls noint plot;
  run;

  /*--- Forecast 30-ahead-forecast ---*/
  forecast lead=30 printall;
  run;
```

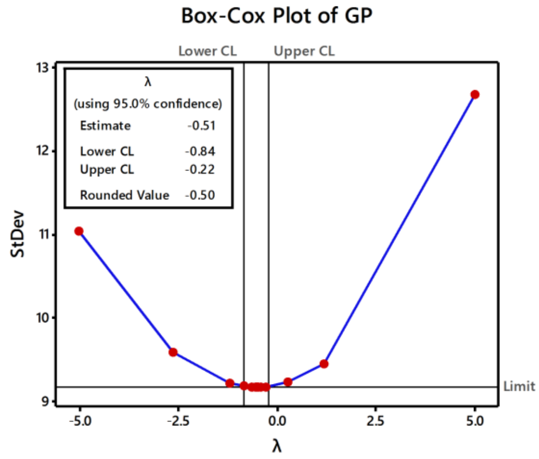
Appendix 7. SAS Studio Syntax for VARI-X Program on Exchange Rate and Gold Price

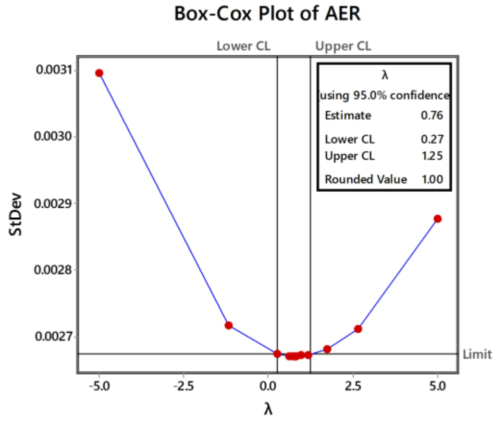
```
/** Differenced variables, corr, parcoef, pcorr, root, causality, minimum AICC and
forecast. **/

proc varmax data=Exam5 plots=(model residual);

model SAER BER AER = GP/p=11 dftest dify(1) difx(1) noint method=ls
print=(corry parcoef pcorr roots) minic=(p=12 q=0);

run;

/** Differenced variables, corr, parcoef, pcorr, root, causality, minimum AICC and
forecast. **/

proc varmax data=Exam5 printall lagmax = 24;

label SAER ='South African Exchange Rate'

        BER = 'Brazilian Exchange Rate'

        AER = 'Australian Exchange Rate'

         GP = 'Gold Price';

model SAER BER AER = GP/p=4 dftest dify(1) difx(1) noint method=ls
print=(corry parcoef pcorr roots) minic=(p=12 q=0);

restrict    ar(1,1,1)=0, ar(1,1,2)=0, ar(1,1,3)=0, ar(2,1,1)=0, ar(2,1,2)=0,
ar(3,1,3)=0, ar(3,1,2)=0, ar(4,3,1)=0, ar(4,1,2)=0,  ar(4,1,3)=0,  ar(2,2,3)=0,
ar(3,2,2)=0,   ar(3,2,3)=0, ar(4,2,1)=0,   ar(4,2,2)=0,  ar(4,2,3)=0,  ar(1,3,2)=0,
ar(2,3,2)=0, ar(3,3,1)=0, ar(3,3,2)=0, ar(3,3,3)=0, ar(4,3,2)=0, ar(4,3,3)=0,
ar(4,1,1)=0;

causal group1=(GP) group2=(SAER BER AER);

causal group1=(BER) group2=(SAER AER);

causal group1=(SAER) group2=(BER AER);

CAUSAL group1=(AER) group2=(SAER BER);

output lead=12;

run;

proc univariate data=Exam5 normal;

var residual;

