TESIS – KS185411

PENAKSIRAN PARAMETER DAN PENGUJIAN HIPOTESIS *GEOGRAPHICALLY WEIGHTED BIVARIATE ZERO INFLATED GENERALIZED POISSON REGRESSION*

(Studi Kasus : Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan Provinsi Jawa Tengah Tahun 2017)

QUROTUL AINI
06211850017001

Dosen Pembimbing
Dr. Purhadi, M.Sc.
Irhamah, M.Si, Ph.D.

Departemen Statistika
Fakultas Sains dan Analitika Data
Institut Teknologi Sepuluh Nopember
2020

TESIS – KS185411

PENAKSIRAN PARAMETER DAN PENGUJIAN HIPOTESIS *GEOGRAPHICALLY WEIGHTED BIVARIATE ZERO INFLATED GENERALIZED POISSON REGRESSION*

(Studi Kasus : Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan Provinsi Jawa Tengah Tahun 2017)

QUROTUL AINI
06211850017001

Dosen Pembimbing
Dr. Purhadi, M.Sc
Irhamah, S.Si, M.Si, Ph.D

Departemen Statistika
Fakultas Sains dan Analitika Data
Institut Teknologi Sepuluh Nopember
2020

LEMBAR PENGESAHAN TESIS

Tesis ini disusun untuk memenuhi salah satu syarat memperoleh gelar
Magister Statistika (M.Stat)
di
Institut Teknologi Sepuluh Nopember
Oleh:
QUROTUL AINI
NRP. 06211850017001

Tanggal Ujian : 31 Januari 2020
Periode Wisuda : Maret 2020

Disetujui oleh:
Pembimbing:

1. Dr. Purhadi, M.Sc.
   NIP. 19620204 198701 1 001 ....

2. Irhamah, M.Si, Ph.D.
   NIP. 19780406 200112 2 002 ....

Penguji:

1. Dr. Sutikno, M.Si.
   NIP. 19710313 199702 1 001 ....

2. Dr. Titi Kanti Lestari, SE, M.Com.
   NIP. 19611231 198312 2 001 ....

KEMENTERIAN PENDIDIKAN DAN KEBUDAYAAN INSTITUT TEKNOLOGI SEPULUH NOPEMBER FAKULTAS SAINS DAN ANALITIKA DATA DEPARTEMEN STATISTIKA

Kepala Departemen Statistika
Fakultas Sains dan Analitika Data

Dr. Kartika Fithriasari, M.Si.
NIP. 19691212 199303 2 002

PENAKSIRAN PARAMETER DAN PENGUJIAN HIPOTESIS GEOGRAPHICALLY WEIGHTED BIVARIATE ZERO INFLATED GENERALIZED POISSON REGRESSION

(Studi Kasus : Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan Provinsi Jawa Tengah Tahun 2017)

Oleh	: Qurotul Aini
Pembimbing	: Dr. Purhadi, M.Sc.
Co-Pembimbing	: Irhamah, S.Si, M.Si, Ph.D

ABSTRAK

Salah satu metode statistika yang telah dikembangkan untuk mengatasi under/overdispersi serta *excess zero* adalah *Zero Inflated Generalized Poisson Regression* (ZIGPR*)*, apabila terdapat dua variabel respon maka analisis regresi yang sesuai adalah *Bivariate ZIGPR* (BZIGPR). Pengembangan ZIGPR dengan memperhitungkan faktor spasial berupa koordinat lintang dan bujur adalah *Geographically Weighted Zero Inflated Generalized Poisson Regression* (GWBZIGPR) yang menghasilkan penaksir parameter yang bersifat lokal untuk setiap lokasi pengamatan. Penelitian ini membahas mengenai penaksiran parameter dan pengujian hipotesis model BZIGPR dan GWBZIGPR. Aplikasi model diterapkan pada data Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan Provinsi Jawa Tengah Tahun 2017. Hasil penelitian menunjukkan bahwa penaksiran parameter menggunakan metode *Maximum Likelihood Estimation* (MLE) diperoleh persamaan yang tidak *close form* sehingga dilanjutkan dengan iterasi numerik *Berndt Hall Hall Hausman* (BHHH). Pengujian parameter secara serentak menggunakan metode *Maximum Likelihood Ratio Test* (MLRT). nilai *Akaike Information Criterion Corrected* (AICc) pada model GWBZIGPR lebih kecil dibanding dengan BZIGPR sehingga dapat diartikan model GWBZIGPR lebih baik dibandingkan dengan model BZIGPR untuk memodelkan Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan Provinsi Jawa Tengah Tahun 2017. Pemodelan menggunakan GWBZIGPR pada model *zero state* menghasilkan 6 kelompok kecamatan berdasarkan kesamaan variabel yang signifikan terhadap Jumlah Kematian Ibu Hamil dan 8 kelompok kecamatan berdasarkan kesamaan variabel yang signifikan terhadap Jumlah Kematian Ibu Nifas. Sedangkan Pemodelan menggunakan GWBZIGPR pada model *poisson state* menghasilkan 6 kelompok kecamatan berdasarkan kesamaan variabel yang signifikan terhadap Jumlah Kematian Ibu Hamil dan 6 kelompok kecamatan berdasarkan kesamaan variabel yang signifikan terhadap Jumlah Kematian Ibu Nifas.

Kata kunci: *BZIGPR, GWBZIGPR, MLE, MLRT,* Angka Kematian Ibu.

(Halaman ini sengaja dikosongkan)

PARAMETERS ESTIMATION AND HYPOTHESIS TESTING OF GEOGRAPHICALLY WEIGHTED BIVARIATE ZERO INFLATED GENERALIZED POISSON REGRESSION

(Case study : The number of pregnant maternal mortality and postpartum maternal mortality in Pekalongan Residency Central Java Province 2017)

Name	: Qurotul Aini
Supervisor	: Dr. Purhadi, M.Sc.
Co- supervisor	: Irhamah, S.Si, M.Si, Ph.D

ABSTRACT

One of the statistical methods that have been developed and resolve under/overdispersion also excess zero is Zero Inflated Generalized Poisson Regression (ZIGPR), if there are two response variables then the corresponding regression analysis is Bivariate ZIGPR (BZIGPR). The development of ZIGPR by taking into account the spatial factor of latitude and longitude coordinates is Geographically Weighted Zero Inflated Generalized Poisson Regression (GWBZIGPR) which generates a parameter estimator of local nature for each observation site. The research discusses the parameter assessment and the BZIGPR and GWBZIGPR model hypothesis testing. Model application applied to the data of the total pregnant maternal mortality and the total postpartum maternal mortality in the Pekalongan residency of the Central Java province in 2017. The results showed that parameter assessment using the Maximum Likelihood Estimation (MLE) method obtained an equation that is not close form so that it is followed by the numerical iteration of Berndt Hall Hall Hausman (BHHH). Simultaneous testing of parameters using the Maximum Likelihood Ratio Test (MLRT) method. The value of Akaike Information Criterion Corrected (AICc) on the GWBZIGPR model is smaller than BZIGPR so that the GWBZIGPR model can be interpreted better than BZIGPR model to model the total pregnant maternal mortality and the total postpartum maternal mortality in the Pekalongan residency of the Central Java province in 2017. Modeling using GWBZIGPR on the zero-state model resulted in 6 sub-districts groups based on a significant variable similarity to pregnant maternal mortality and 8 sub-districts groups based on significant variable similarities to the total postpartum maternal mortality. While modelling using GWBZIGPR on Poisson state models resulted in 6 sub-districts groups based on a significant variable similarity to the total mortality of pregnant women and 6 sub-districts groups based on a significant variable similarity to the total postpartum maternal mortality.

Key words: BZIGPR, GWBZIGPR, MLE, MLRT, Maternal mortality.

(Halaman ini sengaja dikosongkan)

KATA PENGANTAR

Puji syukur penulis panjatkan atas kehadiran Allah SWT yang telah melimpahkan rahmat dan hidayah-Nya sehingga tesis ini dapat diselesaikan tepat pada waktunya. Tesis ini disusun untuk memenuhi salah satu persyaratan dalam rangka menyelesaikan pendidikan pada Program Magister Departemen Statistika Fakultas Sains dan Analitika Data ITS. Tesis ini berjudul: "Penaksiran Parameter dan Pengujian Hipotesis *Geographically Weighted Bivariate Zero Inflated Generalized Poisson Regression* (Studi Kasus : Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan Provinsi Jawa Tengah Tahun 2017)"

Dalam penyusunan tesis ini, penulis banyak memperoleh bimbingan dan petunjuk, serta bantuan dan dukungan dari berbagai pihak baik dari institusi maupun luar institusi. Melalui kesempatan ini penulis mengucapkan terimakasih yang sebesar-besarnya kepada yang terhormat:

1. Bapak Dr. Purhadi, M.Sc selaku dosen pembimbing dan Ibu Irhamah, S.Si, M.Si, Ph.D selaku dosen co pembimbing yang telah meluangkan waktu untuk memberikan arahan dalam menyelesaikan tesis ini.
2. Bapak Dr. Sutikno, M.Si, Dr. Sarpono, S.Si, M.Sc dan Dr. Titi Kanti Lestari, SE, M.Com selaku dosen penguji yang telah memberikan saran serta perbaikan dalam tesis ini.
3. Ibu Dr. Kartika Fithriasari, M.Si selaku Kepala Departemen Statistika FSAD ITS.
4. Bapak Dr.rer.pol. Dedy Dwi Prastyo, S.Si, M.Si selaku Kepala Program Studi Pascasarjana Statistika Departemen Statistika ITS.
5. Bapak Dr.rer.pol. Heri Kuswanto, M.Si selaku dosen wali yang selalu mengarahkan dan memberikan saran.
6. Bapak dan Ibu dosen pengajar Departemen Statistika ITS, terima kasih atas ilmu yang telah diajarkan.
7. Bapak dan Ibu Pegawai Departemen Statistika ITS yang telah banyak membantu penulis selama masa perkuliahan.

8. Kepala BPS RI, Kepala Pusdiklat RI, Kepala BPS Provinsi Kepulauan Bangka Belitung dan Kepala BPS Kabupaten Bangka Selatan beserta jajarannya yang telah memberikan kesempatan dan kepercayaan kepada penulis untuk menempuh tugas belajar serta dukungan moril maupun materiil.
9. Orang tua tercinta, Bapak Hariyanto dan Ibu Sundari serta Mertua, Bapak Suwandi dan Ibu Kasiyati yang selalu memberikan doa dan dukungannya dalam menyelesaikan studi.
10. Suami tercinta, Hadiyanto yang selalu membantu, mendoakan dan memberikan dukungan moril dan materiil serta anak-anak tersayang, Zakiyah Syifaul Qulub, Ahmad Yusuf Faiqul Insan dan Ahmad Yahya Zamzami Syafi sebagai penyemangat dalam penyelesaian studi dan selalu menemani dalam 2 semester meskipun harus berjauhan di semester ketiga.
11. Adik-adik tersayang, Lifa dan Anas yang selalu memberikan dukungan serta meluangkan waktu dalam menjaga anak-anak penulis.
12. Mba Dewi, Mas Rindang, Mas Syahrul, dan mahasiswa S3 Departemen Statistika ITS yang telah membantu dan memberikan saran kepada penulis dalam pengerjaan tesis.
13. Teman-teman seperjuangan tugas belajar BPS Angkatan 2018, Mita, Habi dan Furqon dalam membantu dan memberikan semangat selama masa studi.
14. Teman-teman regular S2 Gaby, Mega, Endah, Istin, dan Luluk dalam membantu pengolahan data tesis ini.
15. Mahasiswa S2 angkatan 2018 semester ganjil dan genap Departemen Statistika ITS dalam memberikan bantuan, dukungan serta doa selama masa studi dan dalam penyelesaian tesis ini.
16. Semua pihak yang tidak dapat saya sebutkan satu per satu dalam penyusunan tesis ini.

Semoga tulisan ini dapat memberikan sumbangan informasi ilmiah dan memberikan manfaat bagi masyarakat. Penulis menyadari bahwa masih banyak kesalahan dalam penulisan tesis ini. Oleh karena itu kritik dan saran dari berbagai pihak sangat penulis harapkan untuk penelitian yang akan datang.

Surabaya, Januari 2020

Qurotul Aini

(Halaman ini sengaja dikosongkan)

DAFTAR ISI

DAFTAR TABEL

(Halaman ini sengaja dikosongkan)

DAFTAR GAMBAR

(Halaman ini sengaja dikosongkan)

DAFTAR LAMPIRAN

(Halaman ini sengaja dikosongkan)

BAB 1
PENDAHULUAN

1.1 Latar Belakang

Distribusi Poisson merupakan suatu distribusi peluang yang menyatakan kemungkinan terjadinya sejumlah peristiwa dalam suatu periode waktu sehingga nilai yang mungkin dari respon adalah bilangan bulat non negatif. Salah satu kelemahan model regresi Poisson adalah adanya asumsi bahwa rata-rata harus sama dengan varian atau equidispersi. Ketika varian sampel lebih besar atau lebih kecil dari rata-rata sampel, maka data dikatakan mengalami overdispersi atau underdispersi. Menurut McCullagh dan Nelder (1989) dan Osmen dan Famoye (2007), jika asumsi equidispersi tidak dipenuhi maka standar error akan menjadi bias dan statistik uji yang berasal dari model menjadi kurang tepat sehingga diperoleh kesimpulan yang kurang valid sehingga dengan adanya masalah overdispersi maupun underdispersi maka model regresi Poisson tidak dapat digunakan. Untuk mengatasi masalah overdispersi maupun underdispersi, pada Tahun 1973 Consul dan Jain memperkenalkan model *Generalized Poisson Distribution* (GPD) yang merupakan bentuk terbatas dari *Generalized Negative Binomial Distribution* (GNBD) yang mampu mengatasi kasus baik overdispersi maupun underdispersi.

Pada beberapa kasus ditemukan suatu peristiwa yang jarang terjadi sehingga respon banyak berisikan data yang bernilai nol *(excess zero)*. Famoye dan Singh (2006) mengembangkan model regresi ZIGP yang merupakan model gabungan antara model regresi ZIP dengan model regresi GP dimana model regresi ZIP merupakan model yang cocok digunakan pada kondisi dimana respon banyak yang bernilai nol, sedangkan model GP merupakan model yang cocok digunakan ketika terjadi pelanggaran asumsi equidispersi pada distribusi Poisson. Famoye dan Singh (2006) memperkirakan proporsi data bernilai nol yang sesuai untuk model regresi ZIGP adalah sebesar 65,7 persen. Pengembangan ZIGP univariat menjadi ZIGP bivariat (BZIGP) dilakukan oleh Zhang dan Huang (2015) yang mengusulkan distribusi BZIGP dengan memasukkan faktor pengali

λ yang disebut sebagai distribusi bivariat $ZIGP_\lambda$ tipe I dan tipe II. Distribusi yang diusulkan oleh Zang dan Huang (2015) memiliki struktur korelasi yang fleksibel yang dapat digunakan baik pada saat korelasi bernilai positif atau negatif maupun ketika terjadi overdispersi atau underdispersi.

Teknik pemodelan statistik yang paling umum digunakan dalam ilmu sosial adalah regresi. Parameter yang dihasilkan dari regresi berlaku global untuk semua lokasi pengamatan sehingga ketika diterapkan pada data spasial, model regresi yang dihasilkan dianggap cocok untuk semua lokasi dimana data tersebut diamati. Dengan kata lain seluruh lokasi pengamatan diasumsikan memiliki karakteristik yang sama. Brunsdon, dkk (1996) dan Fotheringham, dkk (1998) mengembangkan model spasial yang dikenal dengan nama *Geographically Weighted Regression* (GWR). GWR adalah pemodelan spasial dengan pendekatan titik yang bersifat lokal sehingga parameter model yang dihasilkan bersifat lokal untuk setiap titik atau lokasi dimana data tersebut diamati. Hasil akhir dari pemodelan titik adalah perbedaan pengaruh variabel prediktor terhadap respon berdasarkan lokasi. Jika parameter dalam regresi global diasumsikan mempunyai varian yang konstan maka tidak demikian dengan GWR. Model GWR justru menunjukkan adanya variasi spasial yang mungkin terdapat dalam model.

Beberapa penelitian yang menggunakan model GWR adalah Rida, dkk (2014) membahas mengenai *Geographically Weighted Poisson Regression* (GWPR) yang merupakan pengembangan dari regresi poisson dengan memperhatikan faktor spasial. Penelitian mengenai model GWR dengan respon bivariat pernah dilakukan oleh Pangulimang, dkk (2016) yang membahas mengenai estimasi parameter menggunakan MLE dan pengujian hipotesis model *Geographically Weighted Bivariate Zero-Inflated Poisson Regression* (GWBZIPR) menggunakan MLRT. Holgate (1964) melakukan perbandingan antara metode momen dengan MLE dalam menaksir parameter kovarians dan menghasilkan kesimpulan bahwa metode momen hanya efisien pada kovarians sama dengan nol, peningkatan korelasi akan membuat metode moment menjadi semakin tidak efisien.

Salah satu data cacah yang mengikuti distribusi poisson adalah Angka Kematian Ibu (AKI) dimana frekuensi suatu kejadian dari jumlah kejadian tersebut tidak memiliki batas atas dan selalu berupa bilangan bulat non negatif. Agresti (2002) menyatakan bahwa distribusi untuk data cacah yang demikian adalah distribusi Poisson. Penurunan AKI termasuk satu dari tujuh belas tujuan global Sasaran Pembangunan Berkelanjutan atau SDGs (*Sustainable Development Goals*) yang tertuang dalam tujuan ke tiga SDGs dan selanjutnya dijabarkan dalam tiga belas target dengan target pertama yaitu mengurangi rasio kematian ibu menjadi kurang dari 70 kematian per 100.000 kelahiran hidup pada tahun 2030 (United Nations Development Programme, 2016). Dalam rangka pencapaian target SDGs, Pemerintah Indonesia berusaha untuk meningkatkan status kesehatan dan gizi masyarakat Indonesia. Salah satu program prioritas pembangunan kesehatan yang terdapat dalam Rencana Pembangunan Jangka Menengah Nasional (RPJMN) 2015-2019 adalah menurunkan AKI. Hal ini menunjukkan bahwa kematian ibu menjadi salah satu indikator penting dalam pembangunan di bidang kesehatan dan khususnya bagi pencapaian target SDGs.

Provinsi Jawa Tengah merupakan provinsi dengan angka kematian ibu tertinggi di Indonesia pada tahun 2017 yaitu sebesar 68,7 per 100.000 kelahiran hidup dimana Karesidenan Pekalongan memiliki jumlah kematian ibu lebih banyak dibandingkan dengan karesidenan lainnya di Provinsi Jawa Tengah yaitu sebesar 23,38 persen. Data mengenai jumlah kematian ibu hamil dan jumlah kematian ibu nifas di Karesidenan Pekalongan pada tahun 2017 memiliki korelasi yang positip sebesar 0,27 dan mengalami under/over dispersi serta mengandung *excess zero* sehingga penelitian ini akan mengkaji tentang Penaksiran Parameter dan Pengujian Hipotesis *Geographically Weighted Bivariate Zero Inflated Generalized Poisson Regression* pada kasus jumlah kematian ibu hamil dan nifas di Karesidenan Pekalongan di Jawa Tengah Tahun 2017. Hasil kajian diharapkan dapat menentukan faktor-faktor yang berpengaruh signifikan terhadap jumlah kematian ibu hamil dan nifas di Karesidenan Pekalongan Provinsi Jawa Tengah Tahun 2017.

1.2 Perumusan Masalah

Penelitian terakhir mengenai model dengan respon lebih dari satu untuk data cacah berdistribusi Poisson yang memperhitungkan aspek spasial adalah model *Geographically Weighted Multiivariate Poisson Regression* (GWMPR) oleh Triyanto, dkk. (2015), model ini tidak dapat menangani adanya overdispersi maupun underdispersi dan banyaknya nilai nol pada respon. Selain model GWMPR, terdapat juga model *Geographically Weighted Bivariate Zero-Inflated Poisson Regression* (GWBZIPR) yang dikembangkan oleh Pangulimang, dkk. (2016). Model ini sudah memperhitungkan aspek spasial dan dapat menangani banyaknya nilai nol pada respon namun model ini tidak dapat menangani overdispersi maupun underdispersi.

Untuk mengatasi kekurangan pada model-model sebelumnya, maka pada penelitian ini akan dikembangkan model regresi spasial bivariat dengan respon berupa data cacah, yaitu model *Geographically Weighted Bivariat Zero-Inflated Generalized Poisson Regression* (GWBZIGPR). Model ini diharapkan mampu menangani banyaknya nilai nol pada respon dan menangani overdispersi maupun underdispersi sekaligus memperhitungkan aspek spasial dalam hubungan antara respon dan variabel prediktor.

1.3 Tujuan Penelitian

Berdasarkan rumusan masalah di atas, maka tujuan yang ingin dicapai dalam penelitian ini adalah sebagai berikut:

1. Menentukan penaksiran parameter dan pengujian hipotesis untuk model BZIGPR.
2. Menentukan penaksiran parameter dan pengujian hipotesis untuk model GWBZIGPR.
3. Mendapatkan faktor-faktor yang mempengaruhi jumlah kematian ibu hamil dan ibu nifas di Karesidenan Pekalongan Provinsi Jawa Tengah dengan pendekatan GWBZIGPR.

1.4 Manfaat Penelitian

Manfaat penelitian ini:

1. Pengembangan metode statistik yaitu model GWBZIGPR khususnya dalam hal penaksiran parameter dan statistik uji, algoritma program untuk model GWBZIGPR serta penerapan model GWBZIGPR dalam bidang kesehatan sebagai metode alternatif dalam mengetahui faktor-faktor apa saja yang mempengaruhi jumlah kematian ibu hamil dan jumlah kematian ibu nifas.
2. Diharapkan hasil penelitian ini dapat memberi masukan bagi Dinas Kesehatan pada wilayah penelitian dalam membuat kebijakan terkait masalah jumlah kematian ibu hamil dan jumlah kematian ibu nifas.

1.5 Batasan penelitian

Permasalahan dalam penelitian ini dibatasi pada:

1. Ruang lingkup penelitian adalah semua kecamatan yang berada di Karesidenan Pekalongan Provinsi Jawa Tengah tahun 2017.
2. Penaksiran parameter menggunakan metode MLE dan penentuan statistik uji pada uji hipotesis baik serentak maupun parsial menggunakan MLRT
3. Koordinat lokasi untuk wilayah pengamatan adalah pusat pemerintahan kecamatan atau letak kantor camat.
4. Iterasi numerik menggunakan *Berndt Hall Hall Hausman* (BHHH) dalam mengatasi hasil estimasi parameter yang tidak *closed form*.

(Halaman ini sengaja dikosongkan)

BAB 2
LANDASAN TEORI

2.1. Distribusi Poisson

Distribusi Poisson merupakan pendekatan dari distribusi Binomial ketika kejadian yang muncul dari banyak percobaan Bernoulli memiliki peluang sukses yang kecil dalam masing-masing percobaan. Jika $Y \sim B(n, p)$ maka distribusi dari Y pada saat $n \to \infty$ dan $p \to 0$ dimana $\lambda = np$ mengikuti distribusi Poisson $\lim_{n\to\infty} \binom{n}{y} p_n^y (1-p)^{n-y} = e^{-\lambda} \frac{\lambda^y}{y!}$ dengan mean = λ.

Salah satu model regresi non linier yang dapat menggambarkan hubungan antara respon Y dengan variabel prediktor X dimana respon Y merupakan data *count*, sementara variabel prediktor X dapat berbentuk kontinyu, kategorik atau kombinasi antara kontinyu dan kategorik adalah regresi poisson. Respon dalam regresi poisson menyatakan jumlah peristiwa yang terjadi pada interval waktu dan atau ruang tertentu misalnya jumlah kecelakaan di jalan raya yang terjadi dalam satu bulan, jumlah persalinan secara *caesar* di rumah sakit, jumlah penderita kanker yang meninggal di Indonesia selama 1 tahun, jumlah kriminalitas dalam periode waktu tertentu, jumlah bayi yang terserang campak di Indonesia, dan lain sebagainya.

