Rumusan Eksak Osilasi Neutrino dalam Materi Dengan Kerapatan Konstan

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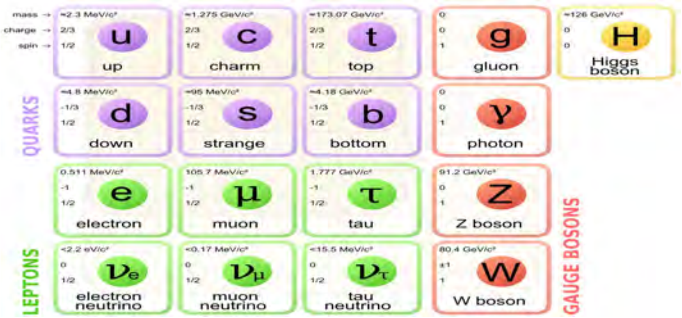
Neutrino

- Neutrino diusulkan oleh Pauli pada tahun 1930, merupakan partikel netral , berspin 1/2 , berinteraksi lemah , dan diasumsikan bermassa nol.
- Model Standar $\longrightarrow m_\nu = 0$.
- Adanya defisit flux neutrino antara percobaan dan teori (*solar neutrino problem*).
- Osilasi Neutrino $\longrightarrow \Delta m_\nu \neq 0$
- Osilasi neutrino dalam vakum dan materi (MSW effect)
- Osilasi dalam materi $\longrightarrow$ rumit $\longrightarrow$ pendekatan.

Partikel Model Standar

- Fermion (kuark dan lepton)
- Boson (pembawa gaya)



Osilasi Neutrino

- tinjau persamaan Schrodinger

$$
\begin{aligned}
i\frac{d}{dt}|\nu_\alpha\rangle &= H|\nu_\alpha\rangle \\
i\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} &= \begin{pmatrix}H_{ee} & H_{e\mu}\\ H_{\mu e} & H_{\mu\mu}\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} \qquad (1)
\end{aligned}
$$

- sisipkan $UU^\dagger = 1$ dan kalikan dengan $U^\dagger$ dari kiri

$$
\begin{aligned}
i\frac{d}{dt}|\nu_\alpha\rangle &= HUU^\dagger|\nu_\alpha\rangle \\
i\frac{d}{dt}U^\dagger|\nu_\alpha\rangle &= U^\dagger HUU^\dagger|\nu_\alpha\rangle \\
i\frac{d}{dt}|\nu'\rangle &= H_d|\nu'\rangle \qquad (2)
\end{aligned}
$$

- dimana

$$
|\nu'\rangle = U^\dagger|\nu_\alpha\rangle \qquad (3)
$$

- **Dalam Vakum**
- Kasus Dua Generasi
  - Tinjau osilasi neutrino elektron ke neutrino muon dengan fungsi gelombang flavor ν_e dan ν_μ,sedangkan keadaan eigen massanya ν_1 dan ν_2

$$\begin{aligned} \nu_e &= \cos\theta\nu_1 + \sin\theta\nu_2 \\ \nu_\mu &= -\sin\theta\nu_1 + \cos\theta\nu_2 \end{aligned} \tag{4}$$

  - Dapat dituliskan

$$|\nu_\alpha\rangle = \sum_{i=1}^{2} U_{\alpha i} |\nu_i\rangle \tag{5}$$

dan

$$\langle\nu_\alpha| = \sum_{i=1}^{2} \langle\nu_i| U^*_{\alpha i} \tag{6}$$

kebergantungan waktu

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^{2} U_{\alpha i} e^{-iE_i t} |\nu_i\rangle \tag{7}$$

- Karena neutrino merupakan partikel relativistik , maka memenuhi hubungan energi momentum relativistik

$$E_i = \sqrt{p^2 + m_i^2} \tag{8}$$

maka ekspansinya

$$\begin{aligned} E_i &= p\sqrt{1 + \frac{m_i^2}{p^2}} \\ &\cong p\left(1 + \frac{m_i^2}{2p^2} + \frac{m_i^4}{4p^4} + ...\right) \end{aligned} \tag{9}$$

karena massa neutrino sangat kecil , mka didapatkan hubungan

$$\begin{aligned} E_i &\cong p\left(1 + \frac{m_i^2}{2p^2}\right) \\ &= p + \frac{m_i^2}{2p} \end{aligned} \tag{10}$$

dengan menganggap momentum hampir sama dengan energinya , sehingga

$$E_i \cong E + \frac{m_i^2}{2E} \tag{11}$$

- Amplitudo survival neutrino elektron ke neutrino elektron adalah

$$\begin{aligned} A_{(\nu_e\to\nu_e)}(t) &= \langle \nu_e | \nu_e(t)\rangle \\ &= \sum_{j=1}^{2} \langle \nu_j | \, U_{ej}^* \left(\sum_{i=1}^{2} U_{ei} e^{-iE_i t} \, |\nu_i\rangle \right) \\ &= \sum_{i,j=1}^{2} U_{ej}^* U_{ei} e^{-iE_i t} \langle \nu_j | \nu_i \rangle \\ &= \sum_i U_{ei}^* U_{ei} e^{-iE_i t} \end{aligned} \tag{12}$$

- Probabilitas survivalnya

$$\begin{aligned} P_{\nu_e\to\nu_e}(t) &= \left| A_{(\nu_e\to\nu_e)}(t) \right|^2 \\ &= \left| \sum_i U_{ei}^* U_{ei} e^{-iE_i t} \right|^2 \\ &= \left(\sum_i U_{ei}^* U_{ei} e^{-iE_i t} \right) \left(\sum_j U_{ej} U_{ej}^* e^{iE_j t} \right) \\ &= \sum_{i,j} |U_{ei}|^2 \, |U_{ej}|^2 \, e^{-i(E_i - E_j)t} \end{aligned} \tag{13}$$

- matrik bauran 2×2

$$U_{\alpha i} = U(\theta) = \begin{pmatrix} U_{e1} & U_{e2} \\ U_{\mu 1} & U_{\mu 2} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad (14)$$

- Selanjutnya diperoleh

$$P_{\nu_e \to \nu_e}(t) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m_{12}^2}{4E} t\right) \quad (15)$$

- karena $t \approx L$, sehingga

$$P_{\nu_e \to \nu_e}(L) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m_{12}^2}{4E} L\right) \quad (16)$$

- Dengan cara yang sama , diperoleh probabilitas transisi neutrino elektron ke neutrino muon

$$P_{\nu_e \to \nu_\mu}(L) = \sin^2 2\theta \sin^2\left(\frac{\Delta m_{12}^2}{4E} L\right) \quad (17)$$

- terlihat bahwa

$$P_{\nu_e \to \nu_e}(L) + P_{\nu_e \to \nu_\mu}(L) = 1 \quad (18)$$

- terlihat bahwa kalau neutrino tidak bermassa, maka nilai probabilitas survival neutrino akan konstan , yang artinya tidak terjadi osilasi pada neutrino. Sehingga dapat dikatakan bahwa osilasi neutrino mensyaratkan neutrino bermassa dan mengalami nondegenerasi massa.

