Design of Sliding-PID Control System for Double Pendulum on Moving Cart

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Abstract *- Most systems that exist in the real world are nonlinear systems so that are difficult to control. Double pendulum is a system that simulates a control mechanism to regulate the stability problem. The main problem in control system design for the double pendulum is to stabilize the pendulum rod in equilibrium by moving train on limited trajectory. In this research, Sliding-PID control system is designed to stabilize the double pendulum. Sliding-PID controller is a combination of PID controller and Sliding Mode Controller. Double pendulum system is modeled by using Matlab Simulink based on the equations of kinematics and dynamics. The results shows the Sliding-PID controller produced better response compared to PID controller. Sliding-PID able to make the double pendulum on moving cart achieves its stability 43.42% faster than classical PID controller.*

Keywords : *double pendulum, PID controller, Siding-PID controller.*

1. INTRODUCTION

Double pendulum on moving cart is a system that simulates a control mechanism to regulate the stability problem. Double pendulum consists of two pendulums interconnected with each other. In real life we can find some mechanism that works according to the principle of double pendulum, such as cranes and robotic arms. Double pendulum is a unstable nonlinear system so that the control becomes complicated when used a conventional control techniques.
Several types of controllers have been applied to obtain the most appropriate control techniques in maintaining the stability of the pendulum system. Among these are the PID controller, neural networks, fuzzy logic controllers, and linear quadratic optimal controller. However, these controllers are not having good resistance to parameter perturbation and external disturbance [1]. It is necessary to develop new methods that produce a better control system, one of using Sliding-PID controller (SPID). Sliding-PID controller is a combination of the PID controller and sliding mode controller (SMC).

2. SLIDING MODE CONTROL

The theory of sliding mode control (SMC) was developed in the 1950's, started by SV Emelyanov. SMC is a robust control that can be applied to nonlinear systems and multi input multi output (MIMO). SMC has been successfully applied to a wide range of applications such as robot manipulators, underwater vehicles, automotive transmissions and engines, high-performance electric motors and power systems.
The main advantage of SMC is insensitive to parameter variations, external disturbance and modeling errors. Another advantage of SMC is having a fast response in achieving stability.
Here is a simple example of SMC application on a system with state variable:

$$x_1 = x, \quad x_2 = \dot{x}$$

The state space of the system is:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(x) + g(x)u$$

x_1 will be stable if

$$\dot{x}_1 = -ax_1 \, , a > 0 \quad (1)$$

While a new target to achieve the stability (x_1, x_2) = (0, 0) is:

$$s = x_2 + ax_1 = 0 \quad (2)$$
$$\dot{x}_1 = x_2 = -ax_1 + s \quad (3)$$

The time derivative of s is:

$$\dot{s} = \dot{x}_2 + a\dot{x}_1 = f(x) + g(x)u + ax_2 \quad (4)$$

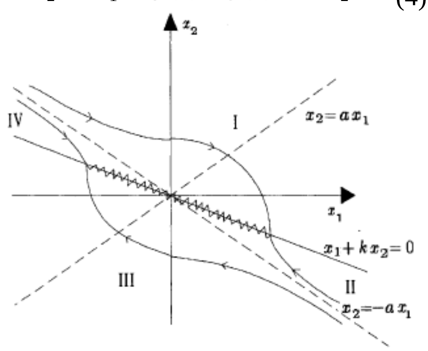

Figure 1: Sliding mode control of second orders system [7]

3. PID CONTROLLER

PID (Proportional-Integral-Derivative) controller is one of controllers that most widely used in control systems application, due to its ability to produce good stability and can be applied to high-order plant. PID controller has several advantages, such as easily

designed, has a low price, maintenance is not expensive, and not requires a special skill for the operator.
The equation of PID controller can be written as follows:

$$u(t) = K_1 e(t) + K_2 \int e dt + K_3 \frac{de}{dt} \qquad (5)$$

PID control system is a simple closed-loop control system which has good performance. However, this controller can not work well if there are uncertainty and nonlinearity on the system. Nevertheless PID control system has compatibility with other control systems, so it can be combined with other control systems such as fuzzy control, adaptive control, sliding mode control, and robust control to produce better performance.
PID controller consists of three kinds of controller, Proportional, Derivative, and Integral controller with each other having advantages and disadvantages. Purpose of merging the three types of controller is to cover the disadvantages and highlight the advantages of each type of controller.