2.1.1 Distribusi Univariat Poisson

Menurut Cameron dan Trivedi (2013) variabel random diskrit Y dikatakan berdistribusi poisson dengan parameter λ jika dan hanya jika fungsi probabilitasnya berbentuk seperti persamaan (2.1). Dimana λ adalah rata-rata suatu kejadian (y) yang bernilai lebih besar dari atau sama dengan nol.

$$f(y) = \begin{cases} \dfrac{e^{-\lambda}\lambda^y}{y!}, y = 0,1,2,\ldots,\lambda > 0 \\ 0, \quad \text{y yang lain} \end{cases} \tag{2.1}$$

Nilai mean dan varians dari distribusi poisson adalah

$$E(Y)=\sum_{y=0}^{\infty} y\, f(y)=\sum_{y=0}^{\infty} y\ \frac{\lambda^{y}e^{-\lambda}}{y!}=\lambda$$

$E(Y^2)=\lambda^2+\lambda$ sehingga $Var(Y)=E\left(Y^2\right)-\left(E(Y)\right)^2=\lambda$

2.1.2 Distribusi Bivariat Poisson

Misalkan variabel random $Y_1=X_1+X_0,\ Y_2=X_2+X_0$ dengan X_0,X_1,X_2 merupakan variabel random yang masing masing berdistribusi poisson dengan parameter $\lambda_0,\lambda_1,\lambda_2$. Jika X_0 dan X_1 saling independen serta X_0 dan X_2 saling independen maka diperoleh $E(Y_1)=\lambda_1+\lambda_0,\ E(Y_2)=\lambda_2+\lambda_0$ setelah diketahui nilai ekspektasi dari masing masing variabel random Y_1 dan Y_2 maka dapat diketahui pula $E(Y_1Y_2)$ adalah $E(Y_1Y_2)=(\lambda_1+\lambda_0)(\lambda_2+\lambda_0)+\lambda_0$ sehingga diperoleh nilai varian dan kovarian sebagai berikut:

$Var(Y_1)=(\lambda_1+\lambda_0),\ Var(Y_2)=(\lambda_2+\lambda_0),\ Cov(Y_1,Y_2)=\lambda_0.$

Dengan fungsi pembangkit momen bersama dari bivariat poisson Y_1,Y_2 adalah :

$$\begin{aligned}
M_{(Y_1Y_2)}(t_1,t_2) &= M_{Y_1}(t_1)M_{Y_2}(t_2)=M_{X_1+X_0}(t_1)M_{X_2+X_0}(t_2)\\
&= E(e^{t_1(X_1+X_0)})E(e^{t_2(X_2+X_0)})=E\left(e^{t_1X_1}\right)E(e^{t_2X_2})E(e^{(t_1+t_2)X_0})\\
&= \exp\left(\lambda_1\left(e^{t_1}-1\right)+\lambda_2\left(e^{t_2}-1\right)+\lambda_0\left(e^{(t_1+t_2)}-1\right)\right)\\
&= \exp\left(-(\lambda_0+\lambda_1+\lambda_2)+\left(\lambda_0e^{(t_1+t_2)}+\lambda_1e^{t_1}+\lambda_2e^{t_2}\right)\right)
\end{aligned}$$

fungsi probabilitas bersama dari variabel random Y_1 dan Y_2 yang berdistribusi bivariat poisson adalah

$$f(y_1,y_2)=\begin{cases} e^{-(\lambda_0+\lambda_1+\lambda_2)}\displaystyle\sum_{k=0}^{S}\frac{\lambda_1^{y_1-k}\lambda_2^{y_2-k}\lambda_0^{k}}{(y_1-k)!(y_2-k)!k!}; & y_1,y_2=0,1,2,\ldots,\lambda_1>0,\lambda_2>0\\ 0 & ;\ y_1,y_2 \text{ yang lain}\end{cases} \qquad (2.2)$$

dimana s = min(y_1,y_2) (Kawamura, 1973).

2.2 Distribusi Generalized Poisson

Variabel random Y akan berdistribusi G*eneralized Poisson* jika memiliki fungsi distribusi peluang sebagai berikut:

$$P\left(Y=y\mid\lambda,\theta\right)=\begin{cases}\dfrac{\lambda\left(\lambda+\theta y\right)^{y-1}e^{-\lambda-\theta y}}{y!} & ,y=0,1,2,\ldots;\quad \lambda>0\\ 0 & ,\text{untuk } y>m \text{ ketika } \theta<0\end{cases}\tag{2.3}$$

dimana θ merupakan parameter dispersi dengan $\max\left(-1,\dfrac{-\lambda}{m}\right)\le\theta\le 1$, dan $m\ge 4$ merupakan bilangan bulat positif terbesar untuk $\lambda+m\theta>0$ ketika $\theta<0$ karena ketika $\max\left(-1,\dfrac{-\lambda}{m}\right)\le\theta\le 0$ total errornya menjadi <0,05 (Consul dan Famoye, 1992). Rata-rata dan varians dari GPD adalah

$$E\left(Y\right)=\mu=\lambda\left(1-\theta\right)^{-1},Var\left(Y\right)=\lambda\left(1-\theta\right)^{-3}=\kappa E\left(Y\right)$$

dimana $\kappa=\dfrac{1}{\left(1-\theta\right)^{2}}$ menyatakan faktor dispersi.

2.3 Distribusi Bivariate Generalized Poisson

Fungsi distribusi peluang dari *Bivariate Generalized Poisson Distribution* adalah

$$f(y_1,y_2)=\lambda_1\lambda_2\lambda_0\exp\left\{-(\lambda_1+\lambda_2+\lambda_0)-y_1\theta_1-y_2\theta_2\right\}\sum_{k=0}^{\min(y_1,y_2)}\frac{1}{(y_1-k)!(y_2-k)!k!}$$

$$\left(\lambda_1+\left(y_1-k\right)\theta_1\right)^{y_1-k-1}\left(\lambda_2+\left(y_2-k\right)\theta_2\right)^{y_2-k-1}\left(\lambda_0+k\theta_0\right)^{k-1}$$

$$\exp\left\{k(\theta_1+\theta_2-\theta_0)\right\}\tag{2.4}$$

Mean dan Varians distribusi *Bivariat Generalized Poisson* adalah sebagai berikut

$$E(Y_1)=\lambda_1\left(1-\theta_1\right)^{-1}+\lambda_0\left(1-\theta_0\right)^{-1},\ Var(Y_1)=\lambda_1\left(1-\theta_1\right)^{2}+\lambda_0\left(1-\theta_0\right)^{2},$$

$$E(Y_2)=\lambda_2\left(1-\theta_2\right)^{-1}+\lambda_0\left(1-\theta_0\right)^{-1},\ Var(Y_2)=\lambda_2\left(1-\theta_2\right)^{2}+\lambda_0\left(1-\theta_0\right)^{2}$$

(Vernic, 1997). (2.5)

2.3.1 Pengujian Distribusi Bivariat Generalized Poisson

Pengujian distribusi bivariat poisson dilakukan untuk mengetahui apakah respon (Y_1 dan Y_2) mengikuti distribusi *bivariate generalized poisson*. Loukas dan Kemp's (1986) dalam Best (1999) melakukan pengujian distribusi bivariat poisson dengan pendekatan *index of dispersion test* (I_B). Hipotesis yang digunakan adalah

H_0 : Respon (Y_1 dan Y_2) mengikuti distribusi *bivariate generalized poisson*

H_1 : Respon (Y_1 dan Y_2) tidak mengikuti distribusi *bivariate generalized poisson*

Statistik uji yang digunakan adalah

$$I_B = \frac{n(\bar{Y}_2 S_{Y_1}^2 - 2m_{11}^2 + \bar{Y}_1 S_{Y_2}^2)}{\left(\bar{Y}_1 \bar{Y}_2 - m_{11}^2\right)} \tag{2.6}$$

dengan.

n = jumlah data pada variabel respon (Y_1 dan Y_2)

$\bar{Y}_1$ = nilai rata rata variabel respon (Y_1)

$\bar{Y}_2$ = nilai rata rata variabel respon (Y_2)

$$S_{Y_1}^2 = \frac{\sum_{i=1}^{n}\left(Y_{1i} - \bar{Y}_1\right)^2}{n}, \quad S_{Y_2}^2 = \frac{\sum_{i=1}^{n}\left(Y_{2i} - \bar{Y}_2\right)^2}{n}, \quad m_{11} = \frac{\sum_{i=1}^{n}\left(Y_{1i} - \bar{Y}_1\right)\left(Y_{2i} - \bar{Y}_2\right)}{n}$$

Daerah penolakan H_0 adalah $\left|I_B\right| > \chi^2_{(\alpha;2n-3)}$ (Best, 1999).

2.4 Regresi Univariat Poisson

Regresi poisson digambarkan dengan adanya hubungan antara respon (Y) yang berdistribusi poisson dan terdapat satu atau lebih variabel prediktor (X) (Agresti, 1990). Regresi Poisson merupakan model regresi yang digunakan untuk menganalisis suatu data *count*. Regresi poisson mengacu pada penggunaan distribusi poisson.

2.4.1 Model Regresi Univariat Poisson

Pada model regresi Poisson, fungsi penghubung yang digunakan berdasarkan keluarga eksponensial adalah fungsi penghubung log karena penggunaan fungsi log menjamin bahwa nilai dari ekspektasi respon selalu bernilai non negatif. Jika diberikan sampel random $Y_i \sim Poisson(\lambda), i = 1, 2, ..., n$ maka model regresi Poisson dinyatakan dalam bentuk seperti berikut:

$\ln\left[E\left(Y_i\right)\right] = \mathbf{x}_i^T\boldsymbol{\beta}$ maka $E\left(Y_i\right) = e^{\mathbf{x}_i^T\boldsymbol{\beta}}$ atau $\lambda_i = e^{\mathbf{x}_i^T\boldsymbol{\beta}}$.

dimana $\mathbf{x}_i = \begin{bmatrix} 1 & \mathbf{x}_{1i} & \mathbf{x}_{2i} & \ldots & \mathbf{x}_{qi} \end{bmatrix}^T$, $\boldsymbol{\beta} = \begin{bmatrix} \beta_0 & \beta_1 & \beta_2 & \ldots & \beta_q \end{bmatrix}^T$.

Penaksiran parameter model regresi univariat poisson menggunakan metode *Maximum Likelihood Estimation* (MLE), yaitu dengan memaksimumkan nilai fungsi *likelihoodnya*. Taksiran parameter regresi poisson dengan metode MLE dilambangkan dengan $\widehat{\boldsymbol{\beta}}$ yang didapat dari turunan pertama fungsi ln *likelihood*. Turunan pertama fungsi ln *likelihood* terhadap $\boldsymbol{\beta}^T$ dan disamadengankan nol dan tidak dapat diselesaikan secara analitik, sehingga perlu digunakan prosedur iteratif, yaitu dengan menggunakan metode iterasi Newton-Rhapson. Berikut ini adalah persamaan iterasi Newton-Rhapson.

$$\hat{\boldsymbol{\beta}}_{(m+1)} = \hat{\boldsymbol{\beta}}_{(m)} - \mathbf{H}^{-1}\left(\hat{\boldsymbol{\beta}}_{(m)}\right)\mathbf{g}\left(\hat{\boldsymbol{\beta}}_{(m)}\right) \quad (2.7)$$

Nilai $\hat{\boldsymbol{\beta}}_{(m)}$ merupakan nilai taksiran parameter pada saat iterasi ke ***m***, merupakan $\mathbf{g}\left(\hat{\boldsymbol{\beta}}_{(m)}\right)$ vektor gradien dengan parameter $\hat{\boldsymbol{\beta}}_{(m)}$, dan $\mathbf{H}\left(\hat{\boldsymbol{\beta}}_{(m)}\right)$ adalah matriks Hessian dengan parameter $\hat{\boldsymbol{\beta}}_{(m)}$. Taksiran awal parameter $\hat{\boldsymbol{\beta}}_{(0)}$ menggunakan metode *Ordinary Least Square* (OLS). Iterasi akan berhenti apabila nilai dari $\left\|\hat{\boldsymbol{\beta}}_{(m+1)} - \hat{\boldsymbol{\beta}}_{(m)}\right\| < \varepsilon, \ \varepsilon > 0$ dan sangat kecil.

2.4.2 Pengujian Parameter Model Regresi Univariat Poisson

Pengujian parameter model regresi univariat poisson dilakukan dengan metode *Maximum Likelihood Ratio Test* (MLRT) dengan hipotesis

H_0: $\beta_1 = \beta_2 = ... = \beta_q = 0$

H_1: paling sedikit ada satu $\beta_r \neq 0$, $\mathrm{r} = 1, 2, ..., q$

Himpunan parameter dibawah populasi adalah

$$\Omega = \left\{\beta_0, \beta_1, \beta_2, \ldots, \beta_q \,\middle|\, -\infty < \beta_r < \infty, r = 1, 2, ..., q\right\}$$

Himpunan parameter dibawah H_0 adalah

$\omega = \left\{ \beta_0 \middle| -\infty < \beta_0 < \infty \right\}$

$L_1\left(\widehat{\Omega}\right)$ adalah nilai maksimum *likelihood* untuk model lengkap dimana melibatkan variabel prediktor. $L_1\left(\hat{\omega}\right)$ adalah nilai maksimum *likelihood* untuk model sederhana tanpa melibatkan prediktor. *Likelihood ratio test* dapat ditulis:

$$D(\widehat{\boldsymbol{\beta}}) = -2\left(\ln L_1\left(\widehat{\omega}\right) - \ln L_1\left(\widehat{\Omega}\right)\right) \text{ atau } D(\widehat{\boldsymbol{\beta}}) = 2\left(\ln L_1\left(\widehat{\Omega}\right) - \ln L_1\left(\widehat{\omega}\right)\right) \tag{2.8}$$

dimana

$$L_1\left(\hat{\omega}\right) = \frac{n\exp\left(-\left(\exp\left(\hat{\beta}_0\right)\right)\right)\left(\prod_{i=1}^{n}\exp\left(\hat{\beta}_0\right)^{y_i}\right)}{\prod_{i=1}^{n} y_i\,!}$$

$$L_1\left(\hat{\Omega}\right) = \frac{\left(\exp\left(\sum_{i=1}^{n}\exp\left(\mathbf{x}_i^T\hat{\boldsymbol{\beta}}\right)\right)\right)\left(\prod_{i=1}^{n}\exp\left(\mathbf{x}_i^T\hat{\boldsymbol{\beta}}\right)^{y_i}\right)}{\prod_{i=1}^{n} y_i\,!}$$

sehingga diperoleh

$$D\left(\hat{\boldsymbol{\beta}}\right) = 2\sum_{i=1}^{n}\left(y_i\mathbf{x}_i^T\hat{\boldsymbol{\beta}} - \exp\left(\mathbf{x}_i^T\hat{\boldsymbol{\beta}}\right) - y_i\hat{\beta}_0 + \exp\left(\hat{\beta}_0\right)\right) \tag{2.9}$$

$D\left(\hat{\boldsymbol{\beta}}\right)$ adalah devians dari model regresi univariat poisson dan mengikuti distribusi χ^2 dengan derajat bebas $\left(a-b\right)$. Dimana *a* adalah jumlah paramater dibawah populasi dan *b* adalah jumlah parameter dibawah H_0. Daerah penolakan H_0 adalah $D\left(\hat{\boldsymbol{\beta}}\right) > \chi^2_{a,(a-b)}$.

Apabila keputusan pengujian secara serentak adalah tolak H_0 maka langkah selanjutnya adalah melakukan pengujian parameter secara parsial untuk mengetahui parameter mana saja yang memberikan pengaruh yang signifikan terhadap model. Hipotesis yang digunakan adalah:

H_0: $\beta_r = 0$

H_1: $\beta_r \neq 0,\ r = 1,2,...,q$

Statistik uji yang digunakan adalah seperti persamaan berikut.

$$Z_{hitung} = \frac{\hat{\beta}_r}{se\left(\hat{\beta}_r\right)}$$

$se\left(\hat{\beta}_r\right)$ merupakan standar error dari $\hat{\beta}_r$. Nilai $se\left(\hat{\beta}_r\right)$ merupakan akar dari elemen diagonal utama pada matriks $\text{v}\hat{\text{a}}\text{r}\left(\hat{\boldsymbol{\beta}}\right)$. Matriks $\text{v}\hat{\text{a}}\text{r}\left(\hat{\boldsymbol{\beta}}\right)$ disebut dengan matriks informasi fisher, dimana nilai $\text{v}\hat{\text{a}}\text{r}\left(\hat{\boldsymbol{\beta}}\right) = -\left(\mathbf{H}^{-1}\left(\hat{\boldsymbol{\beta}}\right)\right)$. Daerah penolakan *H0* adalah $|Z| > Z_{(\alpha/2)}$ dimana α adalah tingkat signifikasi yang digunakan.

2.5 Regresi Bivariat Poisson

Regresi bivariat poisson adalah metode yang digunakan untuk memodelkan sepasang *count data* berdistribusi poisson yang memiliki korelasi dengan beberapa variabel prediktor (Karlis & Ntozoufras, 2005). Variabel prediktor tersebut adalah variabel yang diduga sama-sama berpengaruh untuk kedua respon.

Model regresi Poisson bivariat untuk $\left(Y_{1i}, Y_{2i}\right) \sim$ PB $\left(\lambda_{1i}, \lambda_{2i}, \lambda_0\right)$ dengan

$$\lambda_{1i} = e^{\mathbf{x}_{1i}^T\boldsymbol{\beta}_1},\ \lambda_{2i} = e^{\mathbf{x}_{2i}^T\boldsymbol{\beta}_2},\ \lambda_0 = e^{\boldsymbol{\beta}_0} \text{ dan } E\left(Y_{li}\right) = e^{\mathbf{x}_i^T\boldsymbol{\beta}_l} \quad (2.10)$$

$i = 1, 2, ..., n; l = 1, 2;$

$\boldsymbol{\beta}_0$, $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$ adalah vektor yang sesuai untuk koefisien regresi.

2.5.1 Penaksir Parameter Model Regresi Bivariat Poisson

Penaksiran parameter regresi bivariat poisson dilakukan dengan menggunakan metode *Maksimum Likelihood Estimation* (MLE). Metode MLE yaitu dengan memaksimumkan fungsi *likelihood*nya. Misalkan diberikan *n* sampel random dari variabel random:

$$\left(Y_{1i}, Y_{2i}\right) \sim PB\left(\lambda_{1i}, \lambda_{2i}, \lambda_0\right) \quad (2.11)$$

Fungsi *likelihood* menurut Jung dan Winkelman (1993):

$$L\left(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \lambda_0\right) = \prod_{i=1}^{n}\left(e^{-\left(\lambda_0+\lambda_{1i}+\lambda_{2i}\right)}\sum_{k=0}^{\min\left(y_{1i}, y_{2i}\right)}\frac{\lambda_{1i}^{y_{1i}-k}\lambda_{2i}^{y_{2i}-k}\lambda_0^{k}}{\left(y_{1i}-k\right)!\left(y_{2i}-k\right)!k!}\right) \quad (2.12)$$

Transformasi model regresi persamaan (2.11) ke dalam persamaan (2.12) diperoleh fungsi likelihood

$$L\left(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \lambda_0\right) = \prod_{i=1}^{n} \left(\exp\left(\lambda_0 - e^{\mathbf{x}_i^T \boldsymbol{\beta}_1} - e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} \right) * B_i \right) \tag{2.13}$$

dengan nilai B_i adalah

$$B_i = \sum_{k=0}^{\min\left(y_{1i}, y_{2i}\right)} \frac{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_1} - \lambda_0\right)^{y_{1i}-k} \left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} - \lambda_0\right)^{y_{2i}-k} \lambda_0^k}{\left(y_{1i} - k\right)!\left(y_{2i} - k\right)k!}$$

Fungsi ln *likelihood* adalah sebagai berikut

$$\ln L\left(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \lambda_0\right) = n\lambda_0 - \sum_{i=1}^{n} \exp\left(\mathbf{x}_i^T \boldsymbol{\beta}_1\right) - \sum_{i=1}^{n} \exp\left(\mathbf{x}_i^T \boldsymbol{\beta}_2\right) + \sum_{i=1}^{n} \ln B_i \tag{2.14}$$

untuk mendapatkan penaksir parameter dari model ini maka persamaan (2.14) diturunkan terhadap masing-masing parameternya kemudian di samadengankan nol, namun hasilnya tidak dapat diselesaikan secara analitik sehingga perlu digunakan prosedur iteratif. Dengan cara yang sama pada penaksiran parameter model regresi univariat poisson yaitu mengunakan iterasi numerik Newton Raphson. Taksiran awal parameter $\widehat{\boldsymbol{\theta}}_{(0)}$ menggunakan metode *Ordinary Least Square* (OLS). Iterasi akan berhenti apabila nilai dari $\left\| \widehat{\boldsymbol{\theta}}_{(m+1)} - \widehat{\boldsymbol{\theta}}_{(m)} \right\| < \varepsilon$, $\varepsilon > 0$ dan sangat kecil dimana $\widehat{\boldsymbol{\theta}} = \left[\lambda_0 \; \boldsymbol{\beta}_1 \; \boldsymbol{\beta}_2\right]$.

2.5.2 Pengujian Parameter Model Regresi Bivariat Poisson

Pengujian parameter model regresi bivariat poisson dilakukan dengan metode *Maximum Likelihood Ratio Test* (MLRT) dengan hipotesis

H_0: $\beta_{l1} = \beta_{l2} = ... = \beta_{lr} = 0$

H_1: paling sedikit ada satu $\beta_{lr} \neq 0,\ l = 1, 2;\ \mathrm{r} = 1, 2, ..., q$

Himpunan parameter dibawah populasi adalah $\Omega = \left\{\lambda_0, \beta_{l0}, \beta_{l1}, \ldots, \beta_{lq}\right\}$ dan himpunan parameter dibawah H_0 adalah $\omega = \left\{\lambda_0, \beta_{l0}\right\}$.

Likelihood ratio test dapat ditulis:

$$D(\widehat{\boldsymbol{\theta}}) = -2\left(\ln L_2\left(\widehat{\omega}\right) - \ln L_2\left(\widehat{\Omega}\right)\right) \tag{2.15}$$

$D(\widehat{\boldsymbol{\theta}})$ adalah devians dari model regresi bivariat poisson yang mengikuti distribusi χ^2 dengan derajat bebas $(a-b)$, dimana a adalah jumlah paramater dibawah populasi dan b adalah jumlah parameter dibawah H$_0$. $L_2\left(\widehat{\Omega}\right)$ adalah nilai maksimum *likelihood* untuk model lengkap dimana melibatkan variabel prediktor dan $L_2\left(\widehat{\omega}\right)$ adalah nilai maksimum *likelihood* untuk model sederhana tanpa melibatkan prediktor sehingga *Likelihood ratio test* dapat ditulis:

$$D\left(\widehat{\boldsymbol{\theta}}\right) = 2\left[\left(n\widehat{\lambda}_0 - \sum_{i=1}^{n}\exp\left(\mathbf{x}_i^T\widehat{\boldsymbol{\beta}}_1\right) - \sum_{i=1}^{n}\exp\left(\mathbf{x}_i^T\widehat{\boldsymbol{\beta}}_2\right) + \sum_{i=1}^{n}\ln\widehat{B}_i\right) - C\right.$$

$$\text{dimana C} = n\widehat{\lambda}_0 - \sum_{i=1}^{n}\exp\left(\widehat{\beta}_{1.0}\right) - \sum_{i=1}^{n}\exp\left(\widehat{\beta}_{2.0}\right) + \sum_{i=1}^{n}\ln\widehat{B}_{i.0} \tag{2.16}$$

Daerah penolakan H$_0$ adalah $D\left(\widehat{\boldsymbol{\theta}}\right) > \chi^2_{(\alpha;a-b)}$. Apabila keputusan pengujian secara serentak adalah tolak H$_0$ maka langkah selanjutnya adalah melakukan pengujian parameter secara parsial untuk mengetahui parameter mana saja yang memberikan pengaruh yang signifikan terhadap model.

Hipotesis yang digunakan adalah:

$H_0 = \beta_{lr} = 0 \quad , l = 1, 2;\ \ r = 1, 2, ..., q$

$H_1 = \beta_{lr} \neq 0$

Statistik uji yang digunakan adalah

$$Z = \frac{\widehat{\beta}_{lr}}{se\left(\widehat{\beta}_{lr}\right)}$$

Tolak H$_0$ adalah $\left|Z_{hitung}\right| > Z_{(\alpha/2)}$.