Kasus Tiga Generasi

- neutrino mempunyai tiga flavor yaitu neutrino elektron (ν_e), neutrino muon (ν_μ) , dan neutrino tauon (ν_τ).Keadaan eigen massanya dimisalkan ν_1, ν_2, dan ν_3 atau dalam bentuk matrik dapat dituliskan

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad (19)$$

- Amplitudo transisi dari ν_α ke ν_β, dimana $\alpha, \beta = e, \mu, \tau$ adalah

$$\begin{aligned} A_{\nu_\alpha \to \nu_\beta}(t) &= \langle \nu_\beta | \nu_\alpha(t) \rangle \\ &= \sum_{j=1}^{3} \langle \nu_j | \, U^*_{\beta j} \left(\sum_{i=1}^{3} U_{\alpha i} e^{-iE_i t} | \nu_i \rangle \right) \\ &= \sum_{i,j} U^*_{\beta j} U_{\alpha i} e^{-iE_i t} \langle \nu_j | \nu_i \rangle \\ &= \sum_{i} U^*_{\beta i} U_{\alpha i} e^{-iE_i t} \qquad (20) \end{aligned}$$

- sehingga probabilitas transisinya

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(t) &= \left| A_{\nu_\alpha \to \nu_\beta} \right|^2 \\ &= \langle \nu_\beta | \nu_\alpha(t) \rangle^\dagger \langle \nu_\beta | \nu_\alpha(t) \rangle \\ &= \left(\sum_j U^*_{\beta j} U_{\alpha j} e^{-iE_j t} \right)^\dagger \left(\sum_i U^*_{\beta i} U_{\alpha i} e^{-iE_i t} \right) \\ &= \sum_{i,j} U_{\beta j} U^*_{\alpha j} U^*_{\beta i} U_{\alpha_i} e^{-i(E_i - E_j)t} \end{aligned} \tag{21}$$

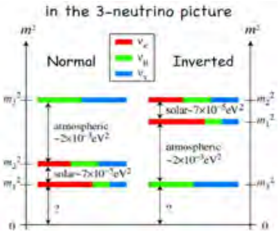
- dengan menggunakan hubungan unitaritas

$$\sum_i |U_{\alpha i}|^2 = |U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 + |U_{\alpha 3}|^2 = 1 \tag{22}$$

- maka diperoleh (dengan menggunakan hirarki normal massa neutrino)

$$\begin{aligned}
P_{\nu_\alpha \to \nu_\alpha}(L) &= 1 - 4\left|U_{\alpha 1}\right|^2 \left|U_{\alpha 2}\right|^2 \sin^2\left(\frac{\Delta m_{21}^2}{4E}L\right) \\
&\quad -4\left|U_{\alpha 1}\right|^2 \left|U_{\alpha 3}\right|^2 \sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \\
&\quad -4\left|U_{\alpha 2}\right|^2 \left|U_{\alpha 3}\right|^2 \sin^2\left(\frac{\Delta m_{32}^2}{4E}L\right) \\
&= 1 - 4\left|U_{\alpha 1}\right|^2 \left|U_{\alpha 3}\right|^2 \sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \\
&\quad -4\left|U_{\alpha 2}\right|^2 \left|U_{\alpha 3}\right|^2 \sin^2\left(\frac{\Delta m_{32}^2}{4E}L\right) \\
&= 1 - 4\left|U_{\alpha 3}\right|^2 \left(\left|U_{\alpha 1}\right|^2 + \left|U_{\alpha 2}\right|^2\right) \sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \\
&= 1 - 4\left|U_{\alpha 3}\right|^2 \left(1 - \left|U_{\alpha 3}\right|^2\right) \sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \qquad (23)
\end{aligned}$$

- hirarki massa neutrino



- dengan $|U_{\alpha 3}|^2 = \cos^2\theta$ sehingga didapatkan

$$\begin{aligned} P_{\nu_\alpha \to \nu_\alpha}(L) &= 1 - 4\cos^2\theta\left(1 - \cos^2\theta\right)\sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \\ &= 1 - 4\cos^2\theta\sin^2\theta\sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \\ &= 1 - \sin^2 2\theta\sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) \end{aligned} \quad (24)$$

- pendekatan hirarki massa $\longrightarrow$ persoalan tiga generasi tereduksi menjadi persoalan dua generasi.

- **Dalam Materi**
- Persamaan Schrodinger

$$i\frac{d\left|\psi\left(t\right)\right\rangle}{dt} = H_0\left|\psi\left(t\right)\right\rangle \quad (25)$$

- Fungsi gelombang bergantung waktu

$$\left|\psi\left(t\right)\right\rangle = \sum_{\alpha} a_{\alpha}\left(t\right)\left|\nu_{\alpha}\right\rangle \quad (26)$$

- Sehingga didapatkan amplitudo transisi neutrino yang bergantung waktu adalah

$$a\left(t\right) = Ue^{-i\frac{m^2}{2p}\left(t-t_0\right)}U^{\dagger}a\left(t_0\right) \quad (27)$$

- Hamiltonian neutrino dalam materi berbentuk

$$H = U\frac{m^2}{2p}U^{\dagger} + \sqrt{2}G_F\rho_e\beta \quad (28)$$

- diagonalisasi Hamiltonian , maka diperloleh

$$\sin 2\theta^m = \frac{\Delta m^2 \sin 2\theta}{\sqrt{\left(\Delta m^2 \cos 2\theta - A\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}} \quad (29)$$

$$\cos 2\theta^m = \frac{\Delta m^2 \cos 2\theta - A}{\sqrt{\left(\Delta m^2 \cos 2\theta - A\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}} \quad (30)$$

