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Rumusan Eksak Osilasi Neutrino dalam Materi dengan Kerapatan Konstan

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Exact Formula of Neutrino Oscillation in Matter with Constant Density

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Rumusan Eksak Osilasi Neutrino dalam Materi dengan Kerapatan Konstan

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ABSTRAK

Neutrino merupakan salah partikel elementer dari keluarga lepton. Ketika dipostulatkan oleh Pauli tahun 1930, neutrino diasumsikan tidak bermassa. Model standar dari fisika partikel juga memprediksi bahwa neutrino juga tidak bermassa. Tetapi , tahun 1998 percobaan di Super-Kamiokande berhasil mengamati osilasi neutrino di alam . Fenomena osilasi neutrino bisa dijelaskan hanya kalau neutrino bermassa.Sampai saat ini diketahui neutrino mempunyai tiga flavor yaitu neutrino elektron , neutrino muon , dan neutrino tauon . Keadaan eigen flavor dan keadaan eigen massa neutrino di hubungkan oleh matrik uniter MNS (Maki-Nakagawa-Sakata), yang mempunyai empat parameter yaitu tiga sudut dan satu fase CP . Dalam penelitian ini akan dikaji probabilitas osilasi neutrino tiga generasi baik dalam vakum maupun materi secara eksak.

Kata kunci : osilasi neutrino , vakum , materi

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Exact Formula of Neutrino Oscillation in Matter with Constant Density

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ABSTRACT

Neutrino is an elementary particle of lepton family. When postulated by Pauli in 1930, neutrinos were assumed massless. The standard model of particle physics predicts that neutrinos are also massless.But, in 1998, Super-Kamiokande experiment successfully observe neutrino oscillations in nature. The phenomenon of neutrino oscillations can be explained only if neutrinos have mass . Currently there are three known neutrino flavor i.e electron neutrinos, mu neutrinos, and tau neutrinos . Eigen state of flavor and eigen state of neutrino masses connected by a unitary matrix MNS (Maki-Nakagawa-Sakata), who has four parameters, namely three angles and the one phase CP . In this study will be assessed both the probability three generation of neutrino oscillation in vacuum and matter exactly.

Kata kunci : neutrino oscillation , vacuum , matter

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Rumusan Eksak Osilasi Neutrino Dalam Materi Dengan Kerapatan Konstan

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Surabaya, 10 Juli 2015

Penulis

Daftar Isi

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Bab 1

Pendahuluan

1.1 Latar belakang

Neutrino merupakan salah satu partikel elementer dari keluarga lepton (yang lain kuark) yang dipercaya sebagai penyusun alam semesta ini. Pertama kali neutrino diusulkan oleh Pauli pada tahun 1930 untuk menjelaskan fenomena peluruhan beta dan tidak bermassa. Pada tahun 1956, Cowan dan Reines menemukan anti neutrino .Neutrino merupakan partikel yang unik karena tidak bermuatan dan hanya berinteraksi lemah saja. Sampai saat ini neutrino diketahui memiliki tiga rasa (flavor) yaitu neutrino elektron, neutrino muon dan neutrino tauon.Selanjutnya model standar fisika partikel juga memprediksi kalau neutrino tidak bermassa , tetapi tahun 1998 percobaan Super-Kamiokande memberikan bukti kuat mengenai kehadiran fenomena osilasi neutrino yang berimplikasi neutrino bermassa [1].Selain itu adanya persoalan neutrino matahari yaitu terjadinya perbedaan fluks neutrino elektron yang di terima ketika sampai di bumi dengan prediksi teoritis. Fluks neutrino elektron yang diterima oleh percobaan ternyata cuma sepertiga dari prediksi teoretis.Persoalan ini bisa diselesaikan dengan osilasi neutrino dan mempunyai massa walaupun kecil.

Jauh sebelumnya osilasi neutrino sebenarnya telah diprediksi oleh Maki, dkk [2] pada tahun 1962. Maki menunjukkan osilasi neutrino dua generasi yaitu neutrino elektron ke neutrino muon dan meprediksi bahwa neutrino bermassa walaupun sangat kecil.Sekarang diketahui neutrino mempunyai tiga generasi yaitu neutrino elektron , neutrino muon , dan neutrino tauon.Karena tiga generasi tersebut , maka matrik MNS nya menjadi 3×3 yang mempunyai empat parameter yaitu tiga sudut $(\theta_{12}, \theta_{23}, \theta_{13})$ dan satu sudut fase CP. Pada penelitian sebelumnya [3–7] menyajikan bagaimana mendapatkan fase CP baik secara teoritis maupun percobaan. Sedangkan pada [8, 9] meneliti bagaimana mendapatkan fase CP dengan menggunakan reaktor neutrino berenergi tinggi, yang mana efek materi tidak dapat diabaikan.Pada tahun 2014 ketiga parameter sudut pada matrik MNS sudah berhasil dikonfirmasi oleh percobaan, tetapi fase CP masih menjadi tantangan tersendiri untuk dikonfirmasi secara percobaan [10]. Untuk kedepannya kajian mengenai neutrino masih menjadi tantangan tersendiri baik dari sisi teoritis maupun percobaan untuk memecahkan masalah terkait dengan neutrino.

1.2 Rumusan masalah

Selama ini dalam merumuskan osilasi neutrino tiga generasi akhirnya digunakan pendekatan limit degenerasi untuk dua generasi. Dalam penelitian ini permasalahan yang akan dibahas rumusan eksak osilasi neutrino tiga generasi.

1.3 Batasan masalah

Permasalahan yang akan dibahas pada penelitian ini dibatasi pada kasus neutrino dalam materi dengan kerapatan konstan.

1.4 Tujuan Penelitian

Selain memperoleh ungkapan osilasi secara eksak , juga mendapatkan bentuk osilasi yang umum. Dalam penelitian ini akan dilakukan penurunan dan eksplorasi secara lengkap bagaimana mendapatkan rumusan eksak osilasi neutrino dalam materi dengan kerapatan konstan.

1.5 Metodologi Penelitian

Penelitian ini merupakan penelitian teoritis yang dilakukan melalui penelaahan berbagai literatur berupa jurnal - jurnal ilmiah dan buku - buku teks (textbook).

1.6 Manfaat Penelitian

Deskripsi rinci mengenai penurunan rumusan eksak osilasi neutrino dalam materi dengan kerapatan konstan dapat memberikan petunjuk dalam menentukan sudut fase CP.

Bab 2

Osilasi Neutrino

Neutrino merupakan salah satu partikel elementer dari keluarga lepton. Awalnya neutrino diprediksi tidak bermassa. Tetapi, eksperimen superkamiokande pada tahun 1998 menunjukkan bahwa neutrino bermassa , walaupun massanya sangat kecil. Hal ini menjadikan neutrino sebagai partikel yang menarik untuk di kaji lebih lanjut. Dalam bab ini akan dijelaskan mengenai model standar elektrolemah dan matriks bauran untuk neutrino, baik dalam vakum maupun materi.

2.1 Model Standar Elektrolemah

2.1.1 Partikel Model Standar

Dalam fisika partikel terdapat empat gaya (interaksi) yaitu gaya gravitasi , gaya lemah, gaya elektromagnetik, dan gaya kuat dan mempunyai grup gauge $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Dari keempat gaya tersebut, tiga sudah berhasil disatukan , yang selanjutnya dikenal dengan Model Standar dari fisika partikel atau Model Standar saja. Model Standar merupakan teori yang menjelaskan penggabungan dari tiga gaya yaitu gaya elektromagnetik, gaya lemah, dan gaya kuat. Dalam Model Standar tersebut terdapat partikel - partikel elementer (partikel dasar) yang berinteraksi satu dengan yang lainnya.Partikel elementer tersebut dibagi menjadi tiga kelompok yaitu kuark (*quark*), lepton , dan boson gauge.

Kuark merupakan partikel elementer yang dipengeruhi gaya kuat dan gaya lemah. Kuark mempunyai 6 jenis (*flavor*) yaitu u (*up*),d (*down*),s (*strange*), c (*charm*),t (*top*), dan b (*bottom*). Selain itu kuark juga memiliki muatan *color* yaitu r (*red*), g (*green*), dan b (*blue*).Kelompok partikel elementer yang lainnya adalah lepton . Terdiri dari 6 jenis lepton yaitu elektron (e^-), muon (μ^-), tauon (τ^-), neutrino elektron (ν_e), neutrion muon (ν_μ), dan neutrino tau (ν_τ).Kelompok yang partikel yang terakhir adalah boson gauge, yang merupakan partikel perantara dari keempat interaksi tersebut. Boson gauge terdiri dari foton , gluon , graviton , $W^\pm$, Z^0 dan boson Higgs yang dipercaya sebagai partikel yang membangkitkan massa dari partikel - partikel lain. Berikut ini ditabelkan sifat dari kuark , lepton , dan boson gauge , meliputi massa, spin dan muatannya. [18]

Partikel	Simbol	Spin	Muatan(e)	Massa
Lepton Generasi I	e^-	$\frac{1}{2}$	-1	0.511 MeV
	ν_e		0	<2 eV
Lepton Generasi II	μ^-	$\frac{1}{2}$	-1	105.66 MeV
	ν_μ		0	<0.19 eV
Lepton Generasi III	τ^-	$\frac{1}{2}$	-1	1776.84 MeV
	ν_τ		0	<18.2 eV

Tabel 2.1: Keluarga Lepton

Partikel	Simbol	Spin	Muatan(e)	Massa
Kuark Generasi I	d	$\frac{1}{2}$	$-\frac{1}{3}$	$4.8^{+0.7}_{-0.3}$ MeV
	u		$+\frac{2}{3}$	$2.3^{+0.7}_{-0.5}$ MeV
Kuark Generasi II	c	$\frac{1}{2}$	$-\frac{1}{3}$	95 ± 5MeV
	s		$+\frac{2}{3}$	1.275 ± 0.025GeV
Kuark Generasi III	b	$\frac{1}{2}$	$-\frac{1}{3}$	4.18 ± 0.03GeV
	t		$+\frac{2}{3}$	173.5 ± 1.0GeV

Tabel 2.2: Keluarga Kuark

Partikel	Simbol	Spin	Muatan(e)	Massa
Foton	γ	1	0	0
Gluon	g	1	0	0
Boson Z	Z^0	1	0	91.1876 ± 0.0023GeV
Boson $W^\pm$	$W^\pm$	1	± 1	80.385 ± 0.015GeV
Graviton	G	2	0	0
Boson Higgs	H^0	0	0	125.7 ± 0.4GeV

Tabel 2.3: Keluarga Boson Gauge

2.1.2 Lagrangian Model Standar Elektrolemah

Model Standar Elektrolemah merupakan teori yang menyatukan gaya/interaksi elektromagnetik dan gaya lemah. Diusulkan oleh Sheldon Glashow, Abdus Salam , dan Steven Weinberg , yang ketiga dianugrahi Nobel fisika pada tahun 1979. Model Standar Elektrolemah di kenal juga dengan sebutan teori GWS.Model Standar ini memiliki grup gauge $SU(2)_L \otimes U(1)_Y$. Lagrangian lengkap untuk teori GWS ini adalah [11] [14]

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_G + \mathcal{L}_H + \mathcal{L}_Y \tag{2.1}$$

dimana $\mathcal{L}_F$ adalah lagrangian fermion yang mempunyai bentuk :

$$\mathcal{L}_F = \bar{L} i \gamma^\mu \left(\partial_\mu - i g_2 \frac{\bar{\tau}}{2} \cdot \bar{A}_\mu + \frac{i}{2} g' B_\mu \right) L + \bar{R} i \gamma^\mu \left(\partial_\mu + i g' B_\mu \right) R \tag{2.2}$$

dimana $L = \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L$,$R = l_R$, dan $l = e^-, \mu^-, \tau^-$. Suku $\left(\partial_\mu - ig_2\frac{\bar{\tau}}{2} \cdot \bar{A}_\mu + \frac{i}{2}g'B_\mu\right)$ pada persamaan (2.2) adalah kovarian derivatif dari $SU\left(2\right)$ dan suku $\left(\partial_\mu + ig'B_\mu\right)$ adalah kovarian derivatif untuk $U\left(1\right)$. Sedangkan $\mathcal{L}_G$ adalah lagrangian untuk medan gauge yang berbentuk :

$$\mathcal{L}_G = -\frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{2.3}$$

dengan

$$\begin{aligned} F^i_{\mu\nu} &= \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\epsilon_{ijk}A^j_\mu A^k_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \tag{2.4}$$

Selanjutnya, $\mathcal{L}_H$ adalah lagrangian untuk medan skalar (lagrangian medan Higgs). Disini dipilih medan skalar yang paling sederhana , yang berbentuk :

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \tag{2.5}$$

dimana φ^+ adalah medan kompleks skalar positif dan φ^0 adalah medan skalar netral. Maka bentuk lagrangian lengkap medan Higgs nya adalah :

$$\mathcal{L}_H = \left(D_\mu\phi\right)^\dagger\left(D^\mu\phi\right) - V\left(\phi^\dagger\phi\right) \tag{2.6}$$

dengan

$$D_\mu\phi = \left(\partial_\mu - ig\frac{\bar{\tau}}{2} \cdot \bar{A}_\mu - \frac{i}{2}g'B_\mu\right)\phi \tag{2.7}$$

dan

$$V\left(\phi^\dagger\phi\right) = \mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2 \qquad\qquad \mu^2 > 0 \tag{2.8}$$

Dan yang terakhir merupakan lagrangian interaksi Yukawa (interaksi medan skalar Higgs dengan medan fermion), yang berbentuk:

$$\mathcal{L}_Y = -G_e\left(\bar{L}\phi R + \bar{R}\phi^\dagger L\right) + h.c \tag{2.9}$$

dimana G_e adalah konstanta kopling Yukawa.

2.1.3 Mekanisme Higgs Pada Model Standar Elektrolemah

Salah satu mekanisme untuk pembangkitan massa partikel - partikel elementer adalah perusakan simetri spontan (*spontaneuos symmetry breaking*) via mekanisme Higgs. Boson skalar Higgs sebagai perantara dalam mekanisme ini sudah terkonfirmasi secara eksperimen di LHC , CERN pada tahun 2012 dan Peter W. Higgs sebagai pengusul mekanisme ini juga memperoleh nobel fisika pada tahun yang sama. Selanjutnya , tinjau potensial $V\left(\phi^{\dagger}\phi\right)$ pada persamaan (2.8) akan memiliki nilai minimum ϕ pada :

$$\phi^{\dagger}\phi = |\phi|^2 = \frac{v^2}{2} \qquad\qquad v = \sqrt{\frac{\mu^2}{\lambda}} \tag{2.10}$$

Selanjutnya nilai VEV pada persamaan (2.5) setelah perusakan simetri spontan adalah

$$\phi_0 = \langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \tag{2.11}$$

Sedangkan generator T_3 , Y , dan Q ketika bekerja pada ϕ_0 menghasilkan

$$\begin{aligned} T_3\phi_0 &= \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \\ &= -\frac{1}{2}\phi_0 \qquad (2.12) \\ Y\phi_0 &= \phi_0 \qquad (2.13) \\ Q\phi_0 &= \left(T_3 + \frac{Y}{2}\right)\phi_0 \\ &= \left[\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \\ &= 0 \qquad (2.14) \end{aligned}$$

Dari hasil diatas diketahui bahwa generator T_3 dan Y merupakan generator yang rusak dan generator Q meupakan generator yang tidak rusak.Dengan kata lain, bahwa teori Elekrolemah $SU\left(2\right)_L \otimes U\left(1\right)_Y$ mengalami perusakan simetri menjadi $U\left(1\right)$ dengan generator Q tidak rusak ($SU\left(2\right)_L \otimes U\left(1\right)_Y \rightarrow U\left(1\right)_{EM}$). Sekarang kita parameterisasi doublet skalar ϕ pada persamaan (2.5) dengan 4 derajat kebebasan medan real baru yang menyatakan pergeseran dari keadaan vakum ϕ_0

$$\begin{aligned} \phi &= \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \\ &= e^{i\frac{\bar{\tau}.\bar{\xi}}{2v}}\begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} \qquad (2.15) \end{aligned}$$

dimana medan skalar kompleks φ^+ dan φ^0 digantikan 4 medan real: 3 boson Goldstone (ξ_i) dan 1 boson Higgs (H). Setelah perusakan simetri spontan pada keadaan unitery gauge dengan menerapkan transfomasi uniter $SU\left(2\right)$

$$U\left(\xi\right) = e^{-i\frac{\bar{\tau}\cdot\bar{\xi}}{2v}} \tag{2.16}$$

didapatkan

$$\begin{aligned}
\phi' &= U(\xi)\,\phi \\
&= \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}(v+H)\,\chi && (2.17) \\
L' &= U(\xi)\,L && (2.18) \\
\bar{\mathbf{A}}'_\mu &= U(\xi)\,\bar{\mathbf{A}}U^{-1}(\xi) - \frac{i}{g}(\partial_\mu u(\xi))\,U^{-1}(\xi) && (2.19)
\end{aligned}$$

dimana $\chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ dan $\bar{\mathbf{A}}_\mu = \bar{A}_\mu \cdot \frac{\bar{\tau}}{2}$. Sedangkan R dan B_μ tidak berubah terhadap transformasi $SU(2)$:

$$\begin{aligned}
R' &= R && (2.20) \\
B'_\mu &= B_\mu && (2.21)
\end{aligned}$$

Akibat transformasi tersebut lagrangian fermion , lagrangian medan gauge , lagrangian medan skalar Higgs dan lagrangian interaksi Yukawa menjadi :

$$\begin{aligned}
\mathcal{L}_F &= \bar{L}i\gamma^\mu\left(\partial_\mu - ig\frac{\bar{\tau}}{2}\cdot\bar{A}'_\mu + \frac{i}{2}g'B'_\mu\right)L' + \bar{R}i\gamma^\mu\left(\partial_\mu + ig'B'_\mu\right)R' && (2.22) \\
\mathcal{L}_G &= -\frac{1}{4}F'^{i}_{\mu\nu}F'^{i\mu\nu} - \frac{1}{4}B'_{\mu\nu}B'^{\mu\nu} && (2.23) \\
\mathcal{L}_H &= (D_\mu\phi')^\dagger(D^\mu\phi') - V\left(\phi'^\dagger\phi'\right) && (2.24) \\
\mathcal{L}_Y &= -G_e\left(\bar{L}\phi' R + \bar{R}\phi'^\dagger L\right) + h.c && (2.25)
\end{aligned}$$

Sekarang perhatikan , jika ϕ' dimasukkan ke dalam lagrangian medan Higgs setelah transformasi tadi akan diperoleh :

$$\begin{aligned}
\mathcal{L}_H &= (D_\mu \phi')^\dagger (D^\mu \phi') - V\left(\phi'^\dagger \phi'\right) \\
&= \left[\left(\partial_\mu - ig\frac{\bar{\tau}}{2}.\bar{A}'_\mu - \frac{i}{2}g' B'_\mu\right)\left(\frac{1}{\sqrt{2}}(v+H)\chi\right)\right]^\dagger \\
&\quad \times \left[\left(\partial_\mu - ig\frac{\bar{\tau}}{2}.\bar{A}'_\mu - \frac{i}{2}g' B'_\mu\right)\left(\frac{1}{\sqrt{2}}(v+H)\chi\right)\right] \\
&\quad - \left[\mu^2\left(\frac{1}{\sqrt{2}}(v+H)\chi\right)^\dagger\left(\frac{1}{\sqrt{2}}(v+H)\chi\right)\right. \\
&\quad \left. +\lambda\left(\left(\frac{1}{\sqrt{2}}(v+H)\chi\right)^\dagger\left(\frac{1}{\sqrt{2}}(v+H)\chi\right)\right)^\dagger\right] \\
&= \frac{1}{2}\left[\frac{2g^2}{4}(v+H)^2 W^-_\mu W^{+\mu} + \underbrace{\partial_\mu v \partial^\mu v}_{=0} + \partial_\mu H \partial^\mu H + \underbrace{2\partial_\mu v \partial^\mu H}_{=0}\right. \\
&\quad \left. + \frac{1}{4}(v+H)^2 \left(gA'_{3\mu} - g'B'_\mu\right)\left(gA'^{3\mu} - g'B'^\mu\right)\right] \\
&\quad -\frac{\mu^2}{2}(v+H)^2 - \frac{\lambda}{4}(v+H)^4
\end{aligned} \tag{2.26}$$

Kemudian didefinisikan medan boson bermuatan baru yaitu

$$W^\pm_\mu = \frac{1}{2}\left(A'_{1\mu} \mp iA'_{2\mu}\right) \tag{2.27}$$

dari lagrangian persamaan (2.26) didapatkan massa boson vektor $W^\pm$ dan boson skalar Higgs adalah

$$M_W = \frac{1}{2}gv \quad \text{dan} \quad \mathrm{M_H} = \sqrt{2\mu^2} \tag{2.28}$$

Dari lagrangaian pada persamaan (2.26) masih tersisa suku $A'_{3\mu}, B'_\mu, A'^{3\mu}$, dan B'^μ. Kemudian suku yang tersisa tersebut di transformasi ortogonal dengan medan baru Z_μ dan A_μ diperoleh

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} A'_{3\mu} \\ B'_\mu \end{pmatrix} \tag{2.29}$$

atau inversnya

$$\begin{pmatrix} A'_{3\mu} \\ B'_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \tag{2.30}$$

dengan θ_W adalah sudut Weinberg (*Weinberg angle*)

$$\frac{(v+H)^2}{8}\begin{pmatrix} A'_{3\mu} & B'_\mu \end{pmatrix}\underbrace{\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}}_{\equiv \text{matriks} \quad \mathrm{G}}\begin{pmatrix} A'_{3\mu} \\ B'^\mu \end{pmatrix} \tag{2.31}$$

Kemudian matriks G didiagonalisasi dengan menggunakan persamaan (2.31) diperoleh :

$$
\begin{aligned}
&\frac{(v+H)^2}{8}\begin{pmatrix} Z_\mu & A_\mu \end{pmatrix}\begin{pmatrix} g^2+g'^2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \\
&= \frac{(v+H)^2}{8}\left(g^2+g'^2\right) Z_\mu Z^\mu \qquad (2.32) \\
\tan\theta_W &= \frac{g'}{g} \qquad (2.33) \\
\sin\theta_W &= \frac{g'}{\sqrt{g^2+g'^2}} \qquad (2.34) \\
\cos\theta_W &= \frac{g}{\sqrt{g^2+g'^2}} \qquad (2.35)
\end{aligned}
$$

Jika persamaan (2.31) disubstitusika ke Lagrangian medan skalar Higgs pada persamaan (2.26) diperoleh lagrangian baru pada keadaan unitary gauge adalah

$$
\begin{aligned}
\mathcal{L}_H &= \frac{1}{2}\partial_\mu H \partial^\mu H - \lambda v H^3 - \frac{\lambda}{4}H^4 + \frac{g^2}{8}\left(H^2+2Hv\right) \\
&\times \left[\frac{1}{\cos^2\theta_W} Z_\mu Z^\mu + 2W_\mu^+ W^{-\mu}\right] + \frac{1}{2}M_Z^2 W_\mu^+ W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu \qquad (2.36)
\end{aligned}
$$

Lagrangian interaksi Yukawa pada persamaan (2.25) pada keadaan unitary gauge nya adalah

$$
\begin{aligned}
\mathcal{L}_Y &= -G_e\left(\bar{L}\phi' R + \bar{R}\phi'^\dagger L\right) + h.c \\
&= -G_e\left[\begin{pmatrix} \bar{\nu}'_L & \bar{e}'_L \end{pmatrix}\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+H) \end{pmatrix} e'_R \right. \\
&\left. +\bar{e}'_R \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}(v+H) \end{pmatrix}\begin{pmatrix} \nu'_L \\ e'_L \end{pmatrix}\right] + h.c \\
&= -G_e\left[\bar{e}'_L \frac{1}{\sqrt{2}}(v+H) e'_R + \bar{e}'_R \frac{1}{\sqrt{2}}(v+H) e'_L\right] + h.c \\
&= -G_e\left[\frac{1}{\sqrt{2}}(v+H)\left(\bar{e}'_L e'_R + \bar{e}'_R e'_L\right)\right] + h.c \\
&= -\frac{G_e v}{\sqrt{2}}\bar{e}' e' - \frac{G_e}{\sqrt{2}} H \bar{e}' e' \qquad (2.37)
\end{aligned}
$$

Dari persamaan (2.37) didapatkan massa elektron sebesar :

$$m_e = \frac{G_e v}{\sqrt{2}} \qquad (2.38)$$

dan

$$\frac{G_e}{\sqrt{2}} \qquad (2.39)$$

adalah kopling interaksi antara Higgs dengan elektron. Dari persamaan (2.37) terlihat bahwa neutrino tidak bermassa , karena tidak ada suku massa medan neutrino pada persamaan tersbut.

2.1.4 Matriks CKM

Kuark merupukan partikel elementer dari keluarga fermion , berspin setengah , bermuatan pecahan muatan elektron dan memiliki enam rasa (flavor). Dalam proses - proses partikel elementer rasa kuark bisa berubah satu ke yang lainnya. Transformasi antar kuark ini di jembatani oleh matriks uniter yang dinamakan matriks CKM. Untuk mengetahui asal mula matriks tersebut, tinjau lagrangian interaksi arus elektromagnetik dan arus lemah bermuatan yang diungkapkan [24]

$$\mathcal{L}_{int} = -eA_\mu J^\mu_{em} - \frac{G_F}{\sqrt{2}} J^{\mu\dagger}_{ch} J^{ch}_\mu \tag{2.40}$$

dimana J^μ_{em} adalah arus elektromagnetik

$$J^\mu_{em} = -\bar{e}\gamma^\mu e + \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d + ... \tag{2.41}$$

J^μ_{ch} adalah arus lemah bermuatan

$$J^\mu_{ch} = \bar{\nu}_e\gamma^\mu (1+\gamma_5) e + \bar{u}\gamma^\mu (1+\gamma_5) d + ... \tag{2.42}$$

dan $G_F \simeq 1,166 \times 10^{-5} GeV^{-2}$ adalah konstanta Fermi. Setelah transformasi $SU(2)_L \times U(1)_Y$ didapatkan

$$\mathcal{L}'_{int} = -\frac{g_2}{\sqrt{8}}\left(W^+_\mu J^\mu_{ch} + W^-_\mu J^{\mu\dagger}_{ch}\right) - g_2 W^3_\mu J^\mu_{W3} - g_1 B_\mu \left(J^\mu_{em} - J^\mu_{W3}\right) \tag{2.43}$$

diaman J^μ_{W3} adalah komponen ketiga arus lemah isospin.

$$\vec{J}^\mu_W = \sum_{\psi_L} \bar{\psi}_L \gamma^\mu \frac{\vec{\tau}}{2}\psi_L \tag{2.44}$$

substitusi B_μ dan W^3_μ didapatkan

$$\mathcal{L}'_{int} = -\frac{g_2}{\sqrt{8}}\left(W^+_\mu J^\mu_{ch} + W^-_\mu J^{\mu\dagger}_{ch}\right) - g_1 \cos\theta_W A_\mu J^\mu_{em} + \mathcal{L}_{intl-wk} \tag{2.45}$$

dimana lagrangian arus lemah netral untuk fermion adalah

$$\begin{aligned} \mathcal{L}^{(f)}_{ntl-wk} &= -\frac{g_2}{2\cos\theta_W} Z^\mu \bar{f}\left(g^{(f)}_v \gamma_\mu + g^{(f)}_a \gamma_\mu\gamma_5\right) f \\ g^{(f)}_v &\equiv T^{(f)}_{W3} - 2\sin^2\theta_W Q^{(f)}_{el}, \qquad g^{(f)}_a \equiv T^{(f)}_{W3} \end{aligned} \tag{2.46}$$

dengan kopling vektor dan vektor axial adalah

$$\begin{aligned} g^{(e,\mu,\tau)}_v &= -\frac{1}{2} + 2\sin^2\theta_W, \qquad g^{(e,\mu,\tau)}_a = -\frac{1}{2} \\ g^{(u,c,t)}_v &= \frac{1}{2} - \frac{4}{3}\sin^2\theta_W, \qquad g^{(u,c,t)}_a = -\frac{1}{2} \\ g^{(d,s,b)}_v &= -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W, \qquad g^{(d,s,b)}_a = -\frac{1}{2} \\ g^{(\nu_e,\nu_\mu,\nu_\tau)}_v &= \frac{1}{2}, \qquad g^{(\nu_e,\nu_\mu,\nu_\tau)}_a = -\frac{1}{2} \end{aligned} \tag{2.47}$$

selanjutnya, untuk bauran antar generasi kuark dalam sistem arus lemah kuark bermuatan mempunyai bentuk

$$\begin{aligned} J^{\mu}_{ch}(qk) &= 2\bar{u}'_{L,\alpha}\gamma^{\mu}d'_{L,\alpha} \\ &= 2\bar{u}_{L,\alpha}\gamma^{\mu}V_{\alpha\beta}d_{L,\beta} \end{aligned} \quad (2.48)$$

dengan

$$V \equiv S_L^{u\dagger}S_L^d \quad (2.49)$$

matriks bauran kuak V adalah perkalian dari dua matrik uniter. Untuk kasus 2 generasi atau disebut juga matriks *Cabibbo* yang berbentuk

$$V = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \quad (2.50)$$

dengan paramater θ_C adalah *sudut Cabibbo*. Secara umum dituliskan

$$\begin{pmatrix} d_C \\ s_C \end{pmatrix} \equiv V \begin{pmatrix} d \\ s \end{pmatrix} \quad (2.51)$$

untuk kasus 3 generasi , matriks bauran ini membaurkan kuark (u,d,c,s,b,t) berbentuk

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad (2.52)$$

atau secara umum dituliskan

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (2.53)$$

matriks bauran tiga generassi ini memiliki tiga paramater yaitu tiga sudut bauran $\theta_{12}, \theta_{23}, \theta_{13}$ dan sebuah fase kompleks δ.Matriks bauran tiga generasi ini disebut juga matriks *Kobayashi-Maskawa* atau matriks CKM.Dalam parameterisasi standar matriks V berbentuk

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (2.54)$$

Sedangkan untuk parametrisasi $Wolfenstein$, juga mempunyai empat parameter yaitu λ, A, ρ, η didefiniskan

$$s_{12} \equiv \lambda, \qquad s_{23} \equiv A\lambda^2, \qquad s_{13}e^{-i\delta} \equiv A\lambda^3(\rho - i\eta) \quad (2.55)$$

matrik V berbentuk

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{4} & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda + \frac{A^2\lambda^5}{2}(1 - 2(\rho + i\eta)) & 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{4}(1 + 4A^2) & \lambda^2 A \\ \lambda^3 A(1 - \bar{\rho} - i\bar{\eta}) & -\lambda A + \frac{A\lambda^4}{2}(1 - 2(\rho + i\eta)) & 1 - \frac{A^2\lambda^2}{2} \end{pmatrix} \quad (2.56)$$

dengan $\bar{\rho} \equiv \rho(1 - \lambda^2/2)$ dan $\bar{\eta} \equiv \eta(1 - \lambda^2/2)$

2.2 Matriks Bauran Untuk Neutrino

Osilasi neutrino dapat menjelaskan defisit neutrino yaitu rasio antara nilai hasil pengamatan dan nilai perkiraan yang kurang dari satu.Berikut ini dijelaskan matrik bauran neutrino untuk dua maupun tiga generasi , baik dalam vakum maupun dalam materi. [13] [17]

2.2.1 Dalam Vakum

Untuk menggambarkan dinamika kuantum suatu partikel digunakan persamaan Schrodinger .Dalam persamaan Schrodinger untuk dua partikel , hamiltonian merupakan operator energi yang berbentuk matriks 2×2 dan fungsi keadaanya berbentuk matriks 2×1. Secara matemaris dituliskan

$$\begin{aligned} i\frac{d}{dt}\left|\nu_\alpha\right\rangle &= H\left|\nu_\alpha\right\rangle \\ i\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} &= \begin{pmatrix} H_{ee} & H_{e\mu} \\ H_{\mu e} & H_{\mu\mu} \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \end{aligned} \tag{2.57}$$

kemudian sisipkan $UU^\dagger = 1$ dan kalikan dengan $U^\dagger$ dari kiri

$$\begin{aligned} i\frac{d}{dt}\left|\nu_\alpha\right\rangle &= HUU^\dagger\left|\nu_\alpha\right\rangle \\ i\frac{d}{dt}U^\dagger\left|\nu_\alpha\right\rangle &= U^\dagger HUU^\dagger\left|\nu_\alpha\right\rangle \\ i\frac{d}{dt}\left|\nu'\right\rangle &= H_d\left|\nu'\right\rangle \end{aligned} \tag{2.58}$$

dimana

$$\left|\nu'\right\rangle = U^\dagger\left|\nu_\alpha\right\rangle \tag{2.59}$$

Untuk kasus dua generasi , tinjau osilasi neutrino elektron ke neutrino muon dengan fungsi gelombang flavor ν_e dan ν_μ.Misalkan , keadaan eigen massanya , ν_1 dan ν_2 , maka didapatkan hubungan :

$$\begin{aligned} \nu_1 &= \cos\theta\nu_e - \sin\theta\nu_\mu \\ \nu_2 &= \sin\theta\nu_e + \cos\theta\nu_\mu \end{aligned} \tag{2.60}$$

dan invernya

$$\begin{aligned} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \\ &= U(\theta)\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \end{aligned} \tag{2.61}$$

atau dapat dituliskan dengan lebih ringkas

$$\left|\nu_\alpha\right\rangle = \sum_{i=1}^{2} U_{\alpha i}\left|\nu_i\right\rangle \tag{2.62}$$

dan

$$\langle \nu_\alpha | = \sum_{i=1}^{2} \langle \nu_i | U_{\alpha i}^* \tag{2.63}$$

kebergantungan waktu

$$|\nu_\alpha (t)\rangle = \sum_{i=1}^{2} U_{\alpha i} e^{-iE_i t} |\nu_i\rangle \tag{2.64}$$

karena neutrino merupakan partikel relativistik , maka memenuhi hubungan energi momentum relativistik , yaitu :

$$E_i = \sqrt{m_i^2 c^4 + p^2 c^2} \tag{2.65}$$

maka ekspansinya

$$\begin{aligned} E_i &= p\sqrt{1 + \frac{m_i^2}{p^2}} \\ &\cong p\left(1 + \frac{m_i^2}{2p^2} + \frac{m_i^4}{4p^4} + ...\right) \end{aligned} \tag{2.66}$$

karena massa neutrino sangat kecil , mka didapatkan hubungan

$$\begin{aligned} E_i &\cong p\left(1 + \frac{m_i^2}{2p^2}\right) \\ &= p + \frac{m_i^2}{2p} \end{aligned} \tag{2.67}$$

dengan menganggap momentum hampir sama dengan energinya , sehingga

$$E_i \cong E + \frac{m_i^2}{2E} \tag{2.68}$$

maka amplitudo survival nutrino elektron ke neutrino elektron adalah

$$\begin{aligned} A_{(\nu_e \to \nu_e)}(t) &= \langle \nu_e | \nu_e (t)\rangle \\ &= \sum_{j=1}^{2} \langle \nu_j | U_{ej}^* \left(\sum_{i=1}^{2} U_{ei} e^{-iE_i t} |\nu_i\rangle \right) \\ &= \sum_{i,j=1}^{2} U_{ej}^* U_{ei} e^{-iE_i t} \langle \nu_j | \nu_i \rangle \\ &= \sum_{i} U_{ei}^* U_{ei} e^{-iE_i t} \end{aligned} \tag{2.69}$$

dan probabilitas survivalnya $P_{\nu_e \to \nu_e}(t)$ adalah

$$\begin{aligned} P_{\nu_e \to \nu_e}(t) &= \left|A_{(\nu_e \to \nu_e)}(t)\right|^2 \\ &= \left|\sum_i U_{ei}^* U_{ei} e^{-iE_i t}\right|^2 \\ &= \left(\sum_i U_{ei}^* U_{ei} e^{-iE_i t}\right)\left(\sum_j U_{ej} U_{ej}^* e^{iE_j t}\right) \\ &= \sum_{i,j} |U_{ei}|^2 |U_{ej}|^2 e^{-i(E_i - E_j)t} \end{aligned} \tag{2.70}$$

jika indeks i dan j dijalankan , diperoleh

$$\begin{aligned} P_{\nu_e \to \nu_e}(t) &= \sum_i \left\{|U_{ei}|^2 |U_{e1}|^2 e^{-i(E_i - E_1)t} + |U_{ei}|^2 |U_{e2}|^2 e^{-i(E_i - E_2)t}\right\} \\ &= |U_{e1}|^2 |U_{e1}|^2 + |U_{e1}|^2 |U_{e2}|^2 e^{-i(E_1 - E_2)t} \\ &+ |U_{e2}|^2 |U_{e1}|^2 e^{-i(E_2 - E_1)t} + |U_{e2}|^2 |U_{e2}|^2 \\ &= |U_{e1}|^4 |U_{e2}|^4 + |U_{e1}|^2 |U_{e2}|^2 \left(e^{-(E_1 - E_2)t} + e^{-(E_2 - E_1)t}\right) \\ &= |U_{e1}|^4 |U_{e2}|^4 + |U_{e1}|^2 |U_{e2}|^2 \left(e^{-(E_1 - E_2)t} + e^{(E_1 - E_2)t}\right) \\ &= |U_{e1}|^4 |U_{e2}|^4 + 2|U_{e1}|^2 |U_{e2}|^2 \cos(E_1 - E_2)t \end{aligned} \tag{2.71}$$

jika

$$U_{\alpha i} = U(\theta) = \begin{pmatrix} U_{e1} & U_{e2} \\ U_{\mu 1} & U_{\mu 2} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2.72}$$

jika persamaan (2.72) dimasukkan ke persamaan (2.71) didapatkan

$$\begin{aligned} P_{\nu_e \to \nu_e}(t) &= \cos^4\theta + \sin^4\theta + 2\cos^2\theta \sin^2\theta \cos\left(\frac{m_1^2 - m_2^2}{2E} t\right) \\ &= \cos^2\theta\left(1 - \sin^2\theta\right) + \sin^2\theta\left(1 - \cos^2\theta\right) + \frac{1}{2}\sin^2 2\theta \cos\left(\frac{\Delta m_{12}^2}{2E} t\right) \\ &= \cos^2\theta - \sin^2\theta\cos^2\theta + \sin^2\theta - \sin^2\theta\cos^2\theta + \frac{1}{2}\sin^2 2\theta \cos\left(\frac{\Delta m_{12}^2}{2E} t\right) \\ &= \cos^2\theta + \sin^2\theta - 2\sin^2\theta\cos^2\theta + \frac{1}{2}\sin^2 2\theta \cos\left(\frac{\Delta m_{12}^2}{2E} t\right) \\ &= 1 - \frac{1}{2}\sin^2 2\theta + \frac{1}{2}\sin^2 2\theta \cos\left(\frac{\Delta m_{12}^2}{2E} t\right) \\ &= 1 - \frac{1}{2}\sin^2 2\theta \left(1 - \cos\left(\frac{\Delta m_{12}^2}{2E} t\right)\right) \\ &= 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m_{12}^2}{4E} t\right) \end{aligned} \tag{2.73}$$

karena $t \approx L$, sehingga

$$P_{\nu_e \to \nu_e}(L) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{12}^2}{4E} L \right) \tag{2.74}$$

sedangkan untuk probabilitas transisi neutrino elektron ke neutrino muon, terlebih dahulu kita hitung amplitudo transisinya , yaitu :

$$\begin{aligned} A_{(\nu_e \to \nu_\mu)}(t) &= \langle \nu_\mu | \nu_e(t) \rangle \\ &= \sum_{j=1}^{2} \langle \nu_j | U_{\mu j}^* \left(\sum_{i=1}^{2} U_{ei} e^{-iE_i t} | \nu_i \rangle \right) \\ &= \sum_{i,j=1}^{2} U_{\mu j}^* U_{ei} e^{-iE_i t} \langle \nu_j | \nu_i \rangle \\ &= \sum_i U_{\mu i}^* U_{ei} e^{-iE_i t} \end{aligned} \tag{2.75}$$

maka probabilitas transisinya $P_{\nu_e \to \nu_\mu}(t)$ adalah

$$\begin{aligned} P_{\nu_e \to \nu_\mu}(t) &= \left| A_{(\nu_e \to \nu_\mu)}(t) \right|^2 \\ &= \left| \sum_i U_{\mu i}^* U_{ei} e^{-iE_i t} \right|^2 \\ &= \left(\sum_i U_{\mu i}^* U_{ei} e^{-iE_i t} \right) \left(\sum_j U_{\mu j} U_{ej}^* e^{iE_j t} \right) \\ &= \sum_{i,j} U_{\mu i}^* U_{ei} U_{\mu j} U_{ej}^* e^{-i(E_i - E_j)t} \end{aligned} \tag{2.76}$$

jika indeks i dan j dijalankan , diperoleh

$$\begin{aligned} P_{\nu_e \to \nu_\mu}(t) &= \sum_i \left(U_{\mu i}^* U_{ei} U_{\mu 1} U_{e1}^* e^{-(E_i - E_1)t} + U_{\mu i}^* U_{ei} U_{\mu 2} U_{e2}^* e^{-(E_i - E_2)t} \right) \\ &= |U_{\mu 1}|^2 |U_{e1}|^2 + U_{\mu 1}^* U_{e1} U_{\mu 2} U_{e2}^* e^{-(E_1 - E_2)t} \\ &+ U_{\mu 2}^* U_{e2} U_{\mu 1} U_{e1}^* e^{-(E_2 - E_1)t} + |U_{\mu 2}|^2 |U_{e2}|^2 \end{aligned} \tag{2.77}$$

dengan memasukkan persamaan (2.72) ke persamaan (2.77) didapatkan

$$\begin{aligned}
P_{\nu_e\to\nu_\mu}(t) &= \sin^2\theta\cos^2\theta + (-\sin\theta)\cos\theta\cos\theta\sin\theta e^{-i(E_1-E_2)t} \\
&= \cos\theta\sin\theta(-\sin\theta)\cos\theta e^{-i(E_2-E_1)t} + \cos^2\theta\sin^2\theta \\
&= 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta\left(e^{-i(E_1-E_2)t} + e^{i(E_1-E_2)t}\right) \\
&= \frac{1}{2}\sin^2 2\theta - \sin^2\theta\cos^2\theta 2\cos\left(\frac{m_1^2-m_2^2}{2E}t\right) \\
&= \frac{1}{2}\sin^2 2\theta\left(1-\cos\left(\frac{\Delta m_{12}^2}{2E}t\right)\right) \\
&= \frac{1}{2}\sin^2 2\theta 2\sin^2\left(\frac{\Delta m_{12}^2}{4E}t\right) \\
&= \sin^2 2\theta\sin^2\left(\frac{\Delta m_{12}^2}{4E}t\right) \qquad (2.78)
\end{aligned}$$

jika $t \cong L$, maka

$$P_{\nu_e\to\nu_\mu}(L) = \sin^2 2\theta\sin^2\left(\frac{\Delta m_{12}^2}{4E}L\right) \qquad (2.79)$$

terlihat bahwa

$$P_{\nu_e\to\nu_e}(L) + P_{\nu_e\to\nu_\mu}(L) = 1 \qquad (2.80)$$

Dari persamaan (2.74) terlihat kalau neutrino tidak bermassa maka nilai probabilitas survival neutrino akan konstan yang artinya tidak terjadi osilasi pada neutrino. Sehingga dapat dikatakan bahwa osilasi neutrino mensyaratkan neutrino bermassa dan mengalami nondegenerasi massa.