2.6. Regresi Univariat Generalized Poisson

Model regresi Poisson merupakan model standar untuk menganalisis data *count*. Salah satu asumsi penting dalam model regresi Poisson adalah bahwa varian dari respon harus sama dengan rata-ratanya. Jika asumsi ini tidak dipenuhi, maka estimasi menggunakan metode MLE akan menjadi bias dan statistik uji dari model menjadi tidak tepat (Ozmen dan Famoye, 2007). Kondisi dimana nilai varian lebih besar dibandingkan nilai rata-ratanya disebut sebagai *overdispersi*, sebaliknya nilai varian yang lebih kecil dibandingkan nilai rata-ratanya disebut sebagai *underdispersi*.

Pelanggaran asumsi *equidispersi* pada model regresi Poisson dapat ditangani dengan model *Generalized Poisson Regression* (GPR). Model GPR ini mirip dengan model regresi Poisson, hanya saja model GPR mengasumsikan bahwa komponen randomnya berdistribusi *Generalized Poisson* (GPD). GPD dibedakan menjadi dua jenis yaitu GPD klasik oleh Consul dan Famoye (1992) dan *restricted* GPD oleh Famoye (1993).

Berbeda dengan distribusi Poisson pada persamaan (2.1) yang hanya memiliki satu parameter yaitu λ, GPD memiliki satu parameter tambahan yaitu θ yang merupakan parameter dispersi. GPD akan menjadi distribusi Poisson jika $\theta = 0$. Jika $\theta > 0$ maka GPD merepresentasikan data *count* yang mengandung kasus overdispersi dan jika $\theta < 0$ maka GPD merepresentasikan data *count* yang mengandung kasus underdispersi.

Jika rata-rata GPD dinyatakan dengan $\mu = \lambda\left(1-\theta\right)^{-1}$, maka fungsi kepadatan distribusi peluang dari GPR berdasarkan GPD klasik adalah (Consul dan Famoye, 1992):

$$P\left(Y_i = y_i \mid \mu_i, \theta\right) = \left(\left(1-\theta\right)\mu_i + \theta y_i\right)^{y_i - 1} \frac{\left(1-\theta\right)\mu_i}{y_i\,!} e^{-\left[\left(1-\theta\right)\mu_i + \theta y_i\right]},\ y_i = 0,1,2,... \quad (2.17)$$

dengan rata-rata dan varians $E\left(Y_i\right) = \mu_i = e^{\mathbf{x}_i^T \boldsymbol{\beta}}$ dan $Var\left(Y_i\right) = \kappa\mu_i$ dimana $\kappa = \left(1-\theta\right)^{-2}$.

Famoye (1993) melakukan transformasi parameter $\theta = \varphi\lambda$ pada persamaan (2.17) yang menyebabkan θ proporsional terhadap λ sehingga parameter θ menjadi terbatasi (*restricted*) dan menghasilkan bentuk GPD terbatas (*restricted* GPD) seperti berikut:

$$P\left(Y_i = y_i \mid \lambda, \varphi\right) = \begin{cases} \dfrac{\lambda^{y_i}\left(1+\varphi y_i\right)^{y_i - 1} e^{-\lambda\left(1+\varphi y_i\right)}}{y_i\,!}; & y_i = 0,1,2,... \\ 0 & ;\text{untuk } y_i > m \text{ ketika } \varphi < 0 \end{cases} \quad (2.18)$$

dimana $\max\left(-\lambda^{-1},-1/4\right)\le\varphi\le\lambda^{-1}$ dan m didefinisikan seperti pada persamaan (2.3). Rata-rata dan varian untuk *restricted* GPD adalah

$$E\left(Y_i\right)=\lambda\left(1-\varphi\lambda\right)^{-1},Var\left(Y_i\right)=\lambda\left(1-\varphi\lambda\right)^{-3} \tag{2.19}$$

Dengan mensubstitusikan persamaan (2.19) ke persamaan (2.18), maka fungsi kepadatan peluang dari *restricted* GPD pada persamaan (2.18) di atas dinyatakan dalam bentuk:

$$P(Y_i=y_i)=\left(\frac{\mu_i}{1+\varphi\mu_i}\right)^{y_i}\frac{\left(1+\varphi y_i\right)^{y_i-1}}{y_i!}\exp\left(\frac{-\mu_i\left(1+\varphi y_i\right)}{1+\varphi\mu_i}\right),y_i=0,1,2,... \tag{2.20}$$

Rata-rata dan varian menjadi $E\left(Y_i\right)=\mu_i=\exp\left(\mathbf{x}_i^T\boldsymbol{\beta}\right),Var\left(Y_i\right)=\mu_i\left(1+\varphi\mu_i\right)^2$

Sehingga pdf dari model GPR adalah:

$$P\left(Y_i=y_i\right)=\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}}}{1+\varphi e^{\mathbf{x}_i^T\boldsymbol{\beta}}}\right)^{y_i}\frac{\left(1+\varphi y_i\right)^{y_i-1}}{y_i!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}}\left(1+\varphi y_i\right)}{1+\varphi e^{\mathbf{x}_i^T\boldsymbol{\beta}}}\right),\ y_i=0,1,2,... \tag{2.21}$$

Ketika $\varphi=0$, maka model *restricted* GPR pada persamaan (2.18) akan menjadi model regresi Poisson dimana $E\left(Y\right)=Var\left(Y\right)$. Jika $\varphi>0$, model *restricted* GPR merepresentasikan data *count* yang mengandung kasus overdispersi dan jika $\varphi<0$ maka model *restricted* GPR merepresentasikan data *count* yang mengandung kasus underdispersi.

2.6.1 Penaksir Parameter Univariate Generalized Poisson Regression

Metode *Maximum Likelihood Estimation* digunakan untuk menduga parameter dalam *Generalized Poisson Regression* yaitu dengan cara memaksimalkan fungsi *likelihood* dari parameter $(\varphi,\boldsymbol{\beta})$. Fungsi *likelihood* dari *Generalized Poisson Regression* adalah:

$$L(\varphi,\boldsymbol{\beta})=\prod_{i=1}^{n}\left(\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}}}{1+\varphi e^{\mathbf{x}_i^T\boldsymbol{\beta}}}\right)^{y_i}\frac{\left(1+\varphi y_i\right)^{y_i-1}}{y_i!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}}\left(1+\varphi y_i\right)}{1+\varphi e^{\mathbf{x}_i^T\boldsymbol{\beta}}}\right)\right) \tag{2.22}$$

Fungsi *ln* likelihood dari *Generalized Poisson Regression* adalah:

$$\ln L(\varphi,\boldsymbol{\beta})=\sum_{i=1}^{n}y_i\ln\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}}}{1+\varphi e^{\mathbf{x}_i^T\boldsymbol{\beta}}}\right)+(y_i-1)\ln\left(1+\varphi y_i\right)-\ln(y_i!)-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}}\left(1+\varphi y_i\right)}{1+\varphi e^{\mathbf{x}_i^T\boldsymbol{\beta}}} \tag{2.23}$$

Proses mendapatkan penaksir parameter dari model ini maka persamaan (2.23) diturunkan terhadap masing-masing parameternya kemudian di samadengankan nol. Namun hasilnya tidak dapat diselesaikan secara analitik, sehingga perlu digunakan prosedur iteratif. Dengan cara yang sama pada penaksiran parameter model GPR yaitu mengunakan iterasi numerik Newton Raphson dengan $\boldsymbol{\theta} = \left[\varphi \ \boldsymbol{\beta}\right]$

$$\hat{\boldsymbol{\theta}}_{m+1} = \hat{\boldsymbol{\theta}}_m - \mathbf{H}^{-1}(\hat{\boldsymbol{\theta}}_m)\mathbf{g}(\hat{\boldsymbol{\theta}}_m) \qquad (2.24)$$

Nilai $\hat{\boldsymbol{\theta}}_{(m)}$ merupakan nilai taksiran parameter pada saat iterasi ke ***m***, $\mathbf{g}\left(\hat{\boldsymbol{\theta}}_{(m)}\right)$ merupakan vektor gradien dengan parameter $\hat{\boldsymbol{\theta}}_{(m)}$, dan $\mathbf{H}\left(\hat{\boldsymbol{\theta}}_{(m)}\right)$ adalah matriks Hessian dengan parameter $\hat{\boldsymbol{\theta}}_{(m)}$. Taksiran awal parameter $\hat{\boldsymbol{\theta}}_{(0)}$ menggunakan metode *Ordinary Least Square* (OLS). Iterasi akan berhenti apabila nilai dari $\left\|\hat{\boldsymbol{\theta}}_{(m+1)} - \hat{\boldsymbol{\theta}}_{(m)}\right\| < \varepsilon$, $\varepsilon > 0$ dan sangat kecil.

2.6.2 Pengujian Hipotesis Univariate Generalized Poisson Regression

Pengujian hipotesis *Univariate Generalized Poisson Regression* dilakukan dengan *Maximum Likelihood Ratio Test* (MLRT) dengan hipotesis:

H_0: $\beta_1 = \beta_2 = ... = \beta_q = 0$

H_1: paling sedikit ada satu $\beta_r \neq 0,\ \mathrm{r} = 1, 2, ..., q$

Statistik uji yang digunakan $D(\hat{\boldsymbol{\beta}}) = -2\left(\ln L_1(\hat{\omega}) - \ln L_1\left(\hat{\Omega}\right)\right), D\left(\hat{\boldsymbol{\beta}}\right)$ adalah devians dari model regresi generalized poisson. $D\left(\hat{\boldsymbol{\beta}}\right)$ mengikuti distribusi χ^2 dengan derajat bebas $(a-b)$. Dimana *a* adalah jumlah paramater dibawah populasi dan *b* adalah jumlah parameter dibawah H_0. Daerah penolakan H_0 adalah $\left|D\left(\hat{\boldsymbol{\beta}}\right)\right| > \chi^2_{(\alpha;a-b)}$

Apabila keputusan pengujian secara serentak adalah tolak *H0* maka langkah selanjutnya adalah melakukan pengujian parameter secara parsial untuk mengetahui parameter mana saja yang memberikan pengaruh yang signifikan terhadap model, tolah *H0* jika $\left|Z_{hitung}\right| > Z_{(\alpha/2)}$.

2.7. Regresi Bivariate Generalized Poisson

Lakshminarayana, Pandit dan Rao (1999) mengembangkan distribusi Poisson bivariat sebagai perkalian dua marjinal Poisson dengan suatu faktor pengali. Metode ini dapat digunakan baik ketika korelasi antara dua variabel random bernilai negatif, positif maupun nol.

Berdasarkan model *restricted* GPD pada persamaan (2.18), metode yang dikembangkan oleh Lakshminarayana, dkk. Famoye (2010) mengembangkan BGPD sebagai perkalian marjinal *Univariat Generalized Poisson* dengan suatu pengali dengan fungsi peluangnya sebagai berikut:

$$P(y_1, y_2) = \prod_{l=1}^{2}\left(\frac{\lambda_l^{y_l}(1+\varphi_l y_l)^{y_l-1}}{y_l!}\exp\left[-\lambda_l(1+\varphi_l y_l)\right]\right)\left(1+\eta\prod_{l=1}^{2}\left(e^{-y_l}-g_l\right)\right);$$
$$y_1, y_2 = 0,1,2,... \tag{2.25}$$

dimana $g_l = E\left(e^{-Y_l}\right) = \exp\left[\lambda_l(t_l - 1)\right]$, $\ln t_l - \varphi_l\lambda_l(t_l - 1) + 1 = 0;\quad l = 1,2$.

Ketika $\varphi_1 = \varphi_2 = 0$, BGPD pada persamaan (2.25) akan menjadi distribusi Poisson bivariat sebagaimana yang dikembangkan oleh Lakshminarayana, dkk (1999).

Rata-rata dan varian dari BGPD adalah

$$\mu_l = \lambda_l(1-\varphi_l\lambda_l)^{-1};\quad l = 1,2,\ \sigma_l^2 = \lambda_l(1-\varphi_l\lambda_l)^{-3};\quad l = 1,2 \tag{2.26}$$

Kovarians dan koefisien korelasi antara Y_1 dan Y_2 adalah:

$$\sigma_{12} = \eta(g_{11} - g_1\mu_1)(g_{22} - g_2\mu_2),\ \rho = \frac{\eta(g_{11} - g_1\mu_1)(g_{22} - g_2\mu_2)}{\sigma_1\sigma_2} \tag{2.27}$$

dimana $g_{ll} = E(Y_l\exp(-Y_l)) = \lambda_l(1-\varphi_l\lambda_l t_l)^{-1}\exp\left(\lambda_l(1+\varphi_l)(t_l - 1) - 1\right)$ dengan $\ln t_l - \varphi_l\lambda_l(t_l - 1) + 1 = 0.$ Koefisien korelasi pada persamaan (2.27) dapat bernilai positif, negatif atau nol tergantung pada nilai parameter faktor pengali η.

Misalkan Y_l adalah respon berbentuk *count* dan $\mathbf{X}_i$ adalah vektor kovariat, dengan menggunakan $E(Y_l \mid \mathbf{x}_i) = \mu_l(\mathbf{x}_i)$ yang merupakan rata-rata marjinal dari distribusi pada persaman (2.25) dan menuliskannya sebagai fungsi kovariat, diperoleh formula (Famoye, 2010):

$$\mu_l\left(\mathbf{x}_i\right)=\frac{\lambda_{li}}{1-\varphi_l\lambda_{li}} \text{ atau } \lambda_{li}=\frac{\mu_{li}}{1+\varphi_l\mu_{li}}; \quad l=1,2 \tag{2.28}$$

Dengan mensubstitusikan persamaan (2.28) dengan BGPD pada persamaan (2.25), maka fungsi peluang dari BGPD adalah sebagai berikut (Famoye, 2012):

$$P\left(y_1,y_2\right)=\prod_{l=1}^{2}\left(\left(\frac{\mu_l}{1+\varphi_l\mu_l}\right)^{y_l}\frac{\left(1+\varphi_l y_l\right)^{y_l-1}}{y_l!}\exp\left(\frac{-\mu_l\left(1+\varphi_l y_l\right)}{1+\varphi_l\mu_l}\right)\right)\left(1+\eta\prod_{l=1}^{2}\left(e^{-y_l}-g_l\right)\right) \tag{2.29}$$

dimana $g_l=\exp\left[\mu_l\left(t_l-1\right)/\left(1+\varphi_l\mu_l\right)\right]$, $\ln t_l-\left[\varphi_l\mu_l\left(t_l-1\right)/\left(1+\varphi_l\mu_l\right)\right]+1=0.$

Rata-rata dan varian marjinal Y_l , $E\left(Y_l \mid \mathbf{x}_i\right)=\mu_{li}$ dan $Var\left(Y_l \mid \mathbf{x}_i\right)=\mu_{li}\left(1+\varphi_l\mu_{li}\right)^2$

Koefisien korelasi antara Y_1 dan Y_2 dinyatakan dalam bentuk:

$$\rho=\frac{\eta\left(g_{11}-g_1\mu_1\right)\left(g_{22}-g_2\mu_2\right)}{\sqrt{\mu_1\mu_{2i}}\left(1+\varphi_1\mu_1\right)\left(1+\varphi_2\mu_2\right)}$$

dimana $g_{ll}=E\left(Y_l e^{-Y_l}\right)=\dfrac{\mu_l}{1-\varphi_l\mu_l\left(t_l-1\right)}\exp\left[\dfrac{\mu_l\left(1+\varphi_l\right)\left(t_l-1\right)}{1+\varphi_l\mu_l}-1\right]$

dengan $\ln t_l-\dfrac{\varphi_l\mu_l\left(t_l-1\right)}{1+\varphi_l\mu_l}+1=0.$

Model regresi dari BGPD dinyatakan dalam bentuk $\mu_{li}\left(x_i\right)=e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}$ sehingga persamaan (2.28) dapat ditulis sebagai $\lambda_{li}=\dfrac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}{1+\varphi_l\, e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}; \quad l=1,2.$

Dengan demikian, pdf dari BGPR adalah:

$$P\left(y_{1i},y_{2i}\right)=\prod_{l=1}^{2}\left(\left(\frac{e^{\mathbf{x}_i^T\beta_l}}{1+\varphi_l e^{\mathbf{x}_i^T\beta_l}}\right)^{y_{li}}\frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}!}\exp\left(\frac{-e^{\mathbf{x}_i^T\beta_l}\left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\mathbf{x}_i^T\beta_l}}\right)\right)*G \tag{2.30}$$

dengan $G=\left(1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right)$

Ketika $\eta=0$, variabel respon Y_1 dan Y_2 akan saling bebas dan masing-masing berdistribusi GPR. Model BGPR memiliki korelasi positif ketika $\eta=0$ dan memiliki korelasi negatif ketika $\eta\neq 0$. Model BGPR akan menjadi model regresi

Poisson bivariat ketika $\varphi_l = 0$. Nilai $\varphi_l < 0$, mengindikasikan adanya underdispersi dan nilai $\varphi_l > 0$ mengindikasikan adanya overdispersi.

2.8. Regresi Univariate Zero Inflated Poisson

Model regresi poisson banyak digunakan untuk menjelaskan hubungan antara respon dengan variabel prediktor dimana respon berbentuk data cacah. Dalam kasus tertentu, ditemukan peristiwa yang jarang terjadi namun pasti terjadi sehingga respon banyak berisikan data bernilai nol. Lambert (1992) menyarankan penggunaan distribusi *Zero Inflated Poisson* (ZIP) untuk menangani banyaknya data respon yang bernilai nol (*excess zero*) yang merupakan kombinasi dari distrbusi poisson dan distribusi logit. Famoye dan Singh (2006) menyebutkan bahwa proporsi data yang bernilai nol adalah sekitar 63,7 persen. Model regresi yang digunakan untuk menjelaskan hubungan antara variabel bebas dengan variabel respon berdistribusi ZIP disebut sebagai model regresi ZIP.

Jika $Y_i, i = 1, 2, ..., n$ adalah variabel random yang saling bebas dan mempunyai distribusi ZIP maka nilai nol pada observasi diduga berasal dari dua *state*. *State* pertama disebut *zero state* yang terjadi dengan peluang sebesar p_i dan hanya menghasilkan observasi bernilai nol sedangkan *state* kedua disebut *Poisson state* yang terjadi dengan peluang sebesar $1 - p_i$ atau dapat dinyatakan dalam bentuk berikut:

$$Y_i \sim \begin{cases} 0, & \text{dengan peluang } p_i \\ Poisson(\lambda_i), & \text{dengan peluang } (1 - p_i) \end{cases}$$

sehingga fungsi kepadatan peluang dari distribusi *Zero Inflated Poisson* (ZIP) untuk variabel random y_i adalah (Lambert, 1992):

$$P(Y_i = y_i) = \begin{cases} p_i + (1 - p_i)e^{-\mu_i} & , y_i = 0 \\ \dfrac{(1 - p_i)e^{-\mu_i}\mu_i^{y_i}}{y_i!} & , y_i > 0 \end{cases} \tag{2.31}$$

dimana μ_i adalah *logistic* fungsi link:

$$\ln \mu_i = \mathbf{x}_i^T \boldsymbol{\beta} \text{ atau } \mu_i = e^{\mathbf{x}_i^T \boldsymbol{\beta}} \tag{2.32}$$

dan p_i adalah *logistic* fungsi link yang didefinisikan $p_i = \frac{\lambda_i}{1+\lambda_i}$ dengan $\ln \lambda_i = \mathbf{x}_i^T \boldsymbol{\gamma}$ atau $\lambda_i = e^{\mathbf{x}_i^T \boldsymbol{\gamma}}$ sehingga:

$$p_i = \frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}}}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}}} \text{ dan } (1 - p_i) = \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}}} \tag{2.33}$$

$\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$ adalah parameter regresi yang akan ditaksir sedangkan **X** adalah matriks kovariat yang terdiri dari variabel prediktor yang mempengaruhi rata-rata poisson pada *Poisson state* dan matriks kovariat yang terdiri dari variabel prediktor yang mempengaruhi probabilitas pada *zero state*.

Model regresi ZIP adalah

$$\ln \mu_i = \mathbf{x}_i^T \boldsymbol{\beta}$$

$$\text{logit } p_i = \mathbf{x}_i^T \boldsymbol{\gamma}$$

Substitusi persaman (2.32) dan (2.33) ke persamaan (2.31) akan menghasilkan fungsi peluang regresi ZIP sebagai berikut:

$$P(Y_i = y_i) = \begin{cases} \dfrac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}} + \left(\exp\left(-e^{\mathbf{x}_i^T \boldsymbol{\beta}}\right)\right)}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}}} & \text{, untuk } y_i = 0 \\ \dfrac{\frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}}}\left(\exp\left(-e^{\mathbf{x}_i^T \boldsymbol{\beta}}\right)\right)\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}}\right)^{y_i}}{y_i!} & \text{, untuk } y_i > 0 \end{cases} \tag{2.34}$$

Nilai rata-rata dan varians dari Y_i adalah:

$$E(Y_i) = (1 - p_i)\mu_i$$

$$\begin{aligned} Var(Y_i) &= (1 - p_i)\left[\mu_i^2 + \mu_i\right] - (1 - p_i)^2 \mu_1^2 \\ &= E(Y_i)\ (1 + p_i \mu_i) \end{aligned}$$

2.8.1. Penaksir Parameter Regresi *Univariate Zero Inflated Poisson*

Penaksiran parameter pada model regresi ZIP menggunakan metode MLE dengan fungsi distribusi peluang Y_i diketahui (Lambert, 1992). Fungi *likelihood* dari model regresi ZIP adalah

$$L(\boldsymbol{\gamma},\boldsymbol{\beta})=\begin{cases}\prod_{I=1}^{n}\dfrac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}}+\exp\left(e^{-\mathbf{x}_i^T\boldsymbol{\beta}}\right)}{1+e^{\mathbf{x}_i^T g}} & ,y_i=0\\ \prod_{i=1}^{n}\dfrac{\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}}}\left(\exp\left(-e^{\mathbf{x}_i^T\boldsymbol{\gamma}}+\mathbf{x}_i^T\boldsymbol{\beta}\right)y_i\right)}{y_i!} & ,y_i>0\end{cases}$$

Fungsi ln likelihood

$$L(\boldsymbol{\gamma},\boldsymbol{\beta})=\begin{cases}\sum_{i=1}^{n}\ln\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}}+e^{-\mathbf{x}_i^T\boldsymbol{\beta}}\right)-\sum_{i=1}^{n}\ln\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}}\right) & ,y_i=0\\ \sum_{i=1}^{n}\left(-e^{\mathbf{x}_i^T\boldsymbol{\beta}}+\left(\mathbf{x}_i^T\boldsymbol{\beta}\right)y_i\right)-\sum_{i=1}^{n}\ln\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}}\right)-\sum_{i=1}^{n}y_i! & ,y_i>0\end{cases}$$

$$L(\boldsymbol{\gamma},\boldsymbol{\beta})=\sum_{y_i=1}^{n}\ln\left(e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\gamma}}+e^{-\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\beta}}\right)-\sum_{y_i=1}^{n}\ln\left(1+e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\gamma}}\right)+\sum_{y_i>1}^{n}\left(-e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\beta}}+\left(\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\beta}\right)y_i\right)-\sum_{y_i>1}^{n}y_i!$$

(2.35)

Persamaan (2.35) disebut *incomplete likelihood* karena nilai nol pada suku pertama tidak diketahui yang mana berasal dari *zero state* dan *poisson state* sehingga persamaan (2.35) diselesaikan dengan mendefinisikan kembali variabel Y_i dengan suatu variabel indikator Z_i yaitu:

$$Z_i=\begin{cases}1 & \text{, jika } y_i \text{ berasal dari zero state}\\ 0 & \text{, jika } y_i \text{ berasal dari poisson state}\end{cases}$$

Jika $y_i>0$ maka nilai $Z_i=0$, tetapi jika $y_i=0$ maka nilai Z_i dapat bernilai 0 atau 1. Untuk mengatasinya, dapat menggunakan algoritma Ekspektasi Maksimalisasi (EM). Algoritma EM merupakan salah satu metode alternatif iteratif untuk memaksimumkan fungsi *likelihood* dari data yang tidak lengkap (*missing*). Setiap iterasi algoritma, EM melalui dua tahap yaitu tahap Ekspektasi dan tahap Maksimalisasi. Tahap Ekspektasi yaitu tahap penghitungan ekspektasi dari fungsi ln *likelihood* dengan memperhatikan data yang tidak lengkap. Tahap Maksimalisasi yaitu tahap penghitungan untuk mencari penaksir parameter yang memaksimumkan fungsi ln *likelihood* dari tahap Ekspektasi sebelumnya.