- Energinya

$$E_1^m = -\frac{1}{4p}\left(\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}\right) \quad (31)$$

$$E_2^m = \frac{1}{4p}\left(\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}\right) \quad (32)$$

- probabilitas osilasi neutrino elektron ke neutrino elektron dalam materi diperoleh

$$P^m_{\nu_e \to \nu_e}(t) = 1 - \frac{(\sin 2\theta)^2}{\left(\cos 2\theta + \frac{2\sqrt{2} G_F \rho_e E}{\Delta m}\right)^2 + \sin^2 2\theta} \sin^2\left\{\frac{\Delta m^2_{12}}{4p}(t - t_0)\right\} \quad (33)$$

- jika $\rho_e = 0$, maka diperoleh

$$P^m_{\nu_e \to \nu_e}(t) = 1 - \sin^2 2\theta \sin^2 \left\{ \frac{\Delta m^2_{12}}{4p} (t - t_0) \right\} \quad (34)$$

karena $(t - t_0) \approx L$ dan $p \approx E$, diperoleh

$$P^m_{\nu_e \to \nu_e}(L) = 1 - \sin^2 2\theta \sin^2 \left\{ \frac{\Delta m^2_{12}}{4E} L \right\} \quad (35)$$

- dengan cara yang sama , probabilitas transisi neutri elektron ke neutrino muon dipeoleh

$$P^m_{\nu_e \to \nu_\mu}(t) = \frac{(\sin 2\theta)^2}{\left(\cos 2\theta + \frac{2\sqrt{2} G_F \rho_e E}{\Delta m}\right)^2 + \sin^2 2\theta} \sin^2 \left\{ \frac{\Delta m^2_{12}}{4p} (t - t_0) \right\} \quad (36)$$

karena $(t - t_0) \approx L$ dan $p \approx E$, diperoleh

$$P^m_{\nu_e \to \nu_\mu}(L) = \sin^2 2\theta \sin^2 \left\{ \frac{\Delta m^2_{12}}{4E} L \right\} \quad (37)$$

- terlihat juga

$$P^m_{\nu_e \to \nu_e}(L) + P^m_{\nu_e \to \nu_\mu}(L) = 1 \quad (38)$$

Simpangan CP

- matrik U bersifat uniter, maka $U^{\dagger}U = UU^{\dagger} = 1$,sehingga terdapat 6 komponen off diagonal matrik U yang sama dengan nol

$$\sum_{i=1,2,3} U_{\alpha i} U^{*}_{\beta i} = \delta_{\alpha\beta} \qquad (\alpha \neq \beta) \tag{39}$$

secara eksplisit

$$(e,\mu) \quad \rightarrow \quad U_{e1}U^{*}_{\mu 1} + U_{e2}U^{*}_{\mu 2} + U_{e3}U^{*}_{\mu 3} = 0 \tag{40}$$

$$(e,\tau) \quad \rightarrow \quad U_{e1}U^{*}_{\tau 1} + U_{e2}U^{*}_{\tau 2} + U_{e3}U^{*}_{\tau 3} = 0 \tag{41}$$

$$(\mu,\tau) \quad \rightarrow \quad U_{\mu 1}U^{*}_{\tau 1} + U_{\mu 2}U^{*}_{\tau 2} + U_{\mu 3}U^{*}_{\tau 3} = 0 \tag{42}$$

sedangkan yang lain

$$\sum_{e,\mu,\tau} U_{\alpha i} U^{*}_{\alpha j} = \delta_{ij} \qquad (i \neq j) \tag{43}$$

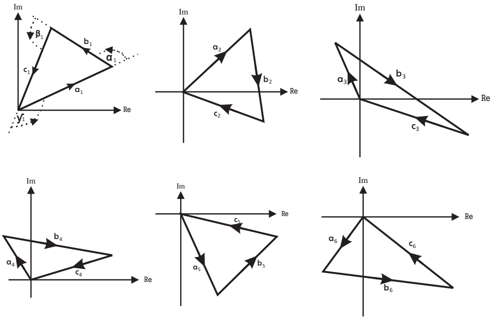
secara eksplisit

$$(\nu_1,\nu_2) \quad \rightarrow \quad U_{e1}U^{*}_{e2} + U_{\mu 1}U^{*}_{\mu 2} + U_{\tau 1}U^{*}_{\tau 2} = 0 \tag{44}$$

$$(\nu_1,\nu_3) \quad \rightarrow \quad U_{e1}U^{*}_{e3} + U_{\mu 1}U^{*}_{\mu 3} + U_{\tau 1}U^{*}_{\tau 3} = 0 \tag{45}$$

$$(\nu_2,\nu_3) \quad \rightarrow \quad U_{e2}U^{*}_{e3} + U_{\mu 2}U^{*}_{\mu 3} + U_{\tau 2}U^{*}_{\tau 3} = 0 \tag{46}$$

- **Segitiga Unitaritas untuk Sektor Lepton**



- keenam segitiga tersebut mempunyai luas yang sama.

Rumusan Eksak Osilasi Neutrino Dalam Materi Dengan Kerapatan Konstan

- **Dalam Vakum**
  - tinjau Hamiltonian

$$H = \begin{pmatrix} H_{ee} & H_{e\mu} & H_{e\tau} \\ H_{\mu e} & H_{\mu\mu} & H_{\mu\tau} \\ H_{\tau e} & H_{\tau\mu} & H_{\tau\tau} \end{pmatrix} \quad (47)$$

  - diagonalisasi Hamiltonian

$$U^\dagger HU = \begin{pmatrix} E_1 & & \\ & E_2 & \\ & & E_2 \end{pmatrix} \quad (48)$$

  - dengan $E_i = E + \frac{m_i^2}{2E}$, maka

$$\begin{aligned} U^\dagger HU &= \begin{pmatrix} E + \frac{m_1^2}{2E} & & \\ & E + \frac{m_2^2}{2E} & \\ & & E + \frac{m_3^2}{2E} \end{pmatrix} \\ &= E + \frac{1}{2E} \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} \end{aligned} \quad (49)$$