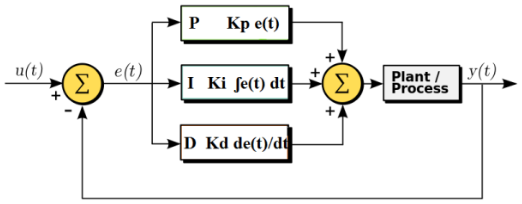

Figure 2: PID control system block diagram

4. MODEL DESIGN

Here is a schematic model of the double pendulum system in this research.

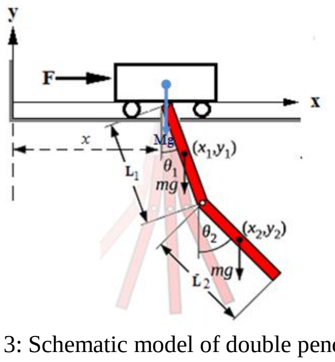

Figure 3: Schematic model of double pendulum

Double pendulum on a moving cart consists of two pendulum rods. The first pendulum mounted on a cart and a second pendulum is connected to the first pendulum rod. Cart can move to the right and left along the trajectory of horizontal (x-axis direction). Control forces F (or can be denoted by u, because the input of the system) works parallel with the direction of the track. The movement of cart is symboled as x.
Train mass is M, the mass of the first pendulum is m_1, and the mass of the second pendulum is m_2. L_1 is the length of the first pendulum, while the L_2 is the length of the second pendulum. θ_1 is the angle of the first pendulum to the vertical axis and θ_2 is the angle of the pendulum to the vertical axis. The moment of inertia of the pendulum rod is denoted by I.
Double pendulum system moves from the initial condition x = 0 meter, $\theta_1 = 0$ radian, and $\theta_2 = 0$ radian. Cart movements caused by the input force, make the pendulum rod swing and deviate from its equilibrium. Design of control system in this research expected able to make the pendulum rod reach its stability by moving the train horizontally as far as a maximum of two meters, either to the right or left (x = ± 2 meters).

4.1. Kinematics equation

Second derivative of the first and second pendulums position are:

$$\ddot{x}_1 = \ddot{x} + \ddot{\theta}_1 . l\cos\theta_1 - \dot{\theta}_1^{\,2} . l\sin\theta_1 \qquad (6)$$

$$\ddot{x}_2 = \ddot{x}_1 + \ddot{\theta}_1 . l\cos\theta_1 - \dot{\theta}_1^{\,2} . l\sin\theta_1 + \ddot{\theta}_2 . l\cos\theta_2 - \dot{\theta}_2^{\,2} . l\sin\theta_2 \qquad (7)$$

$$\ddot{y}_1 = \ddot{\theta}_1 . l\sin\theta_1 + \dot{\theta}_1^{\,2} . l\cos\theta_1 \qquad (8)$$

$$\ddot{y}_2 = \ddot{y}_1 + \ddot{\theta}_1 . l\sin\theta_1 + \dot{\theta}_1^{\,2} . l\cos\theta_1 + \ddot{\theta}_2 . l\sin\theta_2 + \dot{\theta}_2^{\,2} . l\cos\theta_2 \qquad (9)$$

4.2. Dynamics equation

Sum of forces on cart:

$$\frac{1}{M}(F - N_1 - b\dot{x}) = \ddot{x} \qquad (10)$$

Sum of moments and forces on the first pendulum:

$$\frac{1}{I}(N_1 l\cos\theta_1 + P_1 l\sin\theta_1 + N_2 l\cos\theta_1 + P_2 l\sin\theta_1) = \ddot{\theta}_1 \qquad (11)$$

$$N_1 = N_2 + m.\ddot{x}_1 \qquad (12)$$

$$P_1 = P_2 + mg + m.\ddot{y}_1 \qquad (13)$$

Sum of moments and forces on the second pendulum:

$$\frac{1}{I}(N_2 l\cos\theta_2 + P_2 l\sin\theta_2) = \ddot{\theta}_2 \qquad (14)$$

$$N_2 = m.\ddot{x}_2 \qquad (15)$$

$$P_2 = mg + m.\ddot{y}_2 \qquad (16)$$

4.3. Simulink model

Based on the kinematics and dynamics equations we can make a double pendulum system model using Simulink as figure 4.