Untuk kasus tiga generasi, neutrino mempunyai tiga flavor yaitu neutrino elektron (ν_e), neutrino muon (ν_μ) , dan neutrino tauon (ν_τ).Keadaan eigen massanya dimisalkan ν_1, ν_2, dan ν_3 atau dalam bentuk matrik dapat dituliskan

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}\nu_{e1} & \nu_{e2} & \nu_{e3}\\ \nu_{\mu1} & \nu_{\mu2} & \nu_{\mu3}\\ \nu_{\tau1} & \nu_{\tau2} & \nu_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} \qquad (2.81)$$

Serupa dengan kasus dua generasi , amplitudo transisi dari ν_α ke ν_β, dimana $\alpha, \beta = e, \mu, \tau$ adalah

$$\begin{aligned}
A_{\nu_\alpha\to\nu_\beta}(t) &= \langle\nu_\beta|\nu_\alpha(t)\rangle \\
&= \sum_j \langle\nu_j| U_{\beta j}^*\left(\sum_i U_{\alpha i}e^{-iE_it}|\nu_i\rangle\right) \\
&= \sum_{i,j} U_{\beta j}^* U_{\alpha i}e^{-iE_it}\langle\nu_j|\nu_i\rangle \\
&= \sum_i U_{\beta i}^* U_{\alpha i}e^{-iE_it} \qquad (2.82)
\end{aligned}$$

sehingga probabilitas transisinya

$$\begin{aligned} P_{\nu_\alpha\to\nu_\beta}(t) &= \left|A_{\nu_\alpha\to\nu_\beta}\right|^2 \\ &= \langle\nu_\beta|\nu_\alpha(t)\rangle^\dagger \langle\nu_\beta|\nu_\alpha(t)\rangle \\ &= \left(\sum_j U^*_{\beta j}U_{\alpha j}e^{-iE_jt}\right)^\dagger \left(\sum_i U^*_{\beta i}U_{\alpha i}e^{-iE_it}\right) \\ &= \sum_{i,j} U_{\beta j}U^*_{\alpha j}U^*_{\beta i}U_{\alpha_i}e^{-i(E_i-E_j)t} \end{aligned} \quad (2.83)$$

Indeks i dan j dijalankan , jika indeks j dijalankan terlebih dahulu , maka

$$\begin{aligned} P_{\nu_\alpha\to\nu_\beta}(t) &= \sum_i \big(U_{\beta 1}U^*_{\alpha 1}U^*_{\beta i}U_{\alpha i}e^{-i(E_i-E_1)t} \\ &\quad +U_{\beta 2}U^*_{\alpha 2}U^*_{\beta i}U_{\alpha i}e^{-i(E_i-E_2)t} \\ &\quad +U_{\beta 3}U^*_{\alpha 3}U^*_{\beta i}U_{\alpha i}e^{-i(E_i-E_1)t}\big) \\ &= U_{\beta 1}U^*_{\alpha 1}U^*_{\beta 1}U_{\alpha 1}e^{-i(E_1-E_1)t} + U_{\beta 1}U^*_{\alpha 1}U^*_{\beta 2}U_{\alpha 2}e^{-i(E_2-E_1)t} \\ &\quad +U_{\beta 1}U^*_{\alpha 1}U^*_{\beta 3}U_{\alpha 3}e^{-i(E_3-E_1)t} + U_{\beta 2}U^*_{\alpha 2}U^*_{\beta 1}U_{\alpha 1}e^{-i(E_1-E_2)t} \\ &\quad +U_{\beta 2}U^*_{\alpha 2}U^*_{\beta 2}U_{\alpha 2}e^{-i(E_2-E_2)t} + U_{\beta 2}U^*_{\alpha 2}U^*_{\beta 3}U_{\alpha 3}e^{-i(E_3-E_2)t} \\ &\quad +U_{\beta 3}U^*_{\alpha 3}U^*_{\beta 1}U_{\alpha 1}e^{-i(E_1-E_3)t} + U_{\beta 3}U^*_{\alpha 3}U^*_{\beta 2}U_{\alpha 2}e^{-i(E_2-E_3)t} \\ &\quad +U_{\beta 3}U^*_{\alpha 3}U^*_{\beta 3}U_{\alpha 3}e^{-i(E_3-E_3)t} \\ &= \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 \\ &\quad +U_{\beta 1}U^*_{\alpha 1}U^*_{\beta 2}U_{\alpha 2}e^{-i(E_2-E_1)t} + U_{\beta 2}U^*_{\alpha 2}U^*_{\beta 1}U_{\alpha 1}e^{-i(E_1-E_2)t} \\ &\quad +U_{\beta 1}U^*_{\alpha 1}U^*_{\beta 3}U_{\alpha 3}e^{-i(E_3-E_1)t} + U_{\beta 3}U^*_{\alpha 3}U^*_{\beta 1}U_{\alpha 1}e^{-i(E_1-E_3)t} \\ &\quad +U_{\beta 2}U^*_{\alpha 2}U^*_{\beta 3}U_{\alpha 3}e^{-i(E_3-E_2)t} + U_{\beta 3}U^*_{\alpha 2}U^*_{\beta 2}U_{\alpha 2}e^{-i(E_2-E_3)t} \end{aligned} \quad (2.84)$$

maka

$$\begin{aligned} P_{\nu_\alpha\to\nu_\beta}(t) &= \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 \\ &\quad +2Re\left\{U_{\beta 1}U^*_{\alpha 1}U^*_{\beta 2}U_{\alpha 2}e^{-i(E_2-E_1)t}\right\} \\ &\quad +2Re\left\{U_{\beta 1}U^*_{\alpha 1}U^*_{\beta 3}U_{\alpha 3}e^{-i(E_3-E_1)t}\right\} \\ &\quad +2Re\left\{U_{\beta 2}U^*_{\alpha 2}U^*_{\beta 3}U_{\alpha 3}e^{-i(E_3-E_2)t}\right\} \end{aligned}$$

$$
\begin{aligned}
&= \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 \\
&+2Re\left(U_{\beta 1}U_{\alpha 1}^{*}U_{\beta 2}^{*}U_{\alpha 2}\right)\cos\left(E_2-E_1\right)t \\
&+2Im\left(U_{\beta 1}U_{\alpha 1}^{*}U_{\beta 2}^{*}U_{\alpha 2}\right)\sin\left(E_2-E_1\right)t \\
&+2Re\left(U_{\beta 1}U_{\alpha 1}^{*}U_{\beta 3}^{*}U_{\alpha 3}\right)\cos\left(E_3-E_1\right)t \\
&+2Im\left(U_{\beta 1}U_{\alpha 1}^{*}U_{\beta 3}^{*}U_{\alpha 3}\right)\sin\left(E_3-E_1\right)t \\
&+2Re\left(U_{\beta 2}U_{\alpha 2}^{*}U_{\beta 3}^{*}U_{\alpha 3}\right)\cos\left(E_3-E_2\right)t \\
&+2Im\left(U_{\beta 2}U_{\alpha 2}^{*}U_{\beta 3}^{*}U_{\alpha 3}\right)\sin\left(E_3-E_2\right)t
\end{aligned} \tag{2.85}
$$

Untuk $\alpha = \beta$, maka

$$
\begin{aligned}
P_{\nu_\alpha \to \nu_\alpha}(t) &= \sum_i |U_{\alpha i}|^4 \\
&+2|U_{\alpha 1}|^2|U_{\alpha 2}|^2\cos\left(E_2-E_1\right)t \\
&+2|U_{\alpha 1}|^2|U_{\alpha 3}|^2\cos\left(E_3-E_1\right)t \\
&+2|U_{\alpha 2}|^2|U_{\alpha 3}|^2\cos\left(E_3-E_2\right)t
\end{aligned} \tag{2.86}
$$

Sehingga

$$
\begin{aligned}
P_{\nu_\alpha \to \nu_\alpha}(L) &= \sum_i |U_{\alpha i}|^4 \\
&+2|U_{\alpha 1}|^2|U_{\alpha 2}|^2\cos\left\{\left(\frac{m_2^2-m_1^2}{2E}\right)L\right\} \\
&+2|U_{\alpha 1}|^2|U_{\alpha 3}|^2\cos\left\{\left(\frac{m_3^2-m_1^2}{2E}\right)L\right\} \\
&+2|U_{\alpha 2}|^2|U_{\alpha 3}|^2\cos\left\{\left(\frac{m_3^2-m_2^2}{2E}\right)L\right\} \\
&= \sum_i |U_{\alpha i}|^4 \\
&+2|U_{\alpha 1}|^2|U_{\alpha 2}|^2\cos\left\{\left(\frac{\Delta m_{21}^2}{2E}\right)L\right\} \\
&+2|U_{\alpha 1}|^2|U_{\alpha 3}|^2\cos\left\{\left(\frac{\Delta m_{31}^2}{2E}\right)L\right\} \\
&+2|U_{\alpha 2}|^2|U_{\alpha 3}|^2\cos\left\{\left(\frac{\Delta m_{32}^2}{2E}\right)L\right\}
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_i |U_{\alpha i}|^4 \\
&\quad +2|U_{\alpha 1}|^2|U_{\alpha 2}|^2\left\{1-2\sin^2\left(\frac{\Delta m_{21}^2}{4E}L\right)\right\} \\
&\quad +2|U_{\alpha 1}|^2|U_{\alpha 3}|^2\left\{1-2\sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right)\right\} \\
&\quad +2|U_{\alpha 2}|^2|U_{\alpha 3}|^2\left\{1-2\sin^2\left(\frac{\Delta m_{32}^2}{4E}L\right)\right\} \\
&= \sum_i |U_{\alpha i}|^4 \\
&\quad +2|U_{\alpha 1}|^2|U_{\alpha 2}|^2-4|U_{\alpha 1}|^2|U_{\alpha 2}|^2\sin^2\left(\frac{\Delta m_{21}^2}{4E}L\right) \\
&\quad +2|U_{\alpha 1}|^2|U_{\alpha 3}|^2-4|U_{\alpha 1}|^2|U_{\alpha 3}|^2\sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \\
&\quad +2|U_{\alpha 2}|^2|U_{\alpha 3}|^2-4|U_{\alpha 2}|^2|U_{\alpha 3}|^2\sin^2\left(\frac{\Delta m_{32}^2}{4E}L\right) \\
&= \left(\sum_i |U_{\alpha i}|^2\right)\left(\sum_i |U_{\alpha i}|^2\right) \\
&\quad -4|U_{\alpha 1}|^2|U_{\alpha 2}|^2\sin^2\left(\frac{\Delta m_{21}^2}{4E}L\right) \\
&\quad -4|U_{\alpha 1}|^2|U_{\alpha 3}|^2\sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \\
&\quad -4|U_{\alpha 2}|^2|U_{\alpha 3}|^2\sin^2\left(\frac{\Delta m_{32}^2}{4E}L\right)
\end{aligned} \tag{2.87}
$$

dengan menggunakan hubungan unitaritas

$$
\sum_i |U_{\alpha i}|^2 = |U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 + |U_{\alpha 3}|^2 = 1 \tag{2.88}
$$

maka diperoleh (dengan menggunakan hirarki normal massa neutrino)

$$
\begin{aligned}
P_{\nu_\alpha \to \nu_\alpha}(L) &= 1-4|U_{\alpha 1}|^2|U_{\alpha 2}|^2\sin^2\left(\frac{\Delta m_{21}^2}{4E}L\right) \\
&\quad -4|U_{\alpha 1}|^2|U_{\alpha 3}|^2\sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \\
&\quad -4|U_{\alpha 2}|^2|U_{\alpha 3}|^2\sin^2\left(\frac{\Delta m_{32}^2}{4E}L\right)
\end{aligned}
$$

$$\begin{aligned} &= 1-4\left|U_{\alpha1}\right|^2\left|U_{\alpha3}\right|^2\sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \\ &\quad -4\left|U_{\alpha2}\right|^2\left|U_{\alpha3}\right|^2\sin^2\left(\frac{\Delta m_{32}^2}{4E}L\right) \\ &= 1-4\left|U_{\alpha3}\right|^2\left(\left|U_{\alpha1}\right|^2+\left|U_{\alpha2}\right|^2\right)\sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \\ &= 1-4\left|U_{\alpha3}\right|^2\left(1-\left|U_{\alpha3}\right|^2\right)\sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \end{aligned} \quad (2.89)$$

dengan $\left|U_{\alpha3}\right|^2=\cos^2\theta$ sehingga didapatkan

$$\begin{aligned} P_{\nu_\alpha\to\nu_\alpha}(L) &= 1-4\cos^2\theta\left(1-\cos^2\theta\right)\sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \\ &= 1-4\cos^2\theta\sin^2\theta\sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \\ &= 1-\sin^2 2\theta\sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \end{aligned} \quad (2.90)$$

Dari persamaan (2.90) terlihat bahwa persoalan osilasi neutrino tiga generasi bisa tereduksi menjadi dua generasi.

2.2.2 Dalam Materi

Selain berosilasi dalam vakum , neutrino juga bisa berosilasi didalam materi . Contohnya adalah ketika neutrino elektron yang diproduksi dari reaksi termonuklir di inti matahari kemudian berosilasi menembus mantel sampai ke permukaan matahari . Untuk mendapatkan rumusan osilasi neutrino dalam materi , tinjau persamaan Schrodinger

$$i\frac{\partial\left|\psi(t)\right\rangle}{\partial t}=H_0\left|\psi(t)\right\rangle \quad (2.91)$$

dimana H_0 adalah Hamiltonian bebas. Sedangkan $\left|\psi(t)\right\rangle$ dapat diekspansikan terhadap seluruh keadaan total sistem flavor neutrino ν_α dengan momentum $\mathbf{p}$,sehingga

$$\left|\psi(t)\right\rangle=\sum_\alpha a_\alpha(t)\left|\nu_\alpha\right\rangle \quad (2.92)$$

dengan $a_\alpha(t)=\left\langle\nu_\alpha|\psi(t)\right\rangle$ adalah amplitudo probabilitas menemukan ν_α pada keadaan $\psi(t)$.Jika persamaan (2.92) dimasukkan ke persamaan (2.91) diperoleh

$$i\frac{\partial}{\partial t}\sum_\alpha a_\alpha(t)\left|\nu_\alpha\right\rangle=H_0\sum_\alpha a_\alpha(t)\left|\nu_\alpha\right\rangle \quad (2.93)$$

Jika persamaan (2.93) dikalikan dari kiri dengan $\langle \nu_{\alpha'}|$, diperoleh

$$i\frac{\partial}{\partial t}a_{\alpha}(t)\langle \nu_{\alpha'}|\nu_{\alpha}\rangle = \sum_{\alpha}\langle \nu_{\alpha'}|H_0|\nu_{\alpha}\rangle a_{\alpha}(t) \tag{2.94}$$

dan dengan menggunakan hubungan

$$\langle \nu_{\alpha'}|\nu_{\alpha}\rangle = \delta_{\alpha'\alpha} \tag{2.95}$$

$$\sum_i |\nu_i\rangle\langle \nu_i| = 1 \tag{2.96}$$

$$\begin{aligned}\langle \nu_{i'}|\nu_{\alpha}\rangle &= \langle \nu_{i'}|\left(\sum_i U^*_{\alpha i}|\nu_i\rangle\right)\\ &= \sum_i U^*_{\alpha i}\langle \nu_{i'}|\nu_i\rangle\\ &= U^*_{\alpha i}\end{aligned} \tag{2.97}$$

$$\begin{aligned}\langle \nu_{\alpha}|\nu_i\rangle &= \sum_i \langle \nu_{i'}|U_{\alpha i}|\nu_i\rangle\\ &= \sum_i U^*_{\alpha i}\langle \nu_{i'}|\nu_i\rangle\\ &= U_{\alpha i}\end{aligned} \tag{2.98}$$

maka ruas kanannya

$$\begin{aligned}\langle \nu_{\alpha'}|H_0|\nu_{\alpha}\rangle &= \sum_j\sum_i \langle \nu_{\alpha'}|\nu_i\rangle\langle \nu_i|H_0|\nu_j\rangle\langle \nu_j|\nu_{\alpha}\rangle\\ &= \sum_j\sum_i U_{\alpha' i}\langle \nu_i|H_0|\nu_j\rangle U^*_{\alpha j}\\ &= \sum_j\sum_i U_{\alpha' i}E_j\langle \nu_i|\nu_j\rangle U^*_{\alpha j}\\ &= \sum_i U_{\alpha i}E_i U^*_{\alpha i}\end{aligned} \tag{2.99}$$

dimana $E_i = p + \frac{m_i^2}{2p}$, maka

$$\begin{aligned}\langle \nu_{\alpha'}|H_0|\nu_{\alpha}\rangle &= \sum_i U_{\alpha' i}\left(p + \frac{m_i^2}{2p}\right)U^*_{\alpha i}\\ &= \sum_i U_{\alpha' i}pU^*_{\alpha i} + \sum_i U_{\alpha' i}\frac{m_i^2}{2p}U^*_{\alpha i}\\ &= p\sum_i U_{\alpha' i}U^*_{\alpha i} + \sum_i U_{\alpha' i}\frac{m_i^2}{2p}U^*_{\alpha i}\\ &= p\delta_{\alpha'\alpha} + \sum_i U_{\alpha' i}\frac{m_i^2}{2p}U^*_{\alpha i}\end{aligned} \tag{2.100}$$

jika persamaan (2.100) dimasukkan ke persamaan (2.94) didapatkan

$$\begin{aligned} i\frac{\partial}{\partial t}\left(\sum_{\alpha} a_{\alpha}(t)\,\delta_{\alpha'\alpha}\right) &= \sum_{\alpha}\left(p\delta_{\alpha'\alpha} + \sum_{i} U_{\alpha' i}\frac{m_i^2}{2p}U_{\alpha i}^{*}\right) a_{\alpha}(t) \\ &= \sum_{\alpha} p\delta_{\alpha'\alpha}a_{\alpha}(t) + \sum_{\alpha}\sum_{i} U_{\alpha' i}\frac{m_i^2}{2p}U_{\alpha i}^{*}a_{\alpha}(t) \\ i\frac{\partial}{\partial t}a_{\alpha'}(t) &= pa_{\alpha'}(t) + \sum_{\alpha}\sum_{i} U_{\alpha' i}\frac{m_i^2}{2p}U_{\alpha i}^{*}a_{\alpha}(t) \end{aligned} \tag{2.101}$$

untuk menyelesaikan persamaan (2.101) , dimisalkan

$$a_{\alpha'}(t) = e^{-ipt}B_{\alpha'}(t) \tag{2.102}$$

Jika persamaan (2.102) diturunkan terhadap t dan dikalikan dengan i, didapatkan

$$\begin{aligned} i\frac{\partial}{\partial t}a_{\alpha'}(t) &= i(-ip)\,e^{-ipt}B_{\alpha'}(t) + ie^{-ipt}\frac{\partial}{\partial t}B_{\alpha'}(t) \\ i\frac{\partial}{\partial t}a_{\alpha'}(t) &= pe^{-ipt}B_{\alpha'}(t) + ie^{-ipt}\frac{\partial}{\partial t}B_{\alpha'}(t) \\ i\frac{\partial}{\partial t}a_{\alpha'}(t) &= pa_{\alpha'}(t) + ie^{-ipt}\frac{\partial}{\partial t}B_{\alpha'}(t) \end{aligned} \tag{2.103}$$

Kemudian bandingkan suku kedua persamaan (2.103) dan persamaan (2.101), diperoleh

$$\begin{aligned} ie^{-ipt}\frac{\partial}{\partial}B_{\alpha'}(t) &= \sum_{\alpha}\sum_{i} U_{\alpha' i}\frac{m_i^2}{2p}U_{\alpha i}^{*}e^{-ipt}B_{\alpha}(t) \\ i\frac{\partial}{\partial t}B_{\alpha'}(t) &= \sum_{\alpha}\sum_{i} U_{\alpha' i}\frac{m_i^2}{2p}U_{\alpha i}B_{\alpha}(t) \end{aligned} \tag{2.104}$$

dimana : $\alpha, \alpha' = e, \mu, \tau$ maka secara eksplisit , jika $\alpha' = e$ didapatkan

$$\begin{aligned} i\frac{\partial}{\partial t}B_e &= \sum_{\alpha i} U_{ei}\frac{m_i^2}{2p}U_{\alpha i}^{*}B_{\alpha} \\ &= \sum_{i}\left(U_{ei}\frac{m_i^2}{2p}U_{ei}^{*}B_e + U_{ei}\frac{m_i^2}{2p}U_{\mu i}^{*}B_{\mu} + U_{ei}\frac{m_i^2}{2p}U_{\tau i}^{*}B_{\tau}\right) \\ &= U_{e1}\frac{m_1^2}{2p}U_{e1}^{*}B_e + U_{e1}\frac{m_1^2}{2p}U_{\mu 1}^{*}B_{\mu} + U_{e1}\frac{m_1^2}{2p}U_{\tau 1}^{*}B_{\tau} \\ &\quad + U_{e2}\frac{m_2^2}{2p}U_{e2}^{*}B_e + U_{e2}\frac{m_2^2}{2p}U_{\mu 2}^{*}B_{\mu} + U_{e2}\frac{m_2^2}{2p}U_{\tau 2}^{*}B_{\tau} \\ &\quad + U_{e3}\frac{m_3^2}{2p}U_{e3}^{*}B_e + U_{e3}\frac{m_3^2}{2p}U_{\mu 3}^{*}B_{\mu} + U_{e3}\frac{m_3^2}{2p}U_{\tau 3}^{*}B_{\tau} \end{aligned} \tag{2.105}$$

jika $\alpha' = \mu$, didapatkan

$$\begin{aligned}
i\frac{\partial}{\partial t}B_\mu &= \sum_{\alpha i} U_{\mu i}\frac{m_i^2}{2p}U_{\alpha i}^* B_\alpha \\
&= \sum_i \left(U_{\mu i}\frac{m_i^2}{2p}U_{ei}^* B_e + U_{\mu i}\frac{m_i^2}{2p}U_{\mu i}^* B_\mu + U_{\mu i}\frac{m_i^2}{2p}U_{\tau i}^* B_\tau \right) \\
&= U_{\mu 1}\frac{m_1^2}{2p}U_{e1}^* B_e + U_{\mu 1}\frac{m_1^2}{2p}U_{\mu 1}^* B_\mu + U_{\mu 1}\frac{m_1^2}{2p}U_{\tau 1}^* B_\tau \\
&\quad + U_{\mu 2}\frac{m_2^2}{2p}U_{e2}^* B_e + U_{\mu 2}\frac{m_2^2}{2p}U_{\mu 2}^* B_\mu + U_{\mu 2}\frac{m_2^2}{2p}U_{\tau 2}^* B_\tau \\
&\quad + U_{\mu 3}\frac{m_3^2}{2p}U_{e3}^* B_e + U_{\mu 3}\frac{m_3^2}{2p}U_{\mu 3}^* B_\mu + U_{\mu 3}\frac{m_3^2}{2p}U_{\tau 3}^* B_\tau
\end{aligned} \tag{2.106}$$

jika $\alpha' = \tau$, maka didapatkan

$$\begin{aligned}
i\frac{\partial}{\partial t}B_\tau &= \sum_{\alpha i} U_{\tau i}\frac{m_i^2}{2p}U_{\alpha i}^* B_\alpha \\
&= \sum_i \left(U_{\tau i}\frac{m_i^2}{2p}U_{ei}^* B_e + U_{\tau i}\frac{m_i^2}{2p}U_{\mu i}^* B_\mu + U_{\tau i}\frac{m_i^2}{2p}U_{\tau i}^* B_\tau \right) \\
&= U_{\tau 1}\frac{m_1^2}{2p}U_{e1}^* B_e + U_{\tau 1}\frac{m_1^2}{2p}U_{\mu 1}^* B_\mu + U_{\tau 1}\frac{m_1^2}{2p}U_{\tau 1}^* B_\tau \\
&\quad + U_{\tau 2}\frac{m_2^2}{2p}U_{e2}^* B_e + U_{\tau 2}\frac{m_2^2}{2p}U_{\mu 2}^* B_\mu + U_{\tau 2}\frac{m_2^2}{2p}U_{\tau 2}^* B_\tau \\
&\quad + U_{\tau 3}\frac{m_3^2}{2p}U_{e3}^* B_e + U_{\tau 3}\frac{m_3^2}{2p}U_{\mu 3}^* B_\mu + U_{\tau 3}\frac{m_3^2}{2p}U_{\tau 3}^* B_\tau
\end{aligned} \tag{2.107}$$

secara kompak dapat dituliskan

$$i\frac{\partial}{\partial t}\begin{pmatrix} B_e \\ B_\mu \\ B_\tau \end{pmatrix} = \begin{pmatrix} \sum_i U_{ei}\frac{m_i^2}{2p}U_{ei}^* & \sum_i U_{ei}\frac{m_i^2}{2p}U_{\mu i}^* & \sum_i U_{ei}\frac{m_i^2}{2p}U_{\tau i}^* \\ \sum_i U_{\mu i}\frac{m_i^2}{2p}U_{ei}^* & \sum_i U_{\mu i}\frac{m_i^2}{2p}U_{\mu i}^* & \sum_i U_{\mu i}\frac{m_i^2}{2p}U_{\tau i}^* \\ \sum_i U_{\tau i}\frac{m_i^2}{2p}U_{ei}^* & \sum_i U_{\tau i}\frac{m_i^2}{2p}U_{\mu i}^* & \sum_i U_{\tau i}\frac{m_i^2}{2p}U_{\tau i}^* \end{pmatrix}\begin{pmatrix} B_e \\ B_\mu \\ B_\tau \end{pmatrix} \tag{2.108}$$

Sekarang perhatikan matrik U yang berbentuk

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \tag{2.109}$$

dan hermit konjugetnya

$$U^\dagger = \begin{pmatrix} U_{e1}^* & U_{\mu 1}^* & U_{\tau 1}^* \\ U_{e2}^* & U_{\mu 2}^* & U_{\mu 2}^* \\ U_{e3}^* & U_{\mu 3}^* & U_{\tau 3}^* \end{pmatrix} \tag{2.110}$$

maka

$$U\frac{m^2}{2p}U^\dagger = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \frac{m_1^2}{2p} & 0 & 0 \\ 0 & \frac{m_2^2}{2p} & 0 \\ 0 & 0 & \frac{m_3^2}{2p} \end{pmatrix} \begin{pmatrix} U_{e1}^* & U_{\mu 1}^* & U_{\tau 1}^* \\ U_{e2}^* & U_{\mu 2}^* & U_{\mu 2}^* \\ U_{e3}^* & U_{\mu 3}^* & U_{\tau 3}^* \end{pmatrix}$$

$$= \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \frac{m_1^2}{2p}U_{e1}^* & \frac{m_1^2}{2p}U_{\mu 1}^* & \frac{m_1^2}{2p}U_{\tau 1}^* \\ \frac{m_2^2}{2p}U_{e2}^* & \frac{m_1^2}{2p}U_{\mu 2}^* & \frac{m_1^2}{2p}U_{\tau 2}^* \\ \frac{m_3^2}{2p}U_{e3}^* & \frac{m_3^2}{2p}U_{\mu 3}^* & \frac{m_3^2}{2p}U_{\tau 3}^* \end{pmatrix}$$

$$= \begin{pmatrix} A & B & C \\ D & E & F \\ G & H & I \end{pmatrix} \tag{2.111}$$

dengan

$$A = U_{e1}\frac{m_1^2}{2p}U_{e1}^* + U_{e2}\frac{m_2^2}{2p}U_{e2}^* + U_{e3}\frac{m_3^2}{2p}U_{e3}^* \tag{2.112}$$

$$B = U_{e1}\frac{m_1^2}{2p}U_{\mu 1}^* + U_{e2}\frac{m_2^2}{2p}U_{\mu 2}^* + U_{e3}\frac{m_3^2}{2p}U_{\mu 3}^* \tag{2.113}$$

$$C = U_{e1}\frac{m_1^2}{2p}U_{\tau 1}^* + U_{e2}\frac{m_2^2}{2p}U_{\tau 2}^* + U_{e3}\frac{m_3^2}{2p}U_{\tau 3}^* \tag{2.114}$$

$$D = U_{\mu 1}\frac{m_1^2}{2p}U_{e1}^* + U_{\mu 2}\frac{m_2^2}{2p}U_{e2}^* + U_{\mu 3}\frac{m_3^2}{2p}U_{e3}^* \tag{2.115}$$

$$E = U_{\mu 1}\frac{m_1^2}{2p}U_{\mu 1}^* + U_{\mu 2}\frac{m_2^2}{2p}U_{\mu 2}^* + U_{\mu 3}\frac{m_3^2}{2p}U_{\mu 3}^* \tag{2.116}$$

$$F = U_{\mu 1}\frac{m_1^2}{2p}U_{\tau 1}^* + U_{\mu 2}\frac{m_2^2}{2p}U_{\tau 2}^* + U_{\mu 3}\frac{m_3^2}{2p}U_{\tau 3}^* \tag{2.117}$$

$$G = U_{\tau 1}\frac{m_1^2}{2p}U_{e1}^* + U_{\tau 2}\frac{m_2^2}{2p}U_{e2}^* + U_{\tau 3}\frac{m_3^2}{2p}U_{e3}^* \tag{2.118}$$

$$H = U_{\tau 1}\frac{m_1^2}{2p}U_{\mu 1}^* + U_{\tau 2}\frac{m_2^2}{2p}U_{\mu 2}^* + U_{\tau 3}\frac{m_3^2}{2p}U_{\mu 3}^* \tag{2.119}$$

$$I = U_{\tau 1}\frac{m_1^2}{2p}U_{\tau 1}^* + U_{\tau 2}\frac{m_2^2}{2p}U_{\tau 2}^* + U_{\tau 3}\frac{m_3^2}{2p}U_{\tau 3}^* \tag{2.120}$$

terlihat bahwa indeks massa jalan, sehingga persamaan (2.108) diatas dapat ditulis dalam bentuk

$$i\frac{\partial}{\partial t}B = U\frac{m^2}{2p}U^\dagger B \tag{2.121}$$

didefinisikan $B' = U^{\dagger}B$ maka

$$\begin{aligned} iU^{\dagger}\frac{\partial}{\partial t}B &= U^{\dagger}U\frac{m^2}{2p}B' \\ i\frac{\partial}{\partial t}\left(U^{\dagger}B\right) &= \frac{m^2}{2p}B' \\ i\frac{\partial}{\partial t}B' &= \frac{m^2}{2p}B' \end{aligned} \tag{2.122}$$

jika persamaan (2.122) diintegralkan didapatkan

$$\begin{aligned} \frac{d}{dt}B' &= -i\frac{m^2}{2p}B' \\ \int_{B_0}^{B}\frac{dB'}{B'} &= \int_0^t -i\frac{m^2}{2p}dt \\ ln\frac{B'}{B'_0} &= -i\frac{m^2}{2p}t \\ \frac{B'}{B'_0} &= e^{-i\frac{m^2}{2p}t} \\ B' &= e^{-i\frac{m^2}{2p}t}B'_0 \end{aligned} \tag{2.123}$$

jika dikembalikan ke bentuk semula

$$\begin{aligned} U^{\dagger}B &= e^{-i\frac{m^2}{2p}t}B'_0 \\ UU^{\dagger}B &= Ue^{-i\frac{m^2}{2p}t}U^{\dagger}B_0 \\ B &= Ue^{-i\frac{m^2}{2p}t}U^{\dagger}B_0 \end{aligned} \tag{2.124}$$

maka diperoleh ungkapan amplitudo transisi neutrino yang bergantung waktu adalah

$$a\left(t\right) = Ue^{-i\frac{m^2}{2p}\left(t-t_0\right)}U^{\dagger}a\left(t_0\right) \tag{2.125}$$

Untuk mencari probabilitas osilasi neutrino dalam materi (disini diambil kasus dua generasi), perhatikan Hamiltonian neutrino dalam materi yang berbentuk

$$H = U\frac{m^2}{2p}U^{\dagger} + \sqrt{2}G_F\rho_e\beta \tag{2.126}$$

dengan matrik U berbentuk

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2.127}$$

dan hermit konjugetnya

$$U^{\dagger} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{2.128}$$

dimana $\beta_{\nu_e,\nu_e} = 1$, sehingga uraian lengkap Hamiltonian pada persamaan (2.126) adalah

$$\begin{aligned} H &= U\frac{m^2}{2p}U^\dagger + \sqrt{2}G_F\rho_e E\beta \\ &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \frac{m_1^2}{2p} & 0 \\ 0 & \frac{m_2^2}{2p} \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} + \begin{pmatrix} \sqrt{2}G_F\rho_e & 0 \\ 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \frac{m_1^2}{2p}\cos\theta & -\frac{m_1^2}{2p}\sin\theta \\ \frac{m_2^2}{2p}\sin\theta & \frac{m_2^2}{2p}\cos\theta \end{pmatrix} + \begin{pmatrix} \sqrt{2}G_F\rho_e & 0 \\ 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} \frac{m_1^2}{2p}\cos^2\theta + \frac{m_2^2}{2p}\sin^2\theta + \sqrt{2}G_F\rho_e & \left(-\frac{m_1^2}{2p} + \frac{m_2^2}{2p}\right)\sin\theta\cos\theta \\ \left(-\frac{m_1^2}{2p} + \frac{m_2^2}{2p}\right)\sin\theta\cos\theta & \frac{m_1^2}{2p}\sin^2\theta + \frac{m_2^2}{2p}\cos^2\theta \end{pmatrix} \end{aligned} \tag{2.129}$$

kemudian hitung

$$\begin{aligned} \frac{1}{2}TrH &= \frac{1}{2}\left\{\frac{m_1^2}{2p}\left(\cos^2\theta + \sin^2\theta\right) + \frac{m_1^2}{2p}\left(\cos^2\theta + \sin^2\theta\right) + \sqrt{2}G_F\rho_e\right\} \\ &= \frac{1}{4p}\left(m_1^2 + m_2^2\right) + \frac{1}{2}\sqrt{2}G_F\rho_e \end{aligned} \tag{2.130}$$

dalam bentuk matrik

$$\frac{1}{2}TrH = \begin{pmatrix} \frac{1}{4p}\left(m_1^2 + m_2^2\right) + \frac{1}{2}\sqrt{2}G_F\rho_e & 0 \\ 0 & \frac{1}{4p}\left(m_1^2 + m_2^2\right) + \frac{1}{2}\sqrt{2}G_F\rho_e \end{pmatrix} \tag{2.131}$$

Diperkenalkan Hamiltonian efektif yang berbentuk

$$\begin{aligned} H &= \frac{1}{2}TrH + H^m \\ H^m &= H - \frac{1}{2}TrH \end{aligned} \tag{2.132}$$

dalam bentuk matriknya

$$\begin{aligned} H^m &= H - \frac{1}{2}TrH \\ &= \begin{pmatrix} \frac{m_1^2}{2p}\cos^2\theta + \frac{m_2^2}{2p}\sin^2\theta + \sqrt{2}G_F\rho_e & \left(-\frac{m_1^2}{2p} + \frac{m_2^2}{2p}\right)\sin\theta\cos\theta \\ \left(-\frac{m_1^2}{2p} + \frac{m_2^2}{2p}\right)\sin\theta\cos\theta & \frac{m_1^2}{2p}\sin^2\theta + \frac{m_2^2}{2p}\cos^2\theta \end{pmatrix} \\ &\quad - \begin{pmatrix} \frac{1}{4p}\left(m_1^2 + m_2^2\right) + \frac{1}{2}\sqrt{2}G_F\rho_e & 0 \\ 0 & \frac{1}{4p}\left(m_1^2 + m_2^2\right) + \frac{1}{2}\sqrt{2}G_F\rho_e \end{pmatrix} \end{aligned} \tag{2.133}$$

uraian persamaan (2.133) dalam komponen matriknya diperoleh

$$\begin{aligned}
(H^m)_{11} &= \frac{m_1^2}{2p}\cos^2\theta - \frac{1}{4p}m_1^2 + \frac{m_2^2}{2p}\sin^2\theta - \frac{1}{4p}m_2^2 + \frac{1}{2}\sqrt{2}G_F\rho_e \\
&= \frac{1}{4p}\left(2\cos^2\theta - 1\right)m_1^2 + \frac{1}{4p}\left(2\sin^2\theta - 1\right)m_2^2 + \frac{1}{2}\sqrt{2}G_F\rho_e \\
&= \frac{1}{4p}m_2^2\cos 2\theta - \frac{1}{4p}m_1^2\cos 2\theta + \frac{1}{2}\sqrt{2}G_F\rho_e \\
&= -\frac{1}{4p}\left(m_2^2 - m_1^2\right)\cos 2\theta + \frac{1}{2}\sqrt{2}G_F\rho_e \\
&= -\frac{1}{4p}\Delta m^2\cos 2\theta + \frac{1}{2}\sqrt{2}G_F\rho_e \\
&= -\frac{1}{4p}\Delta m^2\cos 2\theta + \frac{1}{4p}2\sqrt{2}G_F\rho_e p \\
&= \frac{1}{4p}\left(-\Delta m^2\cos 2\theta + A\right) \qquad (2.134)
\end{aligned}$$

dimana $\Delta m^2 = m_2^2 - m_1^2$ dan $A = 2\sqrt{2}G_F\rho_e p$.Dengan cara sama didapatkan

$$(H^m)_{22} = \frac{1}{4p}\left(\Delta m^2\cos 2\theta - A\right) \qquad (2.135)$$

sedangkan untuk komponen off diagonalnya adalah

$$\begin{aligned}
(H^m)_{12} &= (H^m)_{21} \\
&= \left(-\frac{m_1^2}{2p} + \frac{m_2^2}{2p}\right)\sin\theta\cos\theta \\
&= \frac{1}{4p}\left(m_2^2 - m_1^2\right)2\sin\theta\cos\theta \\
&= \frac{1}{4p}\Delta m^2\sin 2\theta \qquad (2.136)
\end{aligned}$$

dalam bentuk matriks

$$H^m = \frac{1}{4p}\begin{pmatrix} -\Delta m^2\cos 2\theta + A & \Delta m^2\sin 2\theta \\ \Delta m^2\sin 2\theta & \Delta m^2\cos 2\theta - A \end{pmatrix} \qquad (2.137)$$

Untuk mendapatkan energinya , kita diagonalisasi hamiltonian efektif neutrino dalam materi , sehingga memberikan

$$\begin{aligned}
H^m &= U^m E^m U^{m\dagger} \\
E^m &= U^{m\dagger} H^m U^m \qquad (2.138)
\end{aligned}$$

karena kita mengambil kasus dua generasi , maka kita pilih matrik U^m berbentuk

$$U^m = \begin{pmatrix} \cos\theta^m & \sin\theta^m \\ -\sin\theta^m & \cos\theta^m \end{pmatrix} \qquad (2.139)$$

dan hermit konjugetnya

$$U^{m\dagger} = \begin{pmatrix} \cos\theta^m & -\sin\theta^m \\ \sin\theta^m & \cos\theta^m \end{pmatrix} \tag{2.140}$$

dimana θ^m menyatakan sudut didalam materi , uraiannya dalam matrik

$$\begin{aligned} E^m &= U^{m\dagger} H^m U^m \\ &= \begin{pmatrix} \cos\theta^m & -\sin\theta^m \\ \sin\theta^m & \cos\theta^m \end{pmatrix} \frac{1}{4p} \begin{pmatrix} -\Delta m^2\cos 2\theta + A & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta - A \end{pmatrix} \\ &\times \begin{pmatrix} \cos\theta^m & \sin\theta^m \\ -\sin\theta^m & \cos\theta^m \end{pmatrix} \end{aligned} \tag{2.141}$$

uraian perkomponen

$$\begin{aligned} (E^m)_{11} &= -\Delta m^2 \cos 2\theta \cos^2\theta^m + A\cos^2\theta^m - \Delta m^2 \sin 2\theta \sin\theta^m \cos\theta^m \\ &\quad -\Delta m^2 \sin 2\theta \sin\theta^m\cos\theta^m + \Delta m^2 \cos 2\theta \sin^2\theta^m - A\sin^2\theta^m \\ &= -\Delta m^2 \cos 2\theta \left(\cos^2\theta^m - \sin^2\theta^m\right) + A\left(\cos^2\theta^m - \sin^2\theta^m\right) \\ &\quad -\Delta m^2 \sin 2\theta \sin 2\theta^m \\ &= -\Delta m^2 \cos 2\theta \cos 2\theta^m + A\cos 2\theta^m - \Delta m^2 \sin 2\theta \sin 2\theta^m \end{aligned} \tag{2.142}$$

$$\begin{aligned} (E^m)_{12} &= -\Delta m^2 \cos 2\theta \sin\theta^m\cos\theta^m + A\sin\theta^m\cos\theta^m + \Delta m^2 \sin 2\theta \cos^2\theta^m \\ &\quad -\Delta m^2 \sin 2\theta \sin^2\theta^m - \Delta m^2\cos 2\theta \sin\theta^m\cos\theta^m + A\sin\theta^m\cos\theta^m \\ &= -\Delta m^2 \cos 2\theta \sin 2\theta^m + A\sin 2\theta^m + \Delta m^2 \sin 2\theta \cos 2\theta^m \end{aligned} \tag{2.143}$$

$$\begin{aligned} (E^m)_{21} &= -\Delta m^2 \cos 2\theta \sin\theta^m\cos\theta^m + A\sin\theta^m\cos\theta^m - \Delta m^2 \sin 2\theta \sin^2\theta^m \\ &\quad +\Delta m^2 \sin 2\theta \cos^2\theta^m - \Delta m^2\cos 2\theta \sin\theta^m\cos\theta^m + A\sin\theta^m\cos\theta^m \\ &= -\Delta m^2 \cos 2\theta \sin 2\theta^m + \Delta m^2 \sin 2\theta \cos 2\theta^m + A\sin 2\theta^m \end{aligned} \tag{2.144}$$

$$\begin{aligned} (E^m)_{22} &= -\Delta m^2 \cos 2\theta \sin^2\theta^m + A\sin^2\theta^m + \Delta m^2 \sin 2\theta \sin\theta^m\cos\theta^m \\ &\quad +\Delta m^2 \sin 2\theta \sin\theta^m\cos\theta^m + \Delta m^2\cos 2\theta \cos\theta^m - A\cos^2\theta^m \\ &= \Delta m^2 \cos 2\theta \cos 2\theta^m + \Delta m^2 \sin 2\theta \sin 2\theta^m - A\cos 2\theta^m \end{aligned} \tag{2.145}$$

karena E^m adalah matrik diagonal maka $(E^m)_{12} = (E^m)_{21} = 0$, sehingga diperoleh

$$\begin{aligned} \sin 2\theta^m\left(-\Delta m^2 \cos 2\theta + A\right) + \Delta m^2 \sin 2\theta\cos 2\theta^m &= 0 \\ \sin 2\theta^m\left(-\Delta m^2 \cos 2\theta + A\right) &= -\Delta m^2 \sin 2\theta \cos 2\theta^m \\ \tan 2\theta^m &= \frac{-\Delta m^2 \sin 2\theta}{-\Delta m^2 \cos 2\theta + A} \\ \tan 2\theta^m &= \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A} \end{aligned} \tag{2.146}$$

dan

$$\sin 2\theta^m = \frac{\Delta m^2 \sin 2\theta}{\sqrt{\left(\Delta m^2 \cos 2\theta - A\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}} \tag{2.147}$$

$$\cos 2\theta^m = \frac{\Delta m^2 \cos 2\theta - A}{\sqrt{\left(\Delta m^2 \cos 2\theta - A\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}} \tag{2.148}$$

maka energinya

$$\begin{aligned}
E_1^m &= \frac{1}{4p}\left(-\Delta m^2\cos 2\theta\cos 2\theta^m + A\cos 2\theta^m - \Delta m^2\sin 2\theta\sin 2\theta^m\right)\\
&= \frac{1}{4p}\left(\left(-\Delta m^2\cos 2\theta + A\right)\cos 2\theta^m - \Delta m^2\sin 2\theta\sin 2\theta^m\right)\\
&= -\frac{1}{4p}\left\{\left(\Delta m^2\cos 2\theta - A\right).\frac{\Delta m^2\cos 2\theta - A}{\sqrt{\left(\Delta m^2\cos 2\theta - A\right)^2 + \left(\Delta m^2\sin 2\theta\right)^2}}\right.\\
&\quad\left. + \left(\Delta m^2\sin 2\theta\right)\frac{\Delta m^2\sin 2\theta}{\sqrt{\left(\Delta m^2\cos 2\theta - A\right)^2 + \left(\Delta m^2\sin 2\theta\right)^2}}\right\}\\
&= -\frac{1}{4p}\left\{\frac{\left(\Delta m^2\cos 2\theta - A\right)^2 + \left(\Delta m^2\sin 2\theta\right)^2}{\sqrt{\left(\Delta m^2\cos 2\theta - A\right)^2 + \left(\Delta m^2\sin 2\theta\right)^2}}\right\}\\
&= -\frac{1}{4p}\left(\sqrt{\left(\Delta m^2\cos 2\theta - A\right)^2 + \left(\Delta m^2\sin 2\theta\right)^2}\right) \qquad (2.149)
\end{aligned}$$

dengan cara yang sama didapatkan

$$\begin{aligned}
E_2^m &= \Delta m^2\cos 2\theta\cos 2\theta^m + \Delta m^2\sin 2\theta\sin 2\theta^m - A\cos 2\theta^m\\
&= \frac{1}{4p}\left(\sqrt{\left(\Delta m^2\cos 2\theta - A\right)^2 + \left(\Delta m^2\sin 2\theta\right)^2}\right) \qquad (2.150)
\end{aligned}$$