- keadaan eigen neutrino

$$\left|\nu_\alpha\right\rangle = \sum_i U^*_{\alpha i} \left|\nu_i\right\rangle \tag{50}$$

- kebergantungan terhadap waktunya

$$\left|\nu_\alpha(t)\right\rangle = \sum_i U^*_{\alpha i} e^{-iE_i t} \left|\nu_i\right\rangle \tag{51}$$

- dan

$$\left\langle\nu_\beta\right| = \sum_i \left\langle\nu_\beta\right| U_{\beta i} \tag{52}$$

- sehingga amplitudo transisinya

$$\begin{aligned} \left\langle\nu_\beta|\nu_\alpha(t)\right\rangle &= \sum_j \sum_i U^*_{\alpha i} e^{-iE_i t} U_{\beta j} \left\langle\nu_j|\nu_i\right\rangle \\ &= \sum_i U^*_{\alpha i} e^{-iE_i t} U_{\beta i} \\ &= e^{-iEt} \sum_i U^*_{\alpha i} e^{-i\frac{m_i^2}{2E}t} U_{\beta i} \\ &= e^{-iEL} \sum_i U^*_{\alpha i} e^{-i\frac{m_i^2}{2E}L} U_{\beta i} \end{aligned} \tag{53}$$

- probabilitas neutrino

$$P\left(\nu_\alpha \to \nu_\beta\right) = \left|A\left(\nu_\alpha \to \nu_\beta\right)\right|^2$$
$$= \delta_{\alpha\beta} - 4\sum_{(ij)}^{siklik} ReJ_{\alpha\beta}^{ij}\sin^2\Delta'_{ij} - 2\sum_{(ij)}^{siklik} ImJ_{\alpha\beta}^{ij}\sin 2\Delta'_{ij} \quad (54)$$

dengan

$$J_{\alpha\beta}^{ij} = U_{\alpha i}U_{\beta i}^{*}U_{\alpha j}^{*}U_{\beta j} \quad (55)$$

$$\Delta'_{ij} \equiv \frac{\Delta_{ij}L}{4E} \equiv \frac{\left(m_i^2 - m_j^2\right)L}{4E} \quad (56)$$

- didefinisikan faktor Jarlskog

$$J = ImJ_{e\mu}^{12}$$
$$= Im\left(U_{e1}U_{\mu 1}^{*}U_{e2}^{*}U_{\mu 2}\right) \quad (57)$$

- matrik MNS

$$U_{\alpha i} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (58)$$

- contohnya , kalikan persamaan (40) dengan $U_{e2}^{*}U_{\mu 2}$, ambil imaginernya , didapatkan

$$
\begin{aligned}
U_{e1}U_{\mu 1}^{*}U_{e2}^{*}U_{\mu 2} + |U_{e2}|^2 |U_{\mu 2}|^2 + U_{e3}U_{\mu 3}^{*}U_{e2}^{*}U_{\mu 2} &= 0 \\
Im\left(U_{e1}U_{\mu 1}^{*}U_{e2}^{*}U_{\mu 2}\right) + Im\left(|U_{e2}|^2 |U_{\mu 2}|^2\right) + Im\left(U_{e3}U_{\mu 3}^{*}U_{e2}^{*}U_{\mu 2}\right) &= 0
\end{aligned}
\tag{59}
$$

sehingga

$$
\begin{aligned}
Im\left(U_{e1}U_{\mu 1}^{*}U_{e2}^{*}U_{\mu 2}\right) &= -Im\left(U_{e3}U_{\mu 3}^{*}U_{e2}^{*}U_{\mu 2}\right) \\
J &= -Im\left(U_{e3}U_{\mu 3}^{*}U_{e2}^{*}U_{\mu 2}\right) \\
Im\left(U_{e3}U_{\mu 3}^{*}U_{e2}^{*}U_{\mu 2}\right) &= -J
\end{aligned}
\tag{60}
$$

- sehingga suku ketiga persamaan (54) dapat dituliskan

$$
P\left(\nu_\alpha \rightarrow \nu_\beta\right) = \delta_{\alpha\beta} - 4\sum_{(ij)}^{siklik} ReJ_{\alpha\beta}^{ij}\sin^2\Delta'_{ij} \pm 2\sum_{(ij)}^{siklik} J\sin 2\Delta'_{ij} \tag{61}
$$

Tanda $\pm$ pada suku ketiga persamaan (61), diambil $-(+)$ dalam kasus (α,β) yang merupakan permutasi (anti) siklik dari (e,μ).

- hitung $ReJ^{ij}_{\alpha\beta}$

$$
\begin{aligned}
ReJ^{12}_{e\mu} &= -\left(c_{12}^2 - s_{12}^2\right) J_r \cos\delta + s_{12}^2 c_{12}^2 c_{13}^2 \left(s_{23}^2 s_{13}^2 - c_{23}^2\right) && (62)\\
ReJ^{23}_{e\mu} &= J_r \cos\delta - s_{12}^2 s_{23}^2 s_{13}^2 c_{13}^2 && (63)\\
ReJ^{31}_{e\mu} &= -J_r \cos\delta - c_{12}^2 s_{23}^2 s_{13}^2 c_{13}^2 && (64)\\
ReJ^{12}_{e\tau} &= \left(c_{12}^2 - s_{12}^2\right) J_r \cos\delta + s_{12}^2 c_{12}^2 c_{13}^2 \left(c_{23}^2 s_{13}^2 - s_{23}^2\right) && (65)\\
ReJ^{23}_{e\mu} &= -J_r \cos\delta - s_{12}^2 c_{23}^2 s_{13}^2 c_{13}^2 && (66)\\
ReJ^{31}_{e\tau} &= J_r \cos\delta - c_{12}^2 c_{23}^2 s_{13}^2 c_{13}^2 && (67)\\
ReJ^{12}_{\mu\tau} &= -\left(c_{12}^2 - s_{12}^2\right)\left(c_{23}^2 - s_{23}^2\right) s_{12} c_{12} s_{23} c_{23} s_{13} \left(1 + s_{13}^2\right) \cos\delta \\
&\quad + 2 s_{12}^2 c_{12}^2 s_{23}^2 c_{23}^2 s_{13}^2 \cos 2\delta && (68)\\
ReJ^{23}_{\mu\tau} &= -\left(c_{23}^2 - s_{23}^2\right) J_r \cos\delta + s_{23}^2 c_{23}^2 c_{13}^2 \left(c_{12}^2 s_{13}^2 - s_{12}^2\right) && (69)\\
ReJ^{31}_{\mu\tau} &= \left(c_{23}^2 - s_{23}^2\right) J_r \cos\delta + s_{23}^2 c_{23}^2 c_{13}^2 \left(c_{12}^2 s_{13}^2 - s_{12}^2\right) && (70)
\end{aligned}
$$