run;
```

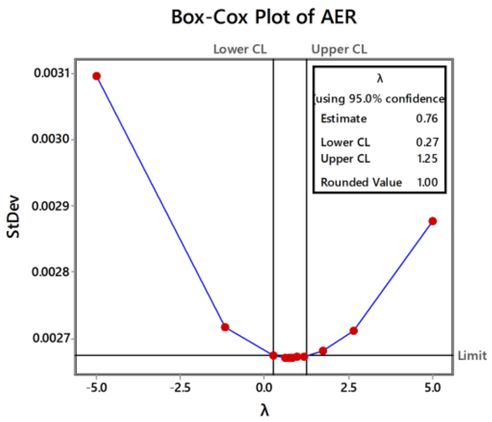
Appendix 8. ARIMA Model Identification on Exchange Rate and Gold Price

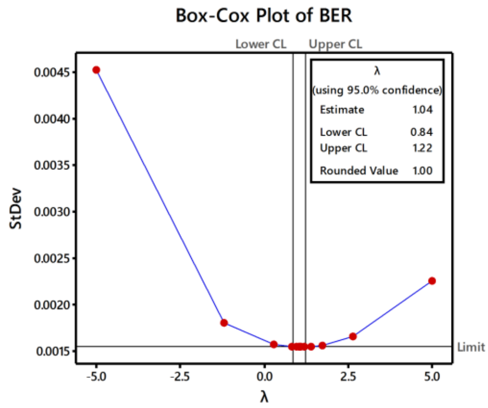


Appendix 9. In-sample Data Box Cox Test before Transformation

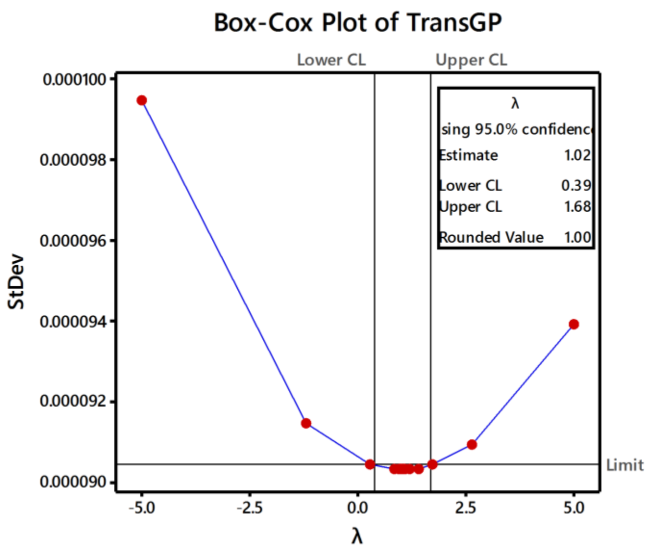


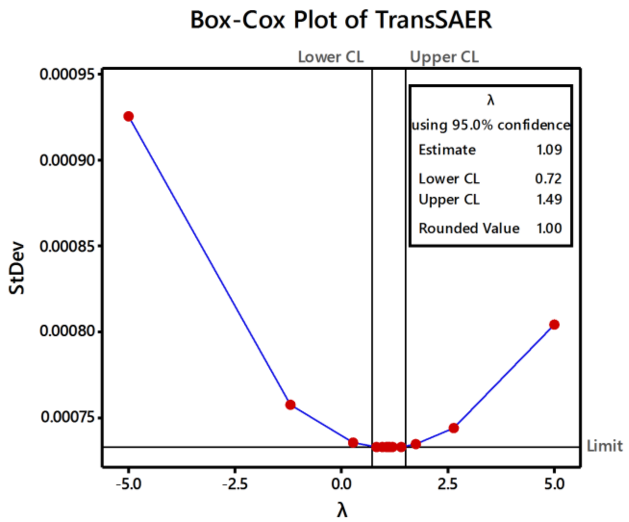




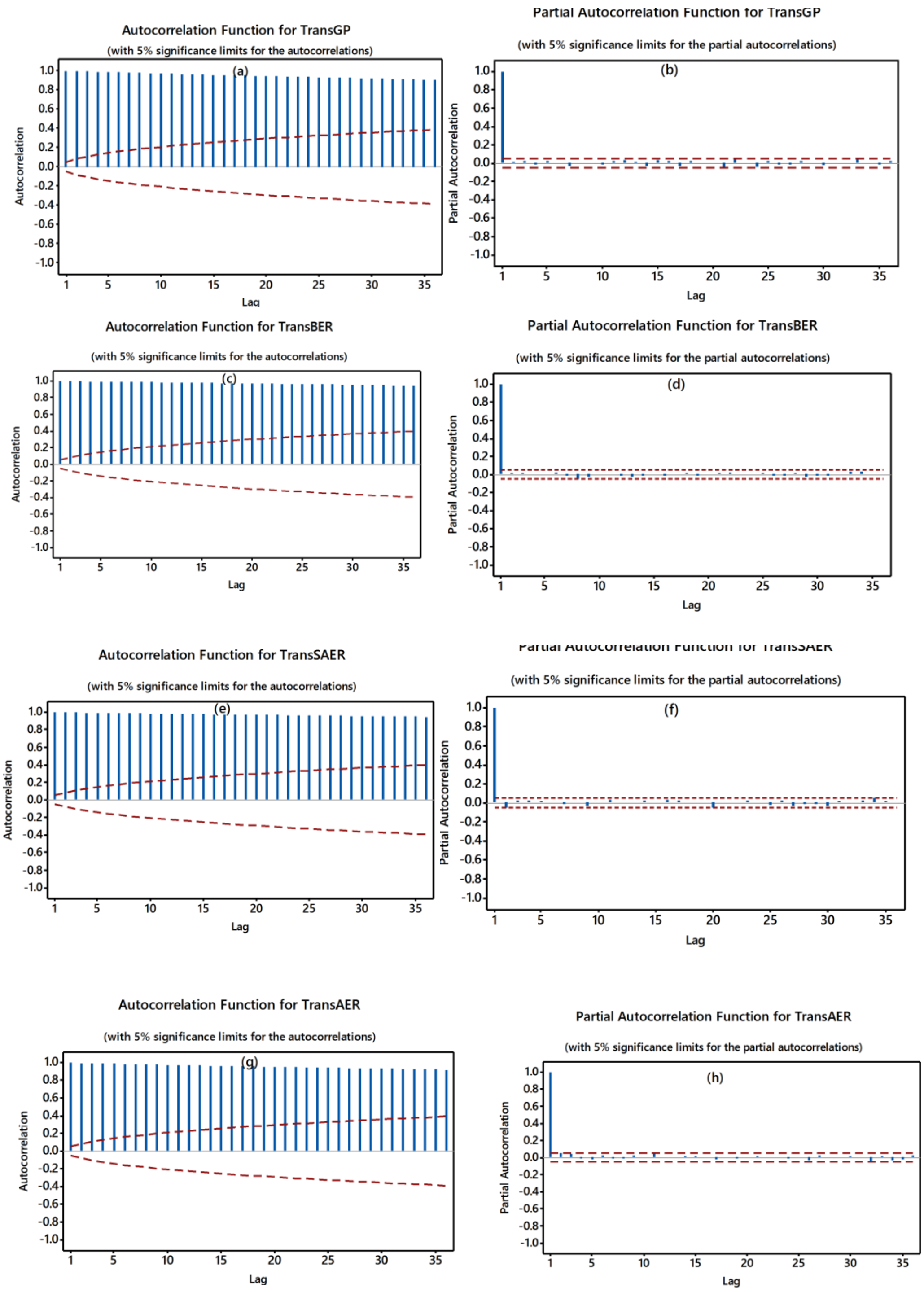


Appendix 10. Box Cox after Transformation

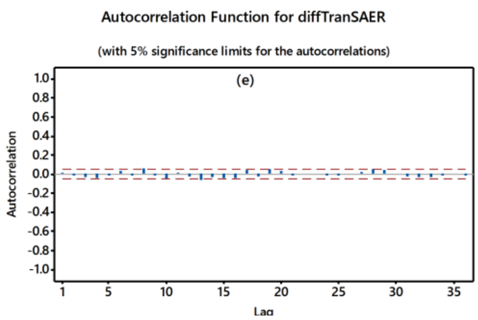


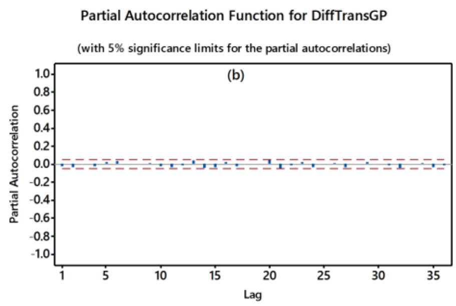


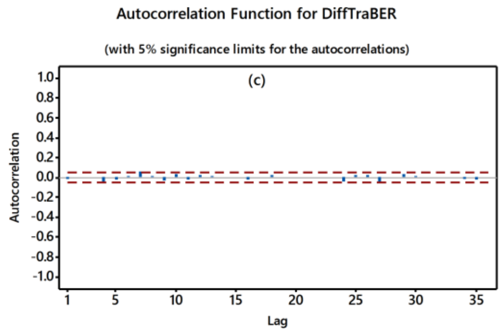
Appendix 11. ACF and PACF of Transformed Data

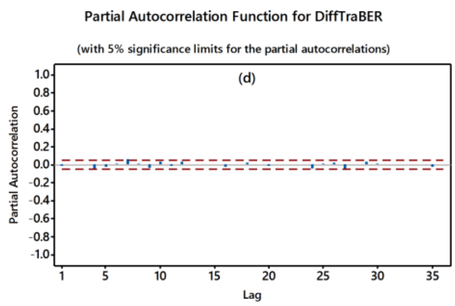


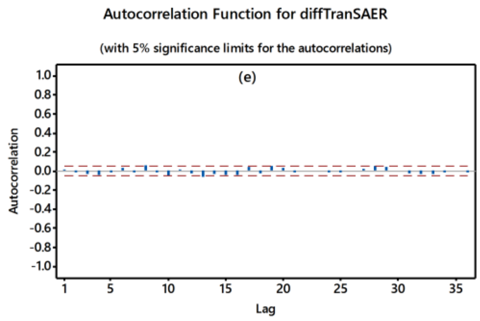
Appendix 12. ACF and PACF after Differencing

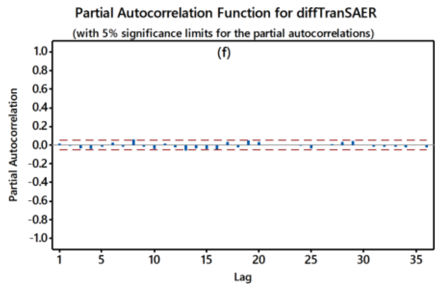


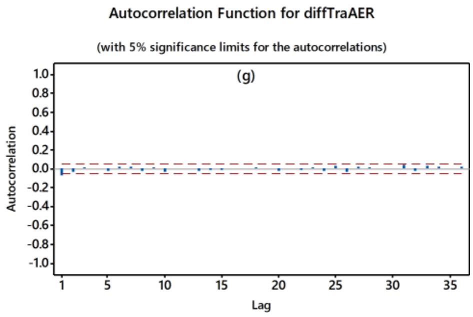


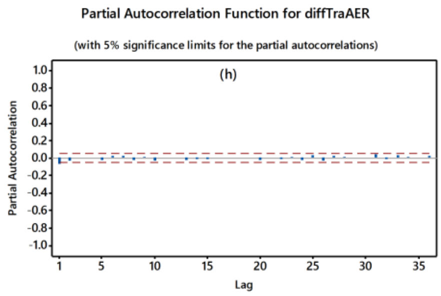












Appendix 13. Transfer Function Parameter Estimate for Gold Price and South African Exchange Rate

Conditional Least Squares Estimation							
Parameter	Estimate	Standard Error	t Value	Approx Pr>|t|	Lag	Variable	Shift
AR1,1	0.04893	0.02467	1.98	0.0475	8	SAER	0
AR1,2	-0.04807	0.02468	-1.95	0.0516	10	SAER	0
AR1,3	-0.06538	0.02472	-2.64	0.0082	13	SAER	0
AR,1,4	-0.04763	0.02472	-1.93	0.0542	15	SAER	0
AR1,5	0.05748	0.02476	2.32	0.0204	19	SAER	0
NUM1	0.64249	0.64249	3.42	0.0006	0	GP	12

Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr>Chisq	Autocorrelations					
6	7.31	1	0.0068	0.019	-0.004	-0.039	-0.029	-0.015	0.039
12	9.22	7	0.2371	-0.012	-0.001	-0.012	-0.001	0.020	-0.022