Selanjutnya kita hitung probabilitas survival neutrino elektron ke neutrino elektron dalam materi dari persamaan (2.125) , diperoleh

$$\begin{aligned}
P^m_{\nu_e\to\nu_e}(t) &= \left|\sum_{i=1}^{2} U^m_{ei} e^{-i\frac{m_i^2}{2p}(t-t_0)} U^{m\dagger}_{ei}\right|^2\\
&= \left|U^m_{e1} e^{-i\frac{m_1^2}{2p}(t-t_0)} U^{m\dagger}_{e1} + U^m_{e2} e^{-i\frac{m_2^2}{2p}(t-t_0)} U^{m\dagger}_{e2}\right|^2\\
&= \left(U^m_{e1} e^{-i\frac{m_1^2}{2p}(t-t_0)} U^{m*}_{e1} + U^m_{e2} e^{-i\frac{m_2^2}{2p}(t-t_0)} U^{m*}_{e2}\right)\\
&\quad \times\left(U^{m*}_{e1} e^{i\frac{m_1^2}{2p}(t-t_0)} U^m_{e1} + U^{m*}_{e2} e^{i\frac{m_2^2}{2p}(t-t_0)} U^m_{e2}\right)\\
&= \left|U^m_{e1}\right|^4 + \left|U^m_{e2}\right|^4 + \left|U^m_{e1}\right|^2\left|U^m_{e2}\right|^2 e^{-i\frac{m_1^2-m_2^2}{2p}(t-t_0)}\\
&\quad + \left|U^m_{e1}\right|^2\left|U^m_{e2}\right|^2 e^{-i\frac{m_2^2-m_1^2}{2p}(t-t_0)}\\
&= \left|U^m_{e1}\right|^4 + \left|U^m_{e2}\right|^4\\
&\quad + \left|U^m_{e1}\right|^2\left|U^m_{e2}\right|^2\left(e^{-i\frac{m_1^2-m_2^2}{2p}(t-t_0)} + e^{i\frac{m_1^2-m_2^2}{2p}(t-t_0)}\right)\\
&= \left|U^m_{e1}\right|^4 + \left|U^m_{e2}\right|^4\\
&\quad + \left|U^m_{e1}\right|^2\left|U^m_{e2}\right|^2 2\cos\left\{\left(\frac{m_1^2-m_2^2}{2p}\right)(t-t_0)\right\} \qquad (2.151)
\end{aligned}$$

Jika dimasukkan nilai - nilainya ,didapatkan

$$
\begin{aligned}
P^m_{\nu_e\to\nu_e}(t) &= \cos^4\theta^m + \sin^4\theta^m \\
&\quad +2\cos^2\theta^m\sin^2\theta^m\cos\left\{\left(\frac{m_1^2-m_2^2}{2p}\right)(t-t_0)\right\} \\
&= \cos^2\theta^m\left(1-\sin^2\theta^m\right)+\sin^2\theta^m\left(1-\cos^2\theta^m\right) \\
&\quad +\frac{1}{2}\sin^2 2\theta^m\cos\left\{\left(\frac{m_1^2-m_2^2}{2p}\right)(t-t_0)\right\} \\
&= \cos^2\theta^m-\sin^2\theta^m\cos^2\theta^m+\sin^2\theta^m-\sin^2\theta^m\cos^2\theta^m \\
&\quad +\frac{1}{2}\sin^2 2\theta^m\cos\left\{\left(\frac{m_1^2-m_2^2}{2p}\right)(t-t_0)\right\} \\
&= \cos^2\theta^m+\sin^2\theta^m-2\sin^2\theta^m\cos^2\theta^m \\
&\quad +\frac{1}{2}\sin^2 2\theta^m\cos\left\{\left(\frac{m_1^2-m_2^2}{2p}\right)(t-t_0)\right\} \\
&= 1-\frac{1}{2}\sin^2 2\theta^m+\frac{1}{2}\sin^2 2\theta^m\cos\left\{\left(\frac{m_1^2-m_2^2}{2p}\right)(t-t_0)\right\} \\
&= 1-\frac{1}{2}\sin^2 2\theta^m\left(1-\cos\left\{\left(\frac{m_1^2-m_2^2}{2p}\right)(t-t_0)\right\}\right) \\
&= 1-\sin^2 2\theta^m\sin^2\left\{\left(\frac{m_1^2-m_2^2}{4p}\right)(t-t_0)\right\} \\
&= 1-\frac{\left(\Delta m^2\sin 2\theta\right)^2}{\left(\Delta m^2\cos 2\theta+A\right)^2+\left(\Delta m^2\sin 2\theta\right)^2}\sin^2\left\{\frac{\Delta m_{12}^2}{4p}(t-t_0)\right\} \\
&= 1-\frac{\left(\sin 2\theta\right)^2}{\left(\cos 2\theta+\frac{2\sqrt{2}G_F\rho_e E}{\Delta m}\right)^2+\sin^2 2\theta}\sin^2\left\{\frac{\Delta m_{12}^2}{4p}(t-t_0)\right\} \quad (2.152)
\end{aligned}
$$

jika $\rho_e = 0$, maka diperoleh

$$P^m_{\nu_e\to\nu_e}(t) = 1-\sin^2 2\theta\sin^2\left\{\frac{\Delta m_{12}^2}{4p}(t-t_0)\right\} \quad (2.153)$$

karena $(t-t_0)\approx L$ dan $p\approx E$, diperoleh

$$P^m_{\nu_e\to\nu_e}(L) = 1-\sin^2 2\theta\sin^2\left\{\frac{\Delta m_{12}^2}{4E}L\right\} \quad (2.154)$$

Seperti pada persamaan (2.74) ,kemudian kita hitung probabilitas transisi neutrino elektron ke neutrino muon dalam materi , yaitu

$$
\begin{aligned}
P^m_{\nu_e\to\nu_\mu}(t) &= \left|\sum_{i=1}^{2} U^m_{ei} e^{-i\frac{m_i^2}{2p}(t-t_0)} U^{m\dagger}_{\mu i}\right|^2 \\
&= \left|U^m_{e1} e^{-i\frac{m_1^2}{2p}(t-t_0)} U^{m\dagger}_{\mu 1} + U^m_{e2} e^{-i\frac{m_2^2}{2p}(t-t_0)} U^{m\dagger}_{\mu 2}\right|^2 \\
&= \left(U^m_{e1} e^{-i\frac{m_1^2}{2p}(t-t_0)} U^{m*}_{\mu 1} + U^m_{e2} e^{-i\frac{m_2^2}{2p}(t-t_0)} U^{m*}_{\mu 2}\right) \\
&\quad \times \left(U^{m*}_{e1} e^{i\frac{m_1^2}{2p}(t-t_0)} U^{m}_{\mu 1} + U^{m*}_{e2} e^{i\frac{m_2^2}{2p}(t-t_0)} U^{m}_{\mu 2}\right) \\
&= \left|U^m_{e1}\right|^2 \left|U^m_{\mu 1}\right|^2 + \left|U^m_{e2}\right|^2 \left|U^m_{\mu 2}\right|^2 \\
&\quad + U^m_{e1} U^{m*}_{e2} U^{m*}_{\mu 1} U^m_{\mu 2} e^{-i\frac{m_1^2-m_2^2}{2p}(t-t_0)} \\
&\quad + U^m_{e2} U^{m*}_{e1} U^{m*}_{\mu 2} U^m_{\mu 1} e^{-i\frac{m_2^2-m_1^2}{2p}(t-t_0)} \qquad (2.155)
\end{aligned}
$$

dengan memasukkan komponen matrik didapatkan

$$
\begin{aligned}
P^m_{\nu_e\to\nu_\mu}(t) &= \cos^2\theta_m \sin^2\theta_m + \sin^2\theta_m \cos^2\theta_m \\
&\quad - \sin^2\theta_m \cos^2\theta_m \left(2\cos\left\{\left(\frac{m_1^2-m_2^2}{2p}\right)(t-t_0)\right\}\right) \\
&= 2\sin^2\theta_m \cos^2\theta_m \left(1-\cos\left\{\left(\frac{m_1^2-m_2^2}{2p}\right)(t-t_0)\right\}\right) \\
&= \frac{1}{2}\sin^2 2\theta_m .2\sin^2\left\{\left(\frac{m_1^2-m_2^2}{4p}\right)(t-t_0)\right\} \\
&= \sin^2 2\theta_m \sin^2\left\{\left(\frac{m_1^2-m_2^2}{4p}\right)(t-t_0)\right\} \qquad (2.156)
\end{aligned}
$$

masukkan nilai $\sin 2\theta_m$ diperoleh

$$
\begin{aligned}
P^m_{\nu_e\to\nu_\mu}(t) &= \frac{\left(\Delta m^2 \sin 2\theta\right)^2}{\left(\Delta m^2\cos 2\theta + A\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}\sin^2\left\{\frac{\Delta m_{12}^2}{4p}(t-t_0)\right\} \\
&= \frac{\left(\sin 2\theta\right)^2}{\left(\cos 2\theta + \frac{2\sqrt{2}G_F\rho_e E}{\Delta m}\right)^2 + \sin^2 2\theta}\sin^2\left\{\frac{\Delta m_{12}^2}{4p}(t-t_0)\right\} \qquad (2.157)
\end{aligned}
$$

jika $\rho_e = 0$, maka diperoleh

$$
P^m_{\nu_e\to\nu_\mu}(t) = \sin^2 2\theta \sin^2\left\{\frac{\Delta m_{12}^2}{4p}(t-t_0)\right\} \qquad (2.158)
$$

karena $(t-t_0)\approx L$ dan $p \approx E$, diperoleh

$$
P^m_{\nu_e\to\nu_\mu}(L) = \sin^2 2\theta \sin^2\left\{\frac{\Delta m_{12}^2}{4E}L\right\} \qquad (2.159)
$$

sama seperti probabilitas transisi dalam vakum pada persamaan (2.79) dan terlihat bahwa

$$P^m_{\nu_e \to \nu_e}(L) + P^m_{\nu_e \to \nu_\mu}(L) = 1 \tag{2.160}$$

Bab 3

Simpangan CP

Hukum - hukum fisika diharapkan invarian terhadap suatu transformasi. Pada bab ini akan dibahas sifat transfomasi C, P , dan T medan spinor , serta segitiga unitaritas untuk sektor lepton.

3.1 Transformasi C,P, dan T Medan Spinor

3.1.1 Transformasi C pada medan spinor

Operator konjugasi muatan (*Charge Conjugation*) akan mengubah muatan q menjadi - q, untuk medan spinor $\psi(x)$ dan $\bar{\psi}(x)$ akan bertransformasi menurut [15]

$$\psi(x) \rightarrow \psi'(x) = \psi^C(x) = \xi_C C \bar{\psi}^T(x) = -\xi_C \gamma^0 C \psi^*(x) \tag{3.1}$$

$$\bar{\psi}(x) \rightarrow \psi'(x) = \bar{\psi^C}(x) = -\xi_C^* \psi^T(x) C^\dagger \tag{3.2}$$

Matrik konjugasi muatan C mempunyai sifat

$$\begin{aligned} C^\dagger &= C^{-1} \\ C^T &= -C \\ C\gamma^{\mu T} C^{-1} &= -\gamma^\mu \\ C\left(\gamma^5\right)^T C^{-1} &= \gamma^5 \end{aligned} \tag{3.3}$$

3.1.2 Transformasi P pada medan spinor

Operator paritas (*Parity*) mengubah ruang $\vec{x}$ menjadi $-\vec{x}$, dalam ruang 4 dimensi , vektor kontravarian akan menjadi kovarian menurut

$$x^\mu = \left(x^0, \vec{x}\right) \rightarrow x_P^\mu = \left(x^0, -\vec{x}\right) = x_\mu \tag{3.4}$$

Sedangkan medan spinor $\psi(x)$ dan $\bar{\psi}(x)$ bertransformasi

$$\psi(x) \rightarrow \psi'(x) = \psi^P(x_P) = \xi_P \gamma^0 \psi(x) \tag{3.5}$$

$$\bar{\psi}(x) \rightarrow \psi'(x) = \bar{\psi}^P(x_P) = \xi_P^* \bar{\psi}(x) \gamma^0 \tag{3.6}$$

3.1.3 Transformasi T pada medan spinor

Operator pembalikkan waktu (*Time Reserval*) akan mengubah t menjadi - t , sebagaimana operator paritas , operator ini akan mengubah vektor kontravarian menjadi kovarian , transformasinya

$$x^{\mu} = \left(x^0, \vec{x}\right) \rightarrow x_T^{\mu} = \left(-x^0, \vec{x}\right) = x_{\mu} \tag{3.7}$$

Medan spinor $\psi(x)$ dan $\bar{\psi}(x)$ bertransformasi menurut

$$\psi(x) \quad \rightarrow \quad \psi'(x) = \psi^T(x_T) = \xi_T \gamma^0 \gamma^5 \bar{\psi}^T(x) = \xi_T \gamma^5 C \psi^*(x) \tag{3.8}$$

$$\bar{\psi}(x) \quad \rightarrow \quad \psi'(x) = \bar{\psi}^T(x_T) = \xi_T^* \psi^T(x) C^{\dagger} \gamma^5 \gamma^0 \tag{3.9}$$

3.1.4 Transformasi CP pada medan spinor

Operator CP merupakan gabungan dari operator konjugasi muatan dan operator paritas , dimana operator paritas bekerja terlebih dahulu kemudian diikuti oleh operator konjugasi muatan. Medan spinor $\psi(x)$ dan $\bar{\psi}(x)$ bertransformasi menurut

$$\begin{aligned} \psi(x) \quad &\rightarrow \quad \psi'(x) = \psi^{CP}(x_P) = \xi_{CP} \gamma^0 C \bar{\psi}^T = -\xi_{CP} C \psi^*(x) \\ \bar{\psi}(x) \quad &\rightarrow \quad \psi'(x) = \bar{\psi}^{CP}(x_P) = -\xi_{CP}^* \psi^T(x) C^{\dagger} \gamma^0 \end{aligned} \tag{3.10}$$

3.1.5 Transformasi CPT pada medan spinor

Operator CPT merupakan gabungan dari ketiga operator tersebut, dimana operator pembalikkan waktu bekerja dulu, kemudian diikuti operator CP. Medan spinor $\psi(x)$ dan $\bar{\psi}(x)$ bertransformasi menurut

$$\begin{aligned} \psi(x) \quad &\rightarrow \quad \psi'(x) = \psi^{CPT}(-x) = \xi_{CPT} \gamma^5 \psi(x) \\ \bar{\psi}(x) \quad &\rightarrow \quad \psi'(x) = \bar{\psi}^{CPT}(-x) = -\xi_{CPT}^* \bar{\psi}(x) \gamma^5 \end{aligned} \tag{3.11}$$

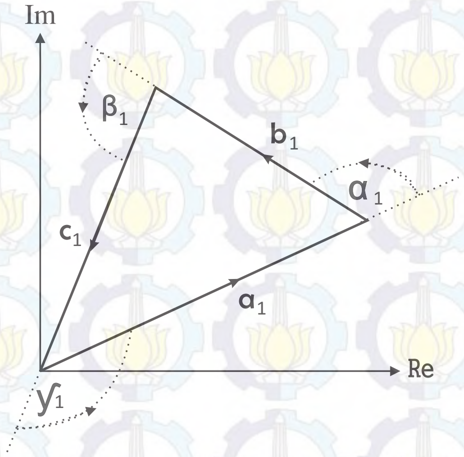
3.2 Segitiga Unitaritas untuk Sektor Lepton

Sifat unitaritas dari matrik bauran untuk tiga generasi menghasilkan sembilan hubungan , tiga yang berasal dari diagonal utama memenuhi kondisi normalisasi, dan enam yang lainnya sama dengan nol. Keenam hubungan tersebut diungkapkan melalui segitiga unitaritas dalama ruang kompleks. Berikut ini gambar keenam segitiga tersebut dalam ruang kompleks. [12]

$$\sum_{i=1}^{3} U_{\alpha i} U_{\beta i}^* = \delta_{\alpha\beta} \qquad (\alpha \neq \beta) \tag{3.12}$$

Maka ,
Untuk : $\alpha = e, \beta = \mu$

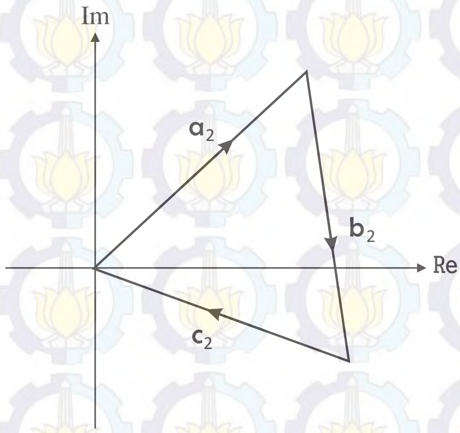
$$U_{e1}U_{\mu 1}^{*} + U_{e2}U_{\mu 2}^{*} + U_{e3}U_{\mu 3}^{*} = 0 \tag{3.13}$$



Gambar 3.1: Segitiga 1

Untuk : $\alpha = e, \beta = \tau$

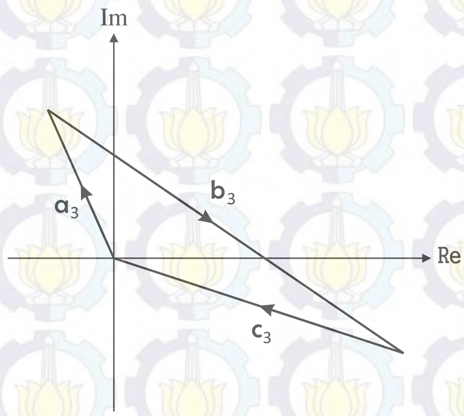
$$U_{e1}U_{\tau 1}^{*} + U_{e2}U_{\tau 2}^{*} + U_{e3}U_{\tau 3}^{*} = 0 \tag{3.14}$$



Gambar 3.2: Segitiga 2

Untuk: $\alpha = \mu, \beta = \tau$

$$U_{\mu 1}U^*_{\tau 1} + U_{\mu 2}U^*_{\tau 2} + U_{\mu 3}U^*_{\tau 3} = 0 \tag{3.15}$$



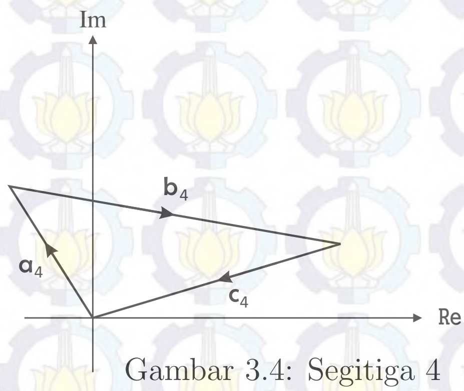
Gambar 3.3: Segitiga 3

$$\sum_{\alpha=e,\mu,\tau} U_{\alpha i}U^*_{\alpha j} = \delta_{ij} \qquad (i \neq j) \tag{3.16}$$

Maka,
Untuk : $i = 1, j = 2$

$$U_{e1}U^*_{e2} + U_{\mu 1}U^*_{\mu 2} + U_{\tau 1}U^*_{\tau 2} = 0 \tag{3.17}$$

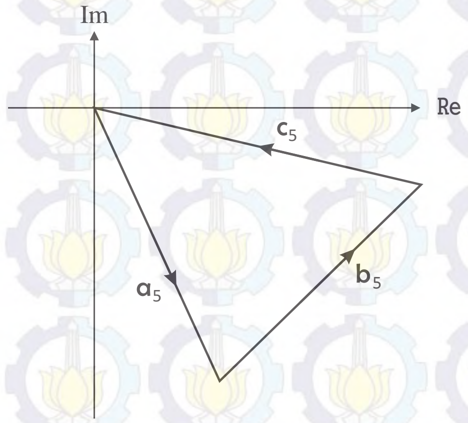
Sedangkan



Gambar 3.4: Segitiga 4

Untuk : $i = 1, j = 3$

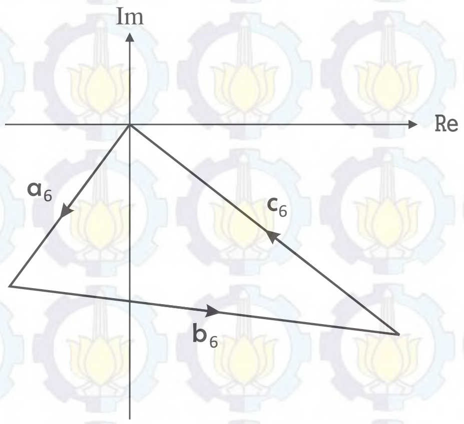
$$U_{e1}U_{e3}^* + U_{\mu1}U_{\mu3}^* + U_{\tau1}U_{\tau3}^* = 0 \tag{3.18}$$



Gambar 3.5: Segitiga 5

Untuk : $i = 2, j = 3$

$$U_{e2}U_{e3}^* + U_{\mu2}U_{\mu3}^* + U_{\tau2}U_{\tau3}^* = 0 \tag{3.19}$$



Gambar 3.6: Segitiga 6

Keenam segitiga tersebut memiliki luas yang sama (lihat lampiran). Karena keenam segitiga tersebut tidak sama dengan nol maka dimungkinkan adanya simpangan CP.

3.3 Faktor Jarlskog

Faktor Jarlskog (Jarlskog Invariant) diusulkan oleh Cecilia Jarlskog pada tahun 1973 dan disebutkan dalam artikel Kobayashi - Maskawa mengenai bauran kuark.

[23].Kemudian dipakai juga dalam bauran neutrino , yang mana matriks bauran untuk kuark maupun neutrino itu kompleks. Faktor Jarlskog didefinisikan

$$\begin{aligned} J &= Im J_{e\mu}^{12} \\ &= Im\left(U_{e1}U_{\mu1}^{*}U_{e2}^{*}U_{\mu2}\right) \end{aligned} \tag{3.20}$$

dimana matrik U bersifat uniter, maka $U^{\dagger}U = UU^{\dagger} = 1$,sehingga terdapat 6 komponen off diagonal matrik U yang sama dengan nol, yaitu

$$\sum_{i=1,2,3} U_{\alpha i}U_{\beta i}^{*} = \delta_{\alpha\beta} \qquad (\alpha \neq \beta) \tag{3.21}$$

secara eksplisit

$$(e,\mu) \rightarrow U_{e1}U_{\mu1}^{*} + U_{e2}U_{\mu2}^{*} + U_{e3}U_{\mu3}^{*} = 0 \tag{3.22}$$
$$(e,\tau) \rightarrow U_{e1}U_{\tau1}^{*} + U_{e2}U_{\tau2}^{*} + U_{e3}U_{\tau3}^{*} = 0 \tag{3.23}$$
$$(\mu,\tau) \rightarrow U_{\mu1}U_{\tau1}^{*} + U_{\mu2}U_{\tau2}^{*} + U_{\mu3}U_{\tau3}^{*} = 0 \tag{3.24}$$

sedangkan yang lain

$$\sum_{e,\mu,\tau} U_{\alpha i}U_{\alpha j}^{*} = \delta_{ij} \qquad (i \neq j) \tag{3.25}$$

secara eksplisit

$$(\nu_1,\nu_2) \rightarrow U_{e1}U_{e2}^{*} + U_{\mu1}U_{\mu2}^{*} + U_{\tau1}U_{\tau2}^{*} = 0 \tag{3.26}$$
$$(\nu_1,\nu_3) \rightarrow U_{e1}U_{e3}^{*} + U_{\mu1}U_{\mu3}^{*} + U_{\tau1}U_{\tau3}^{*} = 0 \tag{3.27}$$
$$(\nu_2,\nu_3) \rightarrow U_{e2}U_{e3}^{*} + U_{\mu2}U_{\mu3}^{*} + U_{\tau2}U_{\tau3}^{*} = 0 \tag{3.28}$$

kemudian kalikan persamaan (3.22) dengan $U_{e2}^{*}U_{\mu2}$, ambil imaginernya , didapatkan

$$\begin{aligned} U_{e1}U_{\mu1}^{*}U_{e2}^{*}U_{\mu2} + |U_{e2}|^2|U_{\mu2}|^2 + U_{e3}U_{\mu3}^{*}U_{e2}^{*}U_{\mu2} &= 0 \\ Im\left(U_{e1}U_{\mu1}^{*}U_{e2}^{*}U_{\mu2}\right) + Im\left(|U_{e2}|^2|U_{\mu2}|^2\right) + Im\left(U_{e3}U_{\mu3}^{*}U_{e2}^{*}U_{\mu2}\right) &= 0 \end{aligned} \tag{3.29}$$

sehingga

$$\begin{aligned} Im\left(U_{e1}U_{\mu1}^{*}U_{e2}^{*}U_{\mu2}\right) &= -Im\left(U_{e3}U_{\mu3}^{*}U_{e2}^{*}U_{\mu2}\right) \\ J &= -Im\left(U_{e3}U_{\mu3}^{*}U_{e2}^{*}U_{\mu2}\right) \\ Im\left(U_{e3}U_{\mu3}^{*}U_{e2}^{*}U_{\mu2}\right) &= -J \end{aligned} \tag{3.30}$$

Selanjutnya,kalikan persamaan (3.26) dengan $U_{\mu1}^{*}U_{\mu2}$ dan ambil imaginernya

$$\begin{aligned} U_{e1}U_{\mu1}^{*}U_{e2}^{*}U_{\mu2} + |U_{\mu1}|^2|U_{\mu2}|^2 + U_{\tau1}U_{\mu1}^{*}U_{\tau2}^{*}U_{\mu2} = 0 \\ Im\left(U_{e1}U_{\mu1}^{*}U_{e2}^{*}U_{\mu2}\right) + Im\left(|U_{\mu1}|^2|U_{\mu2}|^2\right) + Im\left(U_{\tau1}U_{\mu1}^{*}U_{\tau2}^{*}U_{\mu2}\right) = 0 \end{aligned} \tag{3.31}$$

sehingga

$$\begin{aligned} Im\left(U_{e1}U_{\mu1}^{*}U_{e2}^{*}U_{\mu2}\right) &= -Im\left(U_{\tau1}U_{\mu1}^{*}U_{\tau2}^{*}U_{\mu2}\right) \\ J &= -Im\left(U_{\tau1}U_{\mu1}^{*}U_{\tau2}^{*}U_{\mu2}\right) \\ Im\left(U_{\tau1}U_{\mu1}^{*}U_{\tau2}^{*}U_{\mu2}\right) &= -J \end{aligned} \tag{3.32}$$

kalikan persamaan (3.28) dengan $U_{\mu2}^{*}U_{\mu3}$ didapatkan

$$U_{e2}U_{\mu2}^{*}U_{e3}^{*}U_{\mu3} + |U_{\mu2}|^2 |U_{\mu3}|^2 + U_{\tau2}U_{\mu2}^{*}U_{\tau3}^{*}U_{\mu3} = 0 \tag{3.33}$$

kompleks konjugatkan dan ambil imaginernya, maka

$$\begin{aligned} U_{\mu2}U_{e2}^{*}U_{\mu3}^{*}U_{e3} + |U_{\mu2}|^2 |U_{\mu3}|^2 + U_{\mu2}U_{\tau2}^{*}U_{\mu3}^{*}U_{\tau3} &= 0 \\ Im\left(U_{\mu2}U_{e2}^{*}U_{\mu3}^{*}U_{e3}\right) + Im\left(|U_{\mu2}|^2 |U_{\mu3}|^2\right) + Im\left(U_{\mu2}U_{\tau2}^{*}U_{\mu3}^{*}U_{\tau3}\right) &= 0 \end{aligned} \tag{3.34}$$

sehingga

$$\begin{aligned} Im\left(U_{\mu2}U_{e2}^{*}U_{\mu3}^{*}U_{e3}\right) &= -Im\left(U_{\mu2}U_{\tau2}^{*}U_{\mu3}^{*}U_{\tau3}\right) \\ -J &= -Im\left(U_{\mu2}U_{\tau2}^{*}U_{\mu3}^{*}U_{\tau3}\right) \\ Im\left(U_{\mu2}U_{\tau2}^{*}U_{\mu3}^{*}U_{\tau3}\right) &= J \end{aligned} \tag{3.35}$$

kalikan persamaan (3.24) dengan $U_{\mu1}^{*}U_{\tau1}$ dan ambil imaginernya

$$\begin{aligned} |U_{\mu1}|^2 |U_{\tau1}|^2 + U_{\mu2}U_{\tau2}^{*}U_{\mu1}^{*}U_{\tau1} + U_{\mu3}U_{\tau3}^{*}U_{\mu1}^{*}U_{\tau1} &= 0 \\ Im\left(|U_{\mu1}|^2 |U_{\tau1}|^2\right) + Im\left(U_{\mu2}U_{\tau2}^{*}U_{\mu1}^{*}U_{\tau1}\right) + Im\left(U_{\mu3}U_{\tau3}^{*}U_{\mu1}^{*}U_{\tau1}\right) &= 0 \end{aligned} \tag{3.36}$$

sehingga

$$\begin{aligned} Im\left(U_{\mu2}U_{\tau2}^{*}U_{\mu1}^{*}U_{\tau1}\right) &= -Im\left(U_{\mu3}U_{\tau3}^{*}U_{\mu1}^{*}U_{\tau1}\right) \\ -J &= -Im\left(U_{\mu3}U_{\tau3}^{*}U_{\mu1}^{*}U_{\tau1}\right) \\ Im\left(U_{\mu3}U_{\tau3}^{*}U_{\mu1}^{*}U_{\tau1}\right) &= J \end{aligned} \tag{3.37}$$

Terlihat bahwa faktor Jarlskog ini invarian terhadap perubahan fase.

"Halaman ini sengaja dikosongkan"

Bab 4

Rumusan Eksak Osilasi Neutrino dalam Materi dengan Kerapatan Konstan

4.1 Dalam Vakum

Untuk mendapatkan perumusan eksak probabilitas osilasi neutrino dalam vakum, tinjau Hamiltonian

$$H = \begin{pmatrix} H_{ee} & H_{e\mu} & H_{e\tau} \\ H_{\mu e} & H_{\mu\mu} & H_{\mu\tau} \\ H_{\tau e} & H_{\tau\mu} & H_{\tau\tau} \end{pmatrix} \tag{4.1}$$

diagonalisasi Hamiltonian memberikan

$$U^\dagger H U = \begin{pmatrix} E_1 & & \\ & E_2 & \\ & & E_2 \end{pmatrix} \tag{4.2}$$

dengan $E_i = E + \frac{m_i^2}{2E}$, maka

$$\begin{aligned} U^\dagger H U &= \begin{pmatrix} E + \frac{m_1^2}{2E} & & \\ & E + \frac{m_2^2}{2E} & \\ & & E + \frac{m_3^2}{2E} \end{pmatrix} \\ &= E + \frac{1}{2E} \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} \end{aligned} \tag{4.3}$$

selanjutnya

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle \tag{4.4}$$

kebergantungan terhadap waktunya

$$|\nu_\alpha(t)\rangle = \sum_i U^*_{\alpha i} e^{-iE_i t} |\nu_i\rangle \tag{4.5}$$

dan

$$\langle \nu_\beta| = \sum_i \langle \nu_\beta| U_{\beta i} \tag{4.6}$$

sehingga amplitudo transisinya

$$\begin{aligned}
\langle \nu_\beta | \nu_\alpha(t)\rangle &= \sum_j \sum_i U^*_{\alpha i} e^{-iE_i t} U_{\beta j} \langle \nu_j | \nu_i \rangle \\
&= \sum_i U^*_{\alpha i} e^{-iE_i t} U_{\beta i} \\
&= \sum_i U^*_{\alpha i} e^{-i\left(E + \frac{m_i^2}{2E}\right)t} U_{\beta i} \\
&= e^{-iEt} \sum_i U^*_{\alpha i} e^{-i\frac{m_i^2}{2E}t} U_{\beta i} \\
&= e^{-iEL} \sum_i U^*_{\alpha i} e^{-i\frac{m_i^2}{2E}L} U_{\beta i}
\end{aligned} \tag{4.7}$$

sehingga amplitudo probabilitasnya

$$\begin{aligned}
P(\nu_\alpha \to \nu_\beta) &= |A(\nu_\alpha \to \nu_\beta)|^2 \\
&= \delta_{\alpha\beta} - 4 \sum_{(ij)}^{siklik} Re J^{ij}_{\alpha\beta} \sin^2 \Delta'_{ij} - 2 \sum_{(ij)}^{siklik} Im J^{ij}_{\alpha\beta} \sin 2\Delta'_{ij}
\end{aligned} \tag{4.8}$$

dengan

$$J^{ij}_{\alpha\beta} = U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} \tag{4.9}$$

$$\Delta'_{ij} \equiv \frac{\Delta_{ij} L}{4E} \equiv \frac{\left(m_i^2 - m_j^2\right) L}{4E} \tag{4.10}$$

Sehingga suku ketiga persamaan (4.8) dapat dituliskan

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{(ij)}^{siklik} Re J^{ij}_{\alpha\beta} \sin^2 \Delta'_{ij} \pm 2 \sum_{(ij)}^{siklik} J \sin 2\Delta'_{ij} \tag{4.11}$$

Tanda $\pm$ pada suku ketiga persamaan (4.11), diambil $-$ $(+)$ dalam kasus (α, β) yang merupakan permutasi (anti) siklik dari (e, μ). Penjumlahan siklik terhadap seluruh $(ij) = (12), (23), (31)$.

Selanjutnya akan di hitung secara eksplisit $Re J^{ij}_{\alpha\beta} = U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}$ dan faktor Jarlskog $J = Im\left(U_{e1} U^*_{\mu 1} U^*_{e2} U_{\mu 2}\right)$. Matrik MNS mempunyai bentuk eksplisit

$$U_{\alpha i} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{4.12}$$

dengan $s_{ij} \equiv \sin\theta_{ij}, c_{ij} \equiv \cos\theta_{ij}$. Matrik MNS memiliki 4 parameter , yaitu tiga parameter sudut $(\theta_{12}, \theta_{13}, \theta_{23})$ serta satu parameter fasa CP δ.

Sehingga untuk $i = 1, j = 2$

$$\begin{aligned}
J^{12}_{e\mu} &= U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2} \\
&= (c_{12}c_{13})\left(-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta}\right)(s_{12}c_{23})\left(c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta}\right) \\
&= \left(-s_{12}c_{12}c_{13}c_{23} - s_{13}c^2_{12}c_{13}s_{23}e^{-i\delta}\right)\left(s_{12}c_{12}c_{13}c_{23} - s^2_{12}s_{13}c_{13}s_{23}e^{i\delta}\right) \\
&= -s^2_{12}c^2_{12}c^2_{13}c^2_{23} + s^3_{12}s_{13}c_{12}c^2_{13}s_{23}c_{23}e^{i\delta} - s_{12}s_{13}c^3_{12}c^2_{13}s_{23}c_{23}e^{-i\delta} + s^2_{12}c^2_{12}c^2_{13}c^2_{23} \\
&= -s^2_{12}c^2_{12}c^2_{13}c^2_{23} + s^2_{12}c^2_{12}c^2_{13}c^2_{23} \\
&\quad + s^3_{12}s_{13}c_{12}c^2_{13}s_{23}c_{23}(\cos\delta + i\sin\delta) - s_{12}s_{13}c^3_{12}c^2_{13}s_{23}c_{23}(\cos\delta - i\sin\delta)
\end{aligned} \tag{4.13}$$

maka

$$\begin{aligned}
\mathrm{ReJ}^{12}_{\mathrm{e}\mu} &= -\left(c^2_{12} - s^2_{12}\right)s_{12}c_{12}s_{23}c_{23}s_{13}c^2_{13}\cos\delta + s^2_{12}c^2_{12}c^2_{13}\left(s^2_{13}s^2_{23} - c^2_{23}\right) \\
&= -\left(c^2_{12} - s^2_{12}\right)J_r\cos\delta + s^2_{12}c^2_{12}c^2_{13}\left(s^2_{13}s^2_{23} - c^2_{23}\right)
\end{aligned} \tag{4.14}$$

dimana $J_r = s_{12}c_{12}s_{23}c_{23}s_{13}c^2_{13}$, sedangkan

$$\begin{aligned}
ImJ^{12}_{e\mu} &= s^3_{12}s_{13}c_{12}c^2_{13}s_{23}c_{23}\sin\delta + s_{12}s_{13}c^3_{12}c^2_{13}s_{23}c_{23}\sin\delta \\
&= \left(s^2_{12} + c^2_{12}\right)s_{12}s_{13}c_{12}s_{23}c_{23}c^2_{13}\sin\delta \\
&= \left(s^2_{12} + c^2_{12}\right)J_r\sin\delta \\
&= J_r\sin\delta \\
&\equiv J
\end{aligned} \tag{4.15}$$

dengan J adalah faktor Jarlskog, untuk $i = 2, j = 3$

$$\begin{aligned}
\mathrm{J}^{23}_{\mathrm{e}\mu} &= U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3} \\
&= (s_{12}c_{13})\left(c_{12}c_{23} - s_{12}s_{23}s_{13}e^{-i\delta}\right)\left(s_{13}e^{i\delta}\right)(s_{23}c_{13}) \\
&= \left(s_{12}c_{12}c_{13}c_{23} - s^2_{12}s_{13}s_{23}c_{13}e^{-i\delta}\right)\left(s_{13}s_{23}c_{13}e^{i\delta}\right) \\
&= s_{12}s_{13}c_{12}c^2_{13}c_{23}s_{23}e^{i\delta} - s^2_{12}s^2_{13}s^2_{23}c^2_{13}
\end{aligned} \tag{4.16}$$

sehingga

$$\begin{aligned}
\mathrm{ReJ}^{23}_{\mathrm{e}\mu} &= \mathrm{Re}\left(\mathrm{s}_{12}\mathrm{s}_{13}\mathrm{c}_{12}\mathrm{c}^2_{13}\mathrm{c}_{23}\mathrm{s}_{23}\mathrm{e}^{\mathrm{i}\delta} - \mathrm{s}^2_{12}\mathrm{s}^2_{13}\mathrm{s}^2_{23}\mathrm{c}^2_{13}\right) \\
&= s_{12}c_{12}s_{23}c_{23}s_{13}c^2_{13}\cos\delta - s^2_{12}s^2_{13}s^2_{23}c^2_{13} \\
&= J_r\cos\delta - s^2_{12}s^2_{13}s^2_{23}c^2_{13}
\end{aligned} \tag{4.17}$$

untuk $i = 3, j = 1$

$$\begin{aligned}
\mathrm{J}^{31}_{\mathrm{e}\mu} &= U_{e3}U^*_{\mu 3}U^*_{e1}U_{\mu 1} \\
&= \left(s_{13}e^{-i\delta}\right)(s_{23}s_{13})(c_{12}c_{13})\left(-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta}\right) \\
&= -s_{12}s_{13}c_{12}c_{23}s_{23}c^2_{13}e^{-i\delta} - c^2_{12}s^2_{13}c^2_{13}s^2_{23}
\end{aligned} \tag{4.18}$$

maka

$$
\begin{aligned}
\mathrm{ReJ}_{e\mu}^{31} &= \mathrm{Re}\left(-\mathrm{s}_{12}\mathrm{s}_{13}\mathrm{c}_{12}\mathrm{c}_{23}\mathrm{s}_{23}\mathrm{c}_{13}^{2}\mathrm{e}^{-\mathrm{i}\delta} - \mathrm{c}_{12}^{2}\mathrm{s}_{13}^{2}\mathrm{c}_{13}^{2}\mathrm{s}_{23}^{2}\right) \\
&= -s_{12}s_{13}c_{12}c_{23}s_{23}c_{13}^{2}\cos\delta - c_{12}^{2}s_{13}^{2}c_{13}^{2}s_{23}^{2} \\
&= -J_r\cos\delta - c_{12}^{2}s_{13}^{2}c_{13}^{2}s_{23}^{2} \qquad (4.19)
\end{aligned}
$$