- probabilitas transisi neutrino elektron ke neutrino muon

$$P\left(\nu_e \to \nu_\mu\right) = A_{e\mu} \cos\delta + B \sin\delta + C_{e\mu} \quad (71)$$

. dengan $A_{e\mu}$, B, dan $C_{e\mu}$ adalah konstanta.

$$A_{e\mu} = \left\{4\left(c_{12}^2 - s_{12}^2\right) J_r \sin^2\left(\frac{\Delta_{12}}{4E}L\right) - 4J_r \sin^2\left(\frac{\Delta_{23}}{4E}L\right) + 4J_r \sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \quad (72)$$

$$B = \left\{-2J_r\left(\sin\left(\frac{\Delta_{12}}{2E}L\right) + \sin\left(\frac{\Delta_{23}}{2E}L\right) + \sin\left(\frac{\Delta_{31}}{2E}L\right)\right)\right\} \quad (73)$$

$$C_{e\mu} = \left\{-4s_{12}^2 c_{12}^2 c_{13}^2\left(s_{13}^2 s_{23}^2 - c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right) + 4s_{12}^2 s_{13}^2 s_{23}^2 c_{13}^2 \sin^2\left(\frac{\Delta_{23}}{4E}L\right) + 4c_{12}^2 s_{13}^2 c_{13}^2 s_{23}^2 \sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \quad (74)$$

- dengan cara yang sama didapatkan

$$P(\nu_e \to \nu_\tau) = -A_{e\mu}\cos\delta - B\sin\delta + C_{e\tau} \quad (75)$$

$$\begin{aligned} C_{e\tau} = & \left\{ -4s_{12}^2c_{12}^2c_{13}^2\left(s_{13}^2s_{23}^2 - c_{23}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right. \\ & +4s_{12}^2s_{13}^2s_{23}^2c_{13}^2\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\ & \left. +4c_{12}^2s_{13}^2c_{13}^2s_{23}^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \end{aligned} \quad (76)$$

$$P(\nu_\mu \to \nu_\tau) = A_{\mu\tau}\cos\delta + B\sin\delta + C_{\mu\tau} + D\cos 2\delta \quad (77)$$

$$\begin{aligned} A_{\mu\tau} = & \left\{ +4\left(c_{12}^2 - s_{12}^2\right)\left(c_{23}^2 - s_{23}^2\right)s_{12}c_{12}s_{23}c_{23}s_{13}\left(1 + s_{13}^2\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right. \\ & +4\left(c_{23}^2 - s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{23}}{4E}L\right) \\ & \left. -4\left(c_{23}^2 - s_{23}^2\right)J_r\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \end{aligned} \quad (78)$$

$$
\begin{aligned}
C_{\mu\tau} &= \left\{-4s_{12}^2c_{12}^2s_{23}^2c_{23}^2\left(1+s_{13}^2+s_{13}^4\right)\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right. \\
&\quad +4\left(s_{12}^2c_{12}^2+s_{23}^2c_{23}^2\right)s_{13}^2\sin^2\left(\frac{\Delta_{12}}{4E}L\right) \\
&\quad -4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\sin^2\left(\frac{\Delta_{31}}{4E}L\right) \\
&\quad \left.-4s_{23}^2c_{23}^2c_{13}^2\left(c_{12}^2s_{13}^2-s_{12}^2\right)\sin^2\left(\frac{\Delta_{31}}{4E}L\right)\right\} \qquad (79) \\
D &= \left\{-8s_{12}^2c_{12}^2s_{23}^2c_{23}^2s_{13}^2\sin^2\left(\frac{\Delta_{12}}{4E}L\right)\right\} \qquad (80)
\end{aligned}
$$

- probabilitas survival neutrino ,dihitung dengan menggunakan sifat unitaritas dan probabilitas transisi , didapatkan

$$
\begin{aligned}
P(\nu_e \to \nu_e) &= 1 - P(\nu_e \to \nu_\mu) - P(\nu_e \to \nu_\tau) \\
&= 1 - A_{e\mu}\cos\delta - B\sin\delta - C_{e\mu} + A_{e\mu}\cos\delta + B\sin\delta - C_{e\tau} \\
&= 1 - C_{e\mu} - C_{e\tau} \\
&\equiv C_{ee} \qquad (81) \\
P(\nu_\mu \to \nu_\mu) &= 1 - P(\nu_\mu \to \nu_e) - P(\nu_\mu \to \nu_\tau) \\
&= 1 - A_{e\mu}\cos\delta + B\sin\delta - C_{e\mu} - A_{\mu\tau}\cos\delta \\
&\quad - B\sin\delta - C_{\mu\tau} - D\cos 2\delta \\
&= (-A_{e\mu} - A_{\mu\tau})\cos\delta + 1 - C_{e\mu} - C_{e\tau} - D\cos 2\delta \\
&= A_{\mu\mu}\cos\delta + C_{\mu\mu} - D\cos 2\delta \qquad (82) \\
P(\nu_\tau \to \nu_\tau) &= 1 - P(\nu_e \to \nu_\tau) - P(\nu_\mu \to \nu_\tau) \\
&= 1 + A_{e\mu}\cos\delta + B\sin\delta - C_{e\tau} - A_{\mu\tau}\cos\delta \\
&\quad - B\sin\delta - C_{\mu\tau} - D\cos 2\delta \\
&= (A_{e\mu} - A_{\mu\tau})\cos\delta + 1 - C_{e\tau} - C_{\mu\tau} - D\cos 2\delta \\
&= A_{\tau\tau}\cos\delta + C_{\tau\tau} - D\cos 2\delta \qquad (83)
\end{aligned}
$$

Dalam Materi

- tinjau Hamiltonian dalam materi

$$\tilde{H} = \begin{pmatrix} \tilde{H}_{ee} & \tilde{H}_{e\mu} & \tilde{H}_{e\tau} \\ \tilde{H}_{\mu e} & \tilde{H}_{\mu\mu} & \tilde{H}_{\mu\tau} \\ \tilde{H}_{\tau e} & \tilde{H}_{\tau\mu} & \tilde{H}_{\tau\tau} \end{pmatrix} = H + \frac{1}{2E}\begin{pmatrix} A & & \\ & 0 & \\ & & 0 \end{pmatrix} \quad (84)$$

dengan : $A = 2\sqrt{2}G_F\rho_e E$ adalah potensial dalam materi.