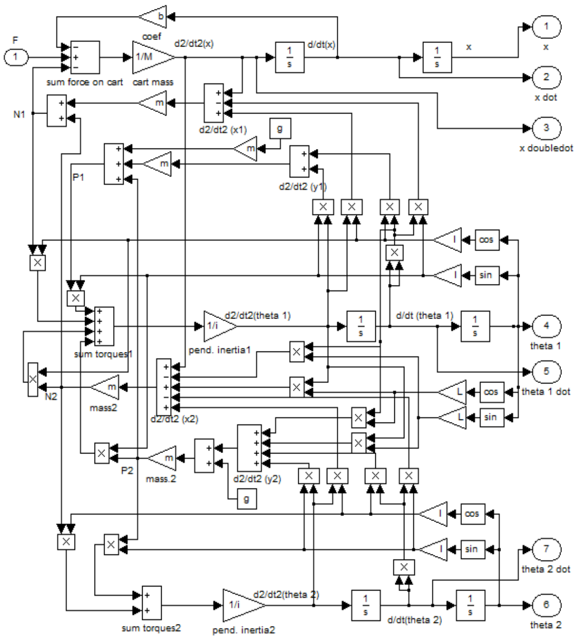

Figure 4: Simulink model of double pendulum

4.4. Model parameters

- Cart mass (M) = 0,5 kg.
- The first pendulum mass (m_1) = 0,2 kg.
- The second pendulum mass (m_2) = 0,2 kg.
- The first pendulum length (L_1) = 0,6 meter.
- The second pendulum length (L_2) = 0,6 meter.
- The moment of inertia of the first pendulum (I_1) = 0,006 kg.m^2.
- The moment of inertia of the second pendulum (I_2) = 0,006 kg.m^2.
- Acceleration of gravity (g) = 9,8 m/s^2.
- Damping coefficient of cart (b) = 0,1 N/m/sec.

5. RESULTS AND DISCUSSIONS

5.1. Analysis of open-loop simulation

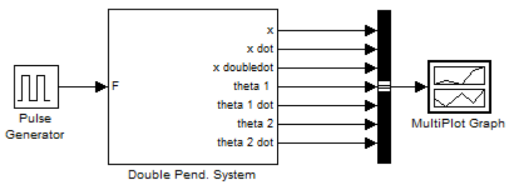

Figure 5: Open-loop system model

Responses of open-loop simulation are cart position (x), cart speed ($\dot{x}$), cart acceleration ($\ddot{x}$), angular position of the first pendulum (θ_1), angular speed of the first pendulum ($\dot{\theta}_1$), angular position of the second pendulum (θ_2), and angular speed of the second pendulum ($\dot{\theta}_2$).