Dengan cara yang sama untuk $\mathrm{J}_{e\tau}^{ij}$ adalah

$$
\begin{aligned}
\mathrm{ReJ}_{e\tau}^{12} &= \mathrm{Re}\left(\mathrm{U}_{e1}\mathrm{U}_{\tau1}^{*}\mathrm{U}_{e2}^{*}\mathrm{U}_{\tau2}\right) \\
&= \mathrm{Re}\left\{(\mathrm{c}_{12}\mathrm{c}_{13})\left(\mathrm{s}_{12}\mathrm{s}_{23} - \mathrm{c}_{12}\mathrm{c}_{23}\mathrm{s}_{13}\mathrm{e}^{-\mathrm{i}\delta}\right)(\mathrm{s}_{12}\mathrm{c}_{13})\left(-\mathrm{c}_{12}\mathrm{s}_{23} - \mathrm{s}_{12}\mathrm{s}_{23}\mathrm{s}_{13}\mathrm{e}^{\mathrm{i}\delta}\right)\right\} \\
&= \left(c_{12}^{2} - s_{12}^{2}\right)J_r\cos\delta + s_{12}^{2}c_{12}^{2}c_{13}^{2}\left(c_{23}^{2}s_{13}^{2} - s_{23}^{2}\right) \qquad (4.20) \\
\mathrm{ReJ}_{e\tau}^{23} &= \mathrm{Re}\left(\mathrm{U}_{e2}\mathrm{U}_{\tau2}^{*}\mathrm{U}_{e3}^{*}\mathrm{U}_{\tau3}\right) \\
&= \left\{(\mathrm{s}_{12}\mathrm{c}_{13})\left(-\mathrm{c}_{12}\mathrm{s}_{23} - \mathrm{s}_{12}\mathrm{c}_{23}\mathrm{s}_{13}\mathrm{e}^{-\mathrm{i}\delta}\right)\left(\mathrm{s}_{13}\mathrm{e}^{-\mathrm{i}\delta}\right)(\mathrm{c}_{23}\mathrm{c}_{13})\right\} \\
&= -J_r\cos\delta - s_{12}^{2}c_{13}^{2}s_{13}^{2}c_{23}^{2} \qquad (4.21) \\
\mathrm{ReJ}_{e\tau}^{31} &= \mathrm{Re}\left(\mathrm{U}_{e3}\mathrm{U}_{\tau3}^{*}\mathrm{U}_{e1}^{*}\mathrm{U}_{\tau1}\right) \\
&= \left\{\left(\mathrm{s}_{13}\mathrm{e}^{-\mathrm{i}\delta}\right)(\mathrm{c}_{23}\mathrm{c}_{13})(\mathrm{c}_{12}\mathrm{c}_{13})\left(\mathrm{s}_{12}\mathrm{s}_{23} - \mathrm{c}_{12}\mathrm{c}_{23}\mathrm{s}_{13}\mathrm{e}^{\mathrm{i}\delta}\right)\right\} \\
&= J_r\cos\delta - c_{12}^{2}c_{23}^{2}s_{13}^{2}c_{13}^{2} \qquad (4.22)
\end{aligned}
$$

dan $\mathrm{J}_{\mu\tau}^{ij}$ adalah

$$
\begin{aligned}
\mathrm{ReJ}_{\mu\tau}^{12} &= \mathrm{Re}\left(\mathrm{U}_{\mu1}\mathrm{U}_{\tau1}^{*}\mathrm{U}_{\mu2}^{*}\mathrm{U}_{\tau2}\right) \\
&= \mathrm{Re}\left\{\left(-\mathrm{s}_{12}\mathrm{c}_{23} - \mathrm{c}_{12}\mathrm{s}_{23}\mathrm{s}_{13}\mathrm{e}^{\mathrm{i}\delta}\right)\left(\mathrm{s}_{12}\mathrm{s}_{23} - \mathrm{c}_{12}\mathrm{c}_{23}\mathrm{s}_{13}\mathrm{e}^{-\mathrm{i}\delta}\right)\left(\mathrm{c}_{12}\mathrm{c}_{23} - \mathrm{s}_{12}\mathrm{s}_{23}\mathrm{s}_{13}\mathrm{e}^{-\mathrm{i}\delta}\right)\right. \\
&\quad \left.\left(-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta}\right)\right\} \\
&= s_{12}^{2}c_{12}^{2}s_{23}^{2}c_{23}^{2}\left(1 + s_{13}^{2} + s_{13}^{4}\right) - \left(s_{12}^{2}c_{12}^{2} + s_{23}^{2}c_{23}^{2}\right)s_{13}^{2} \\
&\quad - \left(c_{12}^{2} - s_{12}^{2}\right)\left(c_{23}^{2} - s_{23}^{2}\right)s_{12}c_{12}s_{23}c_{23}s_{13}\left(1 + s_{13}^{2}\right)\cos\delta \\
&\quad + 2s_{12}^{2}c_{12}^{2}s_{23}^{2}c_{23}^{2}s_{13}^{2}\cos 2\delta \qquad (4.23) \\
\mathrm{ReJ}_{\mu\tau}^{23} &= \mathrm{Re}\left(\mathrm{U}_{\mu2}\mathrm{U}_{\tau2}^{*}\mathrm{U}_{\mu3}^{*}\mathrm{U}_{\tau3}\right) \\
&= \mathrm{Re}\left\{\left(\mathrm{c}_{12}\mathrm{c}_{23} - \mathrm{s}_{12}\mathrm{s}_{23}\mathrm{s}_{13}\mathrm{e}^{\mathrm{i}\delta}\right)\left(-\mathrm{c}_{12}\mathrm{s}_{23} - \mathrm{s}_{12}\mathrm{c}_{23}\mathrm{c}_{13}\mathrm{e}^{-\mathrm{i}\delta}\right)(\mathrm{s}_{23}\mathrm{c}_{13})(\mathrm{c}_{23}\mathrm{c}_{13})\right\} \\
&= -\left(c_{23}^{2} - s_{23}^{2}\right)J_r\cos\delta + s_{23}^{2}c_{23}^{2}c_{13}^{2}\left(c_{12}^{2}s_{13}^{2} - s_{12}^{2}\right) \qquad (4.24) \\
\mathrm{ReJ}_{\mu\tau}^{31} &= \mathrm{Re}\left(\mathrm{U}_{\mu3}\mathrm{U}_{\tau3}^{*}\mathrm{U}_{\mu1}^{*}\mathrm{U}_{\tau1}\right) \\
&= \mathrm{Re}\left\{(\mathrm{s}_{23}\mathrm{c}_{13})(\mathrm{c}_{23}\mathrm{c}_{13})\left(-\mathrm{s}_{12}\mathrm{c}_{23} - \mathrm{c}_{12}\mathrm{s}_{23}\mathrm{s}_{13}\mathrm{e}^{-\mathrm{i}\delta}\right)\left(\mathrm{s}_{12}\mathrm{s}_{23} - \mathrm{c}_{12}\mathrm{c}_{23}\mathrm{s}_{13}\mathrm{e}^{\mathrm{i}\delta}\right)\right\} \\
&= \left(c_{23}^{2} - s_{23}^{2}\right)J_r\cos\delta + s_{23}^{2}c_{23}^{2}c_{13}^{2}\left(c_{12}^{2}s_{13}^{2} - s_{12}^{2}\right) \qquad (4.25)
\end{aligned}
$$

Selanjutnya dengan menggunakan persamaan (4.11) , dihitung secara eksplisit probabilitas transisi neutrino $P\left(\nu_\alpha \to \nu_\beta\right)$ sebagai berikut.

untuk $\alpha = e, \beta = \mu$, diperoleh

$$
\begin{aligned}
P(\nu_e \to \nu_\mu) &= \delta_{e\mu} - 4\sum_{(ij)}^{siklik} ReJ_{e\mu}^{ij}\sin^2\Delta'_{ij} - 2\sum_{(ij)}^{siklik} J\sin 2\Delta'_{ij} \\
&= -4\left(\mathrm{ReJ}_{e\mu}^{12}\sin^2\Delta'_{12} + \mathrm{ReJ}_{e\mu}^{23}\sin^2\Delta'_{23} + \mathrm{ReJ}_{e\mu}^{31}\sin^2\Delta'_{31}\right) \\
&\quad -2J\left(\sin 2\Delta'_{12} + \sin 2\Delta'_{23} + \sin 2\Delta'_{31}\right) \\
&= -4\left(\mathrm{ReJ}_{e\mu}^{12}\sin^2\Delta'_{12} + \mathrm{ReJ}_{e\mu}^{23}\sin^2\Delta'_{23} + \mathrm{ReJ}_{e\mu}^{31}\sin^2\Delta'_{31}\right) \\
&\quad -2J_r\sin\delta\left(\sin 2\Delta'_{12} + \sin 2\Delta'_{23} + \sin 2\Delta'_{31}\right) \\
&= -4\left\{-\left(c_{12}^2 - s_{12}^2\right)J_r\cos\delta + s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2 - c_{23}^2\right)\right\}\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad -4\left\{J_r\cos\delta - s_{12}^2s_{13}^2s_{23}^2c_{13}^2\right\}\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad -4\left\{-J_r\cos\delta - c_{12}^2s_{13}^2c_{13}^2s_{23}^2\right\}\sin^2\left(\frac{\Delta_{31}}{4E}L\right) \\
&\quad -2J_r\sin\delta\left(\sin\left(\frac{\Delta_{12}}{2E}L\right) + \sin\left(\frac{\Delta_{23}}{2E}L\right) + \sin\left(\frac{\Delta_{31}}{2E}L\right)\right) \\
&= 4\left(c_{12}^2 - s_{12}^2\right)J_r\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\cos\delta \\
&\quad -4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2 - c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad -4J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right)\cos\delta \\
&\quad +4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad +4J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\cos\delta \\
&\quad +4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right) \\
&\quad -2J_r\left(\sin\left(\frac{\Delta_{12}}{2E}L\right) + \sin\left(\frac{\Delta_{23}}{2E}L\right) + \sin\left(\frac{\Delta_{31}}{2E}L\right)\right)\sin\delta \\
&= \left\{4\left(c_{12}^2 - s_{12}^2\right)J_r\sin^2\left(\frac{\Delta_{12}}{4E}L\right) - 4J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right)\right. \\
&\quad \left. +4J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\}\cos\delta \\
&\quad +\left\{-2J_r\left(\sin\left(\frac{\Delta_{12}}{2E}L\right) + \sin\left(\frac{\Delta_{23}}{2E}L\right) + \sin\left(\frac{\Delta_{31}}{2E}L\right)\right)\right\}\sin\delta \\
&\quad +\left\{-4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2 - c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right. \\
&\quad +4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad \left. +4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \qquad (4.26)
\end{aligned}
$$

sehingga dapat ditulis

$$P(\nu_e \to \nu_\mu) = A_{e\mu}\cos\delta + B\sin\delta + C_{e\mu} \tag{4.27}$$

dengan $A_{e\mu}, B$, dan $C_{e\mu}$ adalah konstanta.

$$\begin{aligned} A_{e\mu} &= \left\{4\left(c_{12}^2 - s_{12}^2\right) J_r \sin^2\left(\frac{\Delta_{12}}{4E}L\right) - 4J_r \sin^2\left(\frac{\Delta_{23}}{4E}L\right)\right. \\ &\left. +4J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \end{aligned} \tag{4.28}$$

$$B = \left\{-2J_r\left(\sin\left(\frac{\Delta_{12}}{2E}L\right) + \sin\left(\frac{\Delta_{23}}{2E}L\right) + \sin\left(\frac{\Delta_{31}}{2E}L\right)\right)\right\} \tag{4.29}$$

$$\begin{aligned} C_{e\mu} &= \left\{-4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2 - c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right. \\ &+4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\ &\left.+4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \end{aligned} \tag{4.30}$$

Untuk $\alpha = e, \beta = \tau$, diperoleh

$$\begin{aligned} P(\nu_e \to \nu_\tau) &= \delta_{e\tau} - 4\sum_{(ij)}^{siklik} ReJ_{e\tau}^{ij}\sin^2\Delta'_{ij} + 2\sum_{(ij)}^{siklik} J\sin 2\Delta'_{ij} \\ &= -4\left(\mathrm{ReJ}_{e\tau}^{12}\sin^2\Delta'_{12} + \mathrm{ReJ}_{e\tau}^{23}\sin^2\Delta'_{23} + \mathrm{ReJ}_{e\tau}^{31}\sin^2\Delta'_{31}\right) \\ &+2J\left(\sin 2\Delta'_{12} + \sin 2\Delta'_{23} + \sin 2\Delta'_{31}\right) \\ &= -4\left(\mathrm{ReJ}_{e\tau}^{12}\sin^2\Delta'_{12} + \mathrm{ReJ}_{e\tau}^{23}\sin^2\Delta'_{23} + \mathrm{ReJ}_{e\tau}^{31}\sin^2\Delta'_{31}\right) \\ &+2J_r\sin\delta\left(\sin 2\Delta'_{12} + \sin 2\Delta'_{23} + \sin 2\Delta'_{31}\right) \\ &= -4\left\{\left(c_{12}^2 - s_{12}^2\right)J_r\cos\delta + s_{12}^2c_{12}^2c_{13}^2\left(c_{23}^2s_{13}^2 - s_{23}^2\right)\right\}\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\ &-4\left\{-J_r\cos\delta - s_{12}^2c_{13}^2s_{13}^2c_{23}^2\right\}\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\ &-4\left\{J_r\cos\delta - c_{12}^2c_{23}^2s_{13}^2c_{13}^2\right\}\sin^2\left(\frac{\Delta_{31}}{4E}L\right) \\ &+2J_r\sin\delta\left(\sin\left(\frac{\Delta_{12}}{2E}L\right) + \sin\left(\frac{\Delta_{23}}{2E}L\right) + \sin\left(\frac{\Delta_{31}}{2E}L\right)\right) \end{aligned}$$

$$
\begin{aligned}
&= -4\left(c_{12}^2-s_{12}^2\right)J_r\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\cos\delta \\
&\quad -4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2-c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad +4J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right)\cos\delta \\
&\quad +4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad -4J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\cos\delta \\
&\quad +4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right) \\
&\quad +2J_r\left(\sin\left(\frac{\Delta_{12}}{2E}L\right)+\sin\left(\frac{\Delta_{23}}{2E}L\right)+\sin\left(\frac{\Delta_{31}}{2E}L\right)\right)\sin\delta \\
&= -\left\{4\left(c_{12}^2-s_{12}^2\right)J_r\sin^2\left(\frac{\Delta_{12}}{4E}L\right)-4J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right)\right. \\
&\quad \left.+4J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\}\cos\delta \\
&\quad -\left\{-2J_r\left(\sin\left(\frac{\Delta_{12}}{2E}L\right)+\sin\left(\frac{\Delta_{23}}{2E}L\right)+\sin\left(\frac{\Delta_{31}}{2E}L\right)\right)\right\}\sin\delta \\
&\quad +\left\{-4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2-c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right. \\
&\quad +4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad \left.+4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \qquad (4.31)
\end{aligned}
$$

sehingga dapat ditulis

$$
P\left(\nu_e\rightarrow\nu_\tau\right)=-A_{e\mu}\cos\delta-B\sin\delta+C_{e\tau} \qquad (4.32)
$$

dengan $A_{e\mu}, B$, dan $C_{e\tau}$ adalah konstanta.

$$
\begin{aligned}
C_{e\tau} &= \left\{-4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2-c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right. \\
&\quad +4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad \left.+4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \qquad (4.33)
\end{aligned}
$$

untuk $\alpha = \mu, \beta = \tau$, diperoleh

$$
\begin{aligned}
P\left(\nu_\mu \rightarrow \nu_\tau\right) &= \delta_{\mu\tau} - 4\sum_{(ij)}^{siklik} ReJ_{\mu\tau}^{ij}\sin^2\Delta'_{ij} - 2\sum_{(ij)}^{siklik} J\sin 2\Delta'_{ij} \\
&= -4\left(\mathrm{ReJ}_{e\mu}^{12}\sin^2\Delta'_{12} + \mathrm{ReJ}_{e\mu}^{23}\sin^2\Delta'_{23} + \mathrm{ReJ}_{e\mu}^{31}\sin^2\Delta'_{31}\right) \\
&\quad -2J\left(\sin 2\Delta'_{12} + \sin 2\Delta'_{23} + \sin 2\Delta'_{31}\right) \\
&= -4\left(\mathrm{ReJ}_{e\mu}^{12}\sin^2\Delta'_{12} + \mathrm{ReJ}_{e\mu}^{23}\sin^2\Delta'_{23} + \mathrm{ReJ}_{e\mu}^{31}\sin^2\Delta'_{31}\right) \\
&\quad -2J_r\sin\delta\left(\sin 2\Delta'_{12} + \sin 2\Delta'_{23} + \sin 2\Delta'_{31}\right) \\
&= -4\left\{s_{12}^2c_{12}^2s_{23}^2c_{23}^2\left(1+s_{13}^2+s_{13}^4\right) - \left(s_{12}^2c_{12}^2+s_{23}^2c_{23}^2\right)s_{13}^2\right. \\
&\quad -\left(c_{12}^2-s_{12}^2\right)\left(c_{23}^2-s_{23}^2\right)s_{12}c_{12}s_{23}c_{23}s_{13}\left(1+s_{13}^2\right)\cos\delta \\
&\quad \left.+2s_{12}^2c_{12}^2s_{23}^2c_{23}^2s_{13}^2\cos 2\delta\right\}\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad -4\left\{-\left(c_{23}^2-s_{23}^2\right)J_r\cos\delta + s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\right\}\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad -4\left\{\left(c_{23}^2-s_{23}^2\right)J_r\cos\delta + s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\right\}\sin^2\left(\frac{\Delta_{31}}{4E}L\right) \\
&\quad -2J_r\sin\delta\left(\sin\left(\frac{\Delta_{12}}{2E}L\right)+\sin\left(\frac{\Delta_{23}}{2E}L\right)+\sin\left(\frac{\Delta_{31}}{2E}L\right)\right) \\
&= -4s_{12}^2c_{12}^2s_{23}^2c_{23}^2\left(1+s_{13}^2+s_{13}^4\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad +4\left(s_{12}^2c_{12}^2+s_{23}^2c_{23}^2\right)s_{13}^2\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad +4\left(c_{12}^2-s_{12}^2\right)\left(c_{23}^2-s_{23}^2\right)s_{12}c_{12}s_{23}c_{23}s_{13}\left(1+s_{13}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\cos\delta \\
&\quad -8s_{12}^2c_{12}^2s_{23}^2c_{23}^2s_{13}^2\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\cos 2\delta \\
&\quad +4\left(c_{23}^2-s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right)\cos\delta \\
&\quad -4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad -4\left(c_{23}^2-s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\cos\delta \\
&\quad -4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\sin^2\left(\frac{\Delta_{31}}{4E}L\right) \\
&\quad -2J_r\left(\sin\left(\frac{\Delta_{12}}{2E}L\right)+\sin\left(\frac{\Delta_{23}}{2E}L\right)+\sin\left(\frac{\Delta_{31}}{2E}L\right)\right)\sin\delta
\end{aligned}
$$

$$
\begin{aligned}
= & \left\{+4\left(c_{12}^2-s_{12}^2\right)\left(c_{23}^2-s_{23}^2\right)s_{12}c_{12}s_{23}c_{23}s_{13}\left(1+s_{13}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right. \\
& +4\left(c_{23}^2-s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
& \left.-4\left(c_{23}^2-s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\}\cos\delta \\
& +\left\{-2J_r\left(\sin\left(\frac{\Delta_{12}}{2E}L\right)+\sin\left(\frac{\Delta_{23}}{2E}L\right)+\sin\left(\frac{\Delta_{31}}{2E}L\right)\right)\right\}\sin\delta \\
& +\left\{-4s_{12}^2c_{12}^2s_{23}^2c_{23}^2\left(1+s_{13}^2+s_{13}^4\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right. \\
& +4\left(s_{12}^2c_{12}^2+s_{23}^2c_{23}^2\right)s_{13}^2\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
& -4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\sin^2\left(\frac{\Delta_{31}}{4E}L\right) \\
& \left.-4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \\
& +\left\{-8s_{12}^2c_{12}^2s_{23}^2c_{23}^2s_{13}^2\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right\}\cos 2\delta
\end{aligned} \tag{4.34}
$$

sehingga dapat ditulis

$$
P\left(\nu_\mu\rightarrow\nu_\tau\right)=A_{\mu\tau}\cos\delta+B\sin\delta+C_{\mu\tau}+D\cos 2\delta \tag{4.35}
$$

dengan $A_{\mu\tau}$, B,$C_{e\tau}$, dan D adalah konstanta .

$$
\begin{aligned}
A_{\mu\tau} = & \left\{+4\left(c_{12}^2-s_{12}^2\right)\left(c_{23}^2-s_{23}^2\right)s_{12}c_{12}s_{23}c_{23}s_{13}\left(1+s_{13}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right. \\
& +4\left(c_{23}^2-s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
& \left.-4\left(c_{23}^2-s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\}
\end{aligned} \tag{4.36}
$$

$$
\begin{aligned}
C_{\mu\tau} = & \left\{-4s_{12}^2c_{12}^2s_{23}^2c_{23}^2\left(1+s_{13}^2+s_{13}^4\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right. \\
& +4\left(s_{12}^2c_{12}^2+s_{23}^2c_{23}^2\right)s_{13}^2\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
& -4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\sin^2\left(\frac{\Delta_{31}}{4E}L\right) \\
& \left.-4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\}
\end{aligned} \tag{4.37}
$$

$$
D = \left\{-8s_{12}^2c_{12}^2s_{23}^2c_{23}^2s_{13}^2\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right\} \tag{4.38}
$$

Sekarang dihitung probabilitas survival neutrino , dengan menggunakan sifat unitaritas dan probabilitas transisi , didapatkan

$$\begin{aligned} P(\nu_e \to \nu_e) &= 1 - P(\nu_e \to \nu_\mu) - P(\nu_e \to \nu_\tau) \\ &= 1 - A_{e\mu}\cos\delta - B\sin\delta - C_{e\mu} + A_{e\mu}\cos\delta + B\sin\delta - C_{e\tau} \\ &= 1 - C_{e\mu} - C_{e\tau} \\ &\equiv C_{ee} \end{aligned} \tag{4.39}$$

dimana

$$\begin{aligned} C_{ee} &= 1 - C_{e\mu} - C_{e\tau} \\ &= 1 - \left\{ -4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2 - c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \right. \\ &\quad +4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\ &\quad \left. +4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \\ &\quad - \left\{ -4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2 - c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \right. \\ &\quad +4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\ &\quad \left. +4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \end{aligned} \tag{4.40}$$

sedangkan

$$\begin{aligned} P(\nu_\mu \to \nu_\mu) &= 1 - P(\nu_\mu \to \nu_e) - P(\nu_\mu \to \nu_\tau) \\ &= 1 - A_{e\mu}\cos\delta + B\sin\delta - C_{e\mu} - A_{\mu\tau}\cos\delta - B\sin\delta - C_{\mu\tau} - D\cos 2\delta \\ &= (-A_{e\mu} - A_{\mu\tau})\cos\delta + 1 - C_{e\mu} - C_{e\tau} - D\cos 2\delta \\ &= A_{\mu\mu}\cos\delta + C_{\mu\mu} - D\cos 2\delta \end{aligned} \tag{4.41}$$

dengan

$$\begin{aligned} A_{\mu\mu} &= -A_{e\mu} - A_{\mu\tau} \\ &= -\left\{4\left(c_{12}^2 - s_{12}^2\right)J_r\sin^2\left(\frac{\Delta_{12}}{4E}L\right) - 4J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right)\right. \\ &\quad \left. +4J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \\ &\quad -\left\{+4\left(c_{12}^2 - s_{12}^2\right)\left(c_{23}^2 - s_{23}^2\right)s_{12}c_{12}s_{23}c_{23}s_{13}\left(1 + s_{13}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right. \\ &\quad +4\left(c_{23}^2 - s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\ &\quad \left. -4\left(c_{23}^2 - s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \end{aligned} \tag{4.42}$$

dan

$$\begin{aligned}
C_{\mu\mu} &= 1 - C_{e\mu} - C_{\mu\tau} \\
&= 1 - \left\{ -4s_{12}^2 c_{12}^2 c_{13}^2 \left(s_{13}^2 s_{23}^2 - c_{23}^2\right) \sin^2\left(\frac{\Delta_{12}}{4E}L\right) \right. \\
&\quad +4s_{12}^2 s_{13}^2 s_{23}^2 c_{13}^2 \sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad \left. +4c_{12}^2 s_{13}^2 c_{13}^2 s_{23}^2 \sin^2\left(\frac{\Delta_{31}}{4E}L\right) \right\} \\
&\quad - \left\{ -4s_{12}^2 c_{12}^2 s_{23}^2 c_{23}^2 \left(1 + s_{13}^2 + s_{13}^4\right) \sin^2\left(\frac{\Delta_{12}}{4E}L\right) \right. \\
&\quad +4\left(s_{12}^2 c_{12}^2 + s_{23}^2 c_{23}^2\right) s_{13}^2 \sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad -4s_{23}^2 c_{23}^2 c_{13}^2 \left(c_{12}^2 s_{13}^2 - s_{12}^2\right) \sin^2\left(\frac{\Delta_{31}}{4E}L\right) \\
&\quad \left. -4s_{23}^2 c_{23}^2 c_{13}^2 \left(c_{12}^2 s_{13}^2 - s_{12}^2\right) \sin^2\left(\frac{\Delta_{31}}{4E}L\right) \right\} \qquad (4.43)
\end{aligned}$$

sedangkan

$$\begin{aligned}
P\left(\nu_\tau \to \nu_\tau\right) &= 1 - P\left(\nu_e \to \nu_\tau\right) - P\left(\nu_\mu \to \nu_\tau\right) \\
&= 1 + A_{e\mu}\cos\delta + B\sin\delta - C_{e\tau} - A_{\mu\tau}\cos\delta - B\sin\delta - C_{\mu\tau} - D\cos 2\delta \\
&= \left(A_{e\mu} - A_{\mu\tau}\right)\cos\delta + 1 - C_{e\tau} - C_{\mu\tau} - D\cos 2\delta \\
&= A_{\tau\tau}\cos\delta + C_{\tau\tau} - D\cos 2\delta \qquad (4.44)
\end{aligned}$$

dengan

$$\begin{aligned}
A_{\tau\tau} &= A_{e\mu} - A_{\mu\tau} \\
&= \left\{ 4\left(c_{12}^2 - s_{12}^2\right) J_r \sin^2\left(\frac{\Delta_{12}}{4E}L\right) - 4J_r \sin^2\left(\frac{\Delta_{23}}{4E}L\right) \right. \\
&\quad \left. +4J_r \sin^2\left(\frac{\Delta_{31}}{4E}L\right) \right\} \\
&\quad - \left\{ +4\left(c_{12}^2 - s_{12}^2\right)\left(c_{23}^2 - s_{23}^2\right) s_{12}c_{12}s_{23}c_{23}s_{13}\left(1 + s_{13}^2\right) \sin^2\left(\frac{\Delta_{12}}{4E}L\right) \right. \\
&\quad +4\left(c_{23}^2 - s_{23}^2\right) J_r \sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad \left. -4\left(c_{23}^2 - s_{23}^2\right) J_r \sin^2\left(\frac{\Delta_{31}}{4E}L\right) \right\} \qquad (4.45)
\end{aligned}$$

dan

$$
\begin{aligned}
C_{\tau\tau} &= 1 - C_{e\tau} - C_{\mu\tau} \\
&= 1 - \left\{ -4s_{12}^2 c_{12}^2 c_{13}^2 \left(s_{13}^2 s_{23}^2 - c_{23}^2\right) \sin^2\left(\frac{\Delta_{12}}{4E}L\right) \right. \\
&\quad + 4s_{12}^2 s_{13}^2 s_{23}^2 c_{13}^2 \sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad \left. + 4c_{12}^2 s_{13}^2 c_{13}^2 s_{23}^2 \sin^2\left(\frac{\Delta_{31}}{4E}L\right) \right\} \\
&= - \left\{ -4s_{12}^2 c_{12}^2 s_{23}^2 c_{23}^2 \left(1 + s_{13}^2 + s_{13}^4\right) \sin^2\left(\frac{\Delta_{12}}{4E}L\right) \right. \\
&\quad + 4\left(s_{12}^2 c_{12}^2 + s_{23}^2 c_{23}^2\right) s_{13}^2 \sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad - 4s_{23}^2 c_{23}^2 c_{13}^2 \left(c_{12}^2 s_{13}^2 - s_{12}^2\right) \sin^2\left(\frac{\Delta_{31}}{4E}L\right) \\
&\quad \left. - 4s_{23}^2 c_{23}^2 c_{13}^2 \left(c_{12}^2 s_{13}^2 - s_{12}^2\right) \sin^2\left(\frac{\Delta_{31}}{4E}L\right) \right\} \qquad (4.46)
\end{aligned}
$$

4.2 Dalam Materi

Untuk mendapatkan rumusan eksak probabilitas neutrino dalam materi , perhatikan hamiltonian dalam materi berikut ini

$$
\begin{aligned}
\tilde{H} &= \begin{pmatrix} \tilde{H}_{ee} & \tilde{H}_{e\mu} & \tilde{H}_{e\tau} \\ \tilde{H}_{\mu e} & \tilde{H}_{\mu\mu} & \tilde{H}_{\mu\tau} \\ \tilde{H}_{\tau e} & \tilde{H}_{\tau\mu} & \tilde{H}_{\tau\tau} \end{pmatrix} \\
&= H + \frac{1}{2E} \begin{pmatrix} A & & \\ & 0 & \\ & & 0 \end{pmatrix} \qquad (4.47)
\end{aligned}
$$

dengan : $A = 2\sqrt{2}G_F \rho_e E$ adalah potensial dalam materi.
Selanjutnya , diagonalisasi Hamiltonian dalam ini memberikan

$$
\begin{aligned}
\tilde{U}^\dagger \tilde{H} \tilde{U} &= \begin{pmatrix} E_1 & & \\ & E_1 & \\ & & E_2 \end{pmatrix} \\
&= \begin{pmatrix} E + \frac{m_1^2}{2E} & & \\ & E + \frac{m_2^2}{2E} & \\ & & E + \frac{m_3^2}{2E} \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&= E + \frac{1}{2E}\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} \\
&= E + \frac{1}{2E}\begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix} \qquad (4.48)
\end{aligned}
$$

dimana λ_i adalah nilai eigen massa dalam materi , selanjutnya

$$
\begin{aligned}
|\nu_\alpha\rangle &= \sum_i \tilde{U}^*_{\alpha i} |\nu_i\rangle \qquad (4.49) \\
|\nu_\alpha(t)\rangle &= \sum_i \tilde{U}^*_{\alpha i} e^{-iE_i t} |\nu_i\rangle \qquad (4.50) \\
\langle\nu_\beta| &= \sum_i \langle\nu_\beta| \tilde{U}_{\beta i} \qquad (4.51)
\end{aligned}
$$

amplitudo transisinya

$$
\begin{aligned}
\langle\nu_\beta|\nu_\beta(t)\rangle &= \sum_j \sum_i \tilde{U}^*_{\alpha i} e^{-iE_i t} \tilde{U}_{\beta_j} \langle\nu_j|\nu_i\rangle \\
&= \sum_i \tilde{U}^*_{\alpha i} e^{-iE_i t} \tilde{U}_{\beta i} \\
&= \sum_i \tilde{U}^*_{\alpha i} e^{-i\left(E + \frac{m_i^2}{2E}\right)t} \tilde{U}_{\beta i} \\
&= e^{-iEt} \sum_i \tilde{U}^*_{\alpha i} e^{-i\frac{\lambda_i}{2E}t} \tilde{U}_{\beta i} \\
&= e^{-iEL} \sum_i \tilde{U}^*_{\alpha i} e^{-i\frac{\lambda_i}{2E}L} \tilde{U}_{\beta i} \qquad (4.52)
\end{aligned}
$$

Sehingga didapatkan probabilitas neutrino dalam materi adalah seperti pada persamaan (4.11) dengan mengganti $U \to \tilde{U}$,untuk kasus $\alpha \neq \beta$ didapatkan probabilitas osilasi neutrino dalam materi adalah

$$
P(\nu_\alpha \to \nu_\beta) = -4 \sum_{(ij)}^{siklik} Re\tilde{J}^{ij}_{\alpha\beta} \sin^2 \tilde{\Delta}'_{ij} \pm 2 \sum_{(ij)}^{siklik} \tilde{J} \sin 2\tilde{\Delta}_{ij} \qquad (4.53)
$$

Diperkenalkan $\tilde{p}_{\alpha\beta}$ dan $\tilde{q}_{\alpha\beta}$ adalah

$$
\begin{aligned}
\tilde{p}_{\alpha\beta} &= 2E\tilde{H}_{\alpha\beta} \qquad (4.54) \\
\tilde{q}_{\alpha\beta} &= (2E)^2 \tilde{\mathcal{H}}_{\alpha\beta} = (2E)^2 \left(\tilde{H}_{\gamma\beta}\tilde{H}_{\alpha\gamma} - \tilde{H}_{\alpha\beta}\tilde{H}_{\gamma\gamma}\right) \qquad (4.55)
\end{aligned}
$$

dimana $(\alpha\beta\gamma) = (e\mu\tau), (\mu\tau e), (\tau e\mu)$.
maka mulai dari

1. Hubungan unitaritas matriks MNS

$$\begin{aligned}\sum_i \tilde{U}_{\alpha i}\tilde{U}^*_{\beta i} &= \delta_{\alpha\beta}\\ \tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1}+\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \delta_{\alpha\beta}\\ \tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} &= \delta_{\alpha\beta}-\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}-\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3}\end{aligned} \tag{4.56}$$

2. Hubungan kedua $\tilde{H}=\tilde{U}\text{diag}\left(\lambda_1,\lambda_2,\lambda_3\right)\tilde{\mathrm{U}}^\dagger$

$$\begin{aligned}\sum_i \lambda_i\tilde{U}_{\alpha i}\tilde{U}^*_{\beta i} &= \tilde{p}_{\alpha\beta}\\ \lambda_1\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1}+\lambda_2\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\lambda_3\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{p}_{\alpha\beta}\end{aligned} \tag{4.57}$$

3. Hubungan ketiga

$$\begin{aligned}(2E)^2\mathcal{H} &= (2E)^2\tilde{H}^{-1}\left(\det\tilde{\mathrm{H}}\right)\\ &= \tilde{U}\text{diag}\left(\frac{1}{\lambda_1},\frac{1}{\lambda_2},\frac{1}{\lambda_3}\right)\tilde{\mathrm{U}}^\dagger\times\lambda_1\lambda_2\lambda_3\end{aligned} \tag{4.58}$$

didapatkan

$$\begin{aligned}\sum_{ijk}\lambda_j\lambda_k\tilde{U}_{\alpha i}\tilde{U}^*_{\beta i} &= \tilde{q}_{\alpha\beta}\\ \lambda_2\lambda_3\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1}+\lambda_3\lambda_1\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\lambda_1\lambda_2\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta}\end{aligned} \tag{4.59}$$

substitusi persamaan (4.56) ke persamaan (4.57) , didapatkan

$$\begin{aligned}\lambda_1\left(\delta_{\alpha\beta}-\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}-\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3}\right)+\lambda_2\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\lambda_3\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{p}_{\alpha\beta}\\ \lambda_1\delta_{\alpha\beta}-\lambda_1\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}-\lambda_1\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3}+\lambda_2\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\lambda_3\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{p}_{\alpha\beta}\\ \lambda_1\delta_{\alpha\beta}+\left(\lambda_2-\lambda_1\right)\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\left(\lambda_3-\lambda_1\right)\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{p}_{\alpha\beta}\\ \lambda_1\delta_{\alpha\beta}+\Delta_{21}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\Delta_{31}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{p}_{\alpha\beta}\\ \lambda_1\delta_{\alpha\beta}-\Delta_{12}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\Delta_{31}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{p}_{\alpha\beta}\end{aligned} \tag{4.60}$$

substitusi persamaan (4.56) ke persamaan (4.59) , didapatkan

$$\begin{aligned}\lambda_2\lambda_3\left(\delta_{\alpha\beta}-\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}-\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3}\right)+\lambda_3\lambda_1\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\lambda_1\lambda_2\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta}\\ \lambda_2\lambda_3\delta_{\alpha\beta}-\lambda_2\lambda_3\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}-\lambda_2\lambda_3\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3}+\lambda_3\lambda_1\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\lambda_1\lambda_2\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta}\\ \lambda_2\lambda_3\delta_{\alpha\beta}+\left(\lambda_3\lambda_1-\lambda_2\lambda_3\right)\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\left(\lambda_1\lambda_2-\lambda_2\lambda_3\right)\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta}\\ \lambda_2\lambda_3\delta_{\alpha\beta}+\lambda_3\left(\lambda_1-\lambda_2\right)\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\lambda_2\left(\lambda_1-\lambda_3\right)\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta}\\ \lambda_2\lambda_3\delta_{\alpha\beta}+\lambda_3\Delta_{12}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\lambda_2\Delta_{13}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta}\\ \lambda_2\lambda_3\delta_{\alpha\beta}+\lambda_3\Delta_{12}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}-\lambda_2\Delta_{31}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta}\end{aligned} \tag{4.61}$$

dari persamaan (4.60) didapatkan

$$\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} = \frac{\lambda_1\delta_{12} - \tilde{p}_{\alpha\beta} + \tilde{\Delta}_{31}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3}}{\tilde{\Delta}_{12}} \tag{4.62}$$

substitusi persamaan (4.62) ke persamaan (4.61) , didapatkan

$$\begin{aligned}
\lambda_2\lambda_3\delta_{\alpha\beta} + \lambda_3\Delta_{12}\left\{\frac{\lambda_1\delta_{12} - \tilde{p}_{\alpha\beta} + \tilde{\Delta}_{31}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3}}{\tilde{\Delta}_{12}}\right\} - \lambda_2\Delta_{31}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta} \\
\lambda_2\lambda_3\delta_{\alpha\beta} + \lambda_3\lambda_1\delta_{\alpha\beta} - \tilde{p}_{\alpha\beta}\lambda_3 + \lambda_3\tilde{\Delta}_{31}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} - \lambda_2\tilde{\Delta}_{31}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta} \\
\lambda_3\left(\lambda_1 + \lambda_2\right)\delta_{\alpha\beta} - \tilde{p}_{\alpha\beta}\lambda_3 + \left(\lambda_3 - \lambda_2\right)\tilde{\Delta}_{31}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta} \\
\lambda_3\left(\lambda_1 + \lambda_2\right)\delta_{\alpha\beta} - \tilde{p}_{\alpha\beta}\lambda_3 + \tilde{\Delta}_{32}\tilde{\Delta}_{31}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta} \\
\lambda_3\left(\lambda_1 + \lambda_2\right)\delta_{\alpha\beta} - \tilde{p}_{\alpha\beta}\lambda_3 - \tilde{\Delta}_{23}\tilde{\Delta}_{31}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta} \\
\lambda_3\left(\lambda_1 + \lambda_2\right)\delta_{\alpha\beta} - \tilde{p}_{\alpha\beta}\lambda_3 + \tilde{\Delta}_{23}\tilde{\Delta}_{13}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta}
\end{aligned} \tag{4.63}$$

sehingga

$$\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} = \frac{\tilde{p}_{\alpha 3}\lambda_3 + \tilde{q}_{\alpha\beta} - \delta_{\alpha\beta}\lambda_3\left(\lambda_1 + \lambda_2\right)}{\tilde{\Delta}_{23}\tilde{\Delta}_{13}} \tag{4.64}$$

selanjutnya untuk yang kedua dari persamaan (4.56) didapatkan

$$\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} = \delta_{\alpha\beta} - \tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} \tag{4.65}$$

substitusi persamaan (4.65) ke persamaan (4.57) didapatkan

$$\begin{aligned}
\lambda_1\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_2\left(\delta_{\alpha\beta} - \tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3}\right) + \lambda_3\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{p}_{\alpha\beta} \\
\lambda_1\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_2\delta_{\alpha\beta} - \lambda_2\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \lambda_2\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} + \lambda_3\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{p}_{\alpha\beta} \\
\left(\lambda_1 - \lambda_2\right)\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_2\delta_{\alpha\beta} + \left(\lambda_3 - \lambda_2\right)\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{p}_{\alpha\beta} \\
\tilde{\Delta}_{12}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_2\delta_{\alpha\beta} + \tilde{\Delta}_{32}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{p}_{\alpha\beta} \\
\tilde{\Delta}_{12}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_2\delta_{\alpha\beta} - \tilde{\Delta}_{23}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{p}_{\alpha\beta}
\end{aligned} \tag{4.66}$$

substitusi persamaan (4.56) ke persamaan (4.59) didapatkan

$$\begin{aligned}
\lambda_2\lambda_3\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_3\lambda_1\left(\delta_{\alpha\beta} - \tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3}\right) + \lambda_1\lambda_2\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta} \\
\lambda_2\lambda_3\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_3\lambda_1\delta_{\alpha\beta} - \lambda_3\lambda_1\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \lambda_3\lambda_1\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} + \lambda_1\lambda_2\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta} \\
\lambda_3\lambda_1\delta_{\alpha\beta} + \left(\lambda_2\lambda_3 - \lambda_3\lambda_1\right)\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \left(\lambda_1\lambda_2 - \lambda_3\lambda_1\right)\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta} \\
\lambda_3\lambda_1\delta_{\alpha\beta} + \lambda_3\left(\lambda_2 - \lambda_1\right)\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_1\left(\lambda_2 - \lambda_3\right)\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta} \\
\lambda_3\lambda_1\delta_{\alpha\beta} + \lambda_3\tilde{\Delta}_{21}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_1\tilde{\Delta}_{23}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta} \\
\lambda_3\lambda_1\delta_{\alpha\beta} - \lambda_3\tilde{\Delta}_{12}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_1\tilde{\Delta}_{23}\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta}
\end{aligned} \tag{4.67}$$

dari persamaan (4.66) didapatkan

$$\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} = \frac{\lambda_2\delta_{\alpha\beta} + \tilde{\Delta}_{12}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \tilde{p}_{\alpha\beta}}{\tilde{\Delta}_{23}} \tag{4.68}$$

substitusi persamaan (4.68) ke persamaan (4.67) didapatkan

$$\begin{aligned} \lambda_3\lambda_1\delta_{\alpha\beta} - \lambda_3\tilde{\Delta}_{12}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_1\tilde{\Delta}_{23}\left\{\frac{\lambda_2\delta_{\alpha\beta} + \tilde{\Delta}_{12}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \tilde{p}_{\alpha\beta}}{\tilde{\Delta}_{23}}\right\} &= \tilde{q}_{\alpha\beta} \\ \lambda_3\lambda_1\delta_{\alpha\beta} - \lambda_3\tilde{\Delta}_{12}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_1\lambda_2\delta_{\alpha\beta} + \lambda_1\tilde{\Delta}_{12}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \tilde{p}_{\alpha\beta}\lambda_1 &= \tilde{q}_{\alpha\beta} \\ \lambda_1(\lambda_2 + \lambda_3)\delta_{\alpha\beta} - (\lambda_3 - \lambda_2)\tilde{\Delta}_{12}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \tilde{p}_{\alpha\beta}\lambda_1 &= \tilde{q}_{\alpha\beta} \\ \delta_{\alpha\beta}\lambda_1(\lambda_2 + \lambda_3) - \tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \tilde{p}_{\alpha\beta}\lambda_1 &= \tilde{q}_{\alpha\beta} \\ \delta_{\alpha\beta}\lambda_1(\lambda_2 + \lambda_3) + \tilde{\Delta}_{31}\tilde{\Delta}_{21}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \tilde{p}_{\alpha\beta}\lambda_1 &= \tilde{q}_{\alpha\beta} \end{aligned} \tag{4.69}$$

sehingga didapatkan

$$\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} = \frac{\tilde{p}_{\alpha\beta}\lambda_1 + \tilde{q}_{\alpha\beta} - \delta_{\alpha\beta}\lambda_1(\lambda_2 + \lambda_3)}{\tilde{\Delta}_{21}\tilde{\Delta}_{31}} \tag{4.70}$$

dari persamaan (4.56) didapatkan

$$\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} = \delta_{\alpha\beta} - \tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} \tag{4.71}$$

substitusi persamaan (4.71) ke persamaan (4.57) didapatkan

$$\begin{aligned} \lambda_1\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_2\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} + \lambda_3\left(\delta_{\alpha\beta} - \tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}\right) &= \tilde{p}_{\alpha\beta} \\ \lambda_1\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_2\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} + \lambda_3\delta_{\alpha\beta} - \lambda_3\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \lambda_3\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} &= \tilde{p}_{\alpha\beta} \\ \lambda_3\delta_{\alpha\beta} + (\lambda_1 - \lambda_3)\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + (\lambda_2 - \lambda_3)\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} &= \tilde{p}_{\alpha\beta} \\ \lambda_3\delta_{\alpha\beta} + \tilde{\Delta}_{13}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \Delta_{23}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} &= \tilde{p}_{\alpha\beta} \\ \lambda_3\delta_{\alpha\beta} - \tilde{\Delta}_{31}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \Delta_{23}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} &= \tilde{p}_{\alpha\beta} \end{aligned} \tag{4.72}$$

substitusi persamaan (4.71) ke persamaan (4.59) didapatakan

$$\begin{aligned} \lambda_2\lambda_3\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_3\lambda_1\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} + \lambda_1\lambda_2\left(\delta_{\alpha\beta} - \tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}\right) &= \tilde{q}_{\alpha\beta} \\ \lambda_2\lambda_3\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_3\lambda_1\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} + \lambda_1\lambda_2\delta_{\alpha\beta} - \lambda_1\lambda_2\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \lambda_1\lambda_2\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} &= \tilde{q}_{\alpha\beta} \\ \lambda_1\lambda_2\delta_{\alpha\beta} + \lambda_2(\lambda_3 - \lambda_1)\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_1(\lambda_3 - \lambda_2)\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} &= \tilde{q}_{\alpha\beta} \\ \lambda_1\lambda_2\delta_{\alpha\beta} + \lambda_2\tilde{\Delta}_{31}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} + \lambda_1\tilde{\Delta}_{32}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} &= \tilde{q}_{\alpha\beta} \\ \lambda_1\lambda_2\delta_{\alpha\beta} + \lambda_2\tilde{\Delta}_{31}\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} - \lambda_1\tilde{\Delta}_{23}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} &= \tilde{q}_{\alpha\beta} \end{aligned} \tag{4.73}$$

dari persamaan (4.72) didapatkan

$$\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} = \frac{\lambda_3\delta_{\alpha\beta} + \tilde{\Delta}_{23}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}}{\tilde{\Delta}_{31}} \tag{4.74}$$

substitusi persamaan (4.74) ke persamaan (4.73) didapatkan

$$\lambda_1\lambda_2\delta_{\alpha\beta} + \lambda_2\tilde{\Delta}_{31}\left\{\frac{\lambda_3\delta_{\alpha\beta} + \tilde{\Delta}_{23}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}}{\tilde{\Delta}_{31}}\right\} - \lambda_1\tilde{\Delta}_{23}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} = \tilde{q}_{\alpha\beta}$$

$$\lambda_1\lambda_2\delta_{\alpha\beta} + \lambda_2\lambda_3\delta_{\alpha\beta} + \lambda_2\tilde{\Delta}_{23}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} - \tilde{p}_{\alpha\beta}\lambda_2 - \lambda_1\tilde{\Delta}_{23}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} = \tilde{q}_{\alpha\beta}$$

$$\delta_{\alpha\beta}\lambda_2(\lambda_3 + \lambda_1) + (\lambda_2 - \lambda_1)\tilde{\Delta}_{23}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} - \tilde{p}_{\alpha\beta} = \tilde{q}_{\alpha\beta}$$

$$\delta_{\alpha\beta}\lambda_1(\lambda_2 + \lambda_3) + \tilde{\Delta}_{21}\tilde{\Delta}_{23}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} - \tilde{p}_{\alpha\beta} = \tilde{q}_{\alpha\beta}$$

$$\delta_{\alpha\beta}\lambda_1(\lambda_2 + \lambda_3) + \tilde{\Delta}_{12}\tilde{\Delta}_{32}\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} - \tilde{p}_{\alpha\beta} = \tilde{q}_{\alpha\beta} \quad (4.75)$$

sehingga didapatkan

$$\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2} = \frac{\tilde{p}_{\alpha\beta}\lambda_2 + \tilde{q}_{\alpha\beta} - \delta_{\alpha\beta}\lambda_2(\lambda_3 + \lambda_1)}{\tilde{\Delta}_{12}\tilde{\Delta}_{32}} \quad (4.76)$$

atau secara kompak dapat dituliskan

$$\tilde{U}_{\alpha i}\tilde{U}^*_{\beta i} = \frac{\tilde{p}_{\alpha\beta}\lambda_i + \tilde{q}_{\alpha\beta} - \delta_{\alpha\beta}\lambda_i(\lambda_j + \lambda_k)}{\tilde{\Delta}_{ji}\tilde{\Delta}_{ki}} \quad (4.77)$$

sehingga kita punya

$$\tilde{U}_{\alpha i}\tilde{U}^*_{\beta i} = \frac{\tilde{p}_{\alpha\beta}\lambda_i + \tilde{q}_{\alpha\beta} - \delta_{\alpha\beta}\lambda_i(\lambda_j + \lambda_k)}{\tilde{\Delta}_{ji}\tilde{\Delta}_{ki}} \quad (4.78)$$

$$\tilde{U}^*_{\alpha j}\tilde{U}_{\beta j} = \frac{\tilde{p}^*_{\alpha\beta}\lambda_j + \tilde{q}^*_{\alpha\beta} - \delta_{\alpha\beta}\lambda_j(\lambda_k + \lambda_i)}{\tilde{\Delta}_{ij}\tilde{\Delta}_{kj}} \quad (4.79)$$

sekarang akan dihitung

$$\tilde{J}^{ij}_{\alpha\beta} = \tilde{U}_{\alpha i}\tilde{U}^*_{\beta i}\tilde{U}^*_{\alpha j}\tilde{U}_{\beta j} \quad (4.80)$$