- diagonalisasi Hamiltonian

$$\tilde{U}^\dagger\tilde{H}\tilde{U} = \begin{pmatrix} E_1 & & \\ & E_1 & \\ & & E_2 \end{pmatrix} = \begin{pmatrix} E+\frac{m_1^2}{2E} & & \\ & E+\frac{m_2^2}{2E} & \\ & & E+\frac{m_3^2}{2E} \end{pmatrix}$$
$$= E + \frac{1}{2E}\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}$$
$$= E + \frac{1}{2E}\begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix} \quad (85)$$

- keadaan eigen neutrino dalam materi

$$|\nu_\alpha\rangle = \sum_i \tilde{U}^*_{\alpha i} |\nu_i\rangle \tag{86}$$

$$|\nu_\alpha(t)\rangle = \sum_i \tilde{U}^*_{\alpha i} e^{-iE_i t} |\nu_i\rangle \tag{87}$$

$$\langle\nu_\beta| = \sum_i \langle\nu_\beta| \tilde{U}_{\beta i} \tag{88}$$

- amplitudo transisi

$$\begin{aligned}
\langle\nu_\beta|\nu_\beta(t)\rangle &= \sum_j \sum_i \tilde{U}^*_{\alpha i} e^{-iE_i t} \tilde{U}_{\beta_j} \langle\nu_j|\nu_i\rangle \\
&= \sum_i \tilde{U}^*_{\alpha i} e^{-iE_i t} \tilde{U}_{\beta i} \\
&= \sum_i \tilde{U}^*_{\alpha i} e^{-i\left(E+\frac{m_i^2}{2E}\right)t} \tilde{U}_{\beta i} \\
&= e^{-iEt} \sum_i \tilde{U}^*_{\alpha i} e^{-i\frac{\lambda_i}{2E}t} \tilde{U}_{\beta i} \\
&= e^{-iEL} \sum_i \tilde{U}^*_{\alpha i} e^{-i\frac{\lambda_i}{2E}L} \tilde{U}_{\beta i}
\end{aligned} \tag{89}$$

- probabilitas transisi neutrino dalam materi

$$P\left(\nu_\alpha \to \nu_\beta\right) = \delta_{\alpha\beta} - 4\sum_{(ij)}^{siklik} Re\tilde{J}^{ij}_{\alpha\beta} \sin^2 \tilde{\Delta}'_{ij} \pm 2\sum_{(ij)}^{siklik} \tilde{J} \sin 2\tilde{\Delta}_{ij} \tag{90}$$

- didefinisikan $\tilde{p}_{\alpha\beta}$ dan $\tilde{q}_{\alpha\beta}$ adalah

$$\tilde{p}_{\alpha\beta} = 2E\tilde{H}_{\alpha\beta} \tag{91}$$

$$\tilde{q}_{\alpha\beta} = (2E)^2\,\tilde{\mathcal{H}}_{\alpha\beta} = (2E)^2\left(\tilde{H}_{\gamma\beta}\tilde{H}_{\alpha\gamma} - \tilde{H}_{\alpha\beta}\tilde{H}_{\gamma\gamma}\right) \tag{92}$$

dimana $(\alpha\beta\gamma) = (e\mu\tau)\,,(\mu\tau e)\,,(\tau e\mu)$.

- sehingga
  1. hubungan unitaritas

$$\begin{aligned}\sum_i \tilde{U}_{\alpha i}\tilde{U}^*_{\beta i} &= \delta_{\alpha\beta}\\ \tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1}+\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \delta_{\alpha\beta}\\ \tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1} &= \delta_{\alpha\beta}-\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}-\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} \qquad (93)\end{aligned}$$

  2. hubungan kedua $\tilde{H}=\tilde{U}\mathrm{diag}(\lambda_1,\lambda_2,\lambda_3)\,\tilde{\mathrm{U}}^\dagger$

$$\begin{aligned}\sum_i \lambda_i\tilde{U}_{\alpha i}\tilde{U}^*_{\beta i} &= \tilde{p}_{\alpha\beta}\\ \lambda_1\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1}+\lambda_2\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\lambda_3\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{p}_{\alpha\beta} \qquad (94)\end{aligned}$$

  3. hubungan ketiga

$$\begin{aligned}(2E)^2\mathcal{H} &= (2E)^2\tilde{H}^{-1}\left(\mathrm{det}\tilde{\mathrm{H}}\right)\\ &= \tilde{U}\mathrm{diag}\left(\frac{1}{\lambda_1},\frac{1}{\lambda_2},\frac{1}{\lambda_3}\right)\tilde{\mathrm{U}}^\dagger\times\lambda_1\lambda_2\lambda_3 \qquad (95)\end{aligned}$$

$$\begin{aligned}\sum_{ijk}\lambda_j\lambda_k\tilde{U}_{\alpha i}\tilde{U}^*_{\beta i} &= \tilde{q}_{\alpha\beta}\\ \lambda_2\lambda_3\tilde{U}_{\alpha 1}\tilde{U}^*_{\beta 1}+\lambda_3\lambda_1\tilde{U}_{\alpha 2}\tilde{U}^*_{\beta 2}+\lambda_1\lambda_2\tilde{U}_{\alpha 3}\tilde{U}^*_{\beta 3} &= \tilde{q}_{\alpha\beta} \qquad (96)\end{aligned}$$