18	17.15	13	0.1927	0.0003	-0.025	0.003	0.031	0.047	-0.032
24	18.56	19	0.4855	0.001	0.027	0.001	-0.009	-0.004	-0.006
30	28.90	25	0.2681	-0.031	0.007	0.022	-0.048	0.050	0.001
36	31.89	31	0.4220	-0.022	-0.008	-0.026	-0.017	-0.008	-0.014
42	39.19	37	0.3718	-0.008	-0.026	0.001	0.021	-0.017	0.054
48	46.96	43	0.3134	0.011	-0.000	-0.014	0.014	0.038	-0.052

Variance Estimate	1.23E-6
Std Error Estimate	0.001109
AIC	-17592.8
SBC	-17560.4
Number of Residuals	1634

AIC and SBC do not include log determinant

Appendix 14. Transfer Function Parameter Estimate for Gold Price and Brazilian Exchange Rate

Conditional Least Squares Estimation							
Parameter	Estimate	Standard Error	t Value	Approx Pr>|t|	Lag	Variable	Shift
AR1,1	0.06716	0.02486	2.70	0.0070	7	BER	0
NUM1	0.74915	0.40949	1.83	0.0675	0	GP	12
NUM1,1	0.98323	0.40990	2.40	0.0166	2	GP	12
NUM1,2	1.18950	0.40821	2.91	0.0036	5	GP	12
NUM1,3	-1.19711	0.40801	-2.93	0.0034	8	GP	12

Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr>Chisq	Autocorrelations					
6	5.89	5	0.3175	-0.015	0.001	0.000	-0.046	-0.022	0.029
12	10.30	11	0.5035	-0.000	0.015	-0.022	0.033	-0.013	0.027
18	13.39	17	0.7096	0.015	0.005	-0.010	-0.033	0.004	0.020
24	17.13	23	0.8027	0.004	-0.008	0.006	-0.009	0.014	-0.043
30	26.85	29	0.5796	0.032	0.029	-0.050	-0.003	0.039	0.005
36	27.87	35	0.7987	0.006	0.007	-0.000	-0.013	-0.019	-0.002
42	30.50	41	0.8851	-0.003	0.026	-0.003	0.003	-0.002	-0.029
48	37.53	47	0.8367	-0.008	0.033	-0.020	-0.019	-0.048	0.003

Variance Estimate	5.773E-6
Std Error Estimate	0.002403
AIC	-14994