Jika $i = 1, j = 2$ dan $\alpha \neq \beta$, maka

$$\begin{aligned}\tilde{J}^{12}_{\alpha\beta} &= \tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1}\tilde{U}^*_{\alpha 2}\tilde{U}_{\beta 2}\\ &= \left(\frac{\tilde{p}_{\alpha\beta}\lambda_1 + \tilde{q}_{\alpha\beta} - \delta_{\alpha\beta}\lambda_1(\lambda_2 + \lambda_3)}{\tilde{\Delta}_{21}\tilde{\Delta}_{31}}\right)\left(\frac{\tilde{p}^*_{\alpha\beta}\lambda_2 + \tilde{q}^*_{\alpha\beta} - \delta_{\alpha\beta}\lambda_2(\lambda_3 + \lambda_1)}{\tilde{\Delta}_{12}\tilde{\Delta}_{32}}\right)\\ &= \frac{|\tilde{p}_{\alpha\beta}|^2\lambda_1\lambda_2 + |\tilde{q}_{\alpha\beta}|^2 + \tilde{p}_{\alpha\beta}\tilde{q}^*_{\alpha\beta}\lambda_1 + \tilde{p}^*_{\alpha\beta}\tilde{q}_{\alpha\beta}\lambda_2}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \quad (4.81)\end{aligned}$$

dan diambil rielnya, didapatkan

$$\mathrm{Re}\tilde{\mathrm{J}}^{12}_{\alpha\beta} = \frac{|\tilde{p}_{\alpha\beta}|^2\lambda_1\lambda_2 + |\tilde{q}_{\alpha\beta}|^2 + \mathrm{Re}\left(\tilde{\mathrm{p}}_{\alpha\beta}\tilde{\mathrm{q}}^*_{\alpha\beta}\right)(\lambda_1 + \lambda_2)}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \quad (4.82)$$

untuk $i = 2, j = 3$, maka

$$\begin{aligned}\tilde{J}^{23}_{\alpha\beta} &= \tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}\tilde{U}^*_{\alpha 3}\tilde{U}_{\beta 3}\\ &= \left(\frac{\tilde{p}_{\alpha\beta}\lambda_2 + \tilde{q}_{\alpha\beta} - \delta_{\alpha\beta}\lambda_2(\lambda_3 + \lambda_1)}{\tilde{\Delta}_{12}\tilde{\Delta}_{32}}\right)\left(\frac{\tilde{p}^*_{\alpha\beta}\lambda_3 + \tilde{q}^*_{\alpha\beta} - \delta_{\alpha\beta}\lambda_3(\lambda_1 + \lambda_2)}{\tilde{\Delta}_{23}\tilde{\Delta}_{13}}\right)\\ &= \frac{|\tilde{p}_{\alpha\beta}|^2\lambda_2\lambda_3 + |\tilde{q}_{\alpha\beta}|^2 + \tilde{p}_{\alpha\beta}\tilde{q}^*_{\alpha\beta}\lambda_2 + \tilde{p}^*_{\alpha\beta}\tilde{q}_{\alpha\beta}\lambda_3}{\tilde{\Delta}_{12}\tilde{\Delta}_{32}\tilde{\Delta}_{23}\tilde{\Delta}_{13}} \quad (4.83)\end{aligned}$$

dan diambil rielnya , didapatkan

$$\mathrm{Re}\tilde{\mathrm{J}}^{23}_{\alpha\beta} = \frac{|\tilde{p}_{\alpha\beta}|^2 \lambda_2\lambda_3 + \tilde{q}_{\alpha\beta} + \mathrm{Re}\left(\tilde{\mathrm{p}}_{\alpha\beta}\tilde{\mathrm{q}}^*_{\alpha\beta}\right)(\lambda_2+\lambda_3)}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \quad (4.84)$$

untuk $i = 3, j = 1$, maka

$$\begin{aligned} \tilde{J}^{31}_{\alpha\beta} &= \tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3}\tilde{U}^*_{\alpha 1}\tilde{U}_{\beta 1} \\ &= \left(\frac{\tilde{p}_{\alpha\beta}\lambda_3 + \tilde{q}_{\alpha\beta} - \delta_{\alpha\beta}\lambda_3(\lambda_1+\lambda_2)}{\tilde{\Delta}_{21}\tilde{\Delta}_{31}}\right)\left(\frac{\tilde{p}^*_{\alpha\beta}\lambda_1 + \tilde{q}^*_{\alpha\beta} - \delta_{\alpha\beta}\lambda_1(\lambda_2+\lambda_3)}{\tilde{\Delta}_{12}\tilde{\Delta}_{32}}\right) \\ &= \frac{|\tilde{p}_{\alpha\beta}|^2 \lambda_3\lambda_1 + |\tilde{q}_{\alpha\beta}|^2 + \tilde{p}_{\alpha\beta}\tilde{q}^*_{\alpha\beta}\lambda_3 + \tilde{p}^*_{\alpha\beta}\tilde{q}_{\alpha\beta}\lambda_1}{\tilde{\Delta}_{13}\tilde{\Delta}_{23}\tilde{\Delta}_{21}\tilde{\Delta}_{31}} \end{aligned} \quad (4.85)$$

dan diambil rielnya, didapatkan

$$\mathrm{Re}\tilde{\mathrm{J}}^{31}_{\alpha\beta} = \frac{|\tilde{p}_{\alpha\beta}|^2 \lambda_3\lambda_1 + \tilde{q}_{\alpha\beta} + \mathrm{Re}\left(\tilde{\mathrm{p}}_{\alpha\beta}\tilde{\mathrm{q}}^*_{\alpha\beta}\right)(\lambda_3+\lambda_1)}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \quad (4.86)$$

atau secara kompak dapat ditulis

$$\mathrm{Re}\tilde{\mathrm{J}}^{\mathrm{ij}}_{\alpha\beta} = \frac{|\tilde{\mathrm{p}}_{\alpha\beta}|^2 \lambda_{\mathrm{i}}\lambda_{\mathrm{j}} + |\tilde{\mathrm{q}}_{\alpha\beta}|^2 + \mathrm{Re}\left(\tilde{\mathrm{p}}_{\alpha\beta}\tilde{\mathrm{q}}^*_{\alpha\beta}\right)(\lambda_{\mathrm{i}}+\lambda_{\mathrm{j}})}{\tilde{\Delta}_{\mathrm{ij}}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \quad (4.87)$$

Selanjutnya akan dicari faktor Jarlskog dalam materi seperti didalam vakum ,didefinisikan

$$\tilde{J} = \mathrm{Im}\tilde{\mathrm{J}}^{12}_{e\mu} = \mathrm{Im}\left(\tilde{\mathrm{U}}_{e1}\tilde{\mathrm{U}}^*_{\mu 1}\tilde{\mathrm{U}}^*_{e2}\tilde{\mathrm{U}}_{\mu 2}\right) \quad (4.88)$$

dari persamaan (4.78) dan persamaan (4.79) , kita peroleh

$$\tilde{U}_{e1}\tilde{U}^*_{\mu 1} = \frac{\tilde{p}_{e\mu}\lambda_1 + \tilde{q}_{e\mu}}{\tilde{\Delta}_{21}\tilde{\Delta}_{31}} \quad (4.89)$$

$$\tilde{U}^*_{e2}\tilde{U}_{\mu 2} = \frac{\tilde{p}^*_{e\mu}\lambda_2 + \tilde{q}^*_{e\mu}}{\tilde{\Delta}_{12}\tilde{\Delta}_{32}} \quad (4.90)$$

jika kedua persamaan tersebut dikalikan, didapatkan

$$\begin{aligned} \tilde{U}_{e1}\tilde{U}^*_{\mu 1}\tilde{U}^*_{e2}\tilde{U}_{\mu 2} &= \left(\frac{\tilde{p}_{e\mu}\lambda_1 + \tilde{q}_{e\mu}}{\tilde{\Delta}_{21}\tilde{\Delta}_{31}}\right)\left(\frac{\tilde{p}^*_{e\mu}\lambda_2 + \tilde{q}^*_{e\mu}}{\tilde{\Delta}_{12}\tilde{\Delta}_{32}}\right) \\ &= \frac{|\tilde{p}_{e\mu}|^2 \lambda_1\lambda_2 + |\tilde{q}_{e\mu}|^2 + \tilde{p}_{e\mu}\tilde{q}^*_{e\mu}\lambda_1 + \tilde{p}^*_{e\mu}\tilde{q}_{e\mu}\lambda_2}{\tilde{\Delta}_{21}\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{32}} \end{aligned} \quad (4.91)$$

kemudian ambil imajinernya, maka

$$
\begin{aligned}
\mathrm{Im}\left(\tilde{\mathrm{U}}_{e1}\tilde{\mathrm{U}}^*_{\mu1}\tilde{\mathrm{U}}^*_{e2}\tilde{\mathrm{U}}_{\mu2}\right) &= \frac{\mathrm{Im}\left(\tilde{\mathrm{p}}_{e\mu}\tilde{\mathrm{q}}^*_{e\mu}\right)\lambda_1 + \mathrm{Im}\left(\tilde{\mathrm{p}}^*_{e\mu}\tilde{\mathrm{q}}_{e\mu}\right)\lambda_2}{\tilde{\Delta}_{21}\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{32}} \\
&= \frac{\mathrm{Im}\left(\tilde{\mathrm{p}}_{e\mu}\tilde{\mathrm{q}}^*_{e\mu}\right)\lambda_1 - \mathrm{Im}\left(\tilde{\mathrm{p}}_{e\mu}\tilde{\mathrm{q}}^*_{e\mu}\right)\lambda_2 + \mathrm{Im}\left(\tilde{\mathrm{p}}_{e\mu}\tilde{\mathrm{q}}^*_{e\mu}\right)\lambda_2 + \mathrm{Im}\left(\tilde{\mathrm{p}}^*_{e\mu}\tilde{\mathrm{q}}_{e\mu}\right)\lambda_2}{\tilde{\Delta}_{21}\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{32}} \\
&= \frac{\mathrm{Im}\left(\tilde{\mathrm{p}}_{e\mu}\tilde{\mathrm{q}}^*_{e\mu}\right)(\lambda_1 - \lambda_2)}{\tilde{\Delta}_{21}\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{32}} \\
&= \frac{\mathrm{Im}\left(\tilde{\mathrm{p}}_{e\mu}\tilde{\mathrm{q}}^*_{e\mu}\right)\tilde{\Delta}_{12}}{\tilde{\Delta}_{21}\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{32}} \\
\tilde{J} &= \frac{\mathrm{Im}\left(\tilde{\mathrm{p}}_{e\mu}\tilde{\mathrm{q}}^*_{e\mu}\right)}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \qquad (4.92)
\end{aligned}
$$

Selanjutnya akan dihitung probabilitas transisi neutrino dalam materi dengan menggunakan persamaan (4.53) .Sebelumnya , kita hitung dulu kuantitas $\tilde{p}_{e\mu}$ pada persamaan (4.57) dan $\tilde{q}_{e\mu}$ pada persamaan (4.59) . Karena kedua kuantitas tersebut tidak bergantung pada $\tilde{H}_{ee}$, maka keduanya ekivalen dengan $p_{e\mu}$ dan $q_{e\mu}$ seperti dalam vakum. Dari persamaan (4.56) didapatkan hubungan

$$U_{e1}U^*_{\mu_1} = -U_{e2}U^*_{\mu_2} - U_{e3}U^*_{\mu_3} \qquad (4.93)$$

substitusi persamaan (4.93) ke persamaan (4.57) , diperoleh

$$
\begin{aligned}
p_{e\mu} &= -\lambda_1 U_{e2}U^*_{\mu_2} - \lambda_1 U_{e3}U^*_{\mu_3} + \lambda_2 U_{e2}U^*_{\mu_2} + \lambda_3 U_{e3}U^*_{\mu_3} \\
&= (\lambda_2 - \lambda_1)\,U_{e2}U^*_{\mu_2} + (\lambda_3 - \lambda_1)\,U_{e3}U^*_{\mu_2} \\
&= \Delta_{21}U_{e2}U^*_{\mu2} + \Delta_{31}U_{e3}U^*_{\mu3} \qquad (4.94)
\end{aligned}
$$

jika dimasukkan elemen matriks MNS nya didapatkan

$$
\begin{aligned}
p_{e\mu} &= \Delta_{21}(s_{12}c_{13})\left(c_{12}c_{23} - s_{12}s_{23}s_{13}e^{-i\delta}\right) + \Delta_{31}\left(s_{13}e^{-i\delta}\right)(s_{23}c_{13}) \\
&= \Delta_{21}s_{12}c_{13}c_{12}c_{23} - \Delta_{21}s_{12}c_{13}s_{13}s_{23}e^{-i\delta} + \Delta_{21}s_{23}c_{13}s_{13}e^{-i\delta} \\
&= \left(\Delta_{31} - \Delta_{21}s^2_{12}\right)s_{23}s_{13}c_{13}e^{-i\delta} + \Delta_{21}s_{12}c_{12}c_{23}c_{13} \qquad (4.95)
\end{aligned}
$$

sehingga dapat dituliskan

$$p_{e\mu} = p^a_{e\mu}e^{-i\delta} + p^b_{e\mu} \qquad (4.96)$$

dimana

$$
\begin{aligned}
p^a_{e\mu} &= \left(\Delta_{31} - \Delta_{21}s^2_{12}\right)s_{23}s_{13}c_{13} \qquad (4.97) \\
p^b_{e\mu} &= \Delta_{21}s_{12}c_{12}c_{23}c_{13} \qquad (4.98)
\end{aligned}
$$

Dengan cara yang sama , untuk $q_{e\mu}$ didapatkan

$$
\begin{aligned}
q_{e\mu} &= \Delta_{31}\Delta_{21}U_{e1}U^*_{\mu1} \\
&= \Delta_{31}\Delta_{21}(c_{12}c_{13})\left(-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta}\right) \\
&= -\Delta_{31}\Delta_{21}c^2_{12}c_{13}s_{23}s_{13}e^{-i\delta} - \Delta_{31}\Delta_{21}c_{12}c_{13}s_{12}c_{23} \qquad (4.99)
\end{aligned}
$$

sehingga dapat dituliskan

$$q_{e\mu} = q^a_{e\mu} e^{-i\delta} + q^b_{e\mu} \tag{4.100}$$

dimana

$$q^a_{e\mu} = -\Delta_{31}\Delta_{21}c^2_{12}s_{23}s_{13}c_{13} \tag{4.101}$$

$$q^b_{e\mu} = -\Delta_{31}\Delta_{21}s_{12}c_{12}c_{23}c_{13} \tag{4.102}$$

Selanjutnya dihitung probabilitas transisi $P(\nu_e \to \nu_\mu)$. Dari persamaan (4.53) , untuk $\alpha = 2$ dan $\beta = \mu$ serta indeks i,j dijalankan , didapatkan

$$\begin{aligned}
P(\nu_e \to \nu_\mu) &= -4\sum_{(ij)}^{siklik} Re\tilde{J}^{ij}_{e\mu}\sin^2\tilde{\Delta}'_{ij} - 2\sum_{(ij)}^{siklik}\tilde{J}\sin 2\tilde{\Delta}'_{ij} \\
&= -4\left\{Re\tilde{J}^{12}_{e\mu}\sin^2\tilde{\Delta}'_{12} + Re\tilde{J}^{23}_{e\mu}\sin^2\tilde{\Delta}'_{23} + Re\tilde{J}^{31}_{e\mu}\sin^2\tilde{\Delta}'_{31}\right\} \\
&\quad -2\tilde{J}\left(\sin 2\tilde{\Delta}'_{12} + \sin 2\tilde{\Delta}'_{23} + \sin 2\tilde{\Delta}'_{31}\right) \\
&= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{|\tilde{p}_{e\mu}|^2\lambda_1\lambda_2 + |\tilde{q}_{e\mu}|^2 + Re\left(\tilde{p}_{e\mu}\tilde{q}^*_{e\mu}\right)(\lambda_1+\lambda_2)\right\}\sin^2\tilde{\Delta}'_{12} \\
&\quad -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{|\tilde{p}_{e\mu}|^2\lambda_2\lambda_3 + |\tilde{q}_{e\mu}|^2 + Re\left(\tilde{p}_{e\mu}\tilde{q}^*_{e\mu}\right)(\lambda_2+\lambda_3)\right\}\sin^2\tilde{\Delta}'_{23} \\
&\quad -\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{|\tilde{p}_{e\mu}|^2\lambda_3\lambda_1 + |\tilde{q}_{e\mu}|^2 + Re\left(\tilde{p}_{e\mu}\tilde{q}^*_{e\mu}\right)(\lambda_3+\lambda_1)\right\}\sin^2\tilde{\Delta}'_{31} \\
&\quad -2\tilde{J}\left(\sin 2\tilde{\Delta}'_{12} + \sin 2\tilde{\Delta}'_{23} + \sin 2\tilde{\Delta}'_{31}\right)
\end{aligned} \tag{4.103}$$

karena

$$\begin{aligned}
\tilde{p}_{e\mu} &= \tilde{H}_{e\mu} \\
&= H_{e\mu} \\
&= p_{e\mu}
\end{aligned} \tag{4.104}$$

dan

$$\begin{aligned}
\tilde{q}_{e\mu} &= \tilde{H}_{e\tau}\tilde{H}_{\tau\mu} - \tilde{H}_{e\mu}\tilde{H}_{\tau\tau} \\
&= H_{e\tau}H_{\tau\mu} - H_{e\mu}H_{\tau\tau} \\
&= q_{e\mu}
\end{aligned} \tag{4.105}$$

sehingga

$$\begin{aligned}
P(\nu_e \to \nu_\mu) &= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{|p_{e\mu}|^2\lambda_1\lambda_2 + |q_{e\mu}|^2 + Re\left(p_{e\mu}q^*_{e\mu}\right)(\lambda_1+\lambda_2)\right\}\sin^2\tilde{\Delta}'_{12} \\
&\quad -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{|p_{e\mu}|^2\lambda_2\lambda_3 + |q_{e\mu}|^2 + Re\left(p_{e\mu}q^*_{e\mu}\right)(\lambda_2+\lambda_3)\right\}\sin^2\tilde{\Delta}'_{23} \\
&\quad -\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{|p_{e\mu}|^2\lambda_3\lambda_1 + |q_{e\mu}|^2 + Re\left(p_{e\mu}q^*_{e\mu}\right)(\lambda_3+\lambda_1)\right\}\sin^2\tilde{\Delta}'_{31} \\
&\quad -2\tilde{J}\left(\sin 2\tilde{\Delta}'_{12} + \sin 2\tilde{\Delta}'_{23} + \sin 2\tilde{\Delta}'_{31}\right)
\end{aligned} \tag{4.106}$$

sekarang dihitung dulu

$$\begin{aligned} |p_{e\mu}|^2 &= p_{e\mu}p_{e\mu}^* \\ &= \left(p_{e\mu}^a e^{-i\delta} + p_{e\mu}^b\right)\left(p_{e\mu}^a e^{i\delta} + p_{e\mu}^b\right) \\ &= \left(p_{e\mu}^a\right)^2 + p_{e\mu}^a p_{e\mu}^b e^{-i\delta} + p_{e\mu}^a p_{e\mu}^b e^{i\delta} + \left(p_{e\mu}^b\right)^2 \\ &= \left(p_{e\mu}^a\right)^2 + \left(p_{e\mu}^b\right)^2 + p_{e\mu}^a p_{e\mu}^b \left(e^{i\delta} + e^{-i\delta}\right) \\ &= \left(p_{e\mu}^a\right)^2 + \left(p_{e\mu}^b\right)^2 + 2p_{e\mu}^a p_{e\mu}^b \cos\delta \end{aligned} \tag{4.107}$$

hitung

$$\left(p_{e\mu}^a\right)^2 = \left(\Delta_{21} - \Delta_{21}s_{12}^2\right)^2 s_{23}^2 s_{13}^2 c_{13}^2 \tag{4.108}$$

$$\left(p_{e\mu}^b\right)^2 = \Delta_{21}^2 s_{12}^2 c_{12} c_{23}^2 c_{13}^2 \tag{4.109}$$

$$\begin{aligned} p_{e\mu}^a p_{e\mu}^b &= \left\{\left(\Delta_{31} - \Delta_{21}s_{12}^2\right) s_{23}s_{13}c_{13}\right\}\left(\Delta_{21}s_{12}c_{12}c_{23}c_{13}\right) \\ &= \left(\Delta_{31}s_{23}s_{13}c_{13} - \Delta_{21}s_{12}^2 s_{23}s_{13}c_{13}\right)\Delta_{21}s_{12}c_{12}c_{23}c_{13} \\ &= \Delta_{21}\Delta_{31}s_{12}c_{13}s_{23}c_{23}s_{13}c_{13}^2 - \Delta_{21}^2 s_{12}^3 s_{23}s_{13}c_{23}c_{12}c_{13}^2 \\ &= \Delta_{21}\Delta_{31}J_r - \Delta_{21}^2 s_{12}^2 J_r \end{aligned} \tag{4.110}$$

dan

$$\begin{aligned} |q_{e\mu}|^2 &= q_{e\mu}q_{e\mu}^* \\ &= \left(q_{e\mu}^a e^{-i\delta} + q_{e\mu}^b\right)\left(q_{e\mu}^a e^{i\delta} + q_{e\mu}^b\right) \\ &= \left(q_{e\mu}^a\right)^2 + q_{e\mu}^a q_{e\mu}^b e^{-i\delta} + q_{e\mu}^a q_{e\mu}^b e^{i\delta} + \left(q_{e\mu}^b\right)^2 \\ &= \left(q_{e\mu}^a\right)^2 + \left(q_{e\mu}^b\right)^2 + q_{e\mu}^a q_{e\mu}^b \left(e^{i\delta} + e^{-i\delta}\right) \\ &= \left(q_{e\mu}^a\right)^2 + \left(q_{e\mu}^b\right)^2 + 2q_{e\mu}^a q_{e\mu}^b \cos\delta \end{aligned} \tag{4.111}$$

hitung

$$\left(q_{e\mu}^a\right)^2 = \Delta_{31}^2\Delta_{21}^2 c_{12}^4 s_{23}^2 s_{13}^2 c_{13}^2 \tag{4.112}$$

$$\left(q_{e\mu}^b\right)^2 = \Delta_{31}^2\Delta_{21}^2 s_{12}^2 c_{12}^2 c_{23}^2 c_{13}^2 \tag{4.113}$$

$$\begin{aligned} q_{e\mu}^a q_{e\mu}^b &= \left(-\Delta_{31}\Delta_{21}c_{12}^2 s_{23}s_{13}c_{13}\right)\left(-\Delta_{31}\Delta_{21}s_{12}c_{12}c_{23}c_{13}\right) \\ &= \Delta_{31}^2\Delta_{21}^2 s_{12}c_{12}^3 s_{23}c_{23}s_{13}c_{13}^2 \\ &= \Delta_{31}^2\Delta_{21}^2 c_{12}^2 J_r \end{aligned} \tag{4.114}$$

sedangkan

$$\begin{aligned} p_{e\mu}q_{e\mu}^* &= \left(p_{e\mu}^a e^{-i\delta} + p_{e\mu}^b\right)\left(q_{e\mu}^a e^{-i\delta} + q_{e\mu}^b\right) \\ &= p_{e\mu}^a q_{e\mu}^a + p_{e\mu}^a q_{e\mu}^b e^{-i\delta} + p_{e\mu}^b q_{e\mu}^a e^{i\delta} + p_{e\mu}^b q_{e\mu}^b \\ &= p_{e\mu}^a q_{e\mu}^a + p_{e\mu}^b q_{e\mu}^b + p_{e\mu}^a q_{e\mu}^b \left(\cos\delta - i\sin\delta\right) + p_{e\mu}^b q_{e\mu}^a \left(\cos\delta + \sin\delta\right) \\ &= p_{e\mu}^a q_{e\mu}^a + p_{e\mu}^b q_{e\mu}^b + \left(p_{e\mu}^a q_{e\mu}^b + p_{e\mu}^b q_{e\mu}^a\right)\cos\delta + i\left(p_{e\mu}^b q_{e\mu}^a - p_{e\mu}^a q_{e\mu}^b\right)\sin\delta \end{aligned} \tag{4.115}$$

maka

$$Re\left(p_{e\mu}q_{e\mu}^{*}\right) = p_{e\mu}^{a}q_{e\mu}^{a} + p_{e\mu}^{b}q_{e\mu}^{b} + \left(p_{e\mu}^{a}q_{e\mu}^{b} + p_{e\mu}^{b}q_{e\mu}^{a}\right)\cos\delta \tag{4.116}$$

hitung

$$\begin{aligned}
p_{e\mu}^{a}q_{e\mu}^{a} &= \left\{\left(\Delta_{31} - \Delta_{21}s_{12}^{2}\right)s_{23}s_{13}c_{13}\right\}\left(-\Delta_{31}\Delta_{21}c_{12}^{2}s_{23}s_{13}c_{13}\right) \\
&= \left(\Delta_{31}s_{23}s_{13}c_{13} - \Delta_{21}s_{12}^{2}s_{23}s_{13}c_{13}\right)\left(-\Delta_{31}\Delta_{21}c_{12}^{2}s_{23}s_{13}c_{13}\right) \\
&= -\Delta_{31}^{2}\Delta_{21}c_{12}^{2}s_{23}^{2}s_{13}^{2}c_{13} + \Delta_{31}\Delta_{21}^{2}s_{12}^{2}c_{12}^{2}s_{23}^{2}s_{13}^{2}c_{13}^{2} \qquad (4.117)\\
p_{e\mu}^{b}q_{e\mu}^{b} &= \left(\Delta_{21}s_{12}c_{12}c_{23}c_{13}\right)\left(-\Delta_{31}\Delta_{21}s_{12}c_{12}c_{23}c_{13}\right) \\
&= -\Delta_{31}\Delta_{21}^{2}s_{12}^{2}c_{12}^{2}c_{23}^{2}c_{13}^{2} \qquad (4.118)
\end{aligned}$$

dan

$$\begin{aligned}
p_{e\mu}^{a}q_{e\mu}^{b} + p_{e\mu}^{b}q_{e\mu}^{a} &= \left\{\left(\Delta_{31} - \Delta_{21}s_{12}^{2}\right)s_{23}s_{13}c_{13}\right\}\left(-\Delta_{31}\Delta_{21}s_{12}c_{12}c_{23}c_{13}\right) \\
&\quad -\left(\Delta_{21}s_{12}c_{12}c_{23}c_{13}\right)\left(-\Delta_{31}\Delta_{21}c_{12}^{2}s_{23}s_{13}c_{13}\right) \\
&= -\Delta_{31}^{2}\Delta_{21}s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^{2} + \Delta_{21}^{2}\Delta_{31}s_{12}^{3}c_{12}s_{23}c_{23}s_{13}c_{13}^{2} \\
&\quad +\Delta_{21}^{2}\Delta_{31}s_{12}c_{12}^{3}s_{23}c_{23}s_{13}c_{13}^{2} \\
&= -\Delta_{31}^{2}\Delta_{21}s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^{2} \\
&\quad +\Delta_{21}^{2}\Delta_{31}\left(s_{12}^{2} + c_{12}^{2}\right)s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^{2} \\
&= -\Delta_{31}^{2}\Delta_{21}J_{r} + \Delta_{21}^{2}\Delta_{31}J_{r} \\
&= J_{r}\Delta_{31}\Delta_{21}\left(\Delta_{21} - \Delta_{31}\right) \\
&= J_{r}\Delta_{31}\Delta_{21}\left(\lambda_{2} - \lambda_{1} - \lambda_{3} + \lambda_{1}\right) \\
&= J_{r}\Delta_{31}\Delta_{21}\Delta_{23} \\
&= -J_{r}\Delta_{12}\Delta_{23}\Delta_{31} \qquad (4.119)
\end{aligned}$$

ambil imaginernya

$$Im\left(p_{e\mu}q_{e\mu}^{*}\right) = \left(p_{e\mu}^{b}q_{e\mu}^{a} - p_{e\mu}^{a}q_{e\mu}^{b}\right)\sin\delta \tag{4.120}$$

maka faktor Jarlskog-nya dapat dituliskan

$$\begin{aligned}
\tilde{J} &= \frac{Im\left(p_{e\mu}q_{e\mu}^{*}\right)}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \\
&= \frac{\left(p_{e\mu}^{b}q_{e\mu}^{a} - p_{e\mu}^{a}q_{e\mu}^{b}\right)\sin\delta}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \qquad (4.121)
\end{aligned}$$

kemudian dihitung

$$\begin{aligned}
p^b_{e\mu}q^a_{e\mu} - p^a_{e\mu}q^b_{e\mu} &= \left(\Delta_{21}s_{12}c_{12}c_{23}c_{13}\right)\left(-\Delta_{31}\Delta_{21}c^2_{12}s_{23}s_{13}c_{13}\right) \\
&\quad - \left\{\left(\Delta_{31} - \Delta_{21}s^2_{12}\right)s_{23}s_{13}c_{13}\right\}\left(-\Delta_{31}\Delta_{21}s_{12}c_{12}c_{23}c_{13}\right) \\
&= -\Delta^2_{21}\Delta_{31}s_{12}c^3_{12}c^2_{13}s_{13}s_{23}c_{23} + \Delta^2_{31}\Delta_{21}s_{12}c_{12}c^2_{13}s_{13}s_{23}c_{23} \\
&\quad -\Delta^2_{21}\Delta_{31}s^3_{13}c_{12}c^2_{13}s_{13}s_{23}c_{23} \\
&= -\Delta^2_{21}\Delta_{31}\left(c^2_{12} + s^2_{12}\right)c_{12}s_{12}s_{13}s_{23}c_{23}c^2_{13} + \Delta^2_{31}\Delta_{21}J_r \\
&= -\Delta^2_{21}\Delta_{31}J_r + \Delta^2_{31}\Delta_{21}J_r \\
&= -J_r\left(\Delta^2_{21}\Delta_{31} - \Delta^2_{31}\Delta_{21}\right) \\
&= -J_r\Delta_{21}\Delta_{31}\left(\Delta_{21} - \Delta_{31}\right) \\
&= -J_r\Delta_{21}\Delta_{31}\left(\lambda_2 - \lambda_1 - (\lambda_3 - \lambda_1)\right) \\
&= -J_r\Delta_{21}\Delta_{31}\left(\lambda_2 - \lambda_3\right) \\
&= -J_r\Delta_{21}\Delta_{31}\Delta_{23} \\
&= J_r\Delta_{12}\Delta_{23}\Delta_{31} \qquad (4.122)
\end{aligned}$$

sehingga faktor Jarlskognya dapat dituliskan

$$\begin{aligned}
\tilde{J} &= \frac{Im\left(p_{e\mu}q^*_{e\mu}\right)}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \\
&= \frac{J_r\Delta_{12}\Delta_{23}\Delta_{31}\sin\delta}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \qquad (4.123)
\end{aligned}$$

sehingga probabilitas transisinya

$$\begin{aligned}
P\left(\nu_e \to \nu_\mu\right) &= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{\left(p^a_{e\mu}\right)^2 + \left(p^b_{e\mu}\right)^2\right\}\lambda_1\lambda_2 + \left(q^a_{e\mu}\right)^2 + \left(q^b_{e\mu}\right)^2\right. \\
&\quad \left. + \left(p^a_{e\mu}q^a_{e\mu} + p^b_{e\mu}q^b_{e\mu}\right)\left(\lambda_1 + \lambda_2\right)\right]\sin^2\tilde{\Delta}'_{12} \\
&\quad -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^a_{e\mu}p^b_{e\mu}\right)\lambda_1\lambda_2 + 2q^a_{e\mu}q^b_{e\mu} + \left(p^a_{e\mu}q^b_{e\mu} + p^b_{e\mu}q^a_{e\mu}\right)\left(\lambda_1 + \lambda_2\right)\right] \\
&\quad \times\sin^2\tilde{\Delta}'_{12}\cos\delta \\
&\quad -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{\left(p^a_{e\mu}\right)^2 + \left(p^b_{e\mu}\right)^2\right\}\lambda_2\lambda_3 + \left(q^a_{e\mu}\right)^2 + \left(q^b_{e\mu}\right)^2\right. \\
&\quad \left. + \left(p^a_{e\mu}q^a_{e\mu} + p^b_{e\mu}q^b_{e\mu}\right)\left(\lambda_2 + \lambda_3\right)\right]\sin^2\tilde{\Delta}'_{23} \\
&\quad -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^a_{e\mu}p^b_{e\mu}\right)\lambda_2\lambda_3 + 2q^a_{e\mu}q^b_{e\mu} + \left(p^a_{e\mu}q^b_{e\mu} + p^b_{e\mu}q^a_{e\mu}\right)\left(\lambda_2 + \lambda_3\right)\right] \\
&\quad \times\sin^2\tilde{\Delta}'_{23}\cos\delta \\
&\quad -\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{\left(p^a_{e\mu}\right)^2 + \left(p^b_{e\mu}\right)^2\right\}\lambda_3\lambda_1 + \left(q^a_{e\mu}\right)^2 + \left(q^b_{e\mu}\right)^2\right.
\end{aligned}$$

$$
\begin{aligned}
&+\left(p^a_{e\mu}q^a_{e\mu}+p^b_{e\mu}q^b_{e\mu}\right)(\lambda_3+\lambda_1)\big]\sin^2\tilde{\Delta}'_{31}\\
&-\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^a_{e\mu}p^b_{e\mu}\right)\lambda_3\lambda_1+2q^a_{e\mu}q^b_{e\mu}+\left(p^a_{e\mu}q^b_{e\mu}+p^b_{e\mu}q^a_{e\mu}\right)(\lambda_3+\lambda_1)\right]\\
&\times\sin^2\tilde{\Delta}'_{31}\cos\delta\\
&-\frac{2J_r\Delta_{12}\Delta_{23}\Delta_{31}}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left(\sin 2\tilde{\Delta}'_{12}+\sin 2\tilde{\Delta}'_{23}+\sin 2\tilde{\Delta}'_{31}\right)\sin\delta
\end{aligned}
\tag{4.124}
$$

sehingga dapat ditulis

$$
P\left(\nu_e\to\nu_\mu\right)=\tilde{A}_{e\mu}\cos\delta+\tilde{B}\sin\delta+\tilde{C}_{e\mu}
\tag{4.125}
$$

dengan koefisien - koefisiennya adalah

$$
\begin{aligned}
\tilde{A}_{e\mu} &= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^a_{e\mu}p^b_{e\mu}\right)\lambda_1\lambda_2+2q^a_{e\mu}q^b_{e\mu}+\left(p^a_{e\mu}q^b_{e\mu}+p^b_{e\mu}q^a_{e\mu}\right)(\lambda_1+\lambda_2)\right]\\
&\quad\times\sin^2\tilde{\Delta}'_{12}\\
&\quad-\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^a_{e\mu}p^b_{e\mu}\right)\lambda_2\lambda_3+2q^a_{e\mu}q^b_{e\mu}+\left(p^a_{e\mu}q^b_{e\mu}+p^b_{e\mu}q^a_{e\mu}\right)(\lambda_2+\lambda_3)\right]\\
&\quad\times\sin^2\tilde{\Delta}'_{23}\\
&\quad-\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^a_{e\mu}p^b_{e\mu}\right)\lambda_3\lambda_1+2q^a_{e\mu}q^b_{e\mu}+\left(p^a_{e\mu}q^b_{e\mu}+p^b_{e\mu}q^a_{e\mu}\right)(\lambda_3+\lambda_1)\right]\\
&\quad\times\sin^2\tilde{\Delta}'_{31}\\
&= \sum_{(ij)}^{siklik}\left(\tilde{A}_r\right)_{ij}\sin^2\tilde{\Delta}'_{ij}
\end{aligned}
\tag{4.126}
$$

dengan $\left(\tilde{A}_r\right)_{ij}$

$$
\left(\tilde{A}_r\right)_{ij} = -\frac{4}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^a_{e\mu}p^b_{e\mu}\right)\lambda_i\lambda_j+2q^a_{e\mu}q^b_{e\mu}+\left(p^a_{e\mu}q^b_{e\mu}+p^b_{e\mu}q^a_{e\mu}\right)(\lambda_i+\lambda_j)\right]
\tag{4.127}
$$

untuk $\tilde{B}$

$$
\begin{aligned}
\tilde{B} &= -\frac{2J_r\Delta_{12}\Delta_{23}\Delta_{31}}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left(\sin 2\tilde{\Delta}'_{12}+\sin 2\tilde{\Delta}'_{23}+\sin 2\tilde{\Delta}'_{31}\right)\\
&= \sum_{(ij)}^{siklik}\tilde{B}_r\sin 2\tilde{\Delta}'_{ij}
\end{aligned}
\tag{4.128}
$$

dengan $\tilde{B}_r$

$$
\tilde{B}_r=-\frac{2J_r\Delta_{12}\Delta_{23}\Delta_{31}}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}
\tag{4.129}
$$

untuk $\tilde{C}_{e\mu}$

$$\begin{aligned}\tilde{C}_{e\mu} &= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{\left(p^a_{e\mu}\right)^2+\left(p^b_{e\mu}\right)^2\right\}\lambda_1\lambda_2+\left(q^a_{e\mu}\right)^2+\left(q^b_{e\mu}\right)^2\right.\\ &\quad\left.+\left(p^a_{e\mu}q^a_{e\mu}+p^b_{e\mu}q^b_{e\mu}\right)(\lambda_1+\lambda_2)\right]\sin^2\tilde{\Delta}'_{12}\\ &\quad-\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{\left(p^a_{e\mu}\right)^2+\left(p^b_{e\mu}\right)^2\right\}\lambda_2\lambda_3+\left(q^a_{e\mu}\right)^2+\left(q^b_{e\mu}\right)^2\right.\\ &\quad\left.+\left(p^a_{e\mu}q^a_{e\mu}+p^b_{e\mu}q^b_{e\mu}\right)(\lambda_2+\lambda_3)\right]\sin^2\tilde{\Delta}'_{23}\\ &\quad-\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{\left(p^a_{e\mu}\right)^2+\left(p^b_{e\mu}\right)^2\right\}\lambda_3\lambda_1+\left(q^a_{e\mu}\right)^2+\left(q^b_{e\mu}\right)^2\right.\\ &\quad\left.+\left(p^a_{e\mu}q^a_{e\mu}+p^b_{e\mu}q^b_{e\mu}\right)(\lambda_3+\lambda_1)\right]\sin^2\tilde{\Delta}'_{31}\\ &= \sum_{(ij)}^{siklik}\left(\tilde{C}_r\right)_{ij}\sin^2\tilde{\Delta}'_{ij} \qquad (4.130)\end{aligned}$$

dengan $\left(\tilde{C}_r\right)_{ij}$

$$\begin{aligned}\left(\tilde{C}_r\right)_{ij} &= -\frac{4}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{\left(p^a_{e\mu}\right)^2+\left(p^b_{e\mu}\right)^2\right\}\lambda_i\lambda_j+\left(q^a_{e\mu}\right)^2+\left(q^b_{e\mu}\right)^2\right.\\ &\quad\left.+\left(p^a_{e\mu}q^a_{e\mu}+p^b_{e\mu}q^b_{e\mu}\right)(\lambda_i+\lambda_j)\right] \qquad (4.131)\end{aligned}$$