- dari hubungan - hubungan tersebut didapatkan

$$\mathrm{Re}\tilde{J}^{ij}_{\alpha\beta} = \frac{\left|\tilde{p}_{\alpha\beta}\right|^2 \lambda_i \lambda_j + \left|\tilde{q}_{\alpha\beta}\right|^2 + \mathrm{Re}\left(\tilde{p}_{\alpha\beta}\tilde{q}^*_{\alpha\beta}\right)(\lambda_i + \lambda_j)}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \quad (97)$$

- dan

$$\begin{aligned}
\mathrm{Im}\left(\tilde{U}_{e1}\tilde{U}^*_{\mu 1}\tilde{U}^*_{e2}\tilde{U}_{\mu 2}\right) &= \frac{\mathrm{Im}\left(\tilde{p}_{e\mu}\tilde{q}^*_{e\mu}\right)\lambda_1 + \mathrm{Im}\left(\tilde{p}^*_{e\mu}\tilde{q}_{e\mu}\right)\lambda_2}{\tilde{\Delta}_{21}\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{32}} \\
&= \frac{\mathrm{Im}\left(\tilde{p}_{e\mu}\tilde{q}^*_{e\mu}\right)\tilde{\Delta}_{12}}{\tilde{\Delta}_{21}\tilde{\Delta}_{31}\tilde{\Delta}_{12}\tilde{\Delta}_{32}} \\
\tilde{J} &= \frac{\mathrm{Im}\left(\tilde{p}_{e\mu}\tilde{q}^*_{e\mu}\right)}{\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \quad (98)
\end{aligned}$$

- probabilitas transisi osilasi neutrino elektron ke neutrino muon

$$P\left(\nu_e \to \nu_\mu\right) = \tilde{A}_{e\mu} \cos\delta + \tilde{B} \sin\delta + \tilde{C}_{e\mu} \tag{99}$$

$$\begin{aligned} \tilde{A}_{e\mu} &= \sum_{(ijk)}^{siklik} \frac{-8\left[J_r \Delta_{21} \Delta_{31} \lambda_k \left(\lambda_k - \Delta_{31}\right) + \left(\tilde{A}_{e\mu}\right)_k\right]}{\tilde{\Delta}_{jk}^2 \tilde{\Delta}_{ki}^2} \\ &\quad \times \cos\tilde{\Delta}'_{ij} \sin\tilde{\Delta}'_{jk} \sin\tilde{\Delta}'_{ki} \end{aligned} \tag{100}$$

$$\tilde{B} = \frac{8 J_r \Delta_{12} \Delta_{23} \Delta_{31}}{\tilde{\Delta}_{12} \tilde{\Delta}_{23} \tilde{\Delta}_{31}} \cos\tilde{\Delta}'_{12} \sin\tilde{\Delta}'_{23} \sin\tilde{\Delta}'_{31} \tag{101}$$

$$\tilde{C}_{e\mu} = \sum_{(ij)}^{siklik} \frac{-4\left[\Delta_{31}^2 s_{13}^2 s_{23}^2 c_{13}^2 \lambda_i \lambda_j + \left(\tilde{C}_{e\mu}\right)_{ij}\right]}{\tilde{\Delta}_{ij} \tilde{\Delta}_{12} \tilde{\Delta}_{23} \tilde{\Delta}_{31}} \sin^2\tilde{\Delta}'_{ij} \tag{102}$$

- probabilitas transisi neutrino elektron ke neutrino tau

$$P(\nu_e \to \nu_\tau) = -\tilde{A}_{e\mu}\cos\delta - \tilde{B}\sin\delta + \tilde{C}_{e\tau} \tag{103}$$

$$\tilde{C}_{e\tau} = \sum_{(ij)}^{siklik} \frac{-4\left[\Delta_{31}^2 s_{13}^2 c_{23}^2 c_{13}\lambda_i\lambda_j + \left(\tilde{C}_{e\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \sin^2\tilde{\Delta}'_{ij} \tag{104}$$

- probabilitas transisi neutrino muon elektron ke neutrino tau

$$P(\nu_\mu \to \nu_\tau) = \tilde{A}_{\mu\tau}\cos\delta + \tilde{B}\sin\delta + \tilde{C}_{\mu\tau} + \tilde{D}\cos 2\delta \tag{105}$$

$$\begin{aligned}\tilde{A}_{\mu\tau} &= \sum_{(ijk)}^{siklik} \frac{-8\left[J_r\Delta_{21}\Delta_{31}(\lambda_k - A)(\lambda_k - A - \Delta_{31})(c_{23}^2 - s_{23}^2) + \left(\tilde{A}_{\mu\tau}\right)_k\right]}{\tilde{\Delta}_{jk}^2\tilde{\Delta}_{ki}^2} \\ &\quad \times \cos\tilde{\Delta}'_{ij}\sin\tilde{\Delta}'_{jk}\sin\tilde{\Delta}'_{ki}\end{aligned} \tag{106}$$

$$\tilde{C}_{\mu\tau} = \sum_{(ij)}^{siklik} \frac{-4\left[\Delta_{31}^2 s_{23}^2 c_{23}^2 c_{13}^4(\lambda_i - A)(\lambda_j - A) + \left(\tilde{C}_{\mu\tau}\right)_{ij}\right]}{\tilde{\Delta}_{ij}\tilde{\Delta}_{12}\tilde{\Delta}_{23}\tilde{\Delta}_{31}} \sin^2\tilde{\Delta}'_{ij} \tag{107}$$

$$\tilde{D} = \sum_{(ijk)}^{siklik} \frac{-8\Delta_{21}^2\left(\lambda_k - A - \Delta_{31}\right)^2 s_{12}^2 c_{12}^2 s_{23}^2 c_{23}^2 s_{13}^2}{\tilde{\Delta}_{jk}^2 \tilde{\Delta}_{ki}^2} \cos\tilde{\Delta}'_{ij} \sin\tilde{\Delta}'_{jk} \sin\tilde{\Delta}'_{ki} \tag{108}$$