SBC	-14967.1
Number of Residuals	1634

AIC and SBC do not include log determinant

Appendix 15. Transfer Function Parameter Estimate for Gold Price and Australian Exchange Rate

Conditional Least Squares Estimation							
Parameter	Estimate	Standard Error	t Value	Approx Pr>|t|	Lag	Variable	Shift
AR1,1	-0.06860	0.02470	-2.78	0.0055	1	AER	0
NUM1	1.45006	0.66371	2.18	0.0290	0	GP	12

Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr>Chisq	Autocorrelations					
6	3.33	5	0.6499	-0.002	-0.024	0.007	0.003	-0.027	0.026
12	7.24	11	0.7789	0.030	-0.033	0.009	-0.018	0.003	-0.000
18	8.12	17	0.9640	-0.011	-0.010	-0.011	-0.009	-0.009	0.005
24	10.61	23	0.9868	-0.001	-0.025	-0.007	0.001	0.014	-0.026
30	15.25	29	0.9830	0.041	-0.026	0.016	0.009	-0.004	0.010
36	24.42	35	0.9096	0.042	-0.027	0.037	0.032	-0.008	0.023
42	27.58	41	0.9461	0.022	0.021	-0.006	0.029	0.009	0.002
48	35.44	47	0.8919	-0.011	-0.014	-0.035	-0.034	-0.044	0.007

Variance Estimate	0.000015
Std Error Estimate	0.003904
AIC	-13484.2
SBC	-13484.2
Number of Residuals	1634

AIC and SBC do not include log determinant

Appendix 16. White Noise Assumption and Multivariate Normal Distribution

Minimum Information Criterion Based on AICC	
Lag	MA 0
Lag 0	-36.82068
Lag 1	-37.04876
Lag 2	-37.07657
Lag 3	-37.07511
Lag 4	-37.08432
Lag 5	-37.08393
Lag 6	-37.07605
Lag 7	-37.07467
Lag 8	-37.07537
Lag 9	-37.06712
Lag 10	-37.06421
Lag 11	-37.05773
Lag 12	-37.04900

Appendix 17. SAS Studio outputs on Exchange Rate Transfer Function Forecast

South African Exchange Rate Transfer Function Forecast