Estimasi parameter Z_i akan dilakukan dengan algoritma EM, namun sebelumnya akan ditentukan distribusi dari Z_i yaitu

$$P(Z_i = z_i) = \begin{cases} \pi_i, \text{ jika } z_i = 0 \\ 1-\pi_i, \text{ jika } z_i > 0 \end{cases} \quad (2.36)$$

Pada saat $z_i = 0$, peluangnya sama dengan peluang Y_i yang berdistribusi *poisson* dengan parameter π_i yaitu sebesar $\left(1-\pi_i\right)$. Sehingga dapat dikatakan bahwa variabel Z_i berdistribusi Binomial $\left(1,\pi_i\right)$ dengan $E\left(Z_i\right)=\pi_i$ dan $Var\left(Z_i\right)=\pi_i\left(1-\pi_i\right)$. Bentuk distribusi gabungan antara Y_i dan Z_i adalah

$$f\left(y_i,z_i\right) = f\left(z_i\right) f\left(y_i \mid z_i\right) = f\left(z_i \mid 1,\pi_i\right) f\left(y_i \mid z_i,\pi_i\right)$$

$$= \left(\pi_i\right)^{z_i}\left(1-\pi_i\right)^{1-z_i}\left(\frac{\mu_i^{y_i} e^{-\mu_i}}{y_i!}\right)^{1-z_i} \quad (2.37)$$

Substitusikan persamaan (2.32) ke persamaan (2.37)

$$f\left(y_i,z_i\right)=\left(\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}}}\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}}\right)^{z_i}\left(\frac{e^{-\mathbf{x}_i^T\boldsymbol{\beta}}\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}}\right)^{y_i}}{y_i!}\right)^{1-z_i} \quad (2.38)$$

Bentuk fungsi *likelihood* dan ln *likelihood* dari persamaan (2.38)

$$L\left(\boldsymbol{\beta},\boldsymbol{\gamma}\right)=\prod_{i=1}^{n}\left[\left(\frac{1}{1+e^{\mathbf{x_i}^\mathbf{T}\boldsymbol{\gamma}}}\right)\left(e^{\mathbf{x_i}^\mathbf{T}\boldsymbol{\gamma}}\right)^{z_i}\left(\frac{e^{-\mathbf{x_i}^\mathbf{T}\boldsymbol{\beta}}\left(z_i\right)^{y_i}}{y_i!}\right)^{1-z_i}\right]$$

$$\ln L\left(\boldsymbol{\beta},\boldsymbol{\gamma}\right)=\sum_{i=1}^{n}\left(z_i\mathbf{x}_i^T\boldsymbol{\gamma}-\ln\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}}\right)\right)-\sum_{i=1}^{n}\left(1-z_i\right)\ln\left(y_i!\right)+\sum_{i=1}^{n}\left(1-z_i\right)\left(y_i\mathbf{x}_i^T\boldsymbol{\beta}-e^{\mathbf{x}_i^T\boldsymbol{\gamma}}\right)$$

(2.39)

Persamaan diatas disebut sebagai *completed likelihood*. Jika ditulis secara terpisah maka:

$$\ln L\left(\boldsymbol{\beta},\boldsymbol{\gamma}\right)=\ln L\left(\boldsymbol{\beta}\right)+\ln L\left(\boldsymbol{\gamma}\right)+\sum_{i=1}^{n} z_i \ln\left(y_i!\right) \text{ dimana}$$

$$\ln L\left(\boldsymbol{\beta}\right)=\sum_{i=1}^{n}\left(1-z_i\right)\left(y_i\mathbf{x_i}^\mathbf{T}\boldsymbol{\beta}-e^{\mathbf{x_i}^\mathbf{T}\boldsymbol{\beta}}\right) \text{dan } \ln L\left(\boldsymbol{\gamma}\right)=\sum_{i=1}^{n}\left(z_i\mathbf{x_i}^\mathbf{T}\boldsymbol{\gamma}-\ln\left(1+e^{\mathbf{x_i}^\mathbf{T}\boldsymbol{\gamma}}\right)\right) \quad (2.40)$$

Suku terakhir dari $\ln L\left(\boldsymbol{\beta},\boldsymbol{\gamma}\right)$ yaitu $\sum_{i=1}^{n} z_i \ln\left(y_i!\right)$ dapat diabaikan karena tidak mengandung vektor parameter $\boldsymbol{\gamma}$ dan $\boldsymbol{\beta}$.

Langkah-langkah dari algoritma EM:

1. Menghitung nilai ekspektasi dari Z_i

$$E\left(Z_i \mid y_i, \gamma^{(m)}, \beta^{(m)}\right) = Z_i^{(m)}$$

$$Z_i^{(m)} = P\left(Z_i = 1 \mid y_i, \gamma^{(m)}, \beta^{(m)}\right)$$

$$= \begin{cases} P\left(Z_i = 1 \mid y_i, \gamma^{(m)}, \beta^{(m)}\right), \text{untuk } y_i = 0 \\ 0 \qquad\qquad\qquad\qquad\quad , \text{untuk } y_i > 0 \end{cases}$$

Selanjutnya substitusikan hasil ekspektasi tersebut pada persamaan (2.40) sehingga:

$$\ln L\left(\boldsymbol{\beta} \mid \mathbf{y}, \mathbf{Z}^{(m)}\right) = \sum_{i=1}^{n}\left(1 - Z_i^{(m)}\right)\left(y_i \mathbf{x_i}^{\mathbf{T}}\boldsymbol{\beta}\right) \tag{2.41}$$

$$\ln L\left(\boldsymbol{\gamma} \mid \mathbf{y}, \mathbf{Z}^{(m)}\right) = \sum_{i=1}^{n}\left(Z_i^{(m)}\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\gamma} - \ln\left(1 + e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\gamma}}\right)\right) \tag{2.42}$$

2. Maksimalisasi β dengan metode iteratif Newton-Raphson untuk memperoleh $\boldsymbol{\beta}^{(m+1)}$ yang memaksimumkan persamaan (2.41). Vektor gradien (g) dan matriks Hessian (H) yaitu

   $g^{T(m)} = \mathbf{X}^T\mathbf{S}^{(m)}\mathbf{TX}$

   $\mathbf{S}^{(\mathbf{m})}$ adalah matriks diagonal dengan $\left(1\text{-} Z_i^{(m)}\right)$ sebagai elemen diagonal utama dan $\mathbf{T}$ adalah matriks diagonal dengan $\boldsymbol{\mu}$ sebagai diagonal utamanya.

3. Memaksimalisasi γ pada persamaan (2.42), dimana untuk setiap $y_i > 0$ nilai $Z_i^{(m)} = 0$ sehingga menjadi:

$$\ln L\left(\boldsymbol{\gamma} \mid \mathbf{y}, \mathbf{Z}^{(m)}\right) = \sum_{y_i=0}^{y_i=n_0} Z_i^{(m)}\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\gamma} - \sum_{y_i=0}^{y_i=n_0} Z_i^{(m)}\ln\left(1 + e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\gamma}}\right) - \sum_{y_i=0}^{y_i=n_0}\left(1 - Z_i^{(m)}\right)\ln\left(1 + e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\gamma}}\right) \tag{2.43}$$

   Misalkan y_i sampai dengan y_{n_0} adalah 0 atau dapat ditulis $y_1, y_2, \ldots, y_{n_0} = 0$ kemudian didefinisikan matriks diagonal $\mathbf{v}^{(m)}$ dengan elemen diagonal utama:

   $\mathbf{v}_*^T = \left(1 - Z_1^{(m)}, 1 - Z_2^{(m)}, \ldots, 1 - Z_{n+1}^{(m)}, 1 - Z_{n+n_0}^{(m)}\right)$

Didefinisikan juga

$\mathbf{y}_*^T = \left(y_1, y_2, \ldots, y_n, y_{n+1}, y_{n+n_0}\right)$, $\mathbf{x}_*^T = \left(1, x_1^T, x_2^T, \ldots, x_k^T\right)$

$\boldsymbol{\pi}_*^T = \left(\pi_1, \pi_2, \ldots, \pi_n, \pi_{n+1}, \pi_{n+n_0}\right)$

$\mathbf{x}_*^T$ berukuran $\left(k+1\right)$ sehingga dapat ditulis sebagai berikut:

$$\ln L\left(\boldsymbol{\gamma} \mid \mathbf{y}, \mathbf{Z}^{(m)}\right) = \sum_{i=1}^{n+n_0} y_{*i} v_*^{(m)} \mathbf{x}_{*i}^{\mathbf{T}} \boldsymbol{\gamma} - \sum_{i=1}^{n+n_0} v_*^{(m)} \ln\left(1 + e^{\mathbf{x}_{*i}^{\mathbf{T}} \gamma}\right) \tag{2.44}$$

Vektor gradient $(\mathbf{g})$ dan matriks $(\mathbf{H})$ yaitu:

$\mathbf{g}^T = \mathbf{x}^T \mathbf{v}^{(m)} \left(\mathbf{y}_* - \boldsymbol{\pi}_*\right)$ dan $\mathbf{H} = \mathbf{x}_*^T \mathbf{v}^{(m)} \mathbf{Q}_* \mathbf{x}_*$

Dimana $\mathbf{Q}_*$ adalah matriks diagonal dengan elemen diagonal utamanya $\pi_i\left(1-\pi_i\right)$. Langkah-langkah untuk memperoleh $\boldsymbol{\gamma}^{(m+1)}$ ini identik dengan metode iteratif Newton-Raphson, seperti yang dilakukan pada langkah untuk memaksimalkan $\boldsymbol{\beta}$ dengan $\mathbf{y}_*$ sebagai respon dan $\mathbf{x}_*$ sebagai matriks variabel, dan $\mathbf{v}^{(m)}$ sebagai matriks pembobot.

4. Mengganti $\boldsymbol{\beta}$ dan γ dengan $\hat{\boldsymbol{\beta}}^{(m+1)}$ dan $\hat{\boldsymbol{\gamma}}^{(m+1)}$, kemudian lakukan kembali langkah tahap 1 (tahap Ekspektasi)
5. Tahap Ekspektasi dan Maksimalisasi dilakukan secara terus menerus sampai memperoleh penaksir parameter yang konvergen.

2.8.2. Pengujian Parameter Regresi *Univariat Zero Inflated Poisson*

Pengujian parameter pada regresi ZIP dilakukan dengan menggunakan *Maximum Likelihood Ratio Test* (MLRT) sebagai berikut:

1. Pengujian serentak $\boldsymbol{\beta}$

   Parameter yang diuji mencakup seluruh $\boldsymbol{\beta}$ secara bersama-sama dengan hipotesis:

   $\mathrm{H}_0 = \beta_1 = \beta_2 = \ldots = \beta_{\boldsymbol{k}} = 0$

   H_1 : paling sedikit ada satu $\beta_j \neq 0$ dengan $j = 1, 2, \ldots, k$

Sedangkan statistik uji yang digunakan adalah ukuran statistik *likelihood ratio* (*devians*) yang dibentuk dengan menentukan himpunan parameter dibawah populasi (Ω) yaitu $\boldsymbol{\Omega} = \{\boldsymbol{\beta}, \boldsymbol{\gamma}\}$ dan himpunan parameter di bawah H_0 benar (ω) yaitu $\omega = \{\beta_0, \boldsymbol{\gamma}\}$. Masing-masing fungsi *likelihood* $L(\Omega)$ dan $L(\omega)$ yaitu:

$$L(\Omega) = \prod_{i=1}^{n} \left(\left(\frac{1}{1 + e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\gamma}}} \right) \left(e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\gamma}} \right)^{z_i} \left(\frac{e^{-\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\beta}} \left(e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\beta}} \right)^{y_i}}{y_i!} \right)^{1-z_i} \right)$$

$$L(\omega) = \prod_{i=1}^{n} \left(\left(\frac{1}{1 + e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\gamma}}} \right) \left(e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\gamma}} \right)^{z_i} \left(\frac{e^{-\beta_0} \left(e^{\beta_0} \right)^{y_i}}{y_i!} \right)^{1-z_i} \right)$$

Selanjutnya $\hat{\boldsymbol{\beta}}$ dan $\hat{\boldsymbol{\gamma}}$ yang merupakan hasil estimasi parameter pada sub bab 2.2.2 disubstitusikan pada kedua fungsi *likelihood* tersebut serta dibandingkan dalam bentuk devians D

Dimana D berdistribusi $\chi^2_{(k)}$ sehingga pada taraf signifikansi α tolak H_0 jika nilai $\left| D_{hitung} \right| > \chi^2_{(k,\alpha)}$.

2. Pengujian serentak $\boldsymbol{\gamma}$

   Parameter yang diuji mencakup seluruh $\boldsymbol{\gamma}$ secara bersama-sama

   $\text{H}_0 : \gamma_1 = \gamma_2 = ... = \gamma_{\boldsymbol{k}} = 0$

   H_1 : paling sedikit ada satu $\gamma_{\boldsymbol{j}} \neq 0$ dengan $j = 1, 2, \ldots, k$

   Sedangkan statistik uji yang digunakan adalah ukuran statistik *likelihood ratio* (*devians*) yang dibentuk dengan menentukan himpunan parameter dibawah populasi (Ω) yaitu $\boldsymbol{\Omega} = \{\boldsymbol{\beta}, \boldsymbol{\gamma}\}$ dan himpunan parameter di bawah H_0 benar (ω) yaitu $\omega = \{\boldsymbol{\beta}, \gamma_0\}$. Masing-masing fungsi *likelihood* $L(\Omega)$ dan $L(\omega)$ yaitu:

$$L(\Omega)=\prod_{i=1}^{n}\left(\left(\frac{1}{1+e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\gamma}}}\right)\left(e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\gamma}}\right)^{z_i}\left(\frac{e^{-\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\beta}}\left(e^{\mathbf{x_i}^{\mathbf{T}}\boldsymbol{\beta}}\right)^{y_i}}{y_i!}\right)^{1-z_i}\right)$$

$$L(\omega)=\prod_{i=1}^{n}\left(\left(\frac{1}{1+e^{\gamma_0}}\right)\left(e^{\gamma_0}\right)^{z_i}\left(\frac{e^{-\beta_0}\left(e^{\beta_0}\right)^{y_i}}{y_i!}\right)^{1-z_i}\right)$$

Selanjutnya $\hat{\boldsymbol{\beta}}$ dan $\hat{\boldsymbol{\gamma}}$ yang merupakan hasil estimasi parameter pada sub bab 2.2.2 disubstitusikan pada kedua fungsi *likelihood* tersebut serta dibandingkan dalam bentuk devians D

Dimana D berdistribusi $\chi^2_{(k)}$ sehingga pada taraf signifikansi α tolak H_0 jika nilai $\left|D_{hitung}\right|>\chi^2_{(k,\alpha)}$ dengan k adalah jumlah parameter di bawah populasi.

3. Pengujian parsial $\boldsymbol{\beta}$

Parameter yang diuji mencakup seluruh $\boldsymbol{\beta}$ secara bersama-sama dengan hipotesis:

$\mathrm{H}_0:\beta_j=0$

$\mathrm{H}_1:\beta_j\neq 0$ dengan $j=1,2,\ldots,k$

Sedangkan statistik uji yang digunakan adalah ukuran statistik *likelihood ratio* (devians) yang dibentuk dengan menentukan himpunan parameter dibawah populasi (Ω) yaitu $\boldsymbol{\Omega}=\{\boldsymbol{\beta},\boldsymbol{\gamma}\}$ dan himpunan parameter di bawah H_0 benar (ω) yaitu $\omega=\{\boldsymbol{\beta}^*,\boldsymbol{\gamma}\}$, $\boldsymbol{\beta}^*$ adalah $\boldsymbol{\beta}$ dengan $\beta_j=0$.

Selanjutnya $\hat{\boldsymbol{\beta}}$ dan $\hat{\boldsymbol{\gamma}}$ yang merupakan hasil estimasi parameter pada sub bab 2.2.2 disubstitusikan pada kedua fungsi *likelihood* tersebut serta dibandingkan dalam bentuk devians D .

Dimana D berdistribusi $\chi^2_{(1)}$ sehingga pada taraf signifikansi α tolak H_0 jika nilai $\left|D_{hitung}\right|>\chi^2_{(1,\alpha)}$.

4. Pengujian parsial $\boldsymbol{\gamma}$

   $\text{H}_0 : \gamma_j = 0$

   $\text{H}_1 : \gamma_j \neq 0$ dengan $j = 1, 2, \ldots, k$

   Sedangkan statistik uji yang digunakan adalah ukuran statistik *likelihood ratio* (devians) yang dibentuk dengan menentukan himpunan parameter dibawah populasi (Ω) yaitu $\boldsymbol{\Omega} = \{\boldsymbol{\beta}, \boldsymbol{\gamma}\}$ dan himpunan parameter di bawah H_0 benar (ω) yaitu $\omega = \{\boldsymbol{\beta}, \boldsymbol{\gamma}^*\}$, $\boldsymbol{\gamma}^*$ adalah $\boldsymbol{\gamma}$ dengan $\gamma_j = 0$. Selanjutnya $\hat{\boldsymbol{\beta}}$ dan $\hat{\boldsymbol{\gamma}}$ yang merupakan hasil estimasi parameter pada sub bab 2.2.2 disubstitusikan pada kedua fungsi *likelihood* tersebut serta dibandingkan dalam bentuk devians D, tolak H_0 jika nilai $D_{hitung} > \chi^2_{(1,\alpha)}$.

2.9. Regresi Bivariate Zero Inflated Poisson (BZIP)

Menurut Yuen, dkk, 2015, distribusi BZIP (Y_1, Y_2) dibentuk dari gabungan point mass (0,0), dua distribusi poisson univariat dengan parameter λ_1 dan λ_2 dan distribusi *Bivariate Poisson* dengan parameter λ_1, λ_2, dan λ_0 sebagai berikut:

(Y_1, Y_2) ~ $(0,0)$ dengan peluang p_0

~ $(Poisson(\lambda_1), 0)$ dengan peluang p_1

~ $(0, Poisson(\lambda_2))$ dengan peluang p_2

~ $(Poisson \text{ bivariat}(\lambda_{10}, \lambda_{20}, \lambda_{00}))$ dengan peluang $p_{11} = 1 - p_0 - p_1 - p_2$

Fungsi distribusi peluang dari distribusi BZIP (Y_1, Y_2) dinyatakan dalam bentuk sebagai berikut:

$$P(Y_1 = 0, Y_2 = 0) = p_0 + p_1 e^{-\lambda_1} + p_2 e^{-\lambda_2} + p_{11} e^{-\lambda_0} \tag{2.45}$$

$$P(Y_1 = y_1, Y_2 = 0) = \frac{p_1 \lambda_1^{y_1} e^{-\lambda_1} + p_{11} \lambda_{10}^{y_1} e^{-\lambda_0}}{y_1!} \tag{2.46}$$

$$P(Y_1=0,Y_2=y_2)=\frac{p_2\lambda_2^{y_2}e^{-\lambda_2}+p_{11}\lambda_{20}^{y_2}e^{-\lambda_0}}{y_2!} \tag{2.47}$$

$$P(Y_1=0,Y_2=y_2)=p_{11}\sum_{k=0}^{\min(y_1,y_2)}\frac{\lambda_{10}^{y_1-k}\lambda_{20}^{y_2-k}\lambda_{00}^{k}}{(y_1-k)!(y_2-k)!k!}e^{-\lambda_0} \tag{2.48}$$

untuk $y_1, y_2 = 1, 2, ..., n$ dan $\lambda = \lambda_{10}+\lambda_{20}+\lambda_{00}$. Secara ringkas, persaman (2.45) sampai (2.48) dapat dituliskan sebegai berikut:

$$P(Y_1=y_1,Y_2=y_2)$$

$$\begin{cases} p_0+p_1\exp(-\lambda_1)+p_2\exp(\lambda_2)+p_{11}\exp(-\lambda); & y_1=0, y_2=0 \\ \left(\dfrac{p_l\lambda_l^{y_l}\exp(-\lambda_l)+p_{11}\lambda_{10}^{y_l}\exp(-\lambda)}{y_l!}\right); y_l>0\ ; y_{l^*\neq l}=0; \ l,l^*=1,2 \\ p_{11}\displaystyle\sum_{k=0}^{\min(y_1,y_2)}\frac{\lambda_{10}^{y_1-k}\lambda_{20}^{y_2-k}\lambda_{00}^{k}}{(y_1-k)!(y_2-k)!k!}\exp(-\lambda); & y_1>0,\ y_2>0 \end{cases} \tag{2.49}$$

Rata-rata Poisson dapat dituliskan sebagai logaritma dari fungsi kovariat dengan koefisien β yang dinyatakan dalam persamaan:

$$\begin{aligned} &\lambda_1=\exp\left(x_1^T\beta_1\right),\quad \lambda_2=\exp\left(x_2^T\beta_2\right),\quad \lambda_0=\exp(\beta_0) \\ &p_1=p_{11}\exp\left(x_1^T\gamma_1\right),\quad p_2=p_{11}\exp\left(x_2^T\gamma_2\right),\quad p_0=p_{11}\exp(\gamma_0) \\ &p_{11}=\left\{1+\exp\left(x_1^T\gamma_1\right)+\exp\left(x_2^T\gamma_2\right)+\exp(\gamma_0)\right\}^{-1} \end{aligned} \tag{2.50}$$

Substitusi persaman (2.50) ke persamaan (2.45) sampai (2.48) akan menghasilkan fungsi peluang regresi BZIP sebagai berikut:

$$P(Y_1=0,Y_2=0)=p_{11}\exp(\boldsymbol{\gamma}_0)+\left(p_{11}\exp\left(\mathbf{x}_1^T\boldsymbol{\gamma}_1\right)\right)\exp\left(-\exp\left(\mathbf{x}_1^T\boldsymbol{\beta}_1\right)\right)+E_1+E_2 \tag{2.51}$$

dimana $\mathrm{E}_1=\left(p_{11}\exp\left(\mathbf{x}_2^T\boldsymbol{\gamma}_2\right)\right)\exp\left(-\exp\left(\mathbf{x}_2^T\boldsymbol{\beta}_2\right)\right)$

$\mathrm{E}_2=p_{11}\exp\left[-\exp\left(\mathbf{x}_1^T\boldsymbol{\beta}_1\right)-\exp\left(\mathbf{x}_2^T\boldsymbol{\beta}_2\right)-\exp(\boldsymbol{\beta}_0)\right]$

$$P(Y_1=y_1,Y_2=0)=\frac{\left(p_{11}\exp\left(\mathbf{x}_1^T\boldsymbol{\gamma}_1\right)\right)\left(\exp\left(\mathbf{x}_1^T\boldsymbol{\beta}_1\right)\right)^{y_1}\exp\left(-\exp\left(\mathbf{x}_1^T\boldsymbol{\beta}_1\right)\right)}{y_1!}+E_3 \tag{2.52}$$

dimana $E_3 = \dfrac{p_{11}\left(\exp\left(\mathbf{x}_1^T\boldsymbol{\beta}_1\right)\right)^{y_1}\exp\left(-\exp\left(\mathbf{x}_1^T\boldsymbol{\beta}_1\right)-\exp\left(\mathbf{x}_2^T\boldsymbol{\beta}_2\right)-\exp\left(\boldsymbol{\beta}_0\right)\right)}{y_1!}$

$$P\left(Y_1=0, Y_2=y_2\right)=\frac{\left(p_{11}\exp\left(\mathbf{x}_2^T\boldsymbol{\gamma}_2\right)\right)\left(\exp\left(\mathbf{x}_2^T\boldsymbol{\beta}_2\right)\right)^{y_2}\exp\left(-\exp\left(\mathbf{x}_2^T\boldsymbol{\beta}_2\right)\right)}{y_2!}+E_4 \tag{2.53}$$

dimana $E_4 = \dfrac{p_{11}\left(\exp\left(\mathbf{x}_2^T\boldsymbol{\beta}_2\right)\right)^{y_2}\exp\left(-\exp\left(\mathbf{x}_1^T\boldsymbol{\beta}_1\right)-\exp\left(\mathbf{x}_2^T\boldsymbol{\beta}_2\right)-\exp\left(\boldsymbol{\beta}_0\right)\right)}{y_2!}$

$$P\left(Y_1=y_1, Y_2=y_2\right)=p_{11}\sum_{k=0}^{\min(y_1,y_2)}\frac{\exp\left(\mathbf{x}_1^T\boldsymbol{\beta}_1\right)^{y_1-k}\exp\left(\mathbf{x}_2^T\boldsymbol{\beta}_2\right)^{y_2-k}\exp\left(\boldsymbol{\beta}_0\right)^{k}}{\left(y_1-k\right)!\left(y_2-k\right)!k!}*E_5 \tag{2.54}$$

dimana $E_5 = \exp\left(-\exp\left(\mathbf{x}_1^T\boldsymbol{\beta}_1\right)-\exp\left(\mathbf{x}_2^T\boldsymbol{\beta}_2\right)-\exp\left(\boldsymbol{\beta}_0\right)\right)$

$p_{11}=\left\{1+\exp\left(\mathbf{x}_1^T\boldsymbol{\gamma}_1\right)+\exp\left(\mathbf{x}_2^T\boldsymbol{\gamma}_2\right)+\exp\left(\boldsymbol{\gamma}_0\right)\right\}^{-1}.$

Nilai rata-rata, varians dan kovarians dari BZIP adalah:

$E\left(Y_l\right)=\left(1-p_l\right)\left(\lambda_l+\lambda_0\right), Var\left(Y_l\right)=E\left(Y_l\right)\left[1+p\left(\lambda_l+\lambda_0\right)\right], l=1,2$

$E\left(Y_1,Y_2\right)=\left(1-p\right)\left[\left(\lambda_1+\lambda_0\right)\left(\lambda_2+\lambda_0\right)+\lambda_0\right]$

$Cov\left(Y_1,Y_2\right)=\left(1-p\right)\left[\left(\lambda_0+p\left(\lambda_1+\lambda_0\right)\left(\lambda_2+\lambda_0\right)\right)\right]$

2.10. Regresi Univariate Zero Inflated Generalized Poisson

Banyaknya nilai nol pada data dapat mengakibatkan terjadinya overdispersi yang menyebabkan regresi ZIP menjadi kurang akurat karena nilai *standar error* menjadi *underestimate.* Famoye dan Singh (2006) menyarankan penggunaan model regresi *Zero-Inflated Generalized Poisson* (ZIGP) untuk mengatasi kasus overdipersi akibat banyaknya data yang bernilai nol (*zero inflation*). Model regresi ZIGP ini akan lebih baik dibandingkan model regresi ZIP jika terdapat minimal 65,7 persen data yang bernilai nol (Famoye dan Singh, 2006).