- probabilitas survival neutrino dalam materi , dengan mengunakan unitaritas dan probabilitas transisi , didapatkan

$$\begin{aligned} P\left(\nu_e \to \nu_e\right) &= 1 - P\left(\nu_e \to \nu_\mu\right) - P\left(\nu_e \to \nu_\tau\right) \\ &= 1 - \left(\tilde{A}_{e\mu}\cos\delta + \tilde{B}\sin\delta + \tilde{C}_{e\mu}\right) \\ &\quad - \left(-\tilde{A}_{e\mu}\cos\delta - \tilde{B}\sin\delta + \tilde{C}_{e\tau}\right) \\ &= 1 - \tilde{A}_{e\mu}\cos\delta - \tilde{B}\sin\delta - \tilde{C}_{e\mu} \\ &\quad + \tilde{A}_{e\mu}\cos\delta + \tilde{B}\sin\delta - \tilde{C}_{e\tau} \\ &= 1 - \tilde{C}_{e\mu} - \tilde{C}_{e\tau} \\ &\equiv \tilde{C}_{ee} \end{aligned} \tag{109}$$

$$
\begin{aligned}
P\left(\nu_\mu \rightarrow \nu_\mu\right) &= 1 - P\left(\nu_\mu \rightarrow \nu_e\right) - P\left(\nu_\mu \rightarrow \nu_\tau\right) \\
&= 1 - \left(\tilde{A}_{e\mu} \cos\delta - \tilde{B} \sin\delta + \tilde{C}_{e\mu}\right) \\
&\quad - \left(\tilde{A}_{\mu\tau} \cos\delta + \tilde{B} \sin\delta + \tilde{C}_{\mu\tau} + \tilde{D} \cos 2\delta\right) \\
&= 1 - \tilde{A}_{e\mu} \cos\delta + \tilde{B} \sin\delta - \tilde{C}_{e\mu} \\
&\quad - \tilde{A}_{\mu\tau} \cos\delta - \tilde{B} \sin\delta - \tilde{C}_{\mu\tau} - \tilde{D} \cos 2\delta \\
&= \left(-\tilde{A}_{e\mu} - \tilde{A}_{\mu\tau}\right) \cos\delta + \left(1 - \tilde{C}_{e\mu} - \tilde{C}_{\mu\tau}\right) - \tilde{D} \cos 2\delta \\
&= \tilde{A}_{\mu\mu} \cos\delta + \tilde{C}_{\mu\mu} - \tilde{D} \cos 2\delta \qquad (110)
\end{aligned}
$$

$$
\begin{aligned}
P\left(\nu_\tau \rightarrow \nu_\tau\right) &= 1 - P\left(\nu_e \rightarrow \nu_\tau\right) - P\left(\nu_\mu \rightarrow \nu_\tau\right) \\
&= 1 - \left(-\tilde{A}_{e\mu} \cos\delta - \tilde{B} \sin\delta + \tilde{C}_{e\tau}\right) \\
&\quad - \left(\tilde{A}_{\mu\tau} \cos\delta + \tilde{B} \sin\delta + \tilde{C}_{\mu\tau} + \tilde{D} \cos 2\delta\right) \\
&= 1 - \tilde{A}_{e\tau} \cos\delta + \tilde{B} \sin\delta - \tilde{C}_{e\tau} - \tilde{A}_{\mu\tau} \cos\delta \\
&\quad - \tilde{B} \sin\delta - \tilde{C}_{\mu\tau} - \tilde{D} \cos 2\delta \\
&= \left(-\tilde{A}_{e\tau} - \tilde{A}_{\mu\tau}\right) \cos\delta + \left(1 - \tilde{C}_{e\tau} - \tilde{C}_{\mu\tau}\right) - \tilde{D} \cos 2\delta \\
&= \tilde{A}_{\tau\tau} \cos\delta + \tilde{C}_{\tau\tau} - \tilde{D} \cos 2\delta \qquad (111)
\end{aligned}
$$

KESIMPULAN

Probabilitas osilasi neutrino dengan menggunakan matrik MNS kompleks telah dilakukan , baik dalam vakum maupun materi. Diperoleh probabilitas transisi dan survival neutrino dalam vakum adalah

$$
\begin{aligned}
P\left(\nu_e \rightarrow \nu_\mu\right) &= A_{e\mu}\cos\delta + B\sin\delta + C_{e\mu} \\
P\left(\nu_e \rightarrow \nu_\tau\right) &= -A_{e\mu}\cos\delta - B\sin\delta + C_{e\tau} \\
P\left(\nu_\mu \rightarrow \nu_\tau\right) &= A_{\mu\tau}\cos\delta + B\sin\delta + C_{\mu\tau} + D\cos 2\delta \\
P\left(\nu_e \rightarrow \nu_e\right) &= C_{ee} \\
P\left(\nu_\mu \rightarrow \nu_\mu\right) &= A_{\mu\mu}\cos\delta + C_{\mu\mu} - D\cos 2\delta \\
P\left(\nu_\tau \rightarrow \nu_\tau\right) &= A_{\tau\tau}\cos\delta + C_{\tau\tau} - D\cos 2\delta
\end{aligned}
$$

Sedangkan probabilitas transisi maupun survival neutrino dalam materi diperoleh ,

$$
\begin{aligned}
P\left(\nu_e \rightarrow \nu_\mu\right) &= \tilde{A}_{e\mu} \cos\delta + \tilde{B} \sin\delta + \tilde{C}_{e\mu} \\
P\left(\nu_e \rightarrow \nu_\tau\right) &= -\tilde{A}_{e\mu} \cos\delta - \tilde{B} \sin\delta + \tilde{C}_{e\tau} \\
P\left(\nu_\mu \rightarrow \nu_\tau\right) &= \tilde{A}_{\mu\tau} \cos\delta + \tilde{B} \sin\delta + \tilde{C}_{\mu\tau} + \tilde{D} \cos 2\delta \\
P\left(\nu_e \rightarrow \nu_e\right) &= \tilde{C}_{ee} \\
P\left(\nu_\mu \rightarrow \nu_\mu\right) &= \tilde{A}_{\mu\mu} \cos\delta + \tilde{C}_{\mu\mu} - \tilde{D} \cos 2\delta \\
P\left(\nu_\tau \rightarrow \nu_\tau\right) &= \tilde{A}_{\tau\tau} \cos\delta + \tilde{C}_{\tau\tau} - \tilde{D} \cos 2\delta
\end{aligned}
$$

terlihat bahwa probabilitas neutrino dalam vakum maupun materi memiliki bentuk yang sama , hanya dengan memodifikasi koefisien - koefisiennya saja.

TERIMA KASIH

LaFTiFA

the beauty of mind , the art of thinking