Obs	Forecast	Std Error	Lower Limit	Upper Limit
1648	0.2332	0.0011	0.231	0.2354
1649	0.2334	0.0016	0.2303	0.2364
1650	0.2334	0.0019	0.2297	0.2372
1651	0.2335	0.0022	0.2291	0.2378
1652	0.2337	0.0025	0.2288	0.2386
1653	0.2337	0.0027	0.2284	0.239
1654	0.2338	0.0029	0.2281	0.2396
1655	0.2338	0.0031	0.2277	0.24
1656	0.2336	0.0033	0.227	0.2401
1657	0.2336	0.0035	0.2267	0.2406
1658	0.2335	0.0037	0.2262	0.2407
1659	0.2333	0.0039	0.2257	0.2409

Australian Exchange Rate Transfer Function Forecast

obs	Forecast	Std Error	Lower Limit	Upper Limit
1648	0.5381	0.0039	0.5304	0.5457
1649	0.5379	0.0053	0.5274	0.5483
1650	0.538	0.0064	0.5253	0.5506
1651	0.5381	0.0074	0.5237	0.5526
1652	0.5382	0.0082	0.5221	0.5544
1653	0.5383	0.009	0.5206	0.5559
1654	0.5383	0.0097	0.5193	0.5573
1655	0.5384	0.0104	0.5182	0.5587
1656	0.5385	0.011	0.517	0.5600
1657	0.5384	0.0116	0.5157	0.5610
1658	0.5384	0.0121	0.5147	0.5621
1659	0.5382	0.0126	0.5134	0.563

Brazilian Exchange Rate Transfer Function Forecast

obs	Forecast	Std Error	Lower Limit	Upper Limit
1648	0.2285	0.0024	0.2238	0.2332
1649	0.2285	0.0034	0.2218	0.2351
1650	0.2286	0.0042	0.2205	0.2368
1651	0.2293	0.0048	0.2199	0.2387
1652	0.2294	0.0054	0.2189	0.24
1653	0.2294	0.0059	0.2179	0.241
1654	0.2294	0.0064	0.2169	0.2418
1655	0.2294	0.0069	0.2159	0.2428
1656	0.2291	0.0073	0.2147	0.2434
1657	0.2287	0.0078	0.2135	0.2439

Appendix 18. SAS Studio Outputs on Exchange Rate VARI-X Forecast

South African Exchange Rate VARI-X Forecast

Obs	Forecast	std Error	Lower Limit	Upper Limit