Czado, dkk (2007) mendefinisikan distribusi ZIGP sebagai analog dari distribusi ZIP dimana terdapat tambahan parameter p yang merupakan parameter *zero inflation.* Dengan demikian, distribusi ini memiliki tiga parameter yaitu $\lambda, \varphi,$ dan p dan dilambangkan dengan $Y \sim ZIGP\left(\lambda, \varphi, p\right)$. Dengan mengacu pada persaman (2.17) yang merupakan model *restricted* GPD yang dikembangkan oleh Famoye (1993), maka fungsi kepadatan peluang distribusi ZIGP adalah:

$$P\left(Y = y \mid \lambda, \varphi, p_l\right) = \begin{cases} p_l + \left(1 - p_l\right) e^{-\lambda} & \text{, untuk } y = 0 \\ \dfrac{\left(1 - p_l\right)\left(1 + \varphi y\right)^{y-1} \lambda^y e^{-\lambda(1+\varphi y)}}{y!} & \text{, untuk } y > 0 \end{cases} \tag{2.55}$$

dimana $\lambda > \max\left(\dfrac{1}{2}, 1 - \dfrac{\varphi}{m}\right)$ dan m merupakan bilangan bulat terbesar dengan $\varphi + m\left(\lambda - 1\right) > 0$ jika $\lambda < 1$.

Distribusi ZIGP merupakan gabungan dari distribusi ZIP dengan distribusi GP yang didefinisikan oleh Famoye dan Singh (2006) dengan fungsi kepadatan peluang sebagai berikut:

$$P\left(Y_i = y_i\right) = \begin{cases} p_i + \left(1 - p_i\right) P\left(Y_i = y_i \middle| \mu_i, \varphi\right) & \text{, untuk } y_i = 0 \\ \left(1 - p_i\right) P\left(Y_i = y_i \middle| \mu_i, \varphi\right) & \text{, untuk } y > 0 \end{cases} \tag{2.56}$$

dimana $P\left(Y_i = y_i \mid \mu_i, \varphi\right)$ merupakan *restricted* GPD pada persamaan (2.20). Penjabaran distribusi ZIGP pada persamaan (2.56) dapat dituliskan seperti berikut:

$$P\left(Y_i = y_i\right) = \begin{cases} p_i + \left(1 - p_i\right) e^{\left(\frac{-\mu_i}{1+\varphi\mu_i}\right)} & \text{, untuk } y_i = 0 \\ \left(1 - p_i\right)\left(\dfrac{\mu_i}{1 + \varphi\mu_i}\right)^{y_i} \dfrac{\left(1 + \varphi y_i\right)^{y_i - 1}}{y_i!} e^{\left(\frac{-\mu_i\left(1+\varphi y_i\right)}{1+\varphi\mu_i}\right)} & \text{, untuk } y_i > 0 \end{cases} \tag{2.57}$$

dimana $0 < p_i < 1$.

Fungsi $\mu_i = \mu_i\left(\mathbf{x}_i\right)$ dan $p_i = p_i\left(x_i\right)$ dalam persamaan (2.57) memenuhi:

$$\log\left(\mu_i\right) = \mathbf{x}_i^T \boldsymbol{\beta} \text{ dan } \operatorname{logit}\left(p_i\right) = \log\left(\frac{p_i}{1 - p_i}\right) = \mathbf{x}_i^T \boldsymbol{\gamma} \tag{2.58}$$

Dari persamaan (2.58) diperoleh:

$$\mu_i = \exp(\mathbf{x}_i^T\boldsymbol{\beta}),\; p_i = \frac{\exp(\mathbf{x}_i^T\boldsymbol{\gamma})}{1+\exp(\mathbf{x}_i^T\boldsymbol{\gamma})},\; (1-p_i) = \frac{1}{1+\exp(\mathbf{x}_i^T\boldsymbol{\gamma})} \tag{2.59}$$

Rata-rata dan varian dari distribusi ZIGP pada persamaan (2.57) adalah

$$E(Y_i) = (1-p_i)\mu_i,\; Var(Y_i) = E(Y_i)\left[(1+\varphi\mu_i)^2 + p_i\mu_i\right] \tag{2.60}$$

Model ZIGPR akan menjadi model GPR ketika $p_i = 0$ dan akan menjadi model ZIPR seperti yang didefinisikan oleh Lambert (1992) ketika $\varphi = 0$. Nilai p_i yang positif menggambarkan model regresi ZIGP sementara nilai p_i yang negatif menggambarkan model regresi *zero-deflated generalized Poisson.* Namun, dalam prakteknya kasus *zero-deflation* ini jarang terjadi (Famoye dan Singh, 2006).

Jika p_i dan μ_i dipengaruhi oleh matriks kovariat yang sama, maka p_i merupakan fungsi dari μ_i. Model regresi ZIGP dengan log link untuk μ_i dan logit link untuk p_i sebagaimana di definisikan pada persamaan (2.59), akan dituliskan sebagai $ZIGP(\tau)$. Ketika $\tau > 0$ maka kecil kemungkinan untuk *zero state* terjadi dan ketika $\tau < 0$ maka besar kemungkinan untuk *zero state* terjadi (Famoye dan Singh, 2006).

Dengan mensubstitusikan persamaan (2.59) ke persamaan (2.57), maka pdf dari model ZIGPR adalah sebagai berikut:

$$P(Y_i = y_i) = \begin{cases} \dfrac{\exp(\mathbf{x}_i^T\boldsymbol{\gamma})}{1+\exp(\mathbf{x}_i^T\boldsymbol{\gamma})} + \dfrac{1}{1+\exp(\mathbf{x}_i^T\boldsymbol{\gamma})}\exp\left(\dfrac{-\exp(\mathbf{x}_i^T\boldsymbol{\beta})}{1+\varphi\exp(\mathbf{x}_i^T\boldsymbol{\beta})}\right) \\ \text{, untuk } y_i = 0 \\ \dfrac{1}{1+\exp(\mathbf{x}_i^T\boldsymbol{\gamma})}\left(\dfrac{\exp(\mathbf{x}_i^T\boldsymbol{\beta})}{1+\varphi\exp(\mathbf{x}_i^T\boldsymbol{\beta})}\right)^{y_i}\dfrac{(1+\varphi y_i)^{y_i-1}}{y_i!} \\ \exp\left(\dfrac{-\exp(\mathbf{x}_i^T\boldsymbol{\beta})(1+\varphi y_i)}{1+\varphi\exp(\mathbf{x}_i^T\boldsymbol{\beta})}\right) \quad \text{, untuk } y_i > 0 \end{cases} \tag{2.61}$$

Pengujian parameter dapat dilakukan baik secara serentak maupun secara parsial. Uji serentak dilakukan untuk mengetahui parameter regresi yang berpengaruh signifikan terhadap model secara bersama-sama. Hipotesis yang digunakan adalah:

$H_0 : \beta_1 = \beta_2 = ... = \beta_q = 0$

$H_1 :$ minimal ada satu parameter $\beta_r \neq 0; \quad r = 1, 2, ..., q$

dengan statistik uji sebagai berikut:

$$D = -2\left(\ln L\left(\hat{\Omega}\right) - \ln L\left(\hat{\omega}\right)\right)$$

Tolak H_0 jika $D_{hitung} > \chi^2_{\alpha,k}$ atau jika $p - value < \alpha$ maka uji parsial dilakukan untuk menguji signifikansi parameter secara individu.

2.11. Distribusi Bivariate Zero Inflated Generalized Poisson

Zhang dkk (2015) memperkenalkan distribusi bivariat ZIGP untuk menangani masalah overdispersi ketika terdapat banyak nilai nol pada data bivariat yang diamati. Untuk memodelkan banyaknya nilai nol pada data, Zhang, dkk., (2015) membentuk distribusi bivariat ZIGP melalui gabungan antara distribusi Bernoulli dan distribusi bivariat GP dengan $Z_l,\ l = 1, 2$ mengikuti distribusi Bernoulli $(1 - p_l)$ dan $W_l,\ l = 1, 2$ mengikuti distribusi bivariat GP (λ_l, φ_l). Dengan demikian, $Y_l = Z_l W_l$ akan mengikuti distribusi ZIGP $(p_l, \lambda_l, \varphi_l)$ untuk $l = 1, 2$.

Misalkan $(Y_1, Y_2)^T \sim ZIGP(p_1, p_2, \lambda_1, \lambda_2, \varphi_1, \varphi_2, \eta)$ maka peluang bersama dari $(Y_1, Y_2)^T$ adalah $P(Y_1 = y_1, Y_2 = y_2) = P(Z_1 X_1 = y_1, Z_2 X_2 = y_2)$ dengan beberapa kombinasi sebagai berikut (Zhang dkk, 2015):

1. Jika $y_1 = 0$ dan $y_2 = 0$ maka $P(Y_1 = 0, Y_2 = 0)$:

$$\begin{aligned} &= P(Z_1 = 0, Z_2 = 0) + P(Z_1 = 0, Z_2 = 1, X_2 = 0) + P(Z_1 = 1, X_1 = 1, Z_2 = 0) + \\ &\quad P(Z_1 = 1, X_1 = 0, Z_2 = 1, X_2 = 0) \\ &= p_1 p_2 + p_1(1 - p_2)e^{-\lambda_2} + p_2(1 - p_1)e^{-\lambda_1} + (1 - p_1)(1 - p_2)e^{-(\lambda_1 + \lambda_2)}G_1 \end{aligned} \qquad (2.62)$$

2. Jika $y_1 = 0$ dan $y_2 > 0$ maka $P(Y_1 = 0, Y_2 = y_2)$:

$$= P(Z_1 = 0, Z_2 = 1, X_2 = y_2) + P(Z_1 = 1, X_1 = 0, Z_2 = 1, X_2 = y_2)$$

$$=\left(1-p_2\right)\frac{\lambda_2^{y_2}\left(1+\varphi_2 y_2\right)^{y_2-1}}{y_2!}\exp\left[-\lambda_2\left(1+\varphi_2 y_2\right)\right]\left(p_1+\left(1-p_1\right)e^{-\lambda_1}\right)G_2 \tag{2.63}$$

3. Jika $y_1>0$ dan $y_2=0$ maka $P\left(Y_1=y_1,Y_2=0\right)$:

$$=P\left(Z_1=1,X_1=y_1,Z_2=0\right)+P\left(Z_1=1,X_1=y_1,Z_2=1,X_2=0\right)$$

$$=\left(1-p_1\right)\frac{\lambda_1^{y_1}\left(1+\varphi_1 y_1\right)^{y_1-1}}{y_1!}\exp\left[-\lambda_1\left(1+\varphi_1 y_1\right)\right]\left(p_2+\left(1-p_2\right)e^{-\lambda_2}\right)G_3 \tag{2.64}$$

4. Jika $y_1>0$ dan $y_2>0$ maka $P\left(Y_1=y_1,Y_2=y_2\right)$:

$$=P\left(Z_1=1,X_1=y_1,Z_2=1,X_2=y_2\right)$$

$$=\left(1-p_1\right)\left(1-p_2\right)\prod_{l=1}^{2}\frac{\lambda_l^{y_l}\left(1+\varphi_l y_l\right)^{y_l-1}}{y_l!}\exp\left[-\lambda_l\left(1+\varphi_l y_l\right)\right]G_4 \tag{2.65}$$

dimana $G_1=1+\eta\left(1-g_1\right)\left(1-g_2\right)$, $G_2=\left[1+\eta\left(1-g_1\right)\left(e^{-y_2}-g_2\right)\right]$,

$$G_3=\left[1+\eta\left(e^{-y_1}-g_1\right)\left(1-g_2\right)\right],\; G_4=\left[1+\eta\left(e^{-y_1}-g_1\right)\left(e^{-y_2}-g_2\right)\right]$$

Peluang bersama dari Y_1 dan Y_2 di atas dapat dituliskan seperti berikut:

1. Jika $y_1=y_2=0$ maka $P\left(Y_1=0,Y_2=0\right)$:

$$p_1p_2+p_1\left(1-p_2\right)e^{-\lambda_2}+p_2\left(1-p_1\right)e^{-\lambda_1}+\left(1-p_1\right)\left(1-p_2\right)e^{-\lambda_1-\lambda_2}G_1 \tag{2.66}$$

2. Jika $y_1=0,y_2>0$ maka $P\left(Y_1=0,Y_2=y_2\right)$

$$\left(1-p_2\right)GP\left(y_2\mid\lambda_2,\varphi_2\right)\left(p_1+\left(1-p_1\right)e^{-\lambda_1}G_2\right) \tag{2.67}$$

3. Jika $y_1>0,y_2=0$ maka $P\left(Y_1=y_1,Y_2=0\right)$

$$\left(1-p_1\right)GP\left(y_1\mid\lambda_1,\varphi_1\right)\left(p_2+\left(1-p_2\right)e^{-\lambda_2}G_3\right) \tag{2.68}$$

4. Jika $y_1>0,y_2>0$ maka $P\left(Y_1=y_1,Y_2=y_2\right)$

$$\left(1-p_1\right)\left(1-p_2\right)GP\left(y_1\mid\lambda_1,\varphi_1\right)GP\left(y_2\mid\lambda_2,\varphi_2\right)G_4 \tag{2.69}$$

dimana

$g_l=E\left(e^{-Y_l}\right)=\exp\left[\lambda_l\left(t_l-1\right)\right]$ dengan $\ln t_l-\varphi_l\lambda_l\left(t_l-1\right)+1=0;\quad l=1,2$

$GP\left(y_l\mid\lambda_l,\varphi_l\right)$ adalah distribusi GPD pada persamaan (2.17), p_1 dan p_2 adalah parameter *zero inflation*. $p_1=0$ mengindikasikan bahwa hanya Y_2 yang mengikuti distribusi ZIGP. Sebaliknya, $p_2 = 0$ mengindikasikan bahwa hanya Y_1 yang mengikuti distribusi ZIGP.

Rata-rata, varians dan kovarians dari variabel Y adalah sebagai berikut:

$$E(Y_l) = (1-p_l)\frac{\lambda_l}{1-\varphi_l\lambda_l} \sim (1-p_l)\mu_l, Var(Y_l) = (1-p_l)\left[\frac{\lambda_l}{(1-\varphi_l\lambda_l)^3} + p_l\mu_l^2\right], l = 1, 2$$

$$Cov(Y_1, Y_2) = (1-p_1)(1-p_2)\eta(g_{11} - g_1\mu_1)(g_{22} - g_2\mu_2)$$

dimana $g_{ll} = E\left(Y_l e^{-Y_l}\right) = \lambda_l (1-\varphi_l\lambda_l t_l)^{-1} \exp\left(\lambda_l (1+\varphi_l)(t_l - 1) - 1\right)$.

Koefisien korelasi antara Y_1 dan Y_2 adalah:

$$\rho = corr(Y_1, Y_2) = \frac{\sqrt{(1-p_1)(1-p_2)}\,\eta(g_{11} - g_1\mu_1)(g_{22} - g_2\mu_2)}{\sqrt{\left(\frac{\lambda_1}{(1-\varphi_1\lambda_1)^3 + p_1\mu_1^2}\right)\left(\frac{\lambda_2}{(1-\varphi_2\lambda_2)^3 + p_2\mu_2^2}\right)}} \tag{2.70}$$

Dengan mengacu pada persamaan (2.19), maka persamaan (2.66)–(2.69) dapat dituliskan sebagai berikut:

1. Jika $y_1 = y_2 = 0$ maka $P(Y_1 = 0, Y_2 = 0)$:

$$\begin{aligned} &= p_1p_2 + p_1(1-p_2)e^{\frac{-\mu_2}{1+\varphi_2\mu_2}} + p_2(1-p_1)e^{\frac{-\mu_1}{1+\varphi_1\mu_1}} + \\ &\quad (1-p_1)(1-p_2)e^{-\frac{\mu_1}{1+\varphi_1\mu_1} - \frac{\mu_2}{1+\varphi_2\mu_2}}\left[1 + \eta(1-g_1)(1-g_2)\right] \end{aligned} \tag{2.71}$$

2. Jika $y_l > 0$; $y_{l*\neq l} = 0$ maka $P\left(Y_1 = y_1, Y_2 = y_{l^*\neq l}\right)$

$$= (1-p_l)\left(\frac{\mu_l}{1+\varphi_l\mu_l}\right)^{y_l}\frac{(1+\varphi_l y_l)^{y_l - 1}}{y_l!}e^{\frac{-\mu_l(1+\varphi_l y_l)}{1+\varphi_l\mu_l}} * K \tag{2.72}$$

dimana $K = \left(p_{l*} + (1-p_{l*})e^{\frac{-\mu_{l*}}{1+\varphi_{l*}\mu_{l*}}}\left[1 + \eta\left(e^{-y_l} - g_l\right)\left(e^{-y_{l*}} - g_{l*}\right)\right]\right)$

3. Jika $y_1 > 0; y_2 > 0$ maka $P(Y_1 = y_1, Y_2 = y_2)$

$$(1-p_1)(1-p_2)\prod_{l=1}^{2}\left(\frac{\mu_l}{1+\varphi_l\mu_l}\right)^{y_l}\frac{(1+\varphi_l y_l)^{y_l-1}}{y_l!}\exp\left(\frac{-\mu_l(1+\varphi_l y_l)}{1+\varphi_l\mu_l}\right) * L \tag{2.73}$$

dimana $L = 1 + \eta\prod_{l=1}^{2}\left(e^{-y_l} - g_l\right)$

dimana $g_l = \exp\left[\mu_l\left(t_l - 1\right)/\left(1+\varphi_l\mu_l\right)\right]$, $\ln t_l - \left[\varphi_l\mu_l\left(t_l - 1\right)/\left(1+\varphi_l\mu_l\right)\right] + 1 = 0$

2.12 Geographically Weighted Bivariate Zero Inflated Poisson Regression

Geographically Weighted Bivariate Zero Inflated Poisson Regression (GWBZIPR) merupakan model lokal dari BZIPR dengan penaksir parameter yang bersifat lokal.

Masing-masing pengamatan dari respon diambil dari lokasi (u_i, v_i) yang berbeda. Distribusi dari regresi GWBZIP adalah sebagai berikut:

$$f\left(Y_{1i}, Y_{2i}\right) =$$
$$\begin{cases} \left(1-p_i\right) + p_i e^{-\left(\lambda_{1i}+\lambda_{2i}\right)}\left(1+\alpha\left(1-e^{-\lambda_{1i}c}\right)\left(1-e^{-\lambda_{2i}c}\right)\right) & ,\left(y_{1i}, y_{2i}\right) = (0,0) \\ \dfrac{p_i e^{-\left(\lambda_{1i}+\lambda_{2i}\right)}\lambda_{1j}^{\ y_{1i}}\lambda_{2j}^{\ y_{2i}}}{y_{1i}!\,y_{2i}!}\left(1+\alpha\left(e^{-y_{1i}} - e^{-\lambda_{1i}c}\right)\left(e^{-y_{2i}} - e^{-\lambda_{2i}c}\right)\right) & ,\left(y_{1i}, y_{2i}\right) \neq (0,0) \end{cases} \tag{2.74}$$

untuk $c = 1 - \dfrac{1}{e}$

$$\lambda_{1i} = e^{\mathbf{x_i^T\beta_1}(u_i,v_i)},\ \lambda_{2i} = e^{\mathbf{x_i^T\beta_2}(u_i,v_i)},\ p_i = \frac{e^{\mathbf{x_i^T\gamma}(u_i,v_i)}}{1+e^{\mathbf{x_i^T\gamma}(u_i,v_i)}} \text{ dan } \left(1-p_i\right) = \frac{1}{1+e^{\mathbf{x_i^T\gamma}(u_i,v_i)}} \tag{2.75}$$

Substitusikan persamaan (2.82) ke persamaan (2.81) sehingga diperoleh:

$$f\left(Y_{1i}, Y_{2i}\right) = \begin{cases} g_i, & \left(y_{1i}, y_{2i}\right) = (0,0) \\ h_i, & \left(y_{1i}, y_{2i}\right) \neq (0,0) \end{cases} \tag{2.76}$$

dimana

$$g_i = \frac{1}{1+e^{\mathbf{x_i^T\gamma}(u_i,v_i)}} + \frac{e^{\mathbf{x_i^T\gamma}(u_i,v_i)}}{1+e^{\mathbf{x_i^T\gamma}(u_i,v_i)}}\exp\left(-e^{\mathbf{x_i^T\beta_1}(u_i,v_i)} - e^{\mathbf{x_i^T\beta_2}(u_i,v_i)}\right)H_1$$

$$h_i = \frac{\dfrac{e^{\mathbf{x_i^T\gamma}(u_i,v_i)}}{1+e^{\mathbf{x_i^T\gamma}(u_i,v_i)}}\exp\left(-e^{\mathbf{x_i^T\beta_1}(u_i,v_i)} - e^{\mathbf{x_i^T\beta_2}(u_i,v_i)}\right)\left(e^{\mathbf{x_i^T\beta_1}(u_i,v_i)}\right)^{y_{1i}}\left(e^{\mathbf{x_i^T\beta_2}(u_i,v_i)}\right)^{y_{2i}}}{y_{1i}!\,y_{2i}!}H_2$$

Dimana $H_1 = \left(1+\alpha\left(1-\exp\left(-e^{\mathbf{x_i^T\beta_1}(u_i,v_i)}c\right)\right)\left(1-\exp\left(-e^{\mathbf{x_i^T\beta_2}(u_i,v_i)}c\right)\right)\right)$

$$H_2 = \left(1+\alpha\left(e^{-y_{1i}} - \exp\left(-e^{\mathbf{x_i^T\beta_1}(u_i,v_i)}c\right)\right)\left(e^{-y_{2i}} - \exp\left(-e^{\mathbf{x_i^T\beta_2}(u_i,v_i)}c\right)\right)\right)$$

2.12.1 Penaksiran Parameter Model GWBZIPR

Penaksiran parameter GWBZIPR dilakukan dengan metode *Maximum Likelihood Estimator* (MLE). Fungsi *likelihood* berdasarkan fungsi densitas peluang GWBZIPR sesuai persamaan (2.76) adalah sebagai berikut (Pangulimang, dkk., 2016):

$$L\left(\alpha,\boldsymbol{\gamma}\left(u_i,v_i\right),\boldsymbol{\beta_1}\left(u_i,v_i\right),\boldsymbol{\beta_2}\left(u_i,v_i\right),i=1,2,...,n\right)=\prod_{i=1}^{n} f\left(Y_{1i},Y_{2i}\right) \tag{2.77}$$

Fungsi *ln likelihood* dari persamaan (2.77) untuk menaksir lokasi ke-*i** adalah sebagai berikut:

$$\begin{aligned}
\ell &= \ln L\left(\alpha,\boldsymbol{\gamma}\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta_1}\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta_2}\left(u_{i^*},v_{i^*}\right)\right)w_{ii*} \\
&= \ln\prod_{i=1}^{n}\left(g_i\right)^{1-a_i}\left(h_i\right)^{a_i} w_{ii*} \\
&= \sum_{i=1}^{n}\left(1-a_i\right)\ln\left(g_i\right)w_{ii*}+\sum_{j=1}^{n}\left(a_i\right)\ln\left(h_i\right)w_{ii*} \\
&= Pw_{ii*}+Uw_{ii*}
\end{aligned} \tag{2.78}$$

dimana

$$\lambda_{1i}=e^{\mathbf{x_i^T}\boldsymbol{\beta_1}\left(u_{i*},v_{i*}\right)},\ \lambda_{2i}=e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i*},v_{i*}\right)},\ p_i=\frac{e^{\mathbf{x_i^T}\boldsymbol{\gamma}\left(u_{i*},v_{i*}\right)}}{1+e^{\mathbf{x_i^T}\boldsymbol{\gamma}\left(u_{i*},v_{i*}\right)}}\ \text{dan}\ \left(1-p_i\right)=\frac{1}{1+e^{\mathbf{x_i^T}\boldsymbol{\gamma}\left(u_{i*},v_{i*}\right)}}$$

$a_i=1$ jika $\left(y_{1i},y_{2i}\right)\neq\left(0,0\right)$ dan 0 untuk yang lainnya.

w_{ii*} adalah pembobot geografis antara lokasi ke-*i* dan lokasi ke-*i**.