Selanjutnya , dalam kondisi $x+y+z=0$ maka

$$\begin{aligned}\sin 2x+\sin 2y+\sin 2z &= 2\sin(x+y)\cos(x-y)+\sin 2z\\ &= 2\sin(x+y)\cos(x-y)+\sin\{-2(x+y)\}\\ &= 2\sin(x+y)\cos(x-y)-\sin 2(x+y)\\ &= 2\sin(x+y)\cos(x-y)-2\sin(x+y)\cos(x+y)\\ &= 2\sin(x+y)\left[\cos(x-y)-\cos(x+y)\right]\\ &= 2\sin(x+y)\left[2\sin\frac{1}{2}(2x)\sin\frac{1}{2}(2y)\right]\\ &= 2\sin(x+y)(2\sin x\sin y)\\ &= 4\sin(-z)\sin x\sin y\\ &= -4\sin x\sin y\sin z \qquad (4.132)\end{aligned}$$

Sehingga persamaan (4.128) dapat dituliskan

$$\begin{aligned}\tilde{B} &= \sum_{(ij)}^{siklik}\tilde{B}_r\sin 2\tilde{\Delta}'_{ij}\\ &= -4\tilde{B}_r\sin\tilde{\Delta}'_{12}\sin\tilde{\Delta}'_{23}\sin\tilde{\Delta}'_{31}\\ &= -4\left(-\frac{2J_r\Delta_{12}\Delta_{23}\Delta_{31}}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\right)\sin\tilde{\Delta}'_{12}\sin\tilde{\Delta}'_{23}\sin\tilde{\Delta}'_{31}\\ &= \frac{8J_r\Delta_{12}\Delta_{23}\Delta_{31}}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\sin\tilde{\Delta}'_{12}\sin\tilde{\Delta}'_{23}\sin\tilde{\Delta}'_{31} \qquad (4.133)\end{aligned}$$

dalam kondisi

$$
\begin{aligned}
x &= -(y+z) \\
\sin x &= -\sin(x+y) \\
&= -(\sin y \cos z + \cos y \sin z) \\
\sin^2 x &= \sin x \sin x \\
&= -(\sin x \sin y \cos z + \cos x \sin y \sin z) \qquad (4.134)
\end{aligned}
$$

dengan cara yang serupa didapatkan

$$
\begin{aligned}
\sin^2 y &= -(\sin x \sin y \cos z + \cos x \sin y \sin z) \qquad (4.135) \\
\sin^2 z &= -(sinx \cos y \sin z + \cos x \sin y \sin z) \qquad (4.136)
\end{aligned}
$$

sehingga

$$
\begin{aligned}
\tilde{A}_{e\mu} &= \sum_{(ij)}^{siklik} \left(\tilde{A}_r\right)_{ij} \sin^2 \tilde{\Delta}'_{ij} \\
&= \left(\tilde{A}_r\right)_{12} \sin^2 \tilde{\Delta}'_{12} + \left(\tilde{A}_r\right)_{23} \sin^2 \tilde{\Delta}'_{23} + \left(\tilde{A}_r\right)_{31} \sin^2 \tilde{\Delta}'_{31} \\
&= -\left(\tilde{A}_r\right)_{12} \sin \tilde{\Delta}_{12} \sin \tilde{\Delta}_{23} \cos \tilde{\Delta}_{31} \\
&\quad -\left(\tilde{A}_r\right)_{12} \sin \tilde{\Delta}_{12} \cos \tilde{\Delta}_{23} \sin \tilde{\Delta}_{31} \\
&\quad -\left(\tilde{A}_r\right)_{23} \sin \tilde{\Delta}_{12} \sin \tilde{\Delta}_{23} \cos \tilde{\Delta}_{31} \\
&\quad -\left(\tilde{A}_r\right)_{23} \cos \tilde{\Delta}_{12} \sin \tilde{\Delta}_{23} \sin \tilde{\Delta}_{31} \\
&\quad -\left(\tilde{A}_r\right)_{31} \sin \tilde{\Delta}_{12} \cos \tilde{\Delta}_{23} \sin \tilde{\Delta}_{31} \\
&\quad -\left(\tilde{A}_r\right)_{31} \cos \tilde{\Delta}_{12} \sin \tilde{\Delta}_{23} \sin \tilde{\Delta}_{31} \\
&= -\left[\left(\tilde{A}_r\right)_{23} + (A_r)_{31}\right] \cos \tilde{\Delta}_{12} \sin \tilde{\Delta}_{23} \sin \tilde{\Delta}_{31} \\
&\quad -\left[\left(\tilde{A}_r\right)_{31} + (A_r)_{12}\right] \cos \tilde{\Delta}_{23} \sin \tilde{\Delta}_{31} \sin \tilde{\Delta}_{12} \\
&\quad -\left[\left(\tilde{A}_r\right)_{12} + (A_r)_{23}\right] \cos \tilde{\Delta}_{31} \sin \tilde{\Delta}_{12} \sin \tilde{\Delta}_{23} \\
&= -\sum_{(ijk)}^{siklik} \left[\left(\tilde{A}_r\right)_{jk} + (A_r)_{ki}\right] \cos \tilde{\Delta}_{ij} \sin \tilde{\Delta}_{jk} \sin \tilde{\Delta}_{ki} \qquad (4.137)
\end{aligned}
$$

dengan mensubstitusikan , diperoleh

$$
\begin{aligned}
\tilde{A}_{e\mu} &= \sum_{(ijk)}^{siklik} \frac{-8\left[J_r \Delta_{21} \Delta_{31} \lambda_k (\lambda_k - \Delta_{31}) + \left(\tilde{A}_{e\mu}\right)_k\right]}{\tilde{\Delta}_{jk}^2 \tilde{\Delta}_{ki}^2} \\
&\quad \times \cos \tilde{\Delta}'_{ij} \sin \tilde{\Delta}'_{jk} \sin \tilde{\Delta}'_{ki} \qquad (4.138)
\end{aligned}
$$

dengan

$$\left(\tilde{A}_{e\mu}\right)_k = \Delta_{21}^2 J_r \times \left[\Delta_{31}\lambda_k\left(c_{12}^2 - s_{12}^2\right) + \lambda_k^2 s_{12}^2 - \Delta_{31}^2 c_{12}^2\right] \quad (4.139)$$

dan

$$\tilde{B} = \frac{8J_r\Delta_{12}\Delta_{23}\Delta_{31}}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \cos\tilde{\Delta}'_{12}\sin\tilde{\Delta}'_{23}\sin\tilde{\Delta}'_{31} \quad (4.140)$$

dan

$$\tilde{C}_{e\mu} = \sum_{(ij)}^{siklik} \frac{-4\left[\Delta_{31}^2 s_{13}^2 s_{23}^2 c_{13}^2 \lambda_i\lambda_j + \left(\tilde{C}_{e\mu}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \sin^2\tilde{\Delta}'_{ij} \quad (4.141)$$

dengan

$$\begin{aligned}\left(\tilde{C}_{e\mu}\right)_{ij} &= \Delta_{21}s_{13}^2 \times \left[\Delta_{31}\left\{-\lambda_i\left(\lambda_j s_{12}^2 + \Delta_{31}c_{12}^2\right) - \lambda_j\left(\lambda_i s_{12}^2 + \Delta_{31}c_{12}^2\right)\right\} s_{23}^2 c_{13}^2\right] \\ &+ \Delta_{21}^2 \times \left[\left(\lambda_i - \Delta_{31}\right)\left(\lambda_j - \Delta_{31}\right) s_{12}^2 c_{12}^2 c_{23}^2 c_{13}^2\right] \\ &+ \Delta_{21}^2 s_{12}^2 \times \left[\left(\lambda_i s_{12}^2 + \Delta_{31}c_{12}^2\right)\left(\lambda_j s_{12}^2 + \Delta_{31}c_{12}^2\right) c_{23}^2 c_{13}^2\right] \quad (4.142)\end{aligned}$$

selanjutnya dihitung $P\left(\nu_e \rightarrow \nu_\tau\right)$.Dari persamaan (4.56) didapatkan

$$U_{e1}U_{\tau 1}^* = -U_{e2}U_{\tau 2}^* - U_{e3}U_{\tau 3}^* \quad (4.143)$$

dan

$$p_{e\tau} = \lambda_1 U_{e1}U_{\tau 1}^* + \lambda_2 U_{e2}U_{\tau 2}^* + \lambda_3 U_{e3}U_{\tau 3}^* \quad (4.144)$$

substitusi persamaan (4.143) ke persamaan (4.144) , didapatkan

$$\begin{aligned}p_{e\tau} &= -\lambda_1 U_{e1}U_{\tau 1}^* - \lambda_1 U_{e2}U_{\tau 2}^* + \lambda_2 U_{e2}U_{\tau 2}^* + \lambda_3 U_{e3}U_{\tau 3}^* \\ &= \left(\lambda_2 - \lambda_1\right) U_{e2}U_{\tau 2}^* + \left(\lambda_3 - \lambda_1\right) U_{e3}U_{\tau 3}^* \\ &= \Delta_{21}U_{e2}U_{\tau 2}^* + \Delta_{31}U_{e3}U_{\tau 3}^* \quad (4.145)\end{aligned}$$

Jika dimasukkan element matrik MNS , didapatkan

$$\begin{aligned}p_{e\tau} &= \Delta_{21}s_{12}c_{13}\left(-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{-i\delta}\right) + \Delta_{31}\left(s_{13}e^{-i\delta}\right)\left(c_{23}c_{13}\right) \\ &= -\Delta_{21}s_{12}c_{13}c_{12}s_{23} - \Delta_{21}s_{12}^2 c_{13}s_{13}c_{23}e^{-i\delta} + \Delta_{31}s_{13}c_{23}c_{13}e^{-i\delta} \\ &= \left(\Delta_{31} - \Delta_{21}s_{12}^2\right) c_{23}s_{13}c_{13}e^{-i\delta} - \Delta_{21}s_{12}c_{12}s_{23}c_{13} \quad (4.146)\end{aligned}$$

sehingga dapat ditulis

$$p_{e\tau} = p_{e\tau}^a e^{-i\delta} + p_{e\tau}^b \quad (4.147)$$

dimana

$$\begin{aligned}p_{e\tau}^a &= \left(\Delta_{31} - \Delta_{21}s_{12}^2\right) c_{23}s_{13}c_{13} \quad (4.148) \\ p_{e\tau}^b &= -\Delta_{21}s_{12}s_{21}s_{23}c_{13} \quad (4.149)\end{aligned}$$

untuk $q_{e\tau}$

$$\begin{aligned} q_{e\tau} &= \Delta_{31}\Delta_{21}U_{e1}U_{\tau 1}^{*} \\ &= \Delta_{31}\Delta_{21}c_{12}C_{13}\left(s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta}\right) \\ &= -\Delta_{31}\Delta_{21}c_{12}^{2}c_{23}s_{13}c_{13}e^{-i\delta} + \Delta_{31}\Delta_{21}c_{12}c_{13}s_{12}s_{23} \end{aligned} \tag{4.150}$$

sehingga dapat dituliskan

$$q_{e\tau} = q_{e\tau}^{a}e^{-i\delta} + q_{e\tau}^{b} \tag{4.151}$$

dimana

$$q_{e\tau}^{a} = -\Delta_{31}\Delta_{21}c_{12}^{2}c_{23}s_{13}c_{13} \tag{4.152}$$

$$q_{e\tau}^{b} = \Delta_{31}\Delta_{21}s_{12}c_{12}s_{23}c_{13} \tag{4.153}$$

Sekarang dihitung probabilitas $P\left(\nu_e \to \nu_\tau\right)$.Dari persamaan (4.53) didapatkan

$$\begin{aligned} P\left(\nu_e \to \nu_\tau\right) &= -4\sum_{(ij)}^{siklik} Re\tilde{J}_{e\tau}^{ij}\sin^2\tilde{\Delta}_{ij}' + 2\sum_{(ij)}^{siklik}\tilde{J}\sin^2\tilde{\Delta}_{ij}' \\ &= -4\left\{Re\tilde{J}_{e\tau}^{12}\sin^2\tilde{\Delta}_{12}' + Re\tilde{J}_{e\tau}^{23}\sin^2\tilde{\Delta}_{23}' + Re\tilde{J}_{e\tau}^{31}\sin^2\tilde{\Delta}_{31}'\right\} \\ &\quad +2\tilde{J}\left(\sin^2\tilde{\Delta}_{12}' + \sin^2\tilde{\Delta}_{23}' + \sin^2\tilde{\Delta}_{31}'\right) \\ &= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{\left|\tilde{p}_{e\tau}\right|^2\lambda_1\lambda_2 + \left|\tilde{q}_{e\tau}\right|^2 + Re\left(\tilde{p}_{e\tau}\tilde{q}_{e\tau}^{*}\right)\left(\lambda_1+\lambda_2\right)\right\}\sin^2\tilde{\Delta}_{12}' \\ &\quad -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{\left|\tilde{p}_{e\tau}\right|^2\lambda_2\lambda_3 + \left|\tilde{q}_{e\tau}\right|^2 + Re\left(\tilde{p}_{e\tau}\tilde{q}_{e\tau}^{*}\right)\left(\lambda_2+\lambda_3\right)\right\}\sin^2\tilde{\Delta}_{23}' \\ &\quad -\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{\left|\tilde{p}_{e\tau}\right|^2\lambda_3\lambda_1 + \left|\tilde{q}_{e\tau}\right|^2 + Re\left(\tilde{p}_{e\tau}\tilde{q}_{e\tau}^{*}\right)\left(\lambda_3+\lambda_1\right)\right\}\sin^2\tilde{\Delta}_{31}' \\ &\quad +2\tilde{J}\left(\sin^2\tilde{\Delta}_{12}' + \sin^2\tilde{\Delta}_{23}' + \sin^2\tilde{\Delta}_{31}'\right) \end{aligned} \tag{4.154}$$

karena

$$\begin{aligned} \tilde{p}_{e\tau} &= \tilde{H}_{e\tau} \\ &= H_{e\tau} \\ &= p_{e\tau} \end{aligned} \tag{4.155}$$

dan

$$\begin{aligned} \tilde{q}_{e\mu} &= \tilde{H}_{\mu\tau}\tilde{H}_{e\mu} - \tilde{H}_{e\tau}\tilde{H}_{\mu\mu} \\ &= H_{\mu\tau}H_{e\mu} - H_{e\tau}H_{\mu\mu} \\ &= q_{e\tau} \end{aligned} \tag{4.156}$$

sehingga

$$
\begin{aligned}
P(\nu_e \rightarrow \nu_\tau) &= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{|p_{e\tau}|^2 \lambda_1\lambda_2 + |q_{e\tau}|^2 + Re\left(p_{e\tau}q^*_{e\tau}\right)(\lambda_1+\lambda_2)\right\}\sin^2\tilde{\Delta}'_{12} \\
&\quad -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{|p_{e\tau}|^2 \lambda_2\lambda_3 + |q_{e\tau}|^2 + Re\left(p_{e\tau}q^*_{e\tau}\right)(\lambda_2+\lambda_3)\right\}\sin^2\tilde{\Delta}'_{23} \\
&\quad -\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{|p_{e\tau}|^2 \lambda_3\lambda_1 + |q_{e\tau}|^2 + Re\left(p_{e\tau}q^*_{e\tau}\right)(\lambda_3+\lambda_1)\right\}\sin^2\tilde{\Delta}'_{31} \\
&\quad +2\tilde{J}\left(\sin^2\tilde{\Delta}'_{12} + \sin^2\tilde{\Delta}'_{23} + \sin^2\tilde{\Delta}'_{31}\right) \qquad (4.157)
\end{aligned}
$$

sekarang dihitung dulu

$$
\begin{aligned}
|p_{e\tau}|^2 &= p_{e\tau}p^*_{e\tau} \\
&= \left(p^a_{e\tau}e^{-i\delta} + p^b_{e\tau}\right)\left(p^a_{e\tau}e^{i\delta} + p^b_{e\tau}\right) \\
&= \left(p^a_{e\tau}\right)^2 + p^a_{e\tau}p^b_{e\tau}e^{-i\delta} + p^a_{e\tau}p^b_{e\tau}e^{i\delta} + \left(p^b_{e\tau}\right)^2 \\
&= \left(p^a_{e\tau}\right)^2 + \left(p^b_{e\tau}\right)^2 + p^a_{e\tau}p^b_{e\mu}\left(e^{i\delta} + e^{-i\delta}\right) \\
&= \left(p^a_{e\tau}\right)^2 + \left(p^b_{e\tau}\right)^2 + 2p^a_{e\tau}p^b_{e\tau}\cos\delta \qquad (4.158)
\end{aligned}
$$

hitung

$$
\begin{aligned}
\left(p^a_{e\tau}\right)^2 &= \left(\Delta_{31} - \Delta_{21}s^2_{12}\right)^2 c^2_{23}s^2_{13}c^2_{13} \qquad (4.159) \\
\left(p^b_{e\tau}\right)^2 &= \Delta^2_{21}s^2_{12}c^2_{12}s^2_{23}c^2_{13} \qquad (4.160) \\
p^a_{e\tau}p^b_{e\tau} &= \left\{\left(\Delta_{31} - \Delta_{21}s^2_{12}\right)c_{23}s_{13}c_{13}\right\}\left(-\Delta_{21}s_{12}c_{12}s_{23}c_{12}\right) \\
&= \left(\Delta_{31}c_{23}s_{13}c_{13} - \Delta_{21}s^2_{12}c_{23}s_{13}c_{13}\right)\left(-\Delta_{21}s_{12}c_{12}s_{23}c_{12}\right) \\
&= -\Delta_{31}\Delta_{21}s_{12}c_{12}s_{23}c_{23}s_{13}c^2_{13} + \Delta^2_{21}s^3_{12}c_{12}s_{23}c_{23}s_{13}c^2_{13} \\
&= -\Delta_{31}\Delta_{21}J_r + \Delta^2_{21}s^2_{12}J_r \qquad (4.161)
\end{aligned}
$$

dan

$$
\begin{aligned}
|q_{e\tau}|^2 &= q_{e\tau}q^*_{e\tau} \\
&= \left(q^a_{e\tau}e^{-i\delta} + q^b_{e\tau}\right)\left(q^a_{e\tau}e^{i\delta} + q^b_{e\tau}\right) \\
&= \left(q^a_{e\tau}\right)^2 + q^a_{e\tau}q^b_{e\tau}e^{-i\delta} + q^a_{e\tau}q^b_{e\tau}e^{i\delta} + \left(q^b_{e\tau}\right)^2 \\
&= \left(q^a_{e\tau}\right)^2 + \left(q^b_{e\tau}\right)^2 + q^a_{e\tau}q^b_{e\tau}\left(e^{i\delta} + e^{-i\delta}\right) \\
&= \left(q^a_{e\tau}\right)^2 + \left(q^b_{e\tau}\right)^2 + 2q^a_{e\tau}q^b_{e\tau}\cos\delta \qquad (4.162)
\end{aligned}
$$

hitung

$$
\begin{aligned}
\left(q^a_{e\tau}\right)^2 &= \Delta^2_{31}\Delta^2_{21}c^4_{12}c^2_{23}s^2_{13}c^2_{13} \qquad (4.163) \\
\left(q^b_{e\tau}\right)^2 &= \Delta^2_{31}\Delta^2_{21}s^2_{12}c^2_{12}s^2_{23}c^2_{13} \qquad (4.164) \\
q^a_{e\tau}q^b_{e\tau} &= \left(-\Delta_{31}\Delta_{21}c^2_{12}c_{23}s_{13}c_{13}\right)\left(\Delta_{31}\Delta_{21}s_{12}c_{12}s_{23}c_{13}\right) \\
&= -\Delta^2_{31}\Delta^2_{21}s_{12}c^3_{12}s_{23}c_{23}s_{13}c^2_{13} \\
&= -\Delta^2_{31}\Delta^2_{21}c^2_{12}J_r \qquad (4.165)
\end{aligned}
$$

sedangkan

$$\begin{aligned} p_{e\tau}q^*_{e\tau} &= \left(p^a_{e\tau}e^{-i\delta} + p^b_{e\tau}\right)\left(q^a_{e\tau}e^{i\delta} + q^b_{e\tau}\right) \\ &= p^a_{e\tau}q^a_{e\tau} + p^a_{e\tau}q^b_{e\tau}e^{-i\delta} + p^b_{e\tau}q^a_{e\tau}e^{i\delta} + p^b_{e\tau}q^b_{e\tau} \\ &= p^a_{e\tau}q^a_{e\tau} + p^b_{e\tau}q^b_{e\tau} + p^a_{e\tau}q^b_{e\tau}\left(\cos\delta - i\sin\delta\right) + p^b_{e\tau}q^a_{e\tau}\left(\cos\delta + \sin\delta\right) \\ &= p^a_{e\tau}q^a_{e\tau} + p^b_{e\tau}q^b_{e\tau} + \left(p^a_{e\tau}q^b_{e\tau} + p^b_{e\tau}q^a_{e\tau}\right)\cos\delta + i\left(p^b_{e\tau}q^a_{e\tau} - p^a_{e\tau}q^b_{e\tau}\right)\sin\delta \end{aligned} \tag{4.166}$$

maka

$$Re\left(p_{e\tau}q^*_{e\tau}\right) = p^a_{e\tau}q^a_{e\tau} + p^b_{e\tau}q^b_{e\tau} + \left(p^a_{e\tau}q^b_{e\tau} + p^b_{e\tau}q^a_{e\tau}\right)\cos\delta \tag{4.167}$$

hitung

$$\begin{aligned} p^a_{e\tau}q^a_{e\tau} &= \left\{\left(\Delta_{31} - \Delta_{21}s^2_{12}\right)c_{23}s_{13}c_{13}\right\}\left(-\Delta_{31}\Delta_{21}c^2_{12}c_{23}s_{13}c_{13}\right) \\ &= \left(\Delta_{31}c_{23}s_{13}c_{13} - \Delta_{21}s^2_{12}c_{23}s_{13}c_{13}\right)\left(-\Delta_{31}\Delta_{21}c^2_{12}c_{23}s_{13}c_{13}\right) \\ &= -\Delta^2_{31}\Delta_{21}c^2_{12}c^2_{23}s^2_{13}c^2_{13} + \Delta_{31}\Delta^2_{21}s^2_{12}c^2_{12}c^2_{23}c^2_{13}c^2_{13} \end{aligned} \tag{4.168}$$

$$\begin{aligned} p^b_{e\tau}q^b_{e\tau} &= \left(-\Delta_{21}s_{12}c_{12}s_{23}c_{13}\right)\left(\Delta_{31}\Delta_{21}s_{12}c_{12}s_{23}c_{13}\right) \\ &= -\Delta_{31}\Delta^2_{21}s^2_{12}c^2_{12}s^2_{23}c^2_{13} \end{aligned} \tag{4.169}$$

dan

$$\begin{aligned} p^a_{e\tau}q^b_{e\tau} + p^b_{e\tau}q^a_{e\tau} &= \left\{\left(\Delta_{31} - \Delta_{21}s^2_{12}\right)c_{23}s_{13}c_{13}\right\}\left(\Delta_{31}\Delta_{21}s_{12}c_{12}s_{23}c_{13}\right) \\ &\quad + \left(-\Delta_{21}s_{12}c_{12}s_{23}c_{13}\right)\left(-\Delta_{31}\Delta_{21}c^2_{12}c_{23}s_{13}c_{13}\right) \\ &= \Delta^2_{31}\Delta_{21}s_{12}c_{12}c_{23}s_{23}s_{13}c^2_{13} - \Delta_{31}\Delta^2_{21}s^3_{12}c_{12}c_{23}s_{23}s_{13}c^2_{13} \\ &\quad + \Delta_{31}\Delta^2_{21}s_{12}c^3_{12}s_{23}c_{23}s_{13}c^2_{13} \\ &= \Delta^2_{31}\Delta_{21}J_r - \Delta_{31}\Delta^2_{21}J_r\left(s^2_{12} - c^2_{12}\right) \\ &= J_r\Delta_{31}\Delta_{21}\left(\Delta_{31} - \Delta_{21}\left(s^2_{12} - c^2_{12}\right)\right) \end{aligned} \tag{4.170}$$

sedangkan

$$Im\left(p_{e\tau}q^*_{e\tau}\right) = \left(p^b_{e\tau}q^a_{e\tau} - p^a_{e\tau}q^b_{e\tau}\right)\sin\delta \tag{4.171}$$

hitung

$$\begin{aligned} \left(p^b_{e\tau}q^a_{e\tau} - p^a_{e\tau}q^b_{e\tau}\right) &= \left(-\Delta_{21}s_{12}c_{12}s_{23}c_{13}\right)\left(-\Delta_{31}\Delta_{21}c^2_{12}c_{23}s_{13}c_{13}\right) \\ &\quad - \left\{\left(\Delta_{31} - \Delta_{21}s^2_{12}\right)c_{23}s_{13}c_{13}\right\}\left(\Delta_{31}\Delta_{21}s_{12}c_{12}s_{23}c_{13}\right) \\ &= \Delta^2_{21}\Delta_{31}s_{12}c^3_{12}s_{23}s_{13}c^2_{13}c_{23} \\ &\quad - \left(\Delta_{31} - \Delta_{21}s^2_{12}\right)\Delta_{31}\Delta_{21}s_{12}c_{12}s_{13}c^2_{13}c_{23}s_{23} \\ &= \Delta^2_{21}\Delta_{31}s_{12}c^3_{12}s_{23}s_{13}c^2_{13}c_{23} \\ &\quad - \Delta^2_{31}\Delta_{21}s_{12}c_{12}s_{13}c^2_{13}c_{23}s_{23} \\ &\quad + \Delta_{31}\Delta^3_{21}s^3_{12}c_{12}s_{13}c^2_{13}c_{23}s_{23} \end{aligned}$$

$$
\begin{aligned}
&= \Delta_{21}^2\Delta_{31}\left(c_{12}^2+s_{12}^2\right)s_{12}c_{12}s_{23}s_{13}c_{13}^2c_{23} \\
&\quad -\Delta_{31}^2\Delta_{21}s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \\
&= \Delta_{21}^2\Delta_{31}J_r-\Delta_{31}^2\Delta_{21}J_r \\
&= J_r\Delta_{21}\Delta_{31}\left(\Delta_{21}-\Delta_{31}\right) \\
&= J_r\Delta_{21}\Delta_{31}\left(\lambda_2-\lambda_1-\lambda_3+\lambda_1\right) \\
&= J_r\Delta_{21}\Delta_{31}\left(\lambda_2-\lambda_3\right) \\
&= J_r\Delta_{21}\Delta_{31}\Delta_{23} \\
&= -J_r\Delta_{12}\Delta_{23}\Delta_{31} \qquad (4.172)
\end{aligned}
$$

sehingga

$$
Im\left(p_{e\tau}q_{e\tau}^*\right)=-J_r\Delta_{12}\Delta_{23}\Delta_{31}\sin\delta \qquad (4.173)
$$

sehingga probabilitas neutrinonya

$$
\begin{aligned}
P\left(\nu_e\rightarrow\nu_\tau\right) = & -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{\left(p_{e\tau}^a\right)^2+\left(p_{e\tau}^b\right)^2\right\}\lambda_1\lambda_2+\left(q_{e\tau}^a\right)^2+\left(q_{e\tau}^b\right)^2\right. \\
& \left.+\left(p_{e\tau}^a q_{e\tau}^a+p_{e\tau}^b q_{e\tau}^b\right)\left(\lambda_1+\lambda_2\right)\right]\sin^2\tilde{\Delta}'_{12} \\
& -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p_{e\tau}^a p_{e\tau}^b\right)\lambda_1\lambda_2+2q_{e\tau}^a q_{e\tau}^b+\left(p_{e\tau}^a q_{e\tau}^b+p_{e\tau}^b q_{e\tau}^a\right)\left(\lambda_1+\lambda_2\right)\right] \\
& \times\sin^2\tilde{\Delta}'_{12}\cos\delta \\
& -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{\left(p_{e\tau}^a\right)^2+\left(p_{e\tau}^b\right)^2\right\}\lambda_2\lambda_3+\left(q_{e\tau}^a\right)^2+\left(q_{e\tau}^b\right)^2\right. \\
& \left.+\left(p_{e\tau}^a q_{e\tau}^a+p_{e\tau}^b q_{e\tau}^b\right)\left(\lambda_2+\lambda_3\right)\right]\sin^2\tilde{\Delta}'_{23} \\
& -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p_{e\tau}^a p_{e\tau}^b\right)\lambda_2\lambda_3+2q_{e\tau}^a q_{e\tau}^b+\left(p_{e\tau}^a q_{e\tau}^b+p_{e\tau}^b q_{e\tau}^a\right)\left(\lambda_2+\lambda_3\right)\right] \\
& \times\sin^2\tilde{\Delta}'_{23}\cos\delta \\
& -\frac{4}{\Delta_{31}\Delta_{12}\Delta_{23}\Delta_{31}}\left[\left\{\left(p_{e\tau}^a\right)^2+\left(p_{e\tau}^b\right)^2\right\}\lambda_3\lambda_1+\left(q_{e\tau}^a\right)^2+\left(q_{e\tau}^b\right)^2\right. \\
& \left.+\left(p_{e\tau}^a q_{e\tau}^a+p_{e\tau}^b q_{e\tau}^b\right)\left(\lambda_3+\lambda_1\right)\right]\sin^2\tilde{\Delta}'_{31} \\
& -\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p_{e\tau}^a p_{e\tau}^b\right)\lambda_3\lambda_1+2q_{e\tau}^a q_{e\tau}^b+\left(p_{e\tau}^a q_{e\tau}^b+p_{e\tau}^b q_{e\tau}^a\right)\left(\lambda_3+\lambda_1\right)\right] \\
& \times\sin^2\tilde{\Delta}'_{31}\cos\delta \\
& +\frac{2J_r\Delta_{12}\Delta_{23}\Delta_{31}}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left(\sin^2\tilde{\Delta}'_{12}+\sin^2\tilde{\Delta}'_{23}+\sin^2\tilde{\Delta}'_{31}\right)\sin\delta \qquad (4.174)
\end{aligned}
$$

sehingga dapat ditulis

$$
P\left(\nu_e\rightarrow\nu_\tau\right)=\tilde{A}_{e\tau}\cos\delta-\tilde{B}\sin\delta+\tilde{C}_{e\tau} \qquad (4.175)
$$

dengan koefisien - koefisiennya adalah

$$\begin{aligned}
\tilde{A}_{e\tau} &= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^{a}_{e\tau}p^{b}_{e\tau}\right)\lambda_1\lambda_2 + 2q^{a}_{e\tau}q^{b}_{e\tau} + \left(p^{a}_{e\tau}q^{b}_{e\tau} + p^{b}_{e\tau}q^{a}_{e\tau}\right)(\lambda_1+\lambda_2)\right] \\
&\quad \times \sin^2\tilde{\Delta}'_{12} \\
&\quad -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^{a}_{e\tau}p^{b}_{e\tau}\right)\lambda_2\lambda_3 + 2q^{a}_{e\tau}q^{b}_{e\tau} + \left(p^{a}_{e\tau}q^{b}_{e\tau} + p^{b}_{e\tau}q^{a}_{e\tau}\right)(\lambda_2+\lambda_3)\right] \\
&\quad \times \sin^2\tilde{\Delta}'_{23} \\
&\quad -\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^{a}_{e\tau}p^{b}_{e\tau}\right)\lambda_3\lambda_1 + 2q^{a}_{e\tau}q^{b}_{e\tau} + \left(p^{a}_{e\tau}q^{b}_{e\tau} + p^{b}_{e\tau}q^{a}_{e\tau}\right)(\lambda_3+\lambda_1)\right] \\
&\quad \times \sin^2\tilde{\Delta}'_{31} \\
&= -\frac{4}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^{a}_{e\tau}p^{b}_{e\tau}\right)\lambda_i\lambda_j + 2q^{a}_{e\tau}q^{b}_{e\tau} + \left(p^{a}_{e\tau}q^{b}_{e\tau} + p^{b}_{e\tau}q^{a}_{e\tau}\right)(\lambda_i+\lambda_j)\right] \\
&\quad \times \sin^2\tilde{\Delta}'_{ij} \\
&= \sum_{(ij)}^{siklik}\left(\tilde{A}_p\right)_{ij}\sin^2\tilde{\Delta}_{ij} \qquad (4.176)
\end{aligned}$$

substitusikan

$$\begin{aligned}
\tilde{A}_{e\tau} &= \sum_{(ijk)}^{siklik}\frac{8\left[J_r\Delta_{21}\Delta_{31}\lambda_k(\lambda_k-\Delta_{31}) + \left(\tilde{A}_{e\mu}\right)_k\right]}{\tilde{\Delta}^2_{jk}\tilde{\Delta}^2_{ki}} \\
&\quad \times \cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki} \\
&= -\tilde{A}_{e\mu} \qquad (4.177)
\end{aligned}$$

sedangkan untuk $\tilde{C}_{e\tau}$

$$\begin{aligned}
\tilde{C}_{e\tau} &= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{(p^{a}_{e\tau})^2 + (p^{b}_{e\tau})^2\right\}\lambda_1\lambda_2 + (q^{a}_{e\tau})^2 + (q^{b}_{e\tau})^2\right. \\
&\quad \left. + \left(p^{a}_{e\tau}q^{a}_{e\tau} + p^{b}_{e\tau}q^{b}_{e\tau}\right)(\lambda_1+\lambda_2)\right]\sin^2\tilde{\Delta}'_{12} \\
&\quad -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{(p^{a}_{e\tau})^2 + (p^{b}_{e\tau})^2\right\}\lambda_2\lambda_3 + (q^{a}_{e\tau})^2 + (q^{b}_{e\tau})^2\right. \\
&\quad \left. + \left(p^{a}_{e\tau}q^{a}_{e\tau} + p^{b}_{e\tau}q^{b}_{e\tau}\right)(\lambda_2+\lambda_3)\right]\sin^2\tilde{\Delta}'_{23} \\
&\quad -\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{(p^{a}_{e\tau})^2 + (p^{b}_{e\tau})^2\right\}\lambda_3\lambda_1 + (q^{a}_{e\tau})^2 + (q^{b}_{e\tau})^2\right. \\
&\quad \left. + \left(p^{a}_{e\tau}q^{a}_{e\tau} + p^{b}_{e\tau}q^{b}_{e\tau}\right)(\lambda_3+\lambda_1)\right]\sin^2\tilde{\Delta}'_{31} \\
&= -\frac{4}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{(p^{a}_{e\tau})^2 + (p^{b}_{e\tau})^2\right\}\lambda_i\lambda_j + (q^{a}_{e\tau})^2 + (q^{b}_{e\tau})^2\right. \\
&\quad \left. + \left(p^{a}_{e\tau}q^{a}_{e\tau} + p^{b}_{e\tau}q^{b}_{e\tau}\right)(\lambda_i+\lambda_j)\right]\sin^2\tilde{\Delta}'_{ij} \\
&= \sum_{(ij)}^{siklik}\left(\tilde{C}_p\right)_{ij}\sin^2\tilde{\Delta}_{ij} \qquad (4.178)
\end{aligned}$$

dengan mensubstitusikan , didapatkan $\tilde{C}_{e\tau}$ adalah

$$\tilde{C}_{e\tau} = \sum_{(ij)}^{siklik} \frac{-4\left[\Delta_{31}^2 s_{13}^2 c_{23}^2 c_{13}\lambda_i\lambda_j + \left(\tilde{C}_{e\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \sin^2\tilde{\Delta}'_{ij} \tag{4.179}$$

dengan

$$\begin{aligned}\left(\tilde{C}_{\mu\tau}\right)_{ij} &= \Delta_{21}\times\left[\Delta_{31}\left\{\Delta_{31}\left(\lambda_i+\lambda_j-2a\right)\left(c_{13}^2+c_{12}^2\right)-2\left(\lambda_i-a\right)\left(\lambda_j-a\right)c_{12}^2\right\}s_{23}^2 s_{23}^2 c_{13}^2\right]\\ &+ \Delta_{21}s_{13}^2\times\left[\Delta_{31}\left\{-\lambda_{31}\left(\lambda_i+\lambda_j-2a\right)+2\left(\lambda_i-a\right)\left(\lambda_j-a\right)\right\}s_{12}^2 s_{23}^2 c_{23}^2 c_{13}^2\right]\\ &+ \Delta_{21}^2\times\left[\left\{\Delta_{31}c_{13}^2+\left(\lambda_i-a-\Delta_{31}\right)c_{12}^2\right\}\left\{\Delta_{31}c_{13}^2+\left(\lambda_j-a-\Delta_{31}\right)c_{12}^2\right\}s_{23}^2 c_{23}^2\right]\\ &+ \Delta_{21}^2 s_{13}^2\times\left[-\Delta_{31}\left(\lambda_i+\lambda_j-2a-2\Delta_{31}\right)s_{12}^2 s_{23}^2 c_{23}^2 c_{13}^2\right.\\ &\quad +\left(\lambda_i-a-\Delta_{31}\right)\left(\lambda_j-a-\Delta_{31}\right)\\ &\quad \left.\left\{s_{12}^2 c_{12}^2\left(c_{23}^2-s_{23}^2\right)+s_{12}^4 s_{23}^2 c_{23}^2 c_{13}^2\right\}\right]\end{aligned} \tag{4.180}$$

Sehingga dapat ditulis

$$P\left(\nu_e\rightarrow\nu_\tau\right) = -\tilde{A}_{e\mu}\cos\delta-\tilde{B}\sin\delta+\tilde{C}_{e\tau} \tag{4.181}$$

Terakhir , dihitung probabilitas transisi $P\left(\nu_\mu\rightarrow\nu_\tau\right)$. Dari persamaan (4.56) didapatkan

$$U_{\mu1}U_{\tau1}^* = -U_{\mu2}U_{\tau2}^*-U_{\mu3}U_{\tau3}^* \tag{4.182}$$

dan

$$p_{\mu\tau} = \lambda_1 U_{\mu1}U_{\tau1}^*+\lambda_2 U_{\mu2}U_{\tau2}^*+\lambda_3 U_{\mu1}U_{\tau1}^* \tag{4.183}$$

substitusi persamaan (4.182) ke persamaan (4.183) , didapatkan

$$\begin{aligned}p_{\mu\tau} &= -\lambda_1 U_{\mu2}U_{\tau2}^*-\lambda_2 U_{\mu3}U_{\tau3}^*+\lambda_2 U_{\mu2}U_{\tau2}^*+\lambda_3 U_{\mu3}U_{\tau3}^*\\ &= \left(\lambda_2-\lambda_1\right)U_{\mu2}U_{\tau2}^*+\left(\lambda_3-\lambda_2\right)U_{\mu3}U_{\tau3}^*\\ &= \Delta_{21}U_{\mu2}U_{\tau2}^*+\Delta_{31}U_{\mu3}U_{\tau3}^*\end{aligned} \tag{4.184}$$

Jika dimasukkan matriks MNS didapatkan

$$\begin{aligned}p_{\mu\tau} &= \Delta_{21}\left(c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta}\right)\left(-c_{12}s_{23}-s_{12}c_{23}s_{13}e^{-i\delta}\right)\\ &\quad +\Delta_{31}\left(s_{23}c_{13}\right)\left(c_{23}c_{13}\right)\\ &= -\Delta_{21}c_{12}^2 c_{23}s_{23}-\Delta_{21}s_{12}c_{12}c_{23}^2 s_{13}e^{-i\delta}+\Delta_{21}s_{12}c_{12}s_{13}s_{23}^2 e^{i\delta}\\ &\quad +\Delta_{21}s_{12}^2 s_{13}^2 s_{23}c_{23}+\Delta_{31}c_{13}^2 s_{23}c_{23}\end{aligned} \tag{4.185}$$

sehingga dapat ditulis

$$p_{\mu\tau} = p_{\mu\tau}^a e^{-i\delta}+p_{\mu\tau}^b+p_{\mu\tau}^c e^{i\delta} \tag{4.186}$$

dimana

$$\begin{aligned} p^a_{\mu\tau} &= -\Delta_{21}s_{12}c_{12}c^2_{23}s_{13} \\ p^b_{\mu\tau} &= \left[\Delta_{31}c^2_{13} - \Delta_{21}\left(c^2_{12} - s^2_{12}s^2_{13}\right)\right]s_{23}c_{23} \\ p^c_{\mu\tau} &= \Delta_{21}s_{12}c_{12}s^2_{23}s_{13} \end{aligned} \quad (4.187)$$