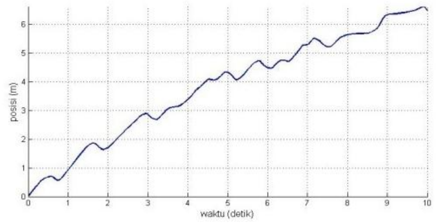

Figure 6: Response of open-loop cart position

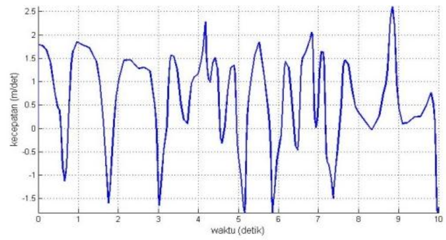

Figure 7: Response of open-loop cart speed

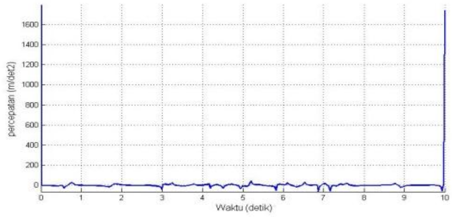

Figure 8: Response of open-loop cart acceleration

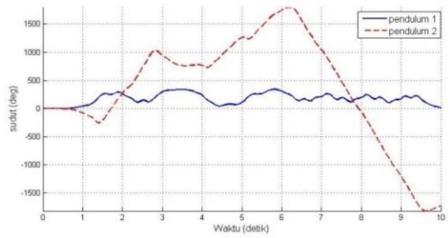

Figure 9: Response of open-loop pendulum angular position

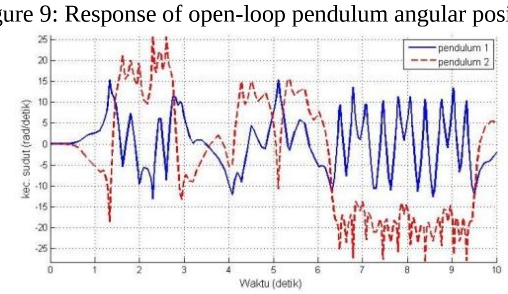

Figure 10: Response of open-loop pendulum angular speed

5.2. Analisys of closed-loop simulation with PID controller and SPID Controller

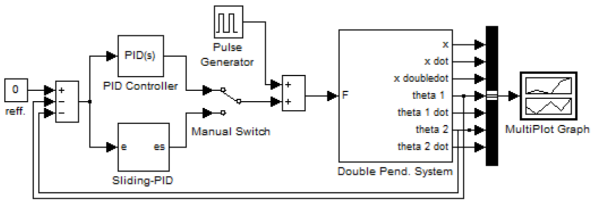

Figure 11: Closed-loop system model

Closed-loop simulations performed by adding PID controller or SPID controller on double pendulum system. PID controller is used to maintain the position of cart in order to meet the desired position, x = ± 2 meters. PID controller is also used to stabilize the movement of the pendulum rod to be able to reach the equilibrium. Sliding mode control is then added to determine the response changes that occur. The response is then compared to determine which control system has better performance.