Misalkan:

$$R=\frac{1}{1+e^{\mathbf{x_i^T}\boldsymbol{\gamma}\left(u_{i*},v_{i*}\right)}}+\frac{e^{\mathbf{x_i^T}\boldsymbol{\gamma}\left(u_{i*},v_{i*}\right)}}{1+e^{\mathbf{x_i^T}\boldsymbol{\gamma}\left(u_{i*},v_{i*}\right)}}\exp\left(-e^{\mathbf{x_i^T}\boldsymbol{\beta_1}\left(u_{i*},v_{i*}\right)}-e^{\mathbf{x_i^T}\boldsymbol{\beta_2}\left(u_{i*},v_{i*}\right)}\right)*R_1$$

dimana $R_1=1+\alpha\left(1-\exp\left(-e^{\mathbf{x_i^T}\boldsymbol{\beta_1}\left(u_{i*},v_{i*}\right)}c\right)\right)\left(1-\exp\left(-e^{\mathbf{x_i^T}\boldsymbol{\beta_2}\left(u_{i*},v_{i*}\right)}c\right)\right)$

$$S=\frac{\dfrac{e^{\mathbf{x_i^T}\boldsymbol{\gamma}\left(u_{i*},v_{i*}\right)}}{1+e^{\mathbf{x_i^T}\boldsymbol{\gamma}\left(u_{i*},v_{i*}\right)}}\exp\left(-e^{\mathbf{x_i^T}\boldsymbol{\beta_1}\left(u_{i*},v_{i*}\right)}-e^{\mathbf{x_i^T}\boldsymbol{\beta_2}\left(u_{i*},v_{i*}\right)}\right)\left(e^{\mathbf{x_i^T}\boldsymbol{\beta_1}\left(u_{i*},v_{i*}\right)}\right)^{y_{1i}}\left(e^{\mathbf{x_i^T}\boldsymbol{\beta_2}\left(u_{i*},v_{i*}\right)}\right)^{y_{2i}}}{y_{1i}!\,y_{2i}!}*S_1$$

dimana $S_1=1+\alpha\left(e^{-y_{1i}}-\exp\left(-e^{\mathbf{x_i^T}\boldsymbol{\beta_1}\left(u_{i*},v_{i*}\right)}c\right)\right)\left(e^{-y_{2i}}-\exp\left(-e^{\mathbf{x_i^T}\boldsymbol{\beta_2}\left(u_{i*},v_{i*}\right)}c\right)\right)$

Turunkan R dan S terhadap parameter $\beta_1, \beta_2, \gamma,$ dan α, kemudian turunkan fungsi P dan U dari fungsi *ln likelihood* pada persamaan (2.78) terhadap $\boldsymbol{\beta}_1^T\left(u_{i*}, v_{i*}\right), \boldsymbol{\beta}_2^T\left(u_{i*}, v_{i*}\right), \boldsymbol{\gamma}^T\left(u_{i*}, v_{i*}\right),$ dan α. Hasil turunan pertama dari setiap parameter tidak *close form* sehingga akan diselesaikan dengan metode iterasi Newton-Raphson. Metode iterasi Newton-Raphson ini dilakukan untuk setiap lokasi sehingga akan diperoleh nilai $\hat{\boldsymbol{\theta}}\left(u_i, v_i\right)$ yang bersifat lokal untuk setiap wilayah penelitian.

2.12.2 Pengujian Hipotesis Model GWBZIPR

Pengujian parameter untuk model GWBZIPR dilakukan dengan menggunakan metode *Maximum Likelihood Ratio Test* (Pangulimang, dkk., 2016). Pengujian serentak parameter model GWBZIPR dilakukan untuk mengetahui signifikansi parameter $\boldsymbol{\beta}\left(u_{i*}, v_{i*}\right)$ dan $\boldsymbol{\gamma}\left(u_{i*}, v_{i*}\right)$ secara bersama-sama. Hipotesis yang digunakan adalah sebagai berikut:

i. Parameter $\boldsymbol{\beta}\left(u_i, v_i\right)$

$$H_0 : \beta_{11}\left(u_i, v_i\right) = ... = \beta_{1q}\left(u_i, v_i\right) = \beta_{21}\left(u_i, v_i\right) = ... = \beta_{2q}\left(u_i, v_i\right) 0, i = 1, 2, ..., n$$
$$H_1 : \text{paling sedikit ada satu } \beta_{lr} \neq 0, l = 1, 2; r = 1, 2, ..., q$$

ii. Parameter $\boldsymbol{\gamma}\left(u_i, v_i\right)$

$$H_0 : \gamma_{11}\left(u_i, v_i\right) = ... = \gamma_{1q}\left(u_i, v_i\right) = \gamma_{21}\left(u_i, v_i\right) = ... = \gamma_{2q}\left(u_i, v_i\right) = 0, i = 1, 2, ..., n$$
$$H_1 : \text{paling sedikit ada satu } \gamma_{lr} \neq 0, l = 1, 2; r = 1, 2, ..., q$$

Statistik uji yang digunakan $D = -2\left(\ln L\left(\hat{\Omega}\right) - \ln L\left(\hat{\omega}\right)\right)$.

Tolak H_0 jika $\left|D_{hitung}\right| > \chi^2_{\alpha;v}$, apabila hasil keputusan uji serentak adalah tolak H_0, maka pengujian dilanjutkan dengan uji parsial untuk mengetahui signifikansi pada masing-masing parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$.

2.13 Uji Korelasi

Koefisien korelasi merupakan suatu nilai yang menggambarkan keeratan hubungan antara dua variabel atau lebih. Pengujian korelasi antar respon dilakukan untuk mengetahui apakah variabel Y_1 dan Y_2 saling berkorelasi. Variabel Y_1 dan Y_2 dikatakan bersifat saling bebas jika matriks korelasi antar variabel membentuk matriks identitas. Untuk menguji korelasi antar variabel respon dapat digunakan uji dengan hipotesis sebagai berikut (Kawamura, 1973):

H_0 : Tidak ada hubungan antara Y_1 dan Y_2

H_1 : Ada hubungan antara Y_1 dan Y_2

Statistik uji yang digunakan adalah:

$$t = \frac{r_{Y_1Y_2}\sqrt{n-2}}{\sqrt{1-\left(r_{Y_1Y_2}\right)^2}} \text{ dimana } r_{Y_1Y_2} = \frac{\sum_{i=1}^{n}\left(y_{1i}-\bar{Y}_1\right)\left(y_{2i}-\bar{Y}_2\right)}{\sqrt{\sum_{i=1}^{n}\left(y_{1i}-\bar{Y}_1\right)^2\sum_{i=1}^{n}\left(y_{2i}-\bar{Y}_2\right)^2}}$$

Tolak H_0 jika $\left|t_{hitung}\right| > t_{\alpha/2;(n-2)}$ yang artinya antar variabel respon berkorelasi atau bersifat *dependent*.

2.14 Multikolinearitas

Syarat yang harus dipenuhi dalam analisis regresi yang menggunakan lebih dari satu variabel prediktor adalah tidak adanya multikolinearitas. Multikolinearitas merupakan suatu kondisi terjadinya korelasi antara satu variabel prediktor dengan variabel prediktor lainnya. Adanya multikolinearitas dalam model regresi menyebabkan taksiran parameter regresi yang dihasilkan memiliki *standar error* yang besar dengan signifikansi yang kecil. Selain itu, multikolinearitas juga dapat mengakibatkan pengujian secara individu menjadi tidak signifikan sekalipun pengujian serentak memberikan hasil yang sangat signifikan.

Variabel prediktor yang berkorelasi tinggi dengan variabel prediktor lain mengakibatkan kedua variabel prediktor mempunyai nilai yang sebanding sehingga matriks dari variabel prediktor tidak memiliki invers. Akibatnya, proses penaksiran dalam model regresi Poisson tidak dapat dilakukan. Oleh karena itu identifikasi multikolinearitas dalam pemodelan regresi Poisson merupakan hal yang penting untuk dilakukan (Pangulimang, dkk. 2016).

Multikolinearitas dapat dideteksi dengan *Variance Inflation Factors* (VIF) yang dinyatakan sebagai berikut (Gujarati,D.N., 2004):

$$VIF = \frac{1}{\left(1 - R_i^2\right)}, \text{ dengan } R_i^2 = \frac{\sum_{i=1}^{n}\left(\hat{x}_i - \overline{x}\right)^2}{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}$$

R_i^2 adalah koefisien determinasi yang diperoleh dengan meregresikan variabel prediktor x_i dengan variabel prediktor lainnya. Nilai R_i^2 berkisar antara 0 sampai 1 sehingga nilai VIF akan naik seiring kenaikan nilai R_i^2. Ketika $R_i^2 = 0$ yang artinya tidak ada korelasi antar variabel prediktor, nilai minimum VIF akan tercapai yaitu VIF = 1. Nilai VIF yang lebih besar dari 10 menunjukkan terjadinya multikolinearitas antar variabel prediktor.

2.15 Efek Spasial

Anselin (1988) menyatakan bahwa data spasial dicirikan dengan adanya dependensi spasial dan heterogenitas spasial yang kemudian disebut dengan efek spasial. Dependensi spasial atau autokorelasi spasial dapat diartikan sebagai adanya hubungan antara kejadian di suatu wilayah dengan kejadian pada wilayah lain di sekitarnya. Sedangkan heterogenitas atau keragaman spasial menggambarkan kejadian yang heterogen (tidak merata) antar wilayah yang disebabkan karena adanya perbedaan karakteristik antara satu wilayah dengan wilayah lainnya.

2.15.1 Heterogenitas Spasial

Sebagaimana telah disebutkan sebelumnya, heterogenitas spasial merupakan suatu kondisi dimana suatu model regresi global tidak dapat menjelaskan hubungan antara variabel-variabel dikarenakan karakteristik antar wilayah amatan yang bervariasi secara spasial.

Hipotesis yang digunakan dalam pengujian heterogenitas spasial adalah sebagai berikut:

H_0 : Tidak terdapat heterogenitas

H_1 : Terdapat heterogenitas

Uji serentak dari model regresi dapat dilakukan dengan metode *Glejser*. Model regresi yang akan diuji adalah:

$$\hat{\varepsilon}_{lj}^2 = \beta_{l0} + \beta_{l1}x_{1i} + \beta_{l2}x_{2i} + ... + \beta_{lq}x_{qi} \quad ; l = 1,2 \quad ; r = 1,2,...,q; \quad i = 1,2,...,n$$

Formula hipotesis untuk uji Glejser adalah (Johnson dan Wichern, 2007):

$H_0 : \beta_{11} = \beta_{12} = ... = \beta_{1q} = \beta_{21} = \beta_{22} = ... = \beta_{2q} = 0$

H_1 : paling sedikit ada satu $\beta_{lr} \neq 0, l = 1,2; r = 1,2,...,q$

Statistik uji yang digunakan adalah:

$$G = \left[n - r - 1 - \frac{1}{2}(l - r + 1) \right] \ln \left(\frac{|\hat{\Sigma}_{\Omega}|}{|\hat{\Sigma}_{\omega}|} \right) \sim \chi^2_{(\alpha, lr)} \tag{2.79}$$

dimana $\hat{\Sigma}_{\omega}$ adalah matriks varian kovarian dibawah H_0 dan $\hat{\Sigma}_{\Omega}$ adalah matriks varian kovarian di bawah populasi. Kriteria pengambilan keputusan adalah tolak H_0 jika $|G_{hitung}| > \chi^2_{\alpha, lr}$.

2.15.2 Pembobot Spasial

Pembobot spasial memiliki peranan penting pada data spasial karena nilai pembobot menggambarkan kedekatan hubungan antara lokasi pengamatan satu dengan lainnya. Elemen matriks pembobot ditentukan berdasarkan kedekatan titik *i* dengan titik *i**. Pada analisis spasial, penaksiran parameter di suatu titik *i* akan lebih dipengaruhi oleh titik-titik yang dekat dengan lokasi *i* daripada titik-titik yang lebih jauh. Dengan memperhitungkan jarak titik koordinat antar lokasi maka lokasi yang berdekatan akan menunjukkan hubungan kemiripan dan sebaliknya. Keragaman spasial antara lokasi yang satu dengan lainnya ditunjukkan dengan adanya matriks pembobot $W(u_i, v_i)$ yang elemen-elemennya merupakan fungsi jarak *Euclidian* antar lokasi.

Pembentukan fungsi pembobot berdasarkan jarak *Euclidean* dapat menggunakan fungsi kernel (kernel function) berikut:

1. Fungsi *Fixed Gaussian Kernel*

$$w_{ii*} = \exp\left(-\frac{1}{2}\left(\frac{d_{ii*}}{h}\right)^2\right)$$

Fungsi *Gaussian kernel* akan memberikan bobot yang semakin kecil mengikuti fungsi *Gaussian* ketika d_{ii*} semakin besar. Oleh karena itu, fungsi *Gaussian kernel* digunakan untuk menaksir parameter dalam model GWR jika fungsi jarak adalah fungsi yang kontinu dan monoton turun.

2. Fungsi *Fixed Bisquare Kernel*

$$w_{ii*} = \begin{cases} \left(1-\left(\frac{d_{ii*}}{h}\right)^2\right)^2 & \text{, jika } d_{ii*} \le h \\ 0 & \text{, jika } d_{ii*} > h \end{cases}$$

2 Adaptive Gaussian Kernel

$$w_{ii*} = \exp\left(-\frac{1}{2}\left(\frac{d_{ii*}}{h_i}\right)^2\right)$$

3 Adaptive Bisquare Kernel

$$w_{ii*} = \begin{cases} \left(1-\left(\frac{d_{ii*}}{h_i}\right)^2\right)^2 & \text{, jika } d_{ii*} \le h_i \\ 0 & \text{, jika } d_{ii*} > h_i \end{cases}$$

dengan $d_{ii*} = \sqrt{(u_i - u_{i*})^2 + (v_i - v_{i*})^2}$ merupakan jarak Euclidean antara lokasi ke-i dan lokasi ke i*

h = parameter non negatif atau yang dikenal sebagai parameter penghalus *fixed* (*fixed bandwidth)*

h_i = *bandwidth* optimum di setiap lokasi ke-*i* yang merupakan radius penaksiran yang berpusat di titik *i* (*adaptive bandwidth*)

Penaksiran parameter tidak sensitif terhadap pemilihan kernel namun sensitif terhadap nilai *bandwidth* terpilih dari kernel. Oleh karena itu Fotheringham, dkk. (2002) menyatakan bahwa pemilihan *bandwidth* yang optimum menjadi sesuatu yang penting karena akan mempengaruhi ketepatan hasil regresi. Nilai *bandwidth* yang terlalu besar akan menyebabkan bias parameter yang besar karena model yang dihasilkan terlalu mulus (*oversmoothing*) sehingga model GWR semakin mendekati model global dan sebaliknya, nilai *bandwidth* yang terlalu kecil akan menyebabkan model terlalu kasar (*undersmoothing*) dimana parameter yang dihasilkan sangat bersifat lokal dengan bias kecil namun memiliki varians yang sangat besar. Pemilihan bandwidth yang optimum dapat dilakukan dengan menggunakan metode *Generalized Cross Validation* (GCV). Metode GCV lebih optimal secara asimtotik dibandingkan metode lainnya dan tidak mengandung parameter populasi. Metode GCV didefinisikan dalam bentuk sebagai berikut (Fotheringham, dkk, 2002):

$$GCV = n\sum_{i=1}^{n} \frac{\left(y_i - \hat{y}_i\left(h\right)\right)^T \left(y_i - \hat{y}_i\left(h\right)\right)}{\left(n - v_1\right)^2} \tag{2.87}$$

dengan:

$\hat{y}_i\left(h\right)$: nilai penaksir y_i

h : *bandwith*

y_i : nilai pengamatan variabel respon ke- i

n : jumlah pengamatan

v_1 : Tr (**S**)

S : matriks yang berisi nilai penaksir GWBZIGPR

$$\mathbf{S} = \mathbf{X}\left(\mathbf{X}^T \mathbf{w}_i \mathbf{X}\right)^{-1} \mathbf{X}^T \mathbf{w}_i$$

2.16 Kriteria Kebaikan Model

Akaike Information Criterion Corrected (AICc) digunakan apabila tujuan dari pemodelan regresi adalah untuk mengidentifikasi faktor-faktor yang berpengaruh terhadap model. Bentuk AICc untuk model regresi bivariat adalah sebagai berikut (Bedrick dan Tsai, 1994):

$$AICc = AIC + \frac{2r\left(r+1\right)}{n-r-1} = -2\ln\left(L\left(\hat{\boldsymbol{\gamma}}_1, \hat{\boldsymbol{\gamma}}_2, \hat{\boldsymbol{\beta}}_1, \hat{\boldsymbol{\beta}}_2, \hat{\varphi}_1, \hat{\varphi}_2, \hat{\eta}\right)\right) + 2r + \frac{2r\left(r+1\right)}{n-r-1}$$

n = banyaknya pengamatan

r = banyaknya parameter yang diestimasi

AICc dapat digunakan sebagai kriteria perbandingan model. Semakin kecil nilai AICc menandakan suatu model semakin baik.

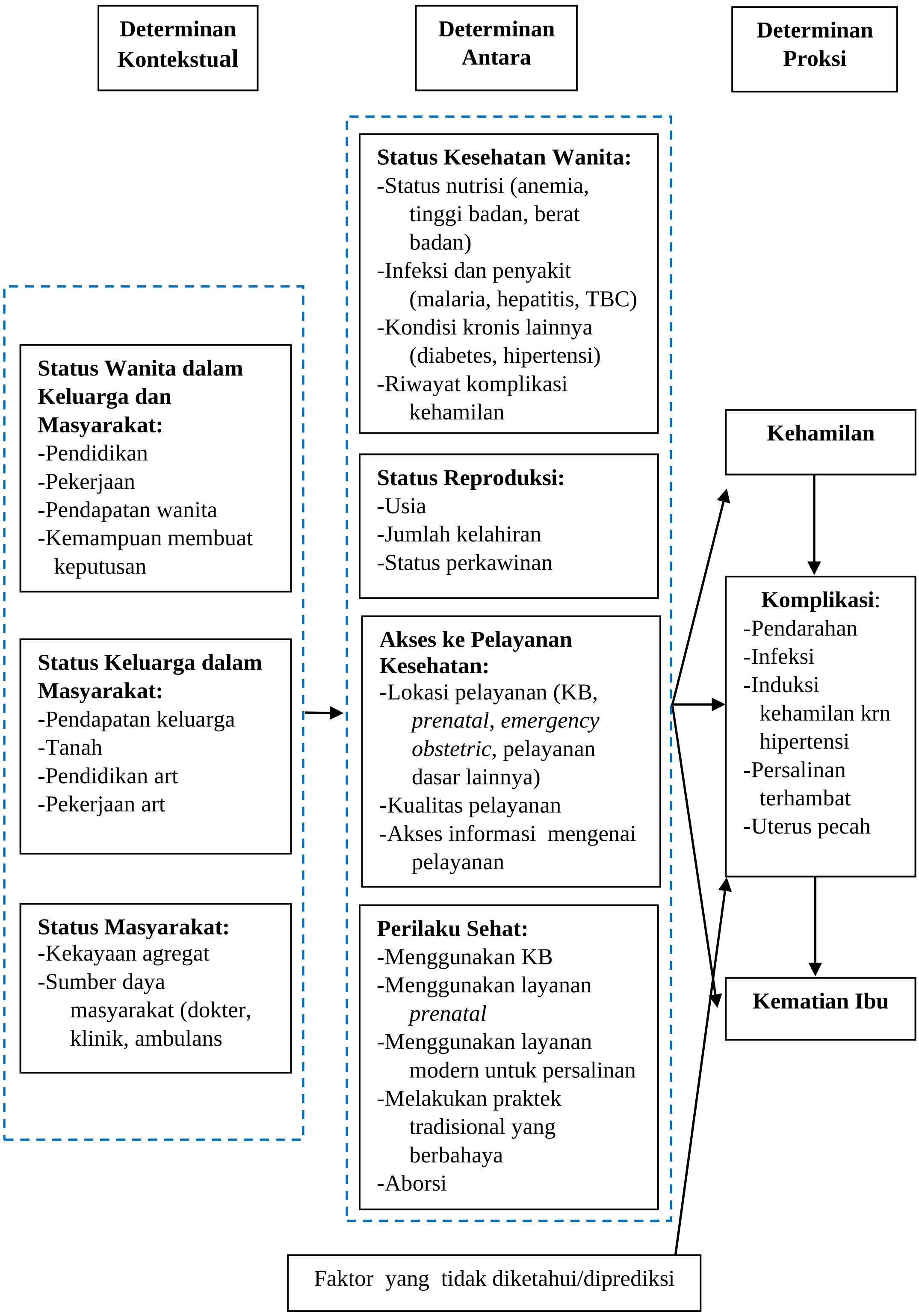
2.17 Kematian Ibu

AKI adalah angka kematian ibu karena kehamilan dan persalinan per 100.000 kelahiran hidup dalam suatu wilayah dan waktu tertentu. Kematian ibu tersebut adalah akibat semua sebab yang berkaitan dengan kehamilan atau penanganannya dan bukan disebabkan oleh kecelakaan, cedera atau alasan insidental lainnya.

Tingginya jumlah kematian ibu di beberapa daerah di dunia mencerminkan adanya ketidakadilan dalam hal akses pada layanan kesehatan. Menurut WHO, 99 persen kematian ibu terjadi di negara berkembang. Rasio kematian ibu di negara berkembang dibandingkan negara maju pada tahun 2015 adalah sebesar 239 per 100.000 kelahiran hidup untuk negara berkembang dan 12 per 100.000 kelahiran hidup di negara maju. Komplikasi dalam kehamilan dan persalinan merupakan penyebab utama kematian remaja perempuan di negara berkembang. Sebagai perbandingan, seorang wanita berusia 15 tahun di negara berkembang memiliki resiko meninggal karena proses *maternal* sebesar 1 banding 180 sementara di negara maju resiko seorang wanita 15 tahun meninggal karena proses maternal hanya 1 banding 4900.