1648	0.23295	0.00111	0.23077	0.23513
1649	0.23271	0.00158	0.2296	0.23581
1650	0.23273	0.00307	0.22671	0.23874
1651	0.23283	0.00609	0.22089	0.24477
1652	0.23292	0.00804	0.21717	0.24867
1653	0.23294	0.00991	0.21352	0.25236
1654	0.23292	0.01151	0.21035	0.25548
1655	0.23296	0.01296	0.20757	0.25836
1656	0.23294	0.01427	0.20498	0.26091
1657	0.23285	0.01552	0.20243	0.26328
1658	0.23286	0.01668	0.20018	0.26555
1659	0.23281	0.0178	0.19793	0.2677

Australian Exchange Rate VARI-X Forecast

Obs	Forecast	std Error	Lower Limit	Upper Limit
1648	0.53843	0.00379	0.53100	0.54586
1649	0.53768	0.00523	0.52742	0.54793
1650	0.5374	0.00638	0.52489	0.54991
1651	0.53782	0.00739	0.52334	0.5523
1652	0.53791	0.00824	0.52177	0.55406
1653	0.53794	0.009	0.52030	0.55558
1654	0.53800	0.00969	0.51900	0.55699
1655	0.53806	0.01033	0.51781	0.55832
1656	0.53806	0.01094	0.51662	0.55951
1657	0.53787	0.01151	0.51530	0.56044
1658	0.53779	0.01206	0.51415	0.56143
1659	0.53766	0.01259	0.51299	0.56233

Australian Exchange Rate VARI-X Forecast

Obs	Forecast	std Error	Lower Limit	Upper Limit
1648	0.2282	0.00221	0.22388	0.23253
1649	0.2280	0.00321	0.2217	0.23429
1650	0.22787	0.00398	0.22007	0.23567
1651	0.22818	0.00466	0.21904	0.23732
1652	0.22837	0.00522	0.21813	0.23861
1653	0.22842	0.00571	0.21724	0.2396
1654	0.22839	0.00614	0.21634	0.24043
1655	0.22836	0.00655	0.21553	0.2412
1656	0.22815	0.00693	0.21456	0.24173
1657	0.22781	0.0073	0.21351	0.2421
1658	0.22778	0.00764	0.2128	0.24276
1659	0.22787	0.00798	0.21224	0.2435

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