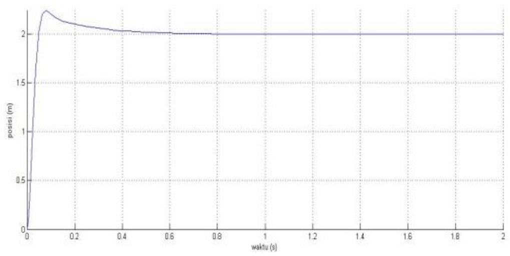

Figure 12: Response of closed-loop cart position

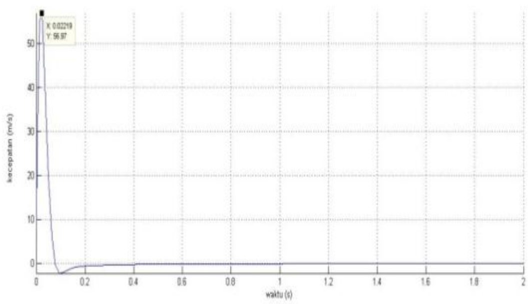

Figure 13: Response of closed-loop cart speed

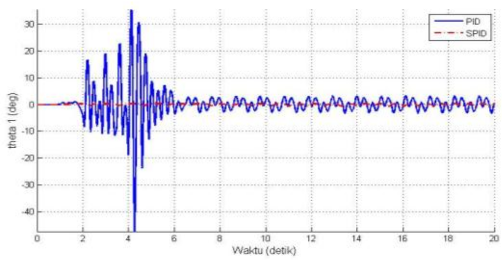

Figure 14: Response of closed-loop 1st pendulum angular position

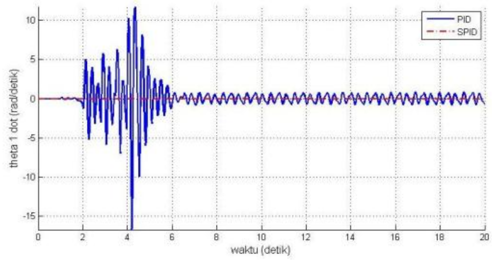

Figure 15: Response of closed-loop 1st pendulum angular speed

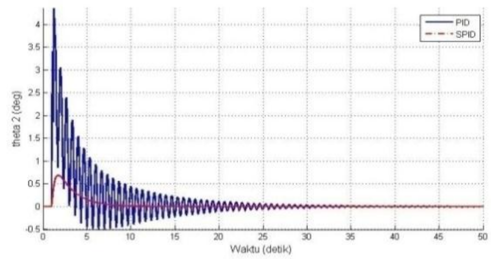

Figure 16: Response of closed-loop 2nd pendulum angular position

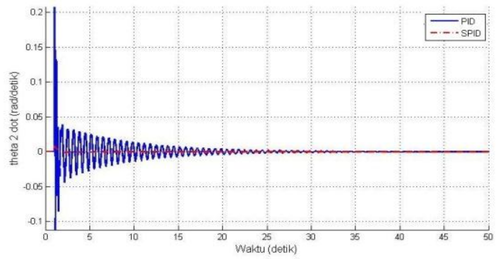

Figure 17: Response of closed-loop 2nd pendulum angular speed

5.3 Comparison of response characteristics between PID controller and SPID controller

Table 1. Response characteristic of the 1st pendulum angle

Response characteristic	PID	SPID
Rise time	1,787 s	2,470 s
Settling time	6,094 s	3,081 s
Steady state error	0,132 %	0,016 %
Maximum overshoot	0,8024 rad	0,00102 rad

Based on the data shown in Table 1, SPID has a faster settling time is equal to 3.081 seconds. While the PID has a settling time of 6.094 seconds. This means that the SPID controller takes 3.013 seconds longer to be able to stabilize the position of the first pendulum. As for the characteristics of the maximum overshoot, SPID is the best among these two types of controller.

The maximum angular displacement of the first pendulum when the PID control signal applied to the system amounted to 0.8024 radians. While the maximum angular displacement of the first pendulum when the SPID signal control applied to the system is equal to 0.00102 radians.

From table 1, we also know that the double pendulum system with PID controller has the fastest rise time, 1.787 seconds. While the double pendulum with SPID controller requires an additional 0.683 seconds. This is because SPID produced a smooth response to reduce the overshoot that occurs in PID control. For the steady state error characteristics, SPID controller has better value than PID controller.

Table 2. Response characteristics of the 2nd pendulum angle

Response characteristic	PID	SPID
Rise time	13,57 s	11,39 s
Settling time	25,70 s	14,54 s
Steady state error	0,0383 %	0,0004 %
Maximum overshoot	0,01525 rad	0,00237 rad

Based on the data shown in table 2, SPID has a faster settling time is equal to 14.54 seconds. While PID has a settling time of 25.70 seconds. This means that the PID control takes 11.16 seconds longer to be able to stabilize the position of the second pendulum. As for the characteristics of the maximum overshoot, SPID controller has the best value among these two types of controller.

The maximum angular displacement of the second pendulum when the PID control signal applied to the system amounted to 0.01525 radians. While the maximum angular displacement of the second pendulum when the SPID signal control applied is equal to 0.00237 radians.

From table 2 also can be seen that the double pendulum system with SPID controller has a faster rise time is 11.39 seconds. While the double pendulum with PID controller requires additional time 2.18 seconds. For the characteristic of steady state error, SPID controller has better value than the PID controller.

From table 1 and table 2 above, we also know that the second pendulum need longer time to reach stability. By using PID controller, the first pendulum takes 6.094 seconds for stable, while the second pendulum may take as long as 25.7 seconds. PID controller is able to produce a better response on the system. After added SPID controller, the first pendulum takes only about 3.081 seconds to stable, while the second pendulum will take approximately 14.54 seconds. Comparison of the settling time between these two types of controller to double pendulum stability is presented in table 3.

Table 3. Comparison of settling time

Settling time	PID	SPID
1st Pendulum	6,094 s	3,081 s
2nd Pendulum	25,70 s	14,54 s
Overall system	25,70 s	14,54 s

In overall, the second pendulum requires a longer time than the first pendulum to achieve stability, this is because movement of the second pendulum is more free than the first pendulum. The first pendulum movement is limited by cart and the second pendulum, while the second pendulum movement is limited only by the first pendulum rod.

Table 3 above shows that the PID controller may take as long as 25.7 seconds to stabilize the double pendulum system as a whole, while the SPID controller only takes about 14.54 seconds. In other words, SPID controller can stabilize the double pendulum system 43.42% faster than PID controller. So it can be said that the SPID control system has better performance than the PID control system.

6. CONCLUSION

From the research has been done, it can be concluded that the use of Sliding-PID controller produces a better response than just using classical PID controller. To stabilize the double pendulum system as a whole, PID controller requires 25.70 seconds. While Sliding-PID controller only requires 14.54 seconds, or 43.42 % faster than PID controller. Additionally, Sliding-PID controller is also able to reduce steady state error and the maximum overshoot occurs at classical PID controller.

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