$\tilde{q}_{\mu\tau}$ memuat H_{ee} , yang tidak konstan dan bergantung secara eksplisit pada potensial materi A, maka

$$\begin{aligned} \frac{1}{2E}\tilde{q}_{\mu\tau} &= \tilde{\mathcal{H}}_{\mu\tau} \\ &= \left\{H_{e\tau}H_{\mu e} - \left(H_{ee} + \frac{A}{2E}\right)H_{\mu\tau}\right\} \\ &= \frac{1}{2E}\left(q_{\mu\tau} - Ap_{\mu\tau}\right) \end{aligned} \quad (4.188)$$

sedangkan untuk $q_{\mu\tau}$

$$\begin{aligned} \tilde{U}_{\mu i}\tilde{U}^*_{\tau i} &= \frac{\tilde{p}_{\mu\tau}\lambda_i + \tilde{q}_{\mu\tau}}{\tilde{\Delta}_{ji}\tilde{\Delta}_{ki}} \\ &= \frac{p_{\mu\tau}\left(\lambda_i - A\right) + q_{\mu\tau}}{\tilde{\Delta}_{ji}\tilde{\Delta}_{ki}} \end{aligned} \quad (4.189)$$

maka

$$\begin{aligned} q_{\mu\tau} &= \Delta_{31}\Delta_{21}U_{\mu 1}U^*_{\tau 1} \\ &= \Delta_{31}\Delta_{21}\left(-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta}\right)\left(s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta}\right) \\ &= -\Delta_{31}\Delta_{21}s^2_{12}s_{23}c_{23} + \Delta_{31}\Delta_{21}s_{12}c_{12}c^2_{23}s_{13}e^{-i\delta} \\ &\quad -\Delta_{31}\Delta_{21}s_{12}c_{12}s^2_{23}s_{13}e^{i\delta} + \Delta_{31}\Delta_{21}c^2_{12}s^2_{13}c_{23}s_{23} \end{aligned} \quad (4.190)$$

sehingga bisa ditulis

$$q_{\mu\tau} = q^a_{\mu\tau}e^{-i\delta} + q^b_{\mu\tau} + q^c_{\mu\tau}e^{i\delta} \quad (4.191)$$

dimana

$$\begin{aligned} q^a_{\mu\tau} &= \Delta_{31}\Delta_{21}s_{12}c_{12}c^2_{23}s_{13} \\ q^b_{\mu\tau} &= \Delta_{31}\Delta_{21}\left(-s^2_{12} + c^2_{12}s^2_{13}\right)s_{23}c_{23} \\ q^c_{\mu\tau} &= -\Delta_{31}\Delta_{21}s_{12}c_{12}s^2_{23}s_{13} \end{aligned} \quad (4.192)$$

Maka probabilitas $P\left(\nu_\mu \to \nu_\tau\right)$ didapatkan

$$
\begin{aligned}
P\left(\nu_\mu \to \nu_\tau\right) &= -4\sum_{(ij)}^{siklik} ReJ_{\mu\tau}^{ij}\sin^2\tilde{\Delta}'_{ij} - 2\sum_{(ij)}^{siklik}\tilde{J}\sin^2\tilde{\Delta}'_{ij} \\
&= -4\left\{ReJ_{\mu\tau}^{12}\sin^2\tilde{\Delta}'_{12} + ReJ_{\mu\tau}^{23}\sin^2\tilde{\Delta}'_{23} + ReJ_{\mu\tau}^{31}\sin^2\tilde{\Delta}'_{31}\right\} \\
&\quad -2\tilde{J}\left(\sin^2\tilde{\Delta}'_{12} + \sin^2\tilde{\Delta}'_{23} + \sin^2\tilde{\Delta}'_{31}\right) \\
&= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{\left|\tilde{p}_{\mu\tau}\right|^2\lambda_1\lambda_2 + \left|\tilde{q}_{\mu\tau}\right|^2 + Re\left(\tilde{p}_{\mu\tau}\tilde{q}_{\mu\tau}^*\right)\left(\lambda_1+\lambda_2\right)\right\}\sin^2\tilde{\Delta}'_{12} \\
&\quad -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{\left|\tilde{p}_{\mu\tau}\right|^2\lambda_2\lambda_3 + \left|\tilde{q}_{\mu\tau}\right|^2 + Re\left(\tilde{p}_{\mu\tau}\tilde{q}_{\mu\tau}^*\right)\left(\lambda_2+\lambda_3\right)\right\}\sin^2\tilde{\Delta}'_{23} \\
&\quad -\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{\left|\tilde{p}_{\mu\tau}\right|^2\lambda_3\lambda_1 + \left|\tilde{q}_{\mu\tau}\right|^2 + Re\left(\tilde{p}_{\mu\tau}\tilde{q}_{\mu\tau}^*\right)\left(\lambda_3+\lambda_1\right)\right\}\sin^2\tilde{\Delta}'_{31} \\
&\quad -2\tilde{J}\left(\sin^2\tilde{\Delta}'_{12} + \sin^2\tilde{\Delta}'_{23} + \sin^2\tilde{\Delta}'_{31}\right) \qquad (4.193)
\end{aligned}
$$

karena

$$
\begin{aligned}
\tilde{p}_{\mu\tau} &= \tilde{H}_{\mu\tau} \\
&= H_{\mu\tau} \\
&= p_{\mu\tau} \qquad (4.194)
\end{aligned}
$$

dan

$$
\begin{aligned}
\tilde{q}_{\mu\tau} &= \tilde{H}_{e\tau}\tilde{H}_{\mu e} - \tilde{H}_{\mu\tau}\tilde{H}_{ee} \\
&= H_{e\tau}H_{\mu e} - H_{\mu\tau}H_{ee} - \frac{A}{2E}H_{\mu\tau} \\
&= q_{e\mu} - \frac{A}{2E}p_{\mu\tau} \qquad (4.195)
\end{aligned}
$$

sehingga

$$
\begin{aligned}
P\left(\nu_\mu \to \nu_\tau\right) &= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{\left|p_{\mu\tau}\right|^2\lambda_1\lambda_2 + \left|q_{\mu\tau}\right|^2 + Re\left(p_{\mu\tau}q_{\mu\tau}^*\right)\left(\lambda_1+\lambda_2\right)\right\}\sin^2\tilde{\Delta}'_{12} \\
&\quad -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{\left|p_{\mu\tau}\right|^2\lambda_2\lambda_3 + \left|q_{\mu\tau}\right|^2 + Re\left(p_{\mu\tau}q_{\mu\tau}^*\right)\left(\lambda_2+\lambda_3\right)\right\}\sin^2\tilde{\Delta}'_{23} \\
&\quad -\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left\{\left|p_{\mu\tau}\right|^2\lambda_3\lambda_1 + \left|q_{\mu\tau}\right|^2 + Re\left(p_{\mu\tau}q_{\mu\tau}^*\right)\left(\lambda_3+\lambda_1\right)\right\}\sin^2\tilde{\Delta}'_{31} \\
&\quad -2\tilde{J}\left(\sin^2\tilde{\Delta}'_{12} + \sin^2\tilde{\Delta}'_{23} + \sin^2\tilde{\Delta}'_{31}\right) \qquad (4.196)
\end{aligned}
$$

sekarang hitung

$$\begin{aligned}
|p_{\mu\tau}|^2 &= p_{\mu\tau}p_{\mu\tau}^* \\
&= \left(p_{\mu\tau}^a e^{-i\delta} + p_{\mu\tau}^b + p_{\mu\tau}^c e^{i\delta}\right)\left(p_{\mu\tau}^a e^{i\delta} + p_{\mu\tau}^b + p_{\mu\tau}^c e^{-i\delta}\right) \\
&= \left(p_{\mu\tau}^a\right)^2 + p_{\mu\tau}^a p_{\mu\tau}^b e^{-i\delta} + p_{\mu\tau}^a p_{\mu\tau}^c e^{-2i\delta} \\
&\quad + p_{\mu\tau}^a p_{\mu\tau}^b e^{i\delta} + \left(p_{\mu\tau}^b\right)^2 + p_{\mu\tau}^b p_{\mu\tau}^c e^{-i\delta} \\
&\quad + p_{\mu\tau}^a p_{\mu\tau}^c e^{2i\delta} + p_{\mu\tau}^b p_{\mu\tau}^c e^{i\delta} + \left(p_{\mu\tau}^c\right)^2 \\
&= \left(p_{\mu\tau}^a\right)^2 + \left(p_{\mu\tau}^b\right)^2 + \left(p_{\mu\tau}^c\right)^2 \\
&\quad + p_{\mu\tau}^a p_{\mu\tau}^c \left(e^{2i\delta} + e^{-2i\delta}\right) \\
&\quad + p_{\mu\tau}^a p_{\mu\tau}^b \left(e^{i\delta} + e^{-i\delta}\right) + p_{\mu\tau}^b p_{\mu\tau}^c \left(e^{i\delta} + e^{-i\delta}\right) \\
&= \left(p_{\mu\tau}^a\right)^2 + \left(p_{\mu\tau}^b\right)^2 + \left(p_{\mu\tau}^c\right)^2 \\
&\quad + 2p_{\mu\tau}^a p_{\mu\tau}^b \cos 2\delta \\
&\quad + 2p_{\mu\tau}^b \left(p_{\mu\tau}^a + p_{\mu\tau}^c\right)\cos\delta
\end{aligned} \tag{4.197}$$

hitung

$$\left(p_{\mu\tau}^a\right)^2 = \Delta_{21}^2 s_{12}^2 c_{12}^2 c_{23}^4 s_{13}^3 \tag{4.198}$$

$$\left(p_{\mu\tau}^b\right)^2 = \left[\Delta_{31}c_{13}^2 - \Delta_{21}\left(c_{12}^2 - s_{12}^2 s_{13}^2\right)\right]^2 s_{23}^2 c_{23}^2 \tag{4.199}$$

$$\left(p_{\mu\tau}^c\right)^2 = \Delta_{21}^2 s_{12}^2 c_{12}^2 s_{23}^4 s_{13}^2 \tag{4.200}$$

$$\begin{aligned}
p_{\mu\tau}^a p_{\mu\tau}^b &= \left(-\Delta_{21}s_{12}c_{12}c_{23}^2 s_{13}\right)\left(\left[\Delta_{31}c_{13}^2 - \Delta_{21}\left(c_{12}^2 - s_{12}^2 s_{13}^2\right)\right]s_{23}c_{23}\right) \\
&= \left(-\Delta_{21}s_{12}c_{12}c_{23}^3 s_{23}s_{13}\right)\left[\Delta_{31}c_{13}^2 - \Delta_{21}\left(c_{12}^2 - s_{12}^2 s_{13}^2\right)\right] \\
&= \left(-\Delta_{21}s_{12}c_{12}c_{23}^3 s_{23}s_{13}\right)\left[\Delta_{31}c_{13}^2 - \Delta_{21}c_{12}^2 + \Delta_{21}s_{12}^2 s_{13}^2\right] \\
&= -\Delta_{21}\Delta_{31}J_r c_{23}^2 + \Delta_{21}^2 s_{12}c_{12}c_{23}^3 s_{23}s_{13}c_{12}^2 \\
&\quad -\Delta_{21}^2 s_{12}^3 c_{12}c_{23}^3 s_{23}s_{13}^3
\end{aligned} \tag{4.201}$$

dan

$$\begin{aligned}
p_{\mu\tau}^b\left(p_{\mu\tau}^a + p_{\mu\tau}^c\right) &= \left\{\left[\Delta_{31}c_{13}^2 - \Delta_{21}\left(c_{12}^2 - s_{12}^2 s_{13}^2\right)\right]s_{23}c_{23}\right\} \\
&\quad \times\left(-\Delta_{21}s_{12}c_{12}c_{23}^2 s_{13} + \Delta_{21}s_{12}c_{12}s_{23}^2 s_{13}\right) \\
&= \left[\Delta_{31}c_{13}^2 s_{23}c_{23} - \Delta_{21}c_{12}^2 s_{23}c_{23} + \Delta_{21}s_{12}^2 s_{13}^2 s_{23}c_{23}\right] \\
&\quad \times\left(-\Delta_{21}s_{12}c_{12}c_{23}^2 s_{13} + \Delta_{21}s_{12}c_{12}s_{23}^2 s_{13}\right) \\
&= -\Delta_{31}\Delta_{21}s_{12}c_{12}s_{23}c_{23}^3 s_{13}c_{13}^2 + \Delta_{31}\Delta_{21}s_{12}c_{12}s_{23}^3 c_{23}s_{13}c_{13}^2 \\
&\quad +\Delta_{21}^2 s_{12}c_{12}^3 c_{23}^3 s_{23}s_{13} - \Delta_{21}^2 s_{12}c_{12}^3 s_{23}^3 s_{13}c_{23} \\
&\quad -\Delta_{21}^2 s_{12}^3 c_{12}c_{23}^3 s_{23}s_{13} + \Delta_{21}^2 s_{12}^3 c_{12}s_{23}^3 s_{13}^3 c_{23} \\
&= -\Delta_{31}\Delta_{21}J_r c_{23}^2 + \Delta_{31}\Delta_{21}J_r s_{23}^2 \\
&\quad +\Delta_{21}^2 s_{12}c_{12}^3 c_{23}^3 s_{23}s_{13} - \Delta_{21}^2 s_{12}c_{12}^3 s_{23}^3 s_{13}c_{23} \\
&\quad -\Delta_{21}^2 s_{12}^3 c_{12}c_{23}^3 s_{23}s_{13} + \Delta_{21}^2 s_{12}^3 c_{12}s_{23}^3 s_{13}^3 c_{23}
\end{aligned} \tag{4.202}$$

sedangkan

$$\begin{aligned}
|q_{\mu\tau}|^2 &= q_{\mu\tau}q_{\mu\tau}^* \\
&= \left(q_{\mu\tau}^a e^{-i\delta} + q_{\mu\tau}^b + q_{\mu\tau}^c e^{i\delta}\right)\left(q_{\mu\tau}^a e^{i\delta} + q_{\mu\tau}^b + q_{\mu\tau}^c e^{-i\delta}\right) \\
&= \left(q_{\mu\tau}^a\right)^2 + q_{\mu\tau}^a q_{\mu\tau}^b e^{-i\delta} + q_{\mu\tau}^a q_{\mu\tau}^c e^{-2i\delta} \\
&\quad + q_{\mu\tau}^a q_{\mu\tau}^b e^{i\delta} + \left(q_{\mu\tau}^b\right)^2 + q_{\mu\tau}^b q_{\mu\tau}^c e^{-i\delta} \\
&\quad + q_{\mu\tau}^a q_{\mu\tau}^c e^{2i\delta} + q_{\mu\tau}^b q_{\mu\tau}^c e^{i\delta} + \left(q_{\mu\tau}^c\right)^2 \\
&= \left(q_{\mu\tau}^a\right)^2 + \left(q_{\mu\tau}^b\right)^2 + \left(q_{\mu\tau}^c\right)^2 \\
&\quad + q_{\mu\tau}^a q_{\mu\tau}^c \left(e^{2i\delta} + e^{-2i\delta}\right) \\
&\quad + q_{\mu\tau}^a q_{\mu\tau}^b \left(e^{i\delta} + e^{-i\delta}\right) + q_{\mu\tau}^b q_{\mu\tau}^c \left(e^{i\delta} + e^{-i\delta}\right) \\
&= \left(q_{\mu\tau}^a\right)^2 + \left(q_{\mu\tau}^b\right)^2 + \left(q_{\mu\tau}^c\right)^2 \\
&\quad + 2q_{\mu\tau}^a q_{\mu\tau}^b \cos 2\delta \\
&\quad + 2q_{\mu\tau}^b \left(q_{\mu\tau}^a + q_{\mu\tau}^c\right)\cos\delta
\end{aligned} \tag{4.203}$$

hitung

$$\left(q_{\mu\tau}^a\right)^2 = \Delta_{31}^2\Delta_{21}^2 s_{12}^2 c_{12}^2 c_{23}^4 s_{13}^2 \tag{4.204}$$

$$\left(q_{\mu\tau}^b\right)^2 = \Delta_{31}^2\Delta_{21}^2 \left(-s_{12}^2 + c_{12}^2 s_{13}^2\right)^2 s_{23}^2 c_{23}^2 \tag{4.205}$$

$$\left(q_{\mu\tau}^c\right)^2 = \Delta_{31}^2\Delta_{21}^2 s_{12}^2 c_{12}^2 s_{23}^4 s_{13}^2 \tag{4.206}$$

$$\begin{aligned}
q_{\mu\tau}^a q_{\mu\tau}^b &= \left(\Delta_{31}\Delta_{21} s_{12} c_{12} c_{23}^2 s_{13}\right)\left\{\Delta_{31}\Delta_{21}\left(-s_{12}^2 + c_{12}^2 s_{13}^2\right) s_{23} c_{23}\right\} \\
&= \left(\Delta_{31}\Delta_{21} s_{12} c_{12} c_{23}^2 s_{13}\right) \\
&\quad \times \left(-\Delta_{31}\Delta_{21} s_{12}^2 s_{23} c_{23} + \Delta_{31}\Delta_{21} c_{12}^2 s_{23} c_{23} s_{13}^2\right) \\
&= -\Delta_{31}^2\Delta_{21}^2 s_{12}^3 c_{12} s_{23} c_{23}^3 s_{13} + \Delta_{31}^2\Delta_{21}^2 s_{12} c_{12}^3 s_{23} c_{23}^3 s_{13}^3
\end{aligned} \tag{4.207}$$

dan

$$\begin{aligned}
q_{\mu\tau}^b \left(q_{\mu\tau}^a + q_{\mu\tau}^c\right) &= \left\{\Delta_{31}\Delta_{21}\left(-s_{12}^2 + c_{12}^2 s_{13}^2\right) s_{23} c_{23}\right\} \\
&\quad \times \left(\Delta_{31}\Delta_{21} s_{12} c_{12} c_{23}^2 s_{13} - \Delta_{31}\Delta_{21} s_{12} c_{12} s_{23}^2 s_{13}\right) \\
&= \left(-\Delta_{31}\Delta_{21} s_{12}^2 s_{23} c_{23} + \Delta_{31}\Delta_{21} c_{12}^2 s_{13}^2 s_{23} c_{23}\right) \\
&\quad \times \left(\Delta_{31}\Delta_{21} s_{12} c_{12} c_{23}^2 s_{13} - \Delta_{31}\Delta_{21} s_{12} c_{12} s_{23}^2 s_{13}\right) \\
&= -\Delta_{31}^2\Delta_{21}^2 s_{12}^3 c_{12} s_{23} c_{23}^3 s_{13} + \Delta_{31}^2\Delta_{21}^2 s_{12}^3 c_{12} s_{23}^3 c_{23} s_{13} \\
&\quad + \Delta_{31}^2\Delta_{21}^2 s_{12} c_{12}^3 c_{23}^3 s_{13}^3 s_{23} - \Delta_{31}^2\Delta_{21}^2 s_{12} c_{12}^3 s_{23}^3 c_{23} s_{13}^3
\end{aligned} \tag{4.208}$$

sedangkan

$$
\begin{aligned}
p_{\mu\tau}q_{\mu\tau}^{*} &= \left(p_{\mu\tau}^{a}e^{-i\delta}+p_{\mu\tau}^{b}+p_{\mu\tau}^{c}e^{i\delta}\right)\left(q_{\mu\tau}^{a}e^{i\delta}+q_{\mu\tau}^{b}+q_{\mu\tau}^{c}e^{-i\delta}\right) \\
&= p_{\mu\tau}^{a}q_{\mu\tau}^{a}+p_{\mu\tau}^{a}q_{\mu\tau}^{b}e^{-i\delta}+p_{\mu\tau}^{a}q_{\mu\tau}^{c}e^{-2i\delta} \\
&\quad +p_{\mu\tau}^{b}q_{\mu\tau}^{a}e^{i\delta}+p_{\mu\tau}^{b}q_{\mu\tau}^{b}+p_{\mu\tau}^{b}q_{\mu\tau}^{c}e^{-i\delta} \\
&\quad +p_{\mu\tau}^{c}q_{\mu\tau}^{a}e^{2i\delta}+p_{\mu\tau}^{c}q_{\mu\tau}^{b}e^{i\delta}+p_{\mu\tau}^{c}q_{\mu\tau}^{c} \\
&= p_{\mu\tau}^{a}q_{\mu\tau}^{a}+p_{\mu\tau}^{b}q_{\mu\tau}^{b}+p_{\mu\tau}^{c}q_{\mu\tau}^{c} \\
&\quad +p_{\mu\tau}^{a}q_{\mu\tau}^{b}\left(\cos\delta-i\sin\delta\right) \\
&\quad +p_{\mu\tau}^{a}q_{\mu\tau}^{c}\left(\cos2\delta-i\sin2\delta\right) \\
&\quad +p_{\mu\tau}^{b}q_{\mu\tau}^{a}\left(\cos\delta+i\sin\delta\right) \\
&\quad +p_{\mu\tau}^{b}q_{\mu\tau}^{c}\left(\cos\delta-i\sin\delta\right) \\
&\quad +p_{\mu\tau}^{c}q_{\mu\tau}^{a}\left(\cos2\delta+i\sin2\delta\right) \\
&\quad +p_{\mu\tau}^{c}q_{\mu\tau}^{b}\left(\cos\delta+i\sin\delta\right) \\
&= p_{\mu\tau}^{a}q_{\mu\tau}^{a}+p_{\mu\tau}^{b}q_{\mu\tau}^{b}+p_{\mu\tau}^{c}q_{\mu\tau}^{c} \\
&\quad +\left(p_{\mu\tau}^{a}q_{\mu\tau}^{b}+p_{\mu\tau}^{b}q_{\mu\tau}^{a}+p_{\mu\tau}^{b}q_{\mu\tau}^{c}+p_{\mu\tau}^{c}q_{\mu\tau}^{b}\right)\cos\delta \\
&\quad +\left(p_{\mu\tau}^{a}q_{\mu\tau}^{c}+p_{\mu\tau}^{c}q_{\mu\tau}^{a}\right)\cos2\delta \\
&\quad +i\left(p_{\mu\tau}^{b}q_{\mu\tau}^{a}+p_{\mu\tau}^{c}q_{\mu\tau}^{b}-p_{\mu\tau}^{a}q_{\mu\tau}^{b}-p_{\mu\tau}^{b}q_{\mu\tau}^{c}\right)\sin\delta \\
&\quad +i\left(p_{\mu\tau}^{c}q_{\mu\tau}^{a}-p_{\mu\tau}^{a}q_{\mu\tau}^{c}\right)\sin2\delta \qquad (4.209)
\end{aligned}
$$

sehingga

$$
\begin{aligned}
Re\left(p_{\mu\tau}q_{\mu\tau}^{*}\right) &= p_{\mu\tau}^{a}q_{\mu\tau}^{a}+p_{\mu\tau}^{b}q_{\mu\tau}^{b}+p_{\mu\tau}^{c}q_{\mu\tau}^{c} \\
&\quad +\left(p_{\mu\tau}^{a}q_{\mu\tau}^{b}+p_{\mu\tau}^{b}q_{\mu\tau}^{a}+p_{\mu\tau}^{b}q_{\mu\tau}^{c}+p_{\mu\tau}^{c}q_{\mu\tau}^{b}\right)\cos\delta \\
&\quad +\left(p_{\mu\tau}^{a}q_{\mu\tau}^{c}+p_{\mu\tau}^{c}q_{\mu\tau}^{a}\right)\cos2\delta \qquad (4.210)
\end{aligned}
$$

hitung

$$
\begin{aligned}
p_{\mu\tau}^{a}q_{\mu\tau}^{a} &= \left(-\Delta_{21}s_{12}c_{12}c_{23}^{2}s_{13}\right)\left(\Delta_{31}\Delta_{21}s_{12}c_{12}c_{23}^{2}s_{13}\right) \\
&= -\Delta_{31}\Delta_{21}^{2}s_{12}^{2}c_{12}^{2}c_{23}^{4}s_{13}^{2} \\
p_{\mu\tau}^{b}q_{\mu\tau}^{b} &= \left\{\left[\Delta_{31}c_{13}^{2}-\Delta_{21}\left(c_{12}^{2}-s_{12}^{2}s_{13}^{2}\right)\right]s_{23}c_{23}\right\} \\
&\quad \times\left\{\Delta_{31}\Delta_{21}\left(-s_{12}^{2}+c_{12}^{2}s_{13}^{2}\right)s_{23}c_{23}\right\}
\end{aligned}
$$

$$
\begin{aligned}
&= \left(\Delta_{31}c_{13}^2 s_{23}c_{23} - \Delta_{21}c_{12}^2 s_{23}c_{23} + \Delta_{21}s_{12}^2 s_{13}^2 s_{23}c_{23}\right) \\
&\quad \times \left(-\Delta_{31}\Delta_{21}s_{12}^2 s_{23}c_{23} + \Delta_{31}\Delta_{21}c_{12}^2 s_{13}^2 s_{23}c_{23}\right) \\
&= -\Delta_{31}^2\Delta_{21}s_{12}^2 c_{13}^2 s_{23}^2 c_{23}^2 + \Delta_{31}^2\Delta_{21}c_{12}^2 s_{13}^2 c_{13}^2 s_{23}^2 c_{23}^2 \\
&\quad +\Delta_{31}\Delta_{21}^2 s_{12}^2 c_{12}^2 s_{23}^2 c_{23}^2 - \Delta_{31}\Delta_{21}^2 c_{12}^4 s_{13}^2 s_{23}^2 c_{23}^2 \\
&\quad -\Delta_{31}\Delta_{21}^2 s_{12}^4 s_{13}^2 s_{23}^2 c_{23}^2 + \Delta_{31}\Delta_{21}^2 c_{12}^2 s_{12}^2 s_{13}^4 s_{23}^2 c_{23}^2 \qquad (4.211)
\end{aligned}
$$

$$
\begin{aligned}
p_{\mu\tau}^c q_{\mu\tau}^c &= \left(\Delta_{21}s_{12}c_{12}s_{23}^2 c_{13}\right)\left(-\Delta_{31}\Delta_{21}s_{12}c_{12}s_{23}^2 s_{13}\right) \\
&= -\Delta_{31}\Delta_{21}^2 s_{12}^2 c_{12}^2 s_{23}^4 s_{13} \qquad (4.212)
\end{aligned}
$$

$$
\begin{aligned}
p_{\mu\tau}^a q_{\mu\tau}^b &= \left(-\Delta_{21}s_{12}c_{12}c_{23}^2 s_{13}\right)\left\{\Delta_{31}\Delta_{21}\left(-s_{12}^2 + c_{12}^2 s_{13}^2\right)s_{23}c_{23}\right\} \\
&= \left(-\Delta_{21}s_{12}c_{12}c_{23}^2 s_{13}\right)\left(-\Delta_{31}\Delta_{21}s_{12}^2 s_{23}c_{23} + \Delta_{31}\Delta_{21}c_{12}^2 s_{13}^2 s_{23}c_{23}\right) \\
&= \Delta_{31}\Delta_{21}^2 s_{12}^3 c_{12}c_{23}^3 s_{23}s_{13} - \Delta_{31}\Delta_{21}^2 s_{12}c_{12}^3 s_{13}^3 c_{23}^3 s_{23} \qquad (4.213)
\end{aligned}
$$

$$
\begin{aligned}
p_{\mu\tau}^b q_{\mu\tau}^a &= \left\{\left[\Delta_{31}c_{13}^2 - \Delta_{21}\left(c_{12}^2 - s_{12}^2 s_{13}^2\right)\right]s_{23}c_{23}\right\}\left(\Delta_{31}\Delta_{21}s_{12}c_{12}c_{23}^2 s_{13}\right) \\
&= \left(\Delta_{31}c_{13}^2 s_{23}c_{23} - \Delta_{21}c_{12}^2 s_{23}c_{23} + \Delta_{21}s_{12}^2 s_{13}^2 s_{23}c_{23}\right)\left(\Delta_{31}\Delta_{21}s_{12}c_{12}c_{23}^2 s_{13}\right) \\
&= \Delta_{31}^2\Delta_{21}s_{12}c_{12}c_{23}^3 s_{23}s_{13}c_{13}^2 - \Delta_{31}\Delta_{21}^2 s_{12}c_{12}^3 c_{23}^3 s_{23}s_{13} \\
&\quad +\Delta_{31}\Delta_{21}^2 s_{12}^3 c_{12}c_{23}^3 s_{23}s_{13}^3 \qquad (4.214)
\end{aligned}
$$

$$
\begin{aligned}
p_{\mu\tau}^b q_{\mu\tau}^c &= \left\{\left[\Delta_{31}c_{13}^2 - \Delta_{21}\left(c_{12}^2 - s_{12}^2 s_{13}^2\right)\right]s_{23}c_{23}\right\}\left(-\Delta_{31}\Delta_{21}s_{12}c_{12}s_{23}^2 s_{13}\right) \\
&= \left(\Delta_{31}c_{13}^2 s_{23}c_{23} - \Delta_{21}c_{12}^2 s_{23}c_{23} + \Delta_{21}s_{12}^2 s_{13}^2 s_{23}c_{23}\right)\left(-\Delta_{31}\Delta_{21}s_{12}c_{12}s_{23}^2 s_{13}\right) \\
&= -\Delta_{31}^2\Delta_{21}s_{12}c_{12}s_{23}^3 c_{23}s_{13}c_{13}^2 + \Delta_{31}\Delta_{21}^2 s_{12}c_{12}^3 s_{23}^3 c_{23}s_{13} \\
&\quad -\Delta_{31}\Delta_{21}^2 s_{12}^3 c_{12}s_{23}^3 c_{23}s_{13}^3 \qquad (4.215)
\end{aligned}
$$

$$
\begin{aligned}
p_{\mu\tau}^c q_{\mu\tau}^b &= \left(\Delta_{21}s_{12}c_{12}s_{23}^2 s_{13}\right)\left\{\Delta_{31}\Delta_{21}\left(-s_{12}^2 + c_{12}^2 s_{13}^2\right)s_{23}c_{23}\right\} \\
&= \left(\Delta_{21}s_{12}c_{12}s_{23}^2 s_{13}\right)\left(-\Delta_{31}\Delta_{21}s_{12}^2 s_{23}c_{23} + \Delta_{31}\Delta_{21}c_{12}^2 s_{13}^2 s_{23}c_{23}\right) \\
&= -\Delta_{31}\Delta_{21}^2 s_{12}^3 c_{12}s_{23}^3 c_{23}s_{13} + \Delta_{31}\Delta_{21}^2 s_{12}c_{12}^3 s_{23}^3 s_{13}^3 c_{23} \qquad (4.216)
\end{aligned}
$$

$$
\begin{aligned}
p_{\mu\tau}^a q_{\mu\tau}^c &= \left(-\Delta_{21}s_{12}c_{12}c_{23}^2 s_{13}\right)\left(-\Delta_{31}\Delta_{21}s_{12}c_{12}s_{23}^2 s_{13}\right) \\
&= \Delta_{31}\Delta_{21}^2 s_{12}^2 c_{12}^2 c_{23}^2 s_{23}^2 s_{13}^2 \qquad (4.217)
\end{aligned}
$$

$$
\begin{aligned}
p_{\mu\tau}^c q_{\mu\tau}^a &= \left(\Delta_{21}s_{12}c_{12}s_{23}^2 s_{13}\right)\left(-\Delta_{31}\Delta_{21}s_{12}c_{12}s_{23}^2 s_{13}\right) \\
&= -\Delta_{31}\Delta_{21}^2 s_{12}^2 c_{12}^2 s_{23}^4 s_{13}^2 \qquad (4.218)
\end{aligned}
$$

Masukkan persamaan (4.197) ,(4.203) , dan (4.210) ke persamaan (4.196)

$$
\begin{aligned}
P\left(\nu_\mu \to \nu_\tau\right) &= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \\
&\quad \times \left\{\left[\left(p_{\mu\tau}^a\right)^2 + \left(p_{\mu\tau}^b\right)^2 + \left(p_{\mu\tau}^c\right)^2 + 2p_{\mu\tau}^a p_{\mu\tau}^b \cos 2\delta + 2p_{\mu\tau}^b\left(p_{\mu\tau}^a + p_{\mu\tau}^c\right)\cos\delta\right]\lambda_1\lambda_2\right. \\
&\quad + \left[\left(q_{\mu\tau}^a\right)^2 + \left(q_{\mu\tau}^b\right)^2 + \left(q_{\mu\tau}^c\right)^2 + 2q_{\mu\tau}^a q_{\mu\tau}^b \cos 2\delta + 2q_{\mu\tau}^b\left(q_{\mu\tau}^a + q_{\mu\tau}^c\right)\cos\delta\right]
\end{aligned}
$$

$$
\begin{aligned}
&+A\left(\lambda_1+\lambda_2\right)\}\\
&\times\sin^2\tilde{\Delta}'_{12}\\
&-\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\\
&\times\left\{\left[\left(p^a_{\mu\tau}\right)^2+\left(p^b_{\mu\tau}\right)^2+\left(p^c_{\mu\tau}\right)^2+2p^a_{\mu\tau}p^b_{\mu\tau}\cos 2\delta+2p^b_{\mu\tau}\left(p^a_{\mu\tau}+p^c_{\mu\tau}\right)\cos\delta\right]\lambda_2\lambda_3\right.\\
&+\left[\left(q^a_{\mu\tau}\right)^2+\left(q^b_{\mu\tau}\right)^2+\left(q^c_{\mu\tau}\right)^2+2q^a_{\mu\tau}q^b_{\mu\tau}\cos 2\delta+2q^b_{\mu\tau}\left(q^a_{\mu\tau}+q^c_{\mu\tau}\right)\cos\delta\right]\\
&+B\left(\lambda_2+\lambda_3\right)\}\\
&\times\sin^2\tilde{\Delta}'_{23}\\
&-\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{31}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\\
&\times\left\{\left[\left(p^a_{\mu\tau}\right)^2+\left(p^b_{\mu\tau}\right)^2+\left(p^c_{\mu\tau}\right)^2+2p^a_{\mu\tau}p^b_{\mu\tau}\cos 2\delta+2p^b_{\mu\tau}\left(p^a_{\mu\tau}+p^c_{\mu\tau}\right)\cos\delta\right]\lambda_3\lambda_1\right.\\
&+\left[\left(q^a_{\mu\tau}\right)^2+\left(q^b_{\mu\tau}\right)^2+\left(q^c_{\mu\tau}\right)^2+2q^a_{\mu\tau}q^b_{\mu\tau}\cos 2\delta+2q^b_{\mu\tau}\left(q^a_{\mu\tau}+q^c_{\mu\tau}\right)\cos\delta\right]\\
&+C\left(\lambda_3+\lambda_1\right)\}\\
&\times\sin^2\tilde{\Delta}'_{31}\\
&-\frac{2J_r\Delta_{12}\Delta_{23}\Delta_{31}}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left(\sin^2\tilde{\Delta}'_{12}+\sin^2\tilde{\Delta}'_{23}+\sin^2\tilde{\Delta}'_{31}\right)\sin\delta
\end{aligned}
\tag{4.219}
$$

dengan

$$
\begin{aligned}
A &= \left[p^a_{\mu\tau}q^a_{\mu\tau}+p^b_{\mu\tau}p^b_{\mu\tau}+p^c_{\mu\tau}p^c_{\mu\tau}+\left(p^a_{\mu\tau}q^b_{\mu\tau}+p^b_{\mu\tau}q^a_{\mu\tau}+p^b_{\mu\tau}q^c_{\mu\tau}+p^a_{\mu\tau}q^b_{\mu\tau}\right)\cos\delta\right.\\
&\quad\left.+\left(p^a_{\mu\tau}q^c_{\mu\tau}+p^c_{\mu\tau}q^a_{\mu\tau}\right)\cos 2\delta\right] \qquad (4.220)\\
B &= \left[p^a_{\mu\tau}q^a_{\mu\tau}+p^b_{\mu\tau}p^b_{\mu\tau}+p^c_{\mu\tau}p^c_{\mu\tau}+\left(p^a_{\mu\tau}q^b_{\mu\tau}+p^b_{\mu\tau}q^a_{\mu\tau}+p^b_{\mu\tau}q^c_{\mu\tau}+p^a_{\mu\tau}q^b_{\mu\tau}\right)\cos\delta\right.\\
&\quad\left.+\left(p^a_{\mu\tau}q^c_{\mu\tau}+p^c_{\mu\tau}q^a_{\mu\tau}\right)\cos 2\delta\right] \qquad (4.221)\\
C &= \left[p^a_{\mu\tau}q^a_{\mu\tau}+p^b_{\mu\tau}p^b_{\mu\tau}+p^c_{\mu\tau}p^c_{\mu\tau}+\left(p^a_{\mu\tau}q^b_{\mu\tau}+p^b_{\mu\tau}q^a_{\mu\tau}+p^b_{\mu\tau}q^c_{\mu\tau}+p^a_{\mu\tau}q^b_{\mu\tau}\right)\cos\delta\right.\\
&\quad\left.+\left(p^a_{\mu\tau}q^c_{\mu\tau}+p^c_{\mu\tau}q^a_{\mu\tau}\right)\cos 2\delta\right] \qquad (4.222)
\end{aligned}
$$

sehingga dapat ditulis

$$P\left(\nu_\mu\to\nu_\tau\right)=\tilde{A}_{\mu\tau}\cos\delta+\tilde{B}\sin\delta+\tilde{C}_{\mu\tau}+\tilde{D}\cos 2\delta \tag{4.223}$$

dengan koefisien - koefisiennya adalah

$$
\begin{aligned}
\tilde{A}_{\mu\tau} &= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[2p^b_{\mu\tau}\left(p^a_{\mu\tau}+p^c_{\mu\tau}\right)\lambda_1\lambda_2+2q^b_{\mu\tau}\left(q^a_{\mu\tau}+q^c_{\mu\tau}\right)\right.\\
&\quad\left.+\left(p^a_{\mu\tau}p^b_{\mu\tau}+p^b_{\mu\tau}p^a_{\mu\tau}+p^b_{\mu\tau}p^c_{\mu\tau}+p^c_{\mu\tau}p^b_{\mu\tau}\right)\left(\lambda_1+\lambda_2\right)\right]\sin^2\tilde{\Delta}_{12}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[2p^b_{\mu\tau}\left(p^a_{\mu\tau}+p^c_{\mu\tau}\right)\lambda_2\lambda_3+2q^b_{\mu\tau}\left(q^a_{\mu\tau}+q^c_{\mu\tau}\right)\right.\\
&\left.+\left(p^a_{\mu\tau}p^b_{\mu\tau}+p^b_{\mu\tau}p^a_{\mu\tau}+p^b_{\mu\tau}p^c_{\mu\tau}+p^c_{\mu\tau}p^b_{\mu\tau}\right)(\lambda_2+\lambda_3)\right]\sin^2\tilde{\Delta}_{23}\\
&-\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[2p^b_{\mu\tau}\left(p^a_{\mu\tau}+p^c_{\mu\tau}\right)\lambda_3\lambda_1+2q^b_{\mu\tau}\left(q^a_{\mu\tau}+q^c_{\mu\tau}\right)\right.\\
&\left.+\left(p^a_{\mu\tau}p^b_{\mu\tau}+p^b_{\mu\tau}p^a_{\mu\tau}+p^b_{\mu\tau}p^c_{\mu\tau}+p^c_{\mu\tau}p^b_{\mu\tau}\right)(\lambda_3+\lambda_1)\right]\sin^2\tilde{\Delta}_{31}
\end{aligned}
\tag{4.224}
$$

dan

$$
\begin{aligned}
\tilde{C}_{\mu\tau} &= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{\left(p^a_{\mu\tau}\right)^2+\left(p^b_{\mu\tau}\right)^2+\left(p^c_{\mu\tau}\right)^2\right\}\lambda_1\lambda_2\right.\\
&\quad+\left\{\left(q^a_{\mu\tau}\right)^2+\left(q^b_{\mu\tau}\right)^2+\left(q^c_{\mu\tau}\right)^2\right\}\\
&\quad\left.+\left(p^a_{\mu\tau}q^a_{\mu\tau}+p^b_{\mu\tau}q^b_{\mu\tau}+p^c_{\mu\tau}q^c_{\mu\tau}\right)(\lambda_1+\lambda_2)\right]\sin^2\tilde{\Delta}'_{12}\\
&= -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{\left(p^a_{\mu\tau}\right)^2+\left(p^b_{\mu\tau}\right)^2+\left(p^c_{\mu\tau}\right)^2\right\}\lambda_2\lambda_3\right.\\
&\quad+\left\{\left(q^a_{\mu\tau}\right)^2+\left(q^b_{\mu\tau}\right)^2+\left(q^c_{\mu\tau}\right)^2\right\}\\
&\quad\left.+\left(p^a_{\mu\tau}q^a_{\mu\tau}+p^b_{\mu\tau}q^b_{\mu\tau}+p^c_{\mu\tau}q^c_{\mu\tau}\right)(\lambda_2+\lambda_3)\right]\sin^2\tilde{\Delta}'_{23}\\
&= -\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left\{\left(p^a_{\mu\tau}\right)^2+\left(p^b_{\mu\tau}\right)^2+\left(p^c_{\mu\tau}\right)^2\right\}\lambda_3\lambda_2\right.\\
&\quad+\left\{\left(q^a_{\mu\tau}\right)^2+\left(q^b_{\mu\tau}\right)^2+\left(q^c_{\mu\tau}\right)^2\right\}\\
&\quad\left.+\left(p^a_{\mu\tau}q^a_{\mu\tau}+p^b_{\mu\tau}q^b_{\mu\tau}+p^c_{\mu\tau}q^c_{\mu\tau}\right)(\lambda_3+\lambda_1)\right]\sin^2\tilde{\Delta}'_{31}
\end{aligned}
\tag{4.225}
$$

dan

$$
\begin{aligned}
\tilde{D} &= -\frac{4}{\tilde{\Delta}_{12}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^a_{\mu\tau}p^b_{\mu\tau}\right)\lambda_1\lambda_2+2q^a_{\mu\tau}q^b_{\mu\tau}\right.\\
&\quad\left.+\left(p^a_{\mu\tau}q^c_{\mu\tau}+p^c_{\mu\tau}q^a_{\mu\tau}\right)(\lambda_1+\lambda_2)\right]\sin^2\tilde{\Delta}'_{12}\\
&= -\frac{4}{\tilde{\Delta}_{23}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^a_{\mu\tau}p^b_{\mu\tau}\right)\lambda_2\lambda_3+2q^a_{\mu\tau}q^b_{\mu\tau}\right.\\
&\quad\left.+\left(p^a_{\mu\tau}q^c_{\mu\tau}+p^c_{\mu\tau}q^a_{\mu\tau}\right)(\lambda_2+\lambda_3)\right]\sin^2\tilde{\Delta}'_{31}\\
&= -\frac{4}{\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\left[\left(2p^a_{\mu\tau}p^b_{\mu\tau}\right)\lambda_3\lambda_1+2q^a_{\mu\tau}q^b_{\mu\tau}\right.\\
&\quad\left.+\left(p^a_{\mu\tau}q^c_{\mu\tau}+p^c_{\mu\tau}q^a_{\mu\tau}\right)(\lambda_3+\lambda_1)\right]\sin^2\tilde{\Delta}'_{31}
\end{aligned}
\tag{4.226}
$$

masuk - masukkan dan ditata ulang , maka koefisiennya menjadi

$$
\begin{aligned}
\tilde{A}_{\mu\tau} &= \sum_{(ijk)}^{siklik}\frac{-8\left[J_r\Delta_{21}\Delta_{31}\left(\lambda_k-A\right)\left(\lambda_k-A-\Delta_{31}\right)\left(c^2_{23}-s^2_{23}\right)+\left(\tilde{A}_{\mu\tau}\right)_k\right]}{\tilde{\Delta}^2_{jk}\tilde{\Delta}^2_{ki}}\\
&\quad\times\cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki}
\end{aligned}
\tag{4.227}
$$

dengan

$$\begin{aligned}\left(\tilde{A}_{\mu\tau}\right)_k &= -\Delta_{21}^2 s_{13}\left(c_{23}^2 - s_{23}^2\right)\\&\quad \times\left[s_{12}c_{12}s_{23}c_{23}\left(\lambda_k - A - \Delta_{31}\right)\right.\\&\quad \left.\left\{\Delta_{31}\left(s_{12}^2 - s_{13}^2 c_{12}^2\right) + \left(\lambda_k - A - \Delta_{31}\right)c_{12}^2\right\}\right] \qquad (4.228)\end{aligned}$$

dan

$$\tilde{C}_{\mu\tau} = \sum_{(ij)}^{siklik} \frac{-4\left[\Delta_{31}^2 s_{23}^2 c_{23}^2 c_{13}^4\left(\lambda_i - A\right)\left(\lambda_j - A\right) + \left(\tilde{C}_{\mu\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \sin^2\tilde{\Delta}'_{ij} \qquad (4.229)$$

dengan

$$\begin{aligned}\left(\tilde{C}_{\mu\tau}\right)_{ij} &= \Delta_{21}\times\left[\Delta_{31}\left\{\Delta_{31}\left(\lambda_i + \lambda_j - 2A\right)\left(c_{13}^2 + c_{12}^2\right) - 2\left(\lambda_i - A\right)\left(\lambda_j - A\right)c_{12}^2\right\}s_{23}^2 s_{23}^2 c_{13}^2\right]\\&+ \Delta_{21}s_{13}^2\times\left[\Delta_{31}\left\{-\lambda_{31}\left(\lambda_i + \lambda_j - 2A\right) + 2\left(\lambda_i - A\right)\left(\lambda_j - A\right)\right\}s_{12}^2 s_{23}^2 c_{23}^2 c_{13}^2\right]\\&+ \Delta_{21}^2\times\left[\left\{\Delta_{31}c_{13}^2 + \left(\lambda_i - A - \Delta_{31}\right)c_{12}^2\right\}\left\{\Delta_{31}c_{13}^2 + \left(\lambda_j - A - \Delta_{31}\right)c_{12}^2\right\}s_{23}^2 c_{23}^2\right]\\&+ \Delta_{21}^2 s_{13}^2\times\left[-\Delta_{31}\left(\lambda_i + \lambda_j - 2A - 2\Delta_{31}\right)s_{12}^2 s_{23}^2 c_{23}^2 c_{13}^2 + \left(\lambda_i - A - \Delta_{31}\right)\right.\\&\quad \left.\left(\lambda_j - A - \Delta_{31}\right)\left\{s_{12}^2 c_{12}^2\left(c_{23}^2 - s_{23}^2\right) + s_{12}^4 s_{23}^2 c_{23}^2 c_{13}^2\right\}\right] \qquad (4.230)\end{aligned}$$

dan

$$\tilde{D} = \sum_{(ijk)}^{siklik} \frac{-8\Delta_{21}^2\left(\lambda_k - A - \Delta_{31}\right)^2 s_{12}^2 c_{12}^2 s_{23}^2 c_{23}^2 s_{13}^2}{\tilde{\Delta}_{jk}^2\tilde{\Delta}_{ki}^2} \cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki} \qquad (4.231)$$