Indikator status kesehatan Indonesia secara keseluruhan mengalami peningkatan selama beberapa tahun terakhir. Hal ini ditunjukkan dengan meningkatnya angka harapan hidup dari 69,81 pada tahun 2010 menjadi 71,06 pada tahun 2017 dan menurunnya angka kematian ibu (AKI). Sekalipun menunjukkan penurunan, AKI di Indonesia masih tergolong tinggi yaitu 305 kematian per 100.000 kelahiran hidup pada tahun 2015. Sebagai perbandingan, pada tahun yang sama AKI di seluruh dunia mencapai angka 216 kematian per 100.000 kelahiran hidup. Dalam lingkup negara-negara ASEAN, Indonesia menduduki peringkat kedua sebagai negara dengan AKI terbanyak. Peringkat pertama diduduki oleh Laos dengan AKI sebesar 357 kematian per 100.000 kelahiran hidup (ASEAN *Secretariat*, 2017). AKI mencerminkan resiko yang dihadapi ibu selama kehamilan sampai paska persalinan yang dipengaruhi oleh status gizi ibu, keadaan sosial ekonomi, keadaan kesehatan yang kurang baik menjelang kehamilan, komplikasi pada kehamilan dan persalinan, tersedianya fasilitas pelayanan kesehatan.

Kematian ibu selain disebabkan oleh kondisi ibu itu sendiri - biasanya terjadi karena tidak adanya akses ke pelayanan kesehatan yang berkualitas, terutama pelayanan kegawatdaruratan tepat waktu yang dilatarbelakangi oleh keterlambatan mengenal tanda bahaya dan mengambil keputusan, terlambat mencapai fasilitas kesehatan, serta terlambat mendapatkan pelayanan di fasilitas kesehatan. Sebab-sebab tersebut dicakup dalam dimensi kematian ibu yang dirumuskan sebagai 4 Terlalu 3 Terlambat yaitu terlalu muda (< 20 tahun), terlalu tua (>35 tahun), terlalu banyak anak (memiliki anak lebih dari tiga), terlalu dekat jarak kelahiran (< 2 tahun), terlambat mencapai fasilitas kesehatan, terlambat mendapatkan pertolongan/penanganan, dan terlambat mengenali tanda bahaya kehamilan dan persalinan (Kemenkes RI, 2016 dan Kemeneg PP dan PA, 2016). Kerangka konsep klasik mengenai determinan kematian ibu yang masih digunakan sampai sekarang adalah kerangka konsep McCarthy dan Maine (1992) seperti pada Gambar 2.1.

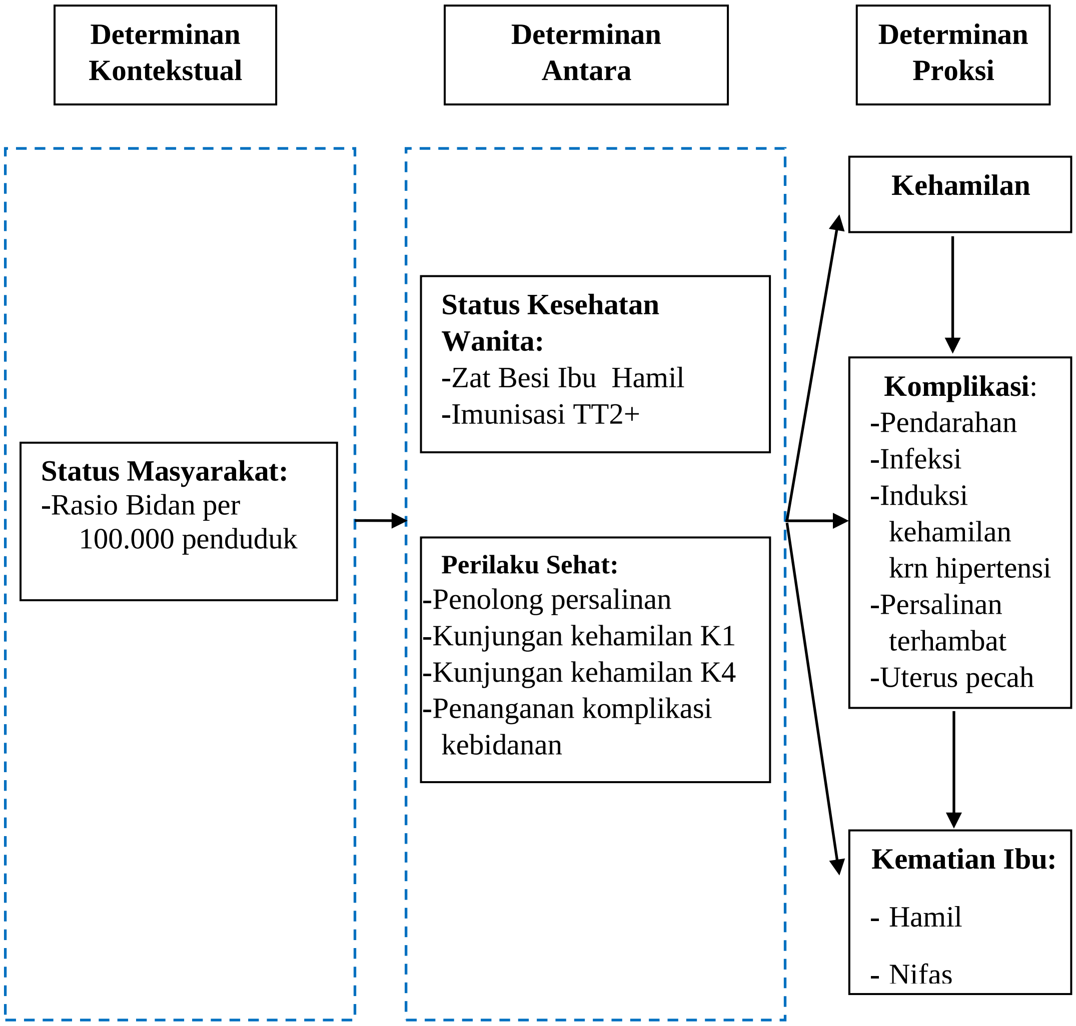


Gambar 2. 1 Kerangka Kerja untuk Analisis Determinan Kematian Ibu Menurut McCarty dan Maine (1992)

Gambar 2.1 menyajikan kerangka kerja untuk menganalisis faktor-faktor penentu kematian ibu. Kerangka kerja ini disusun dalam tiga tahap yaitu determinan kontekstual (determinan jauh/*distant determinants*), determinan antara (*intermediate determinants*), dan determinan proksi (determinan dekat/*outcome*) yang merupakan komponen proses kematian ibu. Kejadian kematian ibu secara langsung dipengaruhi oleh lima komponen pada determinan antara yaitu status kesehatan wanita, status reproduksi, akses pada layanan kesehatan, perilaku perawatan kesehatan dan serangkaian faktor yang tidak diketahui.

Status sosial ekonomi ibu dalam determinan kontekstual tidak memiliki pengaruh secara langsung terhadap kematian ibu. Status sosial ekonomi ibu harus melalui sejumlah faktor dalam determinan antara sebelum mempengaruhi determinan proksi. Setiap faktor dalam kerangka kerja ini dianggap mempengaruhi kematian ibu, oleh karena itu segala upaya untuk mengurangi kematian ibu harus dikaitkan dengan faktor-faktor tersebut (McCarthy dan Maine, 1992).

Berdasarkan beberapa penelitian terkait kematian ibu (Tabel 2.1 dan Gambar 2.1), terdapat beberapa variabel yang dapat mempengaruhi kematian ibu. Dengan pertimbangan ketersediaan data untuk tingkat kecamatan dan kesesuaian hubungan antara respon dengan variabel prediktor, maka tidak semua variabel tersebut digunakan dalam penelitian ini. Kerangka konsep penelitian pada Gambar 2.2 disajikan untuk memberikan gambaran yang lebih detail.



Gambar 2. 2 Modifikasi Model Konseptual McCarty dan Maine (1992) pada Faktor-faktor yang Memengaruhi Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan Tahun 2017

Tabel 2. 1 Beberapa Penelitian Terkait Jumlah Kematian Ibu

No.	Penulis/ Tahun	Judul	Variabel yang digunakan	Hasil Penelitian
1.	Novita, Salamah dan Sutikno (2012)	Pemodelan *Maternal Mortality* di Jawa Timur dengan Pendekatan *Geographically Weighted Poisson Regression* (GWPR)	1. Jumlah kematian ibu 2. Ibu hamil yg melaksanakan K1(%) 3. Persalinan dibantu dukun (%) 4. Ibu hamil yg mendapat Fe1(%) 5. Ibu hamil dengan resiko tinggi/ komplikasi ditangani (%) 6. Ruta hidup sehat (%) 7. Bidan setiap kab/kot (% 8. Sarana kesehatan (%)	Model GWPR lebih baik dibandingkan model regresi Poisson yang ditunjukkan dengan nilai AIC model GWPR yang lebih kecil
2.	Qomariyah, Purnami, dan Pramono (2013)	Pemodelan Faktor-faktor yang Mempengaruhi Jumlah Kematian Ibu di Jawa Timur dengan Pendekatan GWPR (*Geographically Weighted Poisson Regression*) Ditinjau dari Segi Fasilitas Kesehatan	1.Jumlah kematian ibu 2. Ibu hamil dengan K1(%) 3. Persalinan dengan nakes (%) 4. Ibu hamil mendapat Fe (%) 5. Ibu nifas mendapat pelayanan kesehatan (%) 6. Ibu hamil dengan K4 (%) 7. Puskesmas yang melakukan kegiatan P4K (%) 8. Puskesmas yg melakukan kegiatan kemitraan bidan &dukun (%) 9. Puskesmas yg melakukan kegiatan kelas ibu (%) 10. Puskesmas yg melakukan kegiatan pelayanan antenatal terintegrasi (%) 11. Puskesmas yg melakukan kegiatan lengkap kesehatan ibu (%) 12. Puskesmas yg mengikuti pelatihan APN (%)	Model GWPR lebih baik dibandingkan model regresi Poisson yang ditunjukkan dengan nilai R-sq yg lebih besar dan AIC yg lebih kecil. Variabel yg signifikan di seluruh wilayah Jatim adl % kunjungan ibu hamil K1, % ibu nifas mendapat yankes, % puskesmas yg melakukan keg. antenatal terintegrasi & % puskesmas yg memiliki pedoman pencegahan & penanganan malaria pd ibu hamil

Tabel 2.1 Beberapa Penelitian Terkait Jumlah Kematian Ibu

No.	**Penulis/ Tahun**	**Judul**	**Variabel yang digunakan**	**Hasil Penelitian**
2.	Qomariyah, Purnami, dan Pramono (2013)	Pemodelan Faktor-faktor yang Mempengaruhi Jumlah Kematian Ibu di Jawa Timur dengan Pendekatan GWPR (*Geographically Weighted Poisson Regression*) Ditinjau dari Segi Fasilitas Kesehatan	13. Puskesmas yg mengikuti pelatihan PONED (%) 14. Puskesmas yg ikut pelatihan PWS-KIA (%) 15. Puskesmas yg ikut pelatihan lengkap kesehatan ibu (%) 16. Puskesmas memiliki pedoman pelayanan kesehatan maternal & neonatal (%) 17. Puskesmas memiliki pedoman pencegahan & penanganan malaria pada ibu hamil (%) 18. Puskesmas memiliki pedoman operasional pelayanan (%) 19. Puskesmas yg menerima AMP (%)	
3.	Destyanugraha & Kurniawan (2017)	Pemodelan Angka Kematian Ibu di Indonesia dengan Pendekatan *Geographically Weighted Poisson Regression*	1. Jumlah kematian ibu per 100rb wanita 15 thn keatas 2. Rasio sarana kesehatan per 100rb penduduk 3. Rasio bidan per 100rb penduduk 4. Cakupan kunjungan K4 ibu hamil (%) 5. Persalinan dg nakes (%) 6. Ibu hamil mendapat tablet Fe (%)	Model GWPR *kernel fixed Gaussian* lebih baik di bandingkan model regresi Poisson. variabel yg signifikan: % persalinan ditolong nakes, rasio sarana kesehatan per 100rb penduduk, & rasio bidan per 100rb penddk
4.	Fransiska, dkk (2017)	*Analysis of Maternal Mortality Determinants in Bondowoso District, East Java*	1. Jumlah kematian ibu 2. Keterlambatan membuat keputusan 3. Keterlambatan menyiapkan transportasi	Semua variabel signifikan dalam mempengaruhi kematian ibu

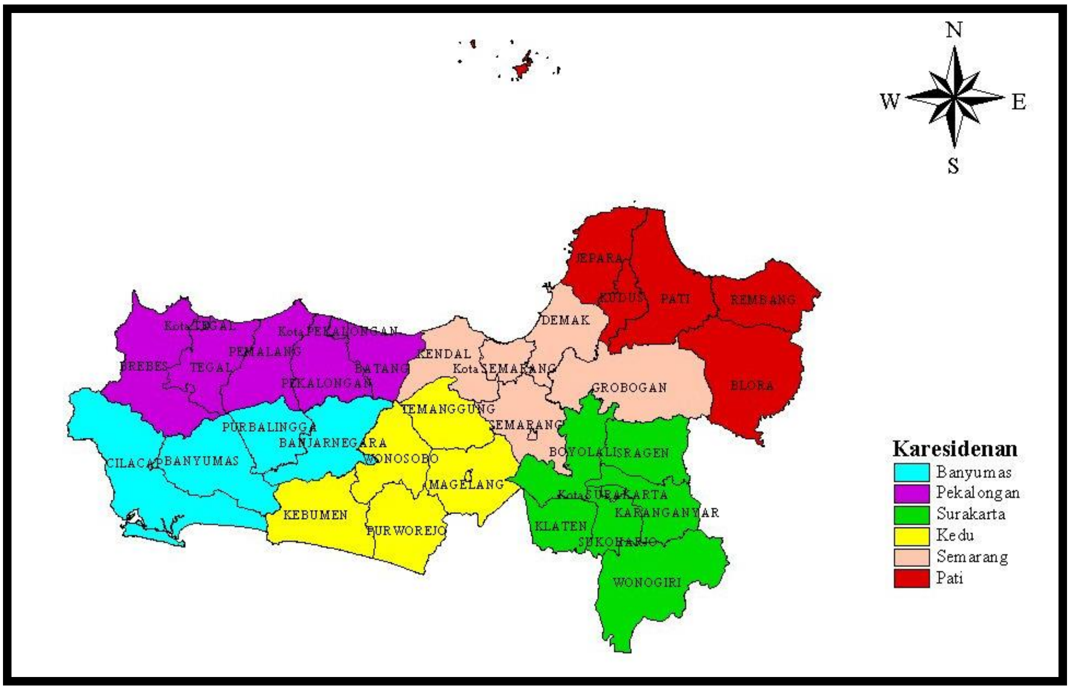
Tabel 2.1 Beberapa Penelitian Terkait Jumlah Kematian Ibu

No.	Penulis/ Tahun	Judul	Variabel yang digunakan	Hasil Penelitian
4.	Fransiska, dkk (2017)	*Analysis of Maternal Mortality Determinants in Bondowoso District, East Java*	4. Keterlambatan penanganan medis 5. Komplikasi kehamilan 6. Kehamilan resiko tinggi (%) 7. Kunjungan antenatal (%) 8. Pendidikan ibu 9. Status kerja ibu	
5.	Girum dan Wasie (2017)	*Correlates of Maternal Mortality in Developing Countries: An Ecological Study in 82 Countries*	1. Rasio kematian ibu 2. Pendapatan nasional bruto 3. Rata-rata pengangguran wanita 4. Pengeluaran untuk kesehatan 5. Tingkat melek huruf 6. Usia kawin pertama 7. Cakupan perawatan antenatal 8. Unmeet need 9. Tingkat prevalensi KB 10. TFR 11. CBR 12. Proporsi dokter, bidan dan perawat 13. Cakupan air bersih dan sanitasi 14. Kejadian TBC 15. Kejadian HIV 16. Kejadian anemia selama kehamilan	Variabel yg signifikan adl usia kawin pertama, cakupan pelayanan antenatal, *unmeet need*, tingkat prevalensi KB, TFR, CBR, proporsi dokter, bidan dan perawat, akses kepada sumber air & sanitasi dan tingkat melek huruf

BAB 3
METODE PENELITIAN

3.1 Sumber Data dan Variabel Penelitian

Data yang digunakan dalam penelitian ini adalah data sekunder yag diperoleh dari Dinas Kesehatan Povinsi Jawa Tengah Tahun 2017 yang berupa data agregat menurut kecamatan. Unit penelitian yang diambil sebanyak 91 kecamatan di Karesidenan Pekalongan Provinsi Jawa Tengah.



Gambar 3. 1 Wilayah Administrasi Provinsi Jawa Tengah menurut Karesidenan

Variabel yang digunakan dalam penelitian ini terdiri dari 2 respon (Y) dan 7 variabel prediktor (X).

Tabel 3. 1 Definisi Operasional Variabel Penelitian

Simbol	Variabel	Definisi Operasional
Y_1	Jumlah kematian ibu hamil	Jumlah ibu di kecamatan yang meninggal pada saat hamil
Y_2	Jumlah kematian ibu nifas	Jumlah ibu di kecamatan yang meninggal pada saat nifas (6 jam sampai 42 hari pasca melahirkan)
X_1	Persentase pemeriksaan kehamilan K1	Persentase ibu hamil di kecamatan yang melakukan kunjungan pertama ke fasilitas kesehatan untuk mendapatkan pelayanan antenatal
X_2	Persentase pemeriksaan kehamilan K4	Persentase ibu hamil di kecamatan yang mendapatkan pelayanan antenatal paling sedikit empat kali kunjungan (sekali pada trimester pertama, sekali pada trimester kedua dan dua kali pada trimester ketiga)
X_3	Persentase persalinan di tolong oleh tenaga kesehatan.	Jumlah persalinan di kecamatan yang ditolong oleh bidan atau tenaga kesehatan yang mempunyai kompetensi kesehatan dibagi dengan jumlah ibu bersalin/nifas dikalikan 100%.
X_4	Persentase imunisasi TT2+ pada ibu hamil.	Jumlah ibu hamil di kecamatan yang mendapatkan imunisasi TT2 sampai TT5 pada wilayah dan kurun waktu tertentu dibagi jumlah ibu hamil pada wilayah dan kurun waktu yang sama dikali 100%.
X_5	Persentase ibu hamil yang mendapatkan Fe3	Hasil bagi antara jumlah ibu hamil di kecamatan yang mendapatkan tablet Fe3 sebanyak 90 tablet dengan jumlah ibu hamil di kecamatan yang bersangkutan dalam waktu tertentu dikalikan 100%.
X_6	Persentase penanganan komplikasi kebidanan.	Ibu hamil, bersalin dan nifas di kecamatan dengan komplikasi yang mendapatkan pelayanan sesuai standar pada tingkat pelayanan dasar dan rujukan dikali 100%.
X_7	Rasio bidan per 100.000 penduduk	Jumlah bidan di kecamatan yang memberikan pelayanan kesehatan di sarana pelayanan kesehatan di kecamatan yang bersangkutan Tahun 2017 dibagi jumlah penduduk di kecamatan tersebut dan waktu yang sama dikali 100.000

Struktur data dalam penelitian ini disajikan dalam Tabel 3.2 berikut.

Tabel 3. 2 Struktur Data

Kecamatan	**Letak Geografis Kantor Camat**		**Respon**		**Variabel Prediktor**		
	Lintang	**Bujur**	**Y_1**	**Y_2**	**X_1**	**…**	**X_7**
1	-7,03	109,61	1	0	100,00	…	63,68
2	-7,06	109,66	1	0	100,00	…	54,81
⋮	⋮	⋮	⋮	⋮	⋮	…	⋮
⋮	⋮	⋮	⋮	⋮	⋮	…	⋮
⋮	⋮	⋮	⋮	⋮	⋮	…	⋮
91	-6,87	109,08	0	1	100,00	…	30,45

Variabel geografis yang menunjukkan lokasi masing-masing kecamatan di kabupaten yang masuk wilayah karesidenan Pekalongan di Provinsi Jawa Tengah ditunjukkan oleh koordinat lintang (*latitude*) yang disimbolkan dengan u_i dan koordinat bujur (*longitude*) yang disimbolkan dengan v_i. Letak titik koordinat pada penelitian ini berdasarkan letak pusat pemerintahan kecamatan.

Kajian teori dibahas mengenai penaksiran parameter model BZIGPR dan GWBZIGPR dan pengujian hipotesis model BZIGPR dan GWBZIGPR secara teori statistik.

3.2 Tahapan Penelitian

Berdasarkan tujuan penelitian maka beikut ini merupakan langkah-langkah analisis untuk menjawab tujuan tersebut.

3.2.1 Langkah-langkah Mendapatkan Penaksir Parameter dan Statistik Uji Parameter Model BZIGPR

Untuk mendapatkan penaksir parameter model BZIGPR dengan algoritma *Berndt Hall Hall Hausman* (BHHH) karena metode iterasi ini hanya memerlukan turunan pertama dengan langkah-langkah sebagai berikut:

1. Menentukan penaksir parameter model BZIGPR

i) Menentukan model BZIGPR dengan mensubtitusikan persamaan (2.66) dengan:

$$\mu_{li} = \exp\left(\mathbf{x}_i^T \boldsymbol{\beta}_l\right), \quad l = 1, 2; \quad i = 1, 2, ..., n$$

$$p_{li} = \frac{\exp\left(\mathbf{x}_i^T \boldsymbol{\gamma}_l\right)}{1 + \exp\left(\mathbf{x}_i^T \boldsymbol{\gamma}_l\right)} \text{ dan } \left(1 - p_{li}\right) = \frac{1}{1 + \exp\left(\mathbf{x}_i^T \boldsymbol{\gamma}_l\right)}$$

pada persamaan (2.71-2.73) maka fungsi kepadatan peluang dari model BZIGPR seperti pada persamaan berikut:

1. Jika $y_1 = 0$ dan $y_2 = 0$ maka $P(Y_1 = 0, Y_2 = 0)$

$$\begin{aligned}&= \frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}} + \frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}.\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)+\\ &\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+\left\{\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left[1+\eta(1-g_1)(1-g_2)\right]\right\}\end{aligned} \tag{3.1}$$

2. Jika $y_1 = 0$ dan $y_2 > 0$ maka $P(Y_1 = 0, Y_2 = y_2)$

$$= \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)^{y_{2i}}\frac{(1+\varphi_2 y_2)^{y_{2i}-1}}{y_2!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}(1+\varphi_2 y_2)}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}+\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left[1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right]\right) \tag{3.2}$$

3. Jika $y_1 > 0$ dan $y_2 = 0$ maka $P(Y_1 = y_1, Y_2 = 0)$

$$= \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)^{y_{1i}}\frac{(1+\varphi_1 y_1)^{y_{1i}-1}}{y_{1i}!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}(1+\varphi_1 y_{1i})}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}+\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left[1+\eta\left(e^{-y_{1i}}-g_1\right)(1-g_2)\right]\right) \tag{3.3}$$

4. Jika $y_1 > 0$ dan $y_2 > 0$ maka $P(Y_1 = y_1, Y_2 = y_2)$

$$= \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\prod_{l=1}^{2}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}\right)^{y_{li}}\frac{(1+\varphi_l y_l)^{y_{li}-1}}{y_{li}!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}(1+\varphi_l y_{li})}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right] \tag{3.4}$$

Penaksiran parameter dan pengujian hipotesis untuk model BZIGPR akan diuraikan pada Bab IV.

ii) Menentukan fungsi *likelihood* dari model BZIGPR

Misalkan $\left(Y_{1i}, Y_{2i}\right) \sim ZIGP\left(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \varphi_1, \varphi_2, \eta\right)$ untuk $i = 1, \ldots, n$ dan $Y_{obs} = \left\{\left(y_{1i}, y_{2i}\right) : i = 1, \ldots, n\right\}$.