Sekarang dihitung probabilitas survival neutrino dalam materi , dengan mengunakan unitaritas dan probabilitas transisi , didapatkan

$$\begin{aligned}P\left(\nu_e \to \nu_e\right) &= 1 - P\left(\nu_e \to \nu_\mu\right) - P\left(\nu_e \to \nu_\tau\right)\\&= 1 - \left(\tilde{A}_{e\mu}\cos\delta + \tilde{B}\sin\delta + \tilde{C}_{e\mu}\right) - \left(-\tilde{A}_{e\mu}\cos\delta - \tilde{B}\sin\delta + \tilde{C}_{e\tau}\right)\\&= 1 - \tilde{A}_{e\mu}\cos\delta - \tilde{B}\sin\delta - \tilde{C}_{e\mu} + \tilde{A}_{e\mu}\cos\delta + \tilde{B}\sin\delta - \tilde{C}_{e\tau}\\&= 1 - \tilde{C}_{e\mu} - \tilde{C}_{e\tau}\\&\equiv \tilde{C}_{ee} \qquad (4.232)\end{aligned}$$

dimana

$$
\begin{aligned}
\tilde{C}_{ee} &= 1 - \tilde{C}_{e\mu} - \tilde{C}_{e\tau} \\
&= 1 - \sum_{(ij)}^{siklik} \frac{-4\left[\Delta_{31}^2 s_{13}^2 s_{23}^2 c_{13}^2 \lambda_i \lambda_j + \left(\tilde{C}_{e\mu}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \sin^2 \tilde{\Delta}'_{ij} \\
&\quad - \sum_{(ij)}^{siklik} \frac{-4\left[\Delta_{31}^2 s_{13}^2 c_{23}^2 c_{13} \lambda_i \lambda_j + \left(\tilde{C}_{e\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \sin^2 \tilde{\Delta}'_{ij} \\
&= 1 + \sum_{(ij)}^{siklik} \frac{4\left[\Delta_{31}^2 s_{13}^2 c_{13}^2 \lambda_i \lambda_j + \left(\tilde{C}_{ee}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \sin^2 \tilde{\delta}'_{ij} \qquad (4.233)
\end{aligned}
$$

dengan

$$
\begin{aligned}
\left(\tilde{C}_{ee}\right)_{ij} &= \Delta_{21} s_{13}^2 \times \left[\Delta_{31}\left\{-\lambda_i\left(\lambda_j s_{12}^2 + \Delta_{31} c_{12}^2\right) - \lambda_j\left(\lambda_i s_{12}^2 + \Delta_{31} c_{12}^2\right)\right\} c_{13}^2\right] \\
&+ \Delta_{21}^2 \times \left[\left(\lambda_i - \Delta_{31}\right)\left(\lambda_j - \Delta_{31}\right) s_{12}^2 c_{12}^2 c_{13}^2\right] \\
&+ \Delta_{21}^2 s_{13}^2 \times \left[\left(\lambda_i s_{12}^2 + \Delta_{31} c_{12}^2\right)\left(\lambda_j s_{12}^2 + \Delta_{31} c_{12}^2\right) c_{13}^2\right] \qquad (4.234)
\end{aligned}
$$

Probabilitas survival muon ke neutrino muon ($P\left(\nu_\mu \to \nu_\mu\right)$) adalah

$$
\begin{aligned}
P\left(\nu_\mu \to \nu_\mu\right) &= 1 - P\left(\nu_\mu \to \nu_e\right) - P\left(\nu_\mu \to \nu_\tau\right) \\
&= 1 - \left(\tilde{A}_{e\mu}\cos\delta - \tilde{B}\sin\delta + \tilde{C}_{e\mu}\right) - \left(\tilde{A}_{\mu\tau}\cos\delta + \tilde{B}\sin\delta + \tilde{C}_{\mu\tau} + \tilde{D}\cos 2\delta\right) \\
&= 1 - \tilde{A}_{e\mu}\cos\delta + \tilde{B}\sin\delta - \tilde{C}_{e\mu} - \tilde{A}_{\mu\tau}\cos\delta - \tilde{B}\sin\delta - \tilde{C}_{\mu\tau} - \tilde{D}\cos 2\delta \\
&= \left(-\tilde{A}_{e\mu} - \tilde{A}_{\mu\tau}\right)\cos\delta + \left(1 - \tilde{C}_{e\mu} - \tilde{C}_{\mu\tau}\right) - \tilde{D}\cos 2\delta \\
&= \tilde{A}_{\mu\mu}\cos\delta + \tilde{C}_{\mu\mu} - \tilde{D}\cos 2\delta \qquad (4.235)
\end{aligned}
$$

dimana

$$
\begin{aligned}
\tilde{A}_{\mu\mu} &= -\tilde{A}_{e\mu} - \tilde{A}_{\mu\tau} \\
&= -\sum_{(ijk)}^{siklik} \frac{-8\left[J_r \Delta_{21}\Delta_{31}\lambda_k\left(\lambda_k - \Delta_{31}\right) + \left(\tilde{A}_{e\mu}\right)_k\right]}{\tilde{\Delta}_{jk}^2 \tilde{\Delta}_{ki}^2} \\
&\quad \times \cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki} \\
&\quad - \sum_{(ijk)}^{siklik} \frac{-8\left[J_r \Delta_{21}\Delta_{31}\left(\lambda_k - A\right)\left(\lambda_k - A - \Delta_{31}\right)\left(c_{23}^2 - s_{23}^2\right) + \left(\tilde{A}_{\mu\tau}\right)_k\right]}{\tilde{\Delta}_{jk}^2 \tilde{\Delta}_{ki}^2} \\
&\quad \times \cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki}
\end{aligned}
$$

$$\begin{aligned}
\tilde{C}_{\mu\mu} &= 1-\tilde{C}_{e\mu}-\tilde{C}_{\mu\tau} \\
&= 1-\sum_{(ij)}^{siklik}\frac{-4\left[\Delta_{31}^2 s_{13}^2 s_{23}^2 c_{13}^2\lambda_i\lambda_j+\left(\tilde{C}_{e\mu}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\sin^2\tilde{\Delta}'_{ij} \\
&\quad -\sum_{(ij)}^{siklik}\frac{-4\left[\Delta_{31}^2 s_{23}^2 c_{23}^2 c_{13}^4\left(\lambda_i-A\right)\left(\lambda_j-A\right)+\left(\tilde{C}_{\mu\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \\
&\quad \times\sin^2\tilde{\Delta}'_{ij} \qquad (4.236)
\end{aligned}$$

Probabilitas survival tauon ke neutrino tauon ($P\left(\nu_\tau\to\nu_\tau\right)$) adalah

$$\begin{aligned}
P\left(\nu_\tau\to\nu_\tau\right) &= 1-P\left(\nu_e\to\nu_\tau\right)-P\left(\nu_\mu\to\nu_\tau\right) \\
&= 1-\left(-\tilde{A}_{e\mu}\cos\delta-\tilde{B}\sin\delta+\tilde{C}_{e\tau}\right)-\left(\tilde{A}_{\mu\tau}\cos\delta+\tilde{B}\sin\delta+\tilde{C}_{\mu\tau}+\tilde{D}\cos 2\delta\right) \\
&= 1-\tilde{A}_{e\tau}\cos\delta+\tilde{B}\sin\delta-\tilde{C}_{e\tau}-\tilde{A}_{\mu\tau}\cos\delta-\tilde{B}\sin\delta-\tilde{C}_{\mu\tau}-\tilde{D}\cos 2\delta \\
&= \left(-\tilde{A}_{e\tau}-\tilde{A}_{\mu\tau}\right)\cos\delta+1-\tilde{C}_{e\tau}-\tilde{C}_{\mu\tau}-\tilde{D}\cos 2\delta \\
&= \tilde{A}_{\tau\tau}\cos\delta+\tilde{C}_{\tau\tau}-\tilde{D}\cos 2\delta \qquad (4.237)
\end{aligned}$$

dimana

$$\begin{aligned}
\tilde{A}_{\tau\tau} &= -\tilde{A}_{e\tau}-\tilde{A}_{\mu\tau} \\
&= -\sum_{(ijk)}^{siklik}\frac{8\left[J_r\Delta_{21}\Delta_{31}\lambda_k\left(\lambda_k-\Delta_{31}\right)+\left(\tilde{A}_{e\mu}\right)_k\right]}{\tilde{\Delta}_{jk}^2\tilde{\Delta}_{ki}^2} \\
&\quad \times\cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki} \\
&\quad -\sum_{(ijk)}^{siklik}\frac{-8\left[J_r\Delta_{21}\Delta_{31}\left(\lambda_k-A\right)\left(\lambda_k-A-\Delta_{31}\right)\left(c_{23}^2-s_{23}^2\right)+\left(\tilde{A}_{\mu\tau}\right)_k\right]}{\tilde{\Delta}_{jk}^2\tilde{\Delta}_{ki}^2} \\
&\quad \times\cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki} \qquad (4.238)
\end{aligned}$$

$$\begin{aligned}
\tilde{C}_{\tau\tau} &= 1-\tilde{C}_{e\tau}-\tilde{C}_{\mu\tau} \\
&= 1-\sum_{(ij)}^{siklik}\frac{-4\left[\Delta_{31}^2 s_{13}^2 c_{23}^2 c_{13}\lambda_i\lambda_j+\left(\tilde{C}_{e\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\sin^2\tilde{\Delta}'_{ij} \\
&\quad -\sum_{(ij)}^{siklik}\frac{-4\left[\Delta_{31}^2 s_{23}^2 c_{23}^2 c_{13}^4\left(\lambda_i-A\right)\left(\lambda_j-A\right)+\left(\tilde{C}_{\mu\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \\
&\quad \times\sin^2\tilde{\Delta}'_{ij} \qquad (4.239)
\end{aligned}$$

Bab 5

Kesimpulan

Probabilitas osilasi neutrino dengan menggunakan matrik MNS komplek telah dilakukan , baik dalam vakum maupun materi.Diperoleh probabilitas transisi dan survival neutrino dalam vakum adalah

$$P(\nu_e \to \nu_\mu) = A_{e\mu}\cos\delta + B\sin\delta + C_{e\mu} \quad (5.1)$$

$$A_{e\mu} = \left\{4\left(c_{12}^2 - s_{12}^2\right)J_r\sin^2\left(\frac{\Delta_{12}}{4E}L\right) - 4J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right) + 4J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \quad (5.2)$$

$$B = \left\{-2J_r\left(\sin\left(\frac{\Delta_{12}}{2E}L\right) + \sin\left(\frac{\Delta_{23}}{2E}L\right) + \sin\left(\frac{\Delta_{31}}{2E}L\right)\right)\right\} \quad (5.3)$$

$$C_{e\mu} = \left\{-4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2 - c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right) + 4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right) + 4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \quad (5.4)$$

$$P(\nu_e \to \nu_\tau) = -A_{e\mu}\cos\delta - B\sin\delta + C_{e\tau} \quad (5.5)$$

$$C_{e\tau} = \left\{-4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2 - c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right) + 4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right) + 4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \quad (5.6)$$

$$P(\nu_\mu \to \nu_\tau) = A_{\mu\tau}\cos\delta + B\sin\delta + C_{\mu\tau} + D\cos 2\delta \quad (5.7)$$

$$
\begin{aligned}
A_{\mu\tau} &= \Bigg\{ +4\left(c_{12}^2 - s_{12}^2\right)\left(c_{23}^2 - s_{23}^2\right) s_{12}c_{12}s_{23}c_{23}s_{13}\left(1 + s_{13}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad +4\left(c_{23}^2 - s_{23}^2\right) J_r \sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad -4\left(c_{23}^2 - s_{23}^2\right) J_r \sin^2\left(\frac{\Delta_{31}}{4E}L\right)\Bigg\} \qquad (5.8) \\
C_{\mu\tau} &= \Bigg\{ -4s_{12}^2c_{12}^2s_{23}^2c_{23}^2\left(1 + s_{13}^2 + s_{13}^4\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad +4\left(s_{12}^2c_{12}^2 + s_{23}^2c_{23}^2\right)s_{13}^2\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad -4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2 - s_{12}^2\right)\sin^2\left(\frac{\Delta_{31}}{4E}L\right) \\
&\quad -4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2 - s_{12}^2\right)\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\Bigg\} \qquad (5.9) \\
D &= \Bigg\{ -8s_{12}^2c_{12}^2s_{23}^2c_{23}^2s_{13}^2\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\Bigg\} \qquad (5.10)
\end{aligned}
$$

$$
\begin{aligned}
P\left(\nu_e \to \nu_e\right) &= C_{ee} \\
&= 1 - C_{e\mu} - C_{e\tau} \\
&= 1 - \Bigg\{ -4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2 - c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad +4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad +4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\Bigg\} \\
&\quad - \Bigg\{ -4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2 - c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad +4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad +4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\Bigg\} \qquad (5.11)
\end{aligned}
$$

$$
\begin{aligned}
P\left(\nu_\mu \to \nu_\mu\right) &= A_{\mu\mu}\cos\delta + C_{\mu\mu} - D\cos 2\delta \\
A_{\mu\mu} &= -A_{e\mu} - A_{\mu\tau} \\
&= -\Bigg\{ 4\left(c_{12}^2 - s_{12}^2\right) J_r \sin^2\left(\frac{\Delta_{12}}{4E}L\right) - 4J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\
&\quad +4J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
& -\left\{+4\left(c_{12}^2-s_{12}^2\right)\left(c_{23}^2-s_{23}^2\right)s_{12}c_{12}s_{23}c_{23}s_{13}\left(1+s_{13}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right.\\
& +4\left(c_{23}^2-s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right)\\
& \left.-4\left(c_{23}^2-s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \qquad (5.12)\\
C_{\mu\mu} &= 1-C_{e\mu}-C_{\mu\tau}\\
&= 1-\left\{-4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2-c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right.\\
& +4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right)\\
& \left.+4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\}\\
& -\left\{-4s_{12}^2c_{12}^2s_{23}^2c_{23}^2\left(1+s_{13}^2+s_{13}^4\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right.\\
& +4\left(s_{12}^2c_{12}^2+s_{23}^2c_{23}^2\right)s_{13}^2\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\\
& -4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\\
& \left.-4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \qquad (5.13)
\end{aligned}
$$

$$
\begin{aligned}
P\left(\nu_\tau\to\nu_\tau\right) &= A_{\tau\tau}\cos\delta+C_{\tau\tau}-D\cos2\delta\\
A_{\tau\tau} &= A_{e\mu}-A_{\mu\tau}\\
&= \left\{4\left(c_{12}^2-s_{12}^2\right)J_r\sin^2\left(\frac{\Delta_{12}}{4E}L\right)-4J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right)\right.\\
& \left.+4J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\}\\
& -\left\{+4\left(c_{12}^2-s_{12}^2\right)\left(c_{23}^2-s_{23}^2\right)s_{12}c_{12}s_{23}c_{23}s_{13}\left(1+s_{13}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right.\\
& +4\left(c_{23}^2-s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right)\\
& \left.-4\left(c_{23}^2-s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\}\\
C_{\mu\mu} &= 1-C_{e\mu}-C_{\mu\tau}
\end{aligned}
$$

$$
\begin{aligned}
&= 1-\Bigg\{-4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2-c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\\
&+4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right)\\
&+4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\Bigg\}\\
&-\Bigg\{-4s_{12}^2c_{12}^2s_{23}^2c_{23}^2\left(1+s_{13}^2+s_{13}^4\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\\
&+4\left(s_{12}^2c_{12}^2+s_{23}^2c_{23}^2\right)s_{13}^2\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\\
&-4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\\
&-4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\Bigg\}
\end{aligned}
\tag{5.14}
$$

Sedangkan probabilitas transisi maupun survival neutrino dalam materi diperoleh ,

$$P\left(\nu_e\to\nu_\mu\right) = \tilde{A}_{e\mu}\cos\delta+\tilde{B}\sin\delta+\tilde{C}_{e\mu} \tag{5.15}$$

$$\tilde{A}_{e\mu} = \sum_{(ijk)}^{siklik}\frac{-8\left[J_r\Delta_{21}\Delta_{31}\lambda_k\left(\lambda_k-\Delta_{31}\right)+\left(\tilde{A}_{e\mu}\right)_k\right]}{\tilde{\Delta}_{jk}^2\tilde{\Delta}_{ki}^2} \times\cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki} \tag{5.16}$$

$$\tilde{B} = \frac{8J_r\Delta_{12}\Delta_{23}\Delta_{31}}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\cos\tilde{\Delta}'_{12}\sin\tilde{\Delta}'_{23}\sin\tilde{\Delta}'_{31} \tag{5.17}$$

$$\tilde{C}_{e\mu} = \sum_{(ij)}^{siklik}\frac{-4\left[\Delta_{31}^2s_{13}^2s_{23}^2c_{13}^2\lambda_i\lambda_j+\left(\tilde{C}_{e\mu}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\sin^2\tilde{\Delta}'_{ij} \tag{5.18}$$

$$P\left(\nu_e\to\nu_\tau\right) = -\tilde{A}_{e\mu}\cos\delta-\tilde{B}\sin\delta+\tilde{C}_{e\tau} \tag{5.19}$$

$$\tilde{C}_{\mu\tau} = \sum_{(ij)}^{siklik}\frac{-4\left[\Delta_{31}^2s_{23}^2c_{23}^2c_{13}^4\left(\lambda_i-A\right)\left(\lambda_j-A\right)+\left(\tilde{C}_{\mu\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \times\sin^2\tilde{\Delta}'_{ij} \tag{5.20}$$

$$P(\nu_\mu \to \nu_\tau) = \tilde{A}_{\mu\tau}\cos\delta + \tilde{B}\sin\delta + \tilde{C}_{\mu\tau} + \tilde{D}\cos 2\delta \tag{5.21}$$

$$\tilde{A}_{\mu\tau} = \sum_{(ijk)}^{siklik} \frac{-8\left[J_r\Delta_{21}\Delta_{31}(\lambda_k - A)(\lambda_k - A - \Delta_{31})(c_{23}^2 - s_{23}^2) + \left(\tilde{A}_{\mu\tau}\right)_k\right]}{\tilde{\Delta}_{jk}^2\tilde{\Delta}_{ki}^2} \times \cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki} \tag{5.22}$$

$$\tilde{C}_{\mu\tau} = \sum_{(ij)}^{siklik} \frac{-4\left[\Delta_{31}^2 s_{23}^2 c_{23}^2 c_{13}^4(\lambda_i - A)(\lambda_j - A) + \left(\tilde{C}_{\mu\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\sin^2\tilde{\Delta}'_{ij} \tag{5.23}$$

$$\tilde{D} = \sum_{(ijk)}^{siklik} \frac{-8\Delta_{21}^2(\lambda_k - A - \Delta_{31})^2 s_{12}^2 c_{12}^2 s_{23}^2 c_{23}^2 s_{13}^2}{\tilde{\Delta}_{jk}^2\tilde{\Delta}_{ki}^2} \times \cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki} \tag{5.24}$$

$$P(\nu_e \to \nu_e) = \tilde{C}_{ee} \tag{5.25}$$

$$\begin{aligned} &= 1 - \tilde{C}_{e\mu} - \tilde{C}_{e\tau} \\ &= 1 - \sum_{(ij)}^{siklik} \frac{-4\left[\Delta_{31}^2 s_{13}^2 s_{23}^2 c_{13}^2\lambda_i\lambda_j + \left(\tilde{C}_{e\mu}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\sin^2\tilde{\Delta}'_{ij} \\ &\quad - \sum_{(ij)}^{siklik} \frac{-4\left[\Delta_{31}^2 s_{13}^2 c_{23}^2 c_{13}\lambda_i\lambda_j + \left(\tilde{C}_{e\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\sin^2\tilde{\Delta}'_{ij} \\ &= 1 + \sum_{(ij)}^{siklik} \frac{4\left[\Delta_{31}^2 s_{13}^2 c_{13}^2\lambda_i\lambda_j + \left(\tilde{C}_{ee}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\sin^2\tilde{\delta}'_{ij} \end{aligned} \tag{5.26}$$

$$P(\nu_\mu \to \nu_\mu) = \tilde{A}_{\mu\mu}\cos\delta + \tilde{C}_{\mu\mu} - \tilde{D}\cos 2\delta$$

$$\begin{aligned} \tilde{A}_{\mu\mu} &= -\tilde{A}_{e\mu} - \tilde{A}_{\mu\tau} \\ &= -\sum_{(ijk)}^{siklik} \frac{-8\left[J_r\Delta_{21}\Delta_{31}\lambda_k(\lambda_k - \Delta_{31}) + \left(\tilde{A}_{e\mu}\right)_k\right]}{\tilde{\Delta}_{jk}^2\tilde{\Delta}_{ki}^2} \\ &\quad \times \cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki} \\ &\quad - \sum_{(ijk)}^{siklik} \frac{-8\left[J_r\Delta_{21}\Delta_{31}(\lambda_k - A)(\lambda_k - A - \Delta_{31})(c_{23}^2 - s_{23}^2) + \left(\tilde{A}_{\mu\tau}\right)_k\right]}{\tilde{\Delta}_{jk}^2\tilde{\Delta}_{ki}^2} \\ &\quad \times \cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki} \end{aligned}$$

$$\tilde{C}_{\mu\mu} = 1 - \tilde{C}_{e\mu} - \tilde{C}_{\mu\tau}$$

$$= \quad 1-\sum_{(ij)}^{siklik}\frac{-4\left[\Delta_{31}^2 s_{13}^2 s_{23}^2 c_{13}^2\lambda_i\lambda_j+\left(\tilde{C}_{e\mu}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\sin^2\tilde{\Delta}'_{ij}$$

$$-\sum_{(ij)}^{siklik}\frac{-4\left[\Delta_{31}^2 s_{23}^2 c_{23}^2 c_{13}^4\left(\lambda_i-A\right)\left(\lambda_j-A\right)+\left(\tilde{C}_{\mu\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\sin^2\tilde{\Delta}'_{ij} \quad (5.27)$$

$$P\left(\nu_\tau\to\nu_\tau\right) = \tilde{A}_{\tau\tau}\cos\delta+\tilde{C}_{\tau\tau}-\tilde{D}\cos 2\delta \quad (5.28)$$

$$\tilde{A}_{\tau\tau} = -\tilde{A}_{e\tau}-\tilde{A}_{\mu\tau}$$

$$= -\sum_{(ijk)}^{siklik}\frac{8\left[J_r\Delta_{21}\Delta_{31}\lambda_k\left(\lambda_k-\Delta_{31}\right)+\left(\tilde{A}_{e\mu}\right)_k\right]}{\tilde{\Delta}_{jk}^2\tilde{\Delta}_{ki}^2}$$

$$\times\cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki}$$

$$-\sum_{(ijk)}^{siklik}\frac{-8\left[J_r\Delta_{21}\Delta_{31}\left(\lambda_k-A\right)\left(\lambda_k-A-\Delta_{31}\right)\left(c_{23}^2-s_{23}^2\right)+\left(\tilde{A}_{\mu\tau}\right)_k\right]}{\tilde{\Delta}_{jk}^2\tilde{\Delta}_{ki}^2}$$

$$\times\cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki} \quad (5.29)$$

$$\tilde{C}_{\tau\tau} = 1-\tilde{C}_{e\tau}-\tilde{C}_{\mu\tau}$$

$$= 1-\sum_{(ij)}^{siklik}\frac{-4\left[\Delta_{31}^2 s_{13}^2 c_{23}^2 c_{13}\lambda_i\lambda_j+\left(\tilde{C}_{e\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\sin^2\tilde{\Delta}'_{ij}$$

$$-\sum_{(ij)}^{siklik}\frac{-4\left[\Delta_{31}^2 s_{23}^2 c_{23}^2 c_{13}^4\left(\lambda_i-A\right)\left(\lambda_j-A\right)+\left(\tilde{C}_{\mu\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}}\sin^2\tilde{\Delta}'_{ij} \quad (5.30)$$

terlihat bahwa probabilitas neutrino dalam vakum maupun materi dengan kerapatan yang konstan memiliki bentuk yang sama , hanya berbeda koefisien - koefisiennya saja.

Dalam kasus osilasi neutrino elektron ke neutrino elektron didapatkan fluks neutrino elektron yang diterima oleh percobaan di bumi jumlahnya sepertiga dari prediksi teoritis.

Lampiran A

Luas Segitiga Unitaritas sektor Lepton

Berikut ini adalah penjelasan gambar bagaimana mendapatkan luas segitiga pada ruang komplek yang dibentuk karena unitaritas matrik MNS pada subbab 3.2. Dari persamaan (3.12) , diperoleh

$$\sum_{i=1}^{3} U_{\alpha i} U^*_{\beta i} = \delta_{\alpha\beta} \qquad (\alpha \neq \beta) \tag{A.1}$$

$\alpha = e, \beta = \mu$

$$U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2} + U_{e3}U^*_{\mu 3} = 0 \tag{A.2}$$

misalkan

$$U_{e1}U^*_{\mu 1} = a_1, \qquad U_{e2}U^*_{\mu 2} = b_1, \qquad U_{e3}U^*_{\mu 3} = c_1 \tag{A.3}$$

maka : $a_1 + b_1 + c_1 = 0$
dengan

$$\begin{aligned} |a_1 \times b_1| &= |a_1|\,|b_1| \sin \alpha_1 \\ |b_1 \times c_1| &= |b_1|\,|c_1| \sin \beta_1 \\ |c_1 \times a_1| &= |c_1|\,|a_1| \sin \gamma_1 \end{aligned} \tag{A.4}$$

$\alpha = e, \beta = \tau$

$$U_{e1}U^*_{\tau 1} + U_{e2}U^*_{\tau 2} + U_{e3}U^*_{\tau 3} = 0 \tag{A.5}$$

misalkan

$$U_{e1}U^*_{\tau 1} = a_1, \qquad U_{e2}U^*_{\tau 2} = b_1, \qquad U_{e3}U^*_{\tau 3} = c_1 \tag{A.6}$$

maka : $a_2 + b_2 + c_2 = 0$
dengan

$$\begin{aligned} |a_2 \times b_2| &= |a_2|\,|b_2| \sin\alpha_2 \\ |b_2 \times c_2| &= |b_2|\,|c_2| \sin\beta_2 \\ |c_2 \times a_2| &= |c_2|\,|a_2| \sin\gamma_2 \end{aligned} \tag{A.7}$$

$\alpha = \mu, \beta = \tau$

$$U_{\mu 1}U^*_{\tau 1} + U_{\mu 2}U^*_{\tau 2} + U_{\mu 3}U^*_{\tau 3} = 0 \tag{A.8}$$

misalkan

$$U_{\mu 1}U^*_{\tau 1} = a_3, \qquad U_{\mu 2}U^*_{\tau 2} = b_3, \qquad U_{\mu 3}U^*_{\tau 3} = c_3 \tag{A.9}$$

maka : $a_3 + b_3 + c_3 = 0$
dengan

$$\begin{aligned} |a_3 \times b_3| &= |a_3|\,|b_3| \sin\alpha_3 \\ |b_3 \times c_3| &= |b_3|\,|c_3| \sin\beta_3 \\ |c_3 \times a_3| &= |c_3|\,|a_3| \sin\gamma_3 \end{aligned} \tag{A.10}$$

Untuk segitiga yang lainnya

$$\sum_{\alpha=e,\mu,\tau} U_{\alpha i}U^*_{\alpha j} = \delta_{ij} \qquad (i \neq j) \tag{A.11}$$

$i = 1, j = 2$

$$U_{e1}U^*_{e2} + U_{\mu 1}U^*_{\mu 2} + U_{\tau 1}U^*_{\tau 2} = 0 \tag{A.12}$$

misalkan

$$U_{e1}U^*_{e2} = a_4, \qquad U_{\mu 1}U^*_{\mu 2} = b_4, \qquad U_{\tau 1}U^*_{\tau 2} = c_4 \tag{A.13}$$

maka : $a_4 + b_4 + c_4 = 0$
dengan

$$\begin{aligned} |a_4 \times b_4| &= |a_4|\,|b_4| \sin\alpha_4 \\ |b_4 \times c_4| &= |b_4|\,|c_4| \sin\beta_4 \\ |c_4 \times a_4| &= |c_4|\,|a_4| \sin\gamma_4 \end{aligned} \tag{A.14}$$

$i = 1, j = 3$

$$U_{e1}U^*_{e3} + U_{\mu 1}U^*_{\mu 3} + U_{\tau 1}U^*_{\tau 3} = 0 \tag{A.15}$$

misalkan

$$U_{e1}U_{e3}^* = a_5, \qquad U_{\mu1}U_{\mu3}^* = b_5, \qquad U_{\tau1}U_{\tau3}^* = c_5 \tag{A.16}$$

maka : $a_5 + b_5 + c_5 = 0$
dengan

$$\begin{aligned} |a_5 \times b_5| &= |a_5|\,|b_5| \sin\alpha_5 \\ |b_5 \times c_5| &= |b_5|\,|c_5| \sin\beta_5 \\ |c_5 \times a_5| &= |c_5|\,|a_5| \sin\gamma_5 \end{aligned} \tag{A.17}$$

$i = 2, j = 3$

$$U_{e2}U_{e3}^* + U_{\mu2}U_{\mu3}^* + U_{\tau2}U_{\tau3}^* = 0 \tag{A.18}$$

misalkan

$$U_{e2}U_{e3}^* = a_6, \qquad U_{\mu2}U_{\mu3}^* = b_6, \qquad U_{\tau2}U_{\tau3}^* = c_6 \tag{A.19}$$

maka : $a_6 + b_6 + c_6 = 0$
dengan

$$\begin{aligned} |a_6 \times b_6| &= |a_6|\,|b_6| \sin\alpha_6 \\ |b_6 \times c_6| &= |b_6|\,|c_6| \sin\beta_6 \\ |c_6 \times a_6| &= |c_6|\,|a_6| \sin\gamma_6 \end{aligned} \tag{A.20}$$

Perhatikan kembali

$$a_1 + b_1 + c_1 = 0 \tag{A.21}$$
$$a_2 + b_2 + c_2 = 0 \tag{A.22}$$
$$a_3 + b_3 + c_3 = 0 \tag{A.23}$$
$$a_4 + b_4 + c_4 = 0 \tag{A.24}$$
$$a_5 + b_5 + c_5 = 0 \tag{A.25}$$
$$a_6 + b_6 + c_6 = 0 \tag{A.26}$$

Kalikan persamaan $(A.21)$ dengan c_1^* dan ambil Im , diperoleh

$$\begin{aligned} a_1c_1^* + b_1c_1^* + |c_1|^2 &= 0 \\ Im\,(a_1c_1^* + b_1c_1^*) + Im\,|c_1|^2 &= 0 \\ Im\,(a_1c_1^*) &= -Im\,(b_1c_1^*) \end{aligned} \tag{A.27}$$

dengan cara yang sama didapatkan

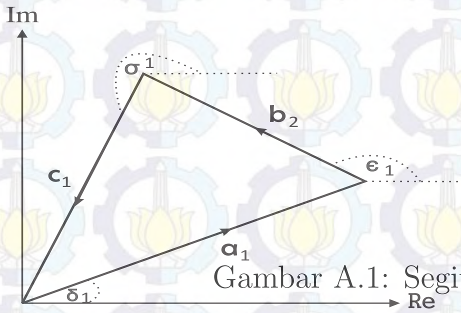
$$A.20 \times a_1^* \rightarrow Im\,(a_1^*b_1) = -Im\,(a_1^*c_1) \tag{A.28}$$
$$A.20 \times b_1^* \rightarrow Im\,(a_1b_1^*) = -Im\,(b_1^*c_1) \tag{A.29}$$

maka

$$|Im\,(a_1c_1^*)| = |Im\,(a_1b_1^*)| = |Im\,(a_1c_1^*)| \tag{A.30}$$

selanjutnya, misalkan

$$\begin{aligned} a_1 &= |a_1|\,e^{i\delta_1} && \text{(A.31)}\\ b_1 &= |b_1|\,e^{i\epsilon_1} && \text{(A.32)}\\ c_1 &= |c_1|\,e^{i\sigma_1} && \text{(A.33)} \end{aligned}$$



Gambar A.1: Segitiga 7

Sehingga

$$\begin{aligned} Im\,(a_1c_1^*) &= |a_1|\,|c_1|\,Im\left(e^{i(\delta_1-\sigma_1)}\right)\\ &= |a_1|\,|c_1|\sin(\delta_1-\sigma_1)\\ &= |a_1|\,|c_1|\sin\gamma_1\\ &= 2L_{\Delta_1} && \text{(A.34)} \end{aligned}$$

sedangkan untuk sudut yang lain , diperoleh

$$\begin{aligned} Im\,(a_1b_1^*) &= |a_1|\,|b_1|\,Im\left(e^{i(\delta_1-\epsilon_1)}\right)\\ &= |a_1|\,|b_1|\sin(\delta_1-\epsilon_1)\\ &= |a_1|\,|b_1|\sin\alpha_1\\ &= 2L_{\Delta_1} && \text{(A.35)} \end{aligned}$$

$$\begin{aligned} Im\,(b_1c_1^*) &= |b_1|\,|c_1|\,Im\left(e^{i(\epsilon_1-\sigma_1)}\right)\\ &= |b_1|\,|c_1|\sin(\epsilon_1-\sigma_1)\\ &= |b_1|\,|c_1|\sin\beta_1\\ &= 2L_{\Delta_1} && \text{(A.36)} \end{aligned}$$

maka

$$2L_{\Delta_1} = Im\,(a_1b_1^*) = Im\,(a_1c_1^*) = Im\,(b_1c_1^*) \tag{A.37}$$

Dengan cara yang sama didapatkan untuk kelima segitiga yang lain,

$$2L_{\Delta_2} = Im\,(a_2 b_2^*) = Im\,(a_2 c_2^*) = Im\,(b_2 c_2^*) \tag{A.38}$$

$$2L_{\Delta_3} = Im\,(a_3 b_3^*) = Im\,(a_3 c_3^*) = Im\,(b_3 c_3^*) \tag{A.39}$$

$$2L_{\Delta_4} = Im\,(a_4 b_4^*) = Im\,(a_4 c_4^*) = Im\,(b_4 c_4^*) \tag{A.40}$$

$$2L_{\Delta_5} = Im\,(a_5 b_5^*) = Im\,(a_5 c_5^*) = Im\,(b_5 c_5^*) \tag{A.41}$$

$$2L_{\Delta_6} = Im\,(a_6 b_6^*) = Im\,(a_6 c_6^*) = Im\,(b_6 c_6^*) \tag{A.42}$$

Perlu diketahui bahwa keenam segitiga di atas adalah sembarang karena dari pemisalan, secara umum a_1 tidak saling berkaitan dengan a_2 atau dengan sisi - sisi segitiga yang lain, sehingga luas segitiga tersebut secara umum berbeda. Terapi apabila a, b, dan c dikembalikan dalam komponen - komponen matriks U , untuk segitiga 1,

$$\begin{aligned} 2L_{\Delta_1} &= |Im\,(a_1 c_1^*)| \\ &= \left|Im\left(U_{e1}U_{\mu 1}^* U_{e3}^* U_{\mu 3}\right)\right| \end{aligned} \tag{A.43}$$

Kemudian untuk

$$\begin{aligned} 2L_{\Delta_5} &= |Im\,(a_5 b_5^*)| \\ &= \left|Im\left(U_{e1}U_{U3}^* U_{\mu 1}^* U_{\mu 3}\right)\right| \\ &= \left|Im\left(U_{e1}U_{\mu 1}^* U_{e3}^* U_{\mu 3}\right)\right| \\ &= 2L_{\Delta_1} \end{aligned} \tag{A.44}$$

ternyata didapat bahwa luas segitiga 1 sama dengan luas segitiga 5,

$$L_{\Delta_1} = L_{\Delta_5} \tag{A.45}$$

$$\begin{aligned} 2L_{\Delta_1} &= |Im\,(a_1 b_1^*)| \\ &= \left|Im\left(U_{e1}U_{\mu 1}^* U_{e2}^* U_{\mu 2}\right)\right| \end{aligned} \tag{A.46}$$

untuk segitiga 4

$$\begin{aligned} 2L_{\Delta_4} &= |Im\,(a_4 b_4^*)| \\ &= \left|Im\left(U_{e1}U_{e2}^* U_{\mu 1}^* U_{\mu 2}\right)\right| \\ &= \left|Im\left(U_{e1}U_{\mu 1}^* U_{e2}^* U_{\mu 2}\right)\right| \\ &= 2L_{\Delta_1} \end{aligned} \tag{A.47}$$

didapatkan bahwa luas segitiga 1 sama dengan luas segitiga 4,sehingga

$$L_{\Delta_1} = L_{\Delta_4} = L_{\Delta_5} \tag{A.48}$$

$$\begin{aligned} 2L_{\Delta_4} &= |Im\,(b_4 c_4^*)| \\ &= \left|Im\left(U_{\mu 1}U_{\mu 2}^* U_{\tau 1}^* U_{\tau 2}\right)\right| \end{aligned} \tag{A.49}$$

untuk segitiga 3

$$\begin{aligned} 2L_{\Delta_3} &= |Im\,(a_3 b_3^*)| \\ &= \left|Im\left(U_{\mu 1}U_{\tau 2}^* U_{\mu 2}^* U_{\tau 2}\right)\right| \\ &= \left|Im\left(U_{\mu 1}U_{\mu 2}^* U_{\tau 1}^* U_{\tau 2}\right)\right| \\ &= 2L_{\Delta_4} \end{aligned} \tag{A.50}$$

didapatkan bahwa luas segitiga 1 sama dengan luas segitiga 4,sehingga

$$L_{\Delta_3} = L_{\Delta_4} \tag{A.51}$$

$$\begin{aligned} 2L_{\Delta_1} &= |Im\,(b_1 c_1^*)| \\ &= \left|Im\left(U_{e2}U_{\mu 2}^* U_{e3}^* U_{\mu 3}\right)\right| \end{aligned} \tag{A.52}$$

untuk segitiga 6

$$\begin{aligned} 2L_{\Delta_6} &= |Im\,(a_6 b_6^*)| \\ &= \left|Im\left(U_{e2}U_{e3}^* U_{\mu 2}^* U_{\mu 3}\right)\right| \\ &= \left|Im\left(U_{e2}U_{\mu 2}^* U_{e3}^* U_{\mu 3}\right)\right| \\ &= 2L_{\Delta_1} \end{aligned} \tag{A.53}$$

didapatkan bahwa luas segitiga 1 sama dengan luas segitiga 4,sehingga

$$L_{\Delta_6} = L_{\Delta_1} \tag{A.54}$$

$$\begin{aligned} 2L_{\Delta_5} &= |(a_5 c_5^*)| \\ &= |Im\,(U_{e1}U_{e3}^* U_{\tau 1}^* U_{\tau 3})| \end{aligned} \tag{A.55}$$

untuk segitiga 2

$$\begin{aligned} 2L_{\Delta_2} &= |Im\,(a_2 c_2^*)| \\ &= |Im\,(U_{e1}U_{\tau 1}^* U_{e3}^* U_{\tau 3})| \\ &= |Im\,(U_{e1}U_{e3}^* U_{\tau 1}^* U_{\tau 3})| \\ &= 2L_{\Delta_5} \end{aligned} \tag{A.56}$$

didapatkan bahwa luas segitiga 5 sama dengan luas segitiga 2. Jadi dari uraian di atas ternyata keenam segitiga tersebut mempunyai luas yang sama ,

$$L_{\Delta_1} = L_{\Delta_2} = L_{\Delta_3} = L_{\Delta_4} = L_{\Delta_5} = L_{\Delta_6} \tag{A.57}$$

Mengingat simpangan CP terjadi jika komponen matriks bauran adalah kompleks. Berarti jika tidak terjadi simpangan CP , maka matriks bauran menjadi riel dan akibatnya luas segitiga di atas akan sama dengan nol atau segitiganya memyusut menjadi sebuah garis. Jadi untuk menguji simpangan CP maka cukup untuk mengukur tiga sudut α,β, dan γ serta melihat apakah jumlah ketiga sudut tersebut adalah 180^o. [12]

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BIODATA PENULIS

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