Didefinisikan

$$\mathbb{I}_0 = \left\{i : y_{1i} = 0,\ y_{2i} = 0,\ i = 1, 2, \ldots, n\right\}, \quad n_0 = \neq \left\{\mathbb{I}_0\right\},$$
$$\mathbb{I}_1 = \left\{i : y_{1i} = 0,\ y_{2i} > 0,\ i = 1, 2, \ldots, n\right\}, \quad n_1 = \neq \left\{\mathbb{I}_1\right\},$$
$$\mathbb{I}_2 = \left\{i : y_{1i} > 0,\ y_{2i} = 0,\ i = 1, 2, \ldots, n\right\}, \quad n_2 = \neq \left\{\mathbb{I}_2\right\},$$
$$\mathbb{I}_3 = \left\{i : y_{1i} > 0,\ y_{2i} > 0,\ i = 1, 2, \ldots, n\right\}, \quad n_3 = \neq \left\{\mathbb{I}_3\right\} = n - n_0 - n_1 - n_2$$

$$L(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \varphi_1, \varphi_2, \eta) = \prod_{i=1}^{n} f\left(y_{1i,} y_{2i}\right)$$

iii) Menetapkan fungsi ln likelihood

$\ln L(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \varphi_1, \varphi_2, \eta)$

iv) Mencari turunan pertama dari fungsi $\ln L(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \varphi_1, \varphi_2, \eta)$ terhadap parameter $\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \varphi_1, \varphi_2, \eta$ kemudian disamadengankan nol.

v) Jika hasil penurunan pertama terhadap masing-masing parameternya menghasilkan bentuk yang tidak *close form* maka untuk menyelesaikannya menggunakan pendekatan numerik dengan iterasi BHHH dengan langkah-langkah sebagai berikut: (Cameron dan Trivedi, 2005)

a. Menentukan nilai awal θ dan m=0 dengan nilai $\varepsilon > 0$ untuk batas toleransi konvergensi. $\hat{\boldsymbol{\theta}} = \left[\hat{\boldsymbol{\gamma}}_{1(0)}^T\ \hat{\boldsymbol{\gamma}}_{2(0)}^T\ \hat{\boldsymbol{\beta}}_{1(0)}^T\ \hat{\boldsymbol{\beta}}_{2(0)}^T\ \hat{\varphi}_1\ \hat{\varphi}_2\ \hat{\eta}\right]^T$ dimana $\hat{\boldsymbol{\gamma}}_{1(0)}^T, \hat{\boldsymbol{\gamma}}_{2(0)}^T, \hat{\boldsymbol{\beta}}_{1(0)}^T$ dan $\hat{\boldsymbol{\beta}}_{2(0)}^T$ diperoleh dari taksiran ZIGPR univariat.

b. Menghitung vektor gradien

$$\mathbf{g}\left(\hat{\boldsymbol{\theta}}_m\right) = \left[\frac{\partial L(\bullet)}{\partial \hat{\boldsymbol{\gamma}}_{1(0)}^T}\ \frac{\partial L(\bullet)}{\partial \hat{\boldsymbol{\gamma}}_{2(0)}^T}\ \frac{\partial L(\bullet)}{\partial \hat{\boldsymbol{\beta}}_{1(0)}^T}\ \frac{\partial L(\bullet)}{\partial \hat{\boldsymbol{\beta}}_{2(0)}^T}\ \frac{\partial L(\bullet)}{\partial \hat{\varphi}_1}\ \frac{\partial L(\bullet)}{\partial \hat{\varphi}_2}\ \frac{\partial L(\bullet)}{\partial \hat{\eta}}\right]^T$$ dengan

$L(\bullet) = \ln L(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \varphi_1, \varphi_2, \eta)$.

c. Membentuk matriks Hessian:

$$\mathbf{H}\left(\hat{\boldsymbol{\theta}}_m\right) = -\sum_{i=1}^{n} \mathbf{g}_i\left(\hat{\boldsymbol{\theta}}_m\right) \mathbf{g}_i\left(\hat{\boldsymbol{\theta}}_m\right)^T$$

d. Mensubtitusikan nilai $\hat{\boldsymbol{\theta}}_m$ pada elemen $\mathbf{g}\left(\hat{\boldsymbol{\theta}}_m\right)$ dan matriks Hessian $\mathbf{H}\left(\hat{\boldsymbol{\theta}}_m\right)$.

e. Lakukan iterasi mulai m=0 dengan persamaan $\hat{\boldsymbol{\theta}}_{m+1} = \hat{\boldsymbol{\theta}}_m - \mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}_m\right)\mathbf{g}\left(\hat{\boldsymbol{\theta}}_m\right)$, iterasi akan berhenti jika $\left\|\hat{\boldsymbol{\theta}}_{m+1} - \hat{\boldsymbol{\theta}}_m\right\| \leq \varepsilon$ dimana ε adalah bilangan positif yang sangat kecil mendekati 0.

f. Ulangi langkah b dan seterusnya dengan m=m+1.

2. Mendapatkan statistik uji untuk model BZIGPR

Langkah-langkah untuk mengkaji bentuk statistik uji model regresi BZIGP dilakukan dengan menggunakan *Maximum Likelihood Ratio Test* (MLRT) sebagai berikut:

i) Pengujian serentak parameter $\boldsymbol{\gamma}$ dan $\boldsymbol{\beta}$

Pada tahap ini, seluruh parameter $\boldsymbol{\gamma}$ dan $\boldsymbol{\beta}$ diuji secara bersama-sama dengan hipotesis:

$H_0 : \beta_{l1} = \beta_{l2} = ... = \beta_{lq} = \gamma_{l1} = \gamma_{l2} = ... = \gamma_{lq} = 0, l = 1, 2$

$H_1 :$ paling sedikit terdapat satu $\beta_{lr} \neq 0, \gamma_{lr} \neq 0, r = 1, 2, ..., q$

a. Menentukan himpunan parameter dibawah populasi (Ω) yaitu $\boldsymbol{\Omega} = \{\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \varphi_1, \varphi_2, \eta\}$ dan himpunan parameter di bawah H_0 yaitu $\omega = \{\gamma_{10}, \gamma_{20}, \beta_{10}, \beta_{20}, \varphi_1, \varphi_2, \eta\}$.

b. Menentukan fungsi likelihood untuk model penuh (*saturated*) pada himpunan parameter di bawah populasi yang melibatkan seluruh variabel prediktor.

c. Menentukan fungsi likelihood pada himpunan parameter di bawah H_0 tanpa melibatkan variabel prediktor.

d. Mendefinisikan $G^2 = -2\ln\left[\dfrac{L(\hat{\omega})}{L\left(\hat{\Omega}\right)}\right]$

e. Menentukan daerah penolakan

ii) Pengujian serentak parameter $\boldsymbol{\gamma}$

Pada tahap ini, seluruh parameter $\boldsymbol{\gamma}$ diuji secara bersama-sama dengan hipotesis:

$H_0 : \gamma_{l1} = \gamma_{l2} = ... = \gamma_{lq} = 0, l = 1, 2$

$H_1 :$ paling sedikit terdapat satu $\gamma_{lr} \neq 0, r = 1, 2, ..., q$

a. Menentukan himpunan parameter dibawah populasi (Ω) yaitu $\Omega = \{\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \varphi_1, \varphi_2, \eta\}$ dan himpunan parameter di bawah H_0 yaitu $\omega = \{\gamma_{10}, \gamma_{20}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \varphi_1, \varphi_2, \eta\}$.

b. Menentukan fungsi likelihood untuk model penuh (*saturated*) pada himpunan parameter di bawah populasi yang melibatkan seluruh variabel prediktor

c. Menentukan fungsi likelihood pada himpunan parameter di bawah H_0 tanpa melibatkan variabel prediktor.

d. Mendefinisikan $G^2 = -2 \ln \left[\frac{L(\hat{\omega})}{L(\hat{\Omega})} \right]$

e. Menentukan daerah penolakan

iii) Pengujian serentak parameter $\boldsymbol{\beta}$

Pada tahap ini, seluruh parameter $\boldsymbol{\beta}$ diuji secara bersama-sama dengan hipotesis:

$H_0 : \beta_{l1} = \beta_{l2} = ... = \beta_{lq} = 0, l = 1, 2$

$H_1 :$ paling sedikit terdapat satu $\beta_{lr} \neq 0, r = 1, 2, ..., q$

a. Menentukan himpunan parameter dibawah populasi (Ω) yaitu $\boldsymbol{\Omega} = \{\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \varphi_1, \varphi_2, \eta\}$ dan himpunan parameter di bawah H_0 yaitu $\omega = \{\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \beta_{10}, \beta_{20}, \varphi_1, \varphi_2, \eta\}$.

b. Menentukan fungsi likelihood untuk model penuh (*saturated*) pada himpunan parameter di bawah populasi yang melibatkan seluruh variabel prediktor.

f. Menentukan fungsi likelihood pada himpunan parameter di bawah H_0 tanpa melibatkan variabel prediktor.

g. Mendefinisikan $G^2 = -2\ln\left[\frac{L(\hat{\omega})}{L(\hat{\Omega})}\right]$

h. Menentukan daerah penolakan

3.2.2 Langkah-langkah Mendapatkan Penaksir dan Statistik Uji Parameter Model GWBZIGPR

Geographically Weighted Bivariat Zero Inflated Generalized Poisson Regression GWBZIGPR merupakan model lokal dari BZIGPR dengan penaksir parameter yang bersifat lokal. Masing-masing pengamatan dari respon diambil dari lokasi (u_i, v_i) yang berbeda. Langkah-langkah untuk mendapatkan penaksir dan statistik uji parameter model GWBZIGPR adalah sebagai berikut:

1. Menentukan penaksir parameter model GWBZIGPR

i) Membentuk fungsi distribusi GWBZIGP sebagaimana persamaan (2.75) sehingga model GWBZIGPR adalah :

$$\mu_{li} = e^{\mathbf{x}_i^T \boldsymbol{\beta}_l(u_i,v_i)}, \quad p_{li} = \frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_l(u_i,v_i)}}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_l(u_i,v_i)}} \text{ dan } (1-p_{li}) = \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_l(u_i,v_i)}} \tag{3.5}$$

Dengan mengacu pada persamaan (3.1) sampai (3.4), maka pdf dari regresi GWBZIGP adalah sebagai berikut:

1. Jika $y_1 = 0$ dan $y_2 = 0$ maka $P\left(Y_1 = 0, Y_2 = 0\right)$

$$= \frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_i,v_i)}}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_i,v_i)}} \frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_i,v_i)}}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_i,v_i)}} + \frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_i,v_i)}}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_i,v_i)}} \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_i,v_i)}} .\exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_i,v_i)}}{1+\varphi_2\left(u_i,v_i\right)e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_i,v_i)}} \right) + M_{11} + M_{12} \tag{3.6}$$

dimana $M_{11} = \frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_i,v_i)}}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_i,v_i)}} \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_i,v_i)}} \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_i,v_i)}}{1+\varphi_1\left(u_i,v_i\right)e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_i,v_i)}} \right)$

$$M_{12} = \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_i,v_i)}} \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_i,v_i)}} \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_i,v_i)}}{1+\varphi_2\left(u_i,v_i\right)e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_i,v_i)}} - \frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_i,v_i)}}{1+\varphi_2\left(u_i,v_i\right)e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_i,v_i)}} \right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

2. Jika $y_1 = 0$ dan $y_2 > 0$ maka $P\left(Y_1 = 0, Y_2 = y_2\right)$

$$= \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_i,v_i)}} \left(\frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_i,v_i)}}{1+\varphi_2\left(u_i,v_i\right)e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_i,v_i)}} \right)^{y_{2i}} \frac{\left(1+\varphi_2\left(u_i,v_i\right)y_2\right)^{y_{2i}-1}}{y_2 !} \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_i,v_i)}\left(1+\varphi_2\left(u_i,v_i\right)y_2\right)}{1+\varphi_2\left(u_i,v_i\right)e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_i,v_i)}} \right) * M_2 \tag{3.7}$$

dimana $M_2 = \left(\frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_i,v_i)}}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_i,v_i)}} + \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_i,v_i)}} \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_i,v_i)}}{1+\varphi_1\left(u_i,v_i\right)e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_i,v_i)}} \right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}} - g_2\right)\right) \right)$

3. Jika $y_1 > 0$ dan $y_2 = 0$ maka $P\left(Y_1 = y_1, Y_2 = 0\right)$

$$= \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_i,v_i)}} \left(\frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_i,v_i)}}{1+\varphi_1\left(u_i,v_i\right)e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_i,v_i)}} \right)^{y_{1i}} \frac{\left(1+\varphi_1\left(u_i,v_i\right)y_1\right)^{y_{1i}-1}}{y_{1i}!} \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_i,v_i)}\left(1+\varphi_1\left(u_i,v_i\right)y_{1i}\right)}{1+\varphi_1\left(u_i,v_i\right)e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_i,v_i)}} \right) * M_3 \tag{3.8}$$

$$\text{dimana } M_3 = \left(\frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_i,v_i)}}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_i,v_i)}} + \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_i,v_i)}} \exp\left(\frac{-e^{\mathbf{x}_i^{\mathbf{T}} \boldsymbol{\beta}_2(u_i,v_i)}}{1+\varphi_2\left(u_i,v_i\right)e^{\mathbf{x}_i^{\mathbf{T}} \boldsymbol{\beta}_2(u_i,v_i)}} \right) \left(1+\eta\left(e^{-y_{1i}} - g_1\right)\left(1-g_2\right)\right) \right)$$

4. Jika $y_1 > 0$ dan $y_2 > 0$ maka $P\left(Y_1 = y_1, Y_2 = y_2\right)$

$$= \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_i,v_i)}} \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_i,v_i)}} \prod_{l=1}^{2} \left(\left(\frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_l(u_i,v_i)}}{1+\varphi_l\left(u_i,v_i\right)e^{\mathbf{x}_i^T \boldsymbol{\beta}_l(u_i,v_i)}} \right)^{y_{li}} \frac{\left(1+\varphi_l\left(u_i,v_i\right)y_l\right)^{y_{li}-1}}{y_{li}!} \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_l(u_i,v_i)}\left(1+\varphi_l\left(u_i,v_i\right)y_{li}\right)}{1+\varphi_l\left(u_i,v_i\right)e^{\mathbf{x}_i^T \boldsymbol{\beta}_l(u_i,v_i)}} \right) \left(1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}} - g_l\right)\right) \right) \tag{3.9}$$

Penaksiran parameter dan pengujian hipotesis untuk model GWBZIGPR akan diuraikan pada Bab IV.

ii) Menentukan fungsi *likelihood* untuk lokasi ke–*i* pada model GWBZIGPR

$$L\left(\boldsymbol{\gamma}_l\left(u_i,v_i\right),\boldsymbol{\beta}_l\left(u_i,v_i\right),\varphi_l,\eta,l=1,2;i=1,...,n\right)=\prod_{i=1}^{n} f\left(y_{1i},y_{2i}\right)$$

iii) Menentukan pembobot $w_{i*}\left(u_i,v_i\right)$ untuk setiap lokasi ke- *i*, dengan lokasi ke- $i^* \neq i, \quad i=1,2,...,n$ yang dirumuskan sebagai :

$$w_{i*}\left(u_i,v_i\right)=\exp\left(-\frac{1}{2}\left(\frac{d_{ii*}}{h}\right)^2\right) \text{ dengan } d_{ii*}=\sqrt{\left(u_i-u_{i*}\right)^2+\left(v_i-v_{i*}\right)^2}$$

Nilai *bandwith* optimum *h* ditentukan dengan metode *Generalized Cross-Validation Criterion* (GCV) yang dirumuskan seperti pada persamaan (2.87)

iv) Menentukan fungsi *local ln likelihood* untuk lokasi ke- *i* model GWBZIGPR:

$$\ln L\left(\boldsymbol{\gamma}_l\left(u_i,v_i\right),\boldsymbol{\beta}_l\left(u_i,v_i\right),\varphi_l,\eta;l=1,2\right)=\sum_{i^*=1}^{n} w_{ii*}\ln f\left(y_{1i*},y_{2i*}\right)$$

vi) Mencari turunan pertama dari fungsi $\ln L\left(\boldsymbol{\gamma}_l\left(u_i,v_i\right),\boldsymbol{\beta}_l\left(u_i,v_i\right),\varphi_l,\eta\right)$ terhadap parameter $\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\varphi_1,\varphi_2,\eta$ kemudian disamadengankan nol.

vii) Mendapatkan penaksir parameter $\hat{\boldsymbol{\gamma}}_l\left(u_i,v_i\right)$, $\hat{\boldsymbol{\beta}}_l\left(u_i,v_i\right)$, $\hat{\varphi}_l$ dan $\hat{\eta}$ melalui algoritma BHHH jika hasil penurunan pertama terhadap masing-masing parameternya menghasilkan bentuk yang tidak *close form* dengan langkah-langkah sebagai berikut:

a. Menentukan nilai awal $\hat{\boldsymbol{\theta}}$ dan m=0 dengan nilai $\varepsilon>0$ untuk batas toleransi konvergensi.

$$\hat{\boldsymbol{\theta}}=\left[\hat{\boldsymbol{\gamma}}_{1(0)}^T\left(u_i,v_i\right)\ \hat{\boldsymbol{\gamma}}_{2(0)}^T\left(u_i,v_i\right)\ \hat{\boldsymbol{\beta}}_{1(0)}^T\left(u_i,v_i\right)\ \hat{\boldsymbol{\beta}}_{2(0)}^T\left(u_i,v_i\right)\ \hat{\varphi}_1\ \ \hat{\varphi}_2\ \hat{\eta}\right]^T$$

dimana

$\hat{\boldsymbol{\gamma}}_{1(0)}^T\left(u_i,v_i\right),\hat{\boldsymbol{\gamma}}_{2(0)}^T\left(u_i,v_i\right),\ \hat{\boldsymbol{\beta}}_{1(0)}^T\left(u_i,v_i\right),\hat{\boldsymbol{\beta}}_{2(0)}^T\left(u_i,v_i\right),\hat{\varphi}_1,\hat{\varphi}_2,$ dan $\hat{\eta}$

diperoleh dari taksiran BZIGPR univariat.

b. Menghitung vektor gradien $\mathbf{g}\left(\hat{\boldsymbol{\theta}}_m\right)$

$$\left[\frac{\partial L(\bullet)}{\partial \hat{\boldsymbol{\gamma}}_{1(0)}^T(u_i,v_i)}\ \frac{\partial L(\bullet)}{\partial \hat{\boldsymbol{\gamma}}_{2(0)}^T(u_i,v_i)}\ \frac{\partial L(\bullet)}{\partial \hat{\boldsymbol{\beta}}_{1(0)}^T(u_i,v_i)}\ \frac{\partial L(\bullet)}{\partial \hat{\boldsymbol{\beta}}_{2(0)}^T(u_i,v_i)}\ \frac{\partial L(\bullet)}{\partial \hat{\varphi}_1}\ \frac{\partial L(\bullet)}{\partial \hat{\varphi}_2}\ \frac{\partial L(\bullet)}{\partial \hat{\eta}}\right]^T$$

dengan $L(\bullet) = \ln L(\boldsymbol{\gamma}_1(u_i,v_i), \boldsymbol{\gamma}_2(u_i,v_i), \boldsymbol{\beta}_1(u_i,v_i), \boldsymbol{\beta}_2(u_i,v_i), \varphi_1, \varphi_2, \eta)$.

c. Membentuk matriks Hessian $\mathbf{H}\left(\hat{\boldsymbol{\theta}}_m\right) = -\sum_{i=1}^{n} \mathbf{g}_i\left(\hat{\boldsymbol{\theta}}_m\right)\mathbf{g}_i\left(\hat{\boldsymbol{\theta}}_m\right)^T$

d. Mensubtitusikan nilai $\hat{\boldsymbol{\theta}}_m$ pada elemen $\mathbf{g}\left(\hat{\boldsymbol{\theta}}_m\right)$ dan matriks Hessian.

e. Lakukan iterasi mulai m=0 dengan persamaan $\hat{\boldsymbol{\theta}}_{m+1} = \hat{\boldsymbol{\theta}}_m - \mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}_m\right)\mathbf{g}\left(\hat{\boldsymbol{\theta}}_m\right)$, iterasi akan berhenti jika $\left\|\hat{\boldsymbol{\theta}}_{m+1} - \hat{\boldsymbol{\theta}}_m\right\| \le \varepsilon$ dimana ε adalah bilangan positif yang sangat kecil mendekati 0.

f. Ulangi langkah b dan seterusnya dengan m=m+1.

2. Menentukan nilai statistik uji untuk pengujian model GWBZIGPR

Langkah-langkah pengujian parameter model GWBZIGPR dapat dilakukan dengan tahapan sebagai berikut:

i) Pengujian serentak model GWBZIGPR

Pengujian serentak parameter model GWBZIGPR dilakukan untuk mengetahui signifikansi parameter $\boldsymbol{\beta}(u_i,v_i)$ dan $\boldsymbol{\gamma}(u_i,v_i)$ secara bersama-sama dengan hipotesis sebagai berikut:

i.1) Pengujian serentak parameter $\boldsymbol{\gamma}(u_i,v_i)$ dan $\boldsymbol{\beta}(u_i,v_i)$

Pada tahap ini, seluruh parameter $\boldsymbol{\gamma}(u_i,v_i)$ dan $\boldsymbol{\beta}(u_i,v_i)$ diuji secara bersama-sama dengan hipotesis:

$$\begin{aligned} H_0 &: \beta_{l1}(u_i,v_i) = \beta_{l2}(u_i,v_i) = \ldots = \beta_{lq}(u_i,v_i) = \gamma_{l1}(u_i,v_i) \\ &= \gamma_{l2}(u_i,v_i) = \ldots = \gamma_{lq}(u_i,v_i) = 0, l = 1,2; i = 1,2,\ldots,n \end{aligned}$$

H_1 : paling sedikit terdapat satu $\beta_{lr}(u_i,v_i) \neq 0, \gamma_{lr}(u_i,v_i) \neq 0, r = 1,2,\ldots,q$

a. Menentukan himpunan parameter dibawah populasi (Ω) yaitu $\Omega = \left\{\boldsymbol{\gamma}_1(u_i,v_i), \boldsymbol{\gamma}_2(u_i,v_i), \boldsymbol{\beta}_1(u_i,v_i), \boldsymbol{\beta}_2(u_i,v_i), \varphi_1, \varphi_2, \eta\right\}$ dan himpunan parameter di bawah H_0 yaitu

$$\omega = \left\{\gamma_{10}(u_i,v_i), \gamma_{20}(u_i,v_i), \beta_{10}(u_i,v_i), \beta_{20}(u_i,v_i), \varphi_1, \varphi_2, \eta, i = 1,2,\ldots,n\right\}.$$

b. Menentukan fungsi likelihood untuk model penuh (*saturated*) pada himpunan parameter di bawah populasi yang melibatkan seluruh variabel prediktor.

c. Menentukan fungsi likelihood pada himpunan parameter di bawah H_0 tanpa melibatkan variabel prediktor.

d. Mendefinisikan $G^2 = -2\ln\left[\frac{L(\hat{\omega})}{L(\hat{\Omega})}\right]$

e. Menentukan daerah penolakan

i.2) Pengujian serentak parameter $\boldsymbol{\gamma}(u_i, v_i)$

Pada tahap ini, seluruh parameter $\gamma(u_i, v_i)$ diuji secara bersama-sama dengan hipotesis:

$H_0 : \gamma_{l1}(u_i, v_i) = \gamma_{l2}(u_i, v_i) = ... = \gamma_{lq}(u_i, v_i) = 0, l = 1, 2, i = 1, 2, ..., n$

$H_1 :$ paling sedikit terdapat satu $\gamma_{lr}(u_i, v_i) \neq 0, r = 1, 2, ..., q$

a. Menentukan himpunan parameter dibawah populasi (Ω) yaitu

$\Omega = \left\{\boldsymbol{\gamma}_1(u_i, v_i), \boldsymbol{\gamma}_2(u_i, v_i), \boldsymbol{\beta}_1(u_i, v_i), \boldsymbol{\beta}_2(u_i, v_i), \varphi_1, \varphi_2, \eta, i = 1, 2, ..., n\right\}$

dan himpunan parameter di bawah H_0 yaitu

$\omega = \left\{\gamma_{10}(u_i, v_i), \gamma_{20}(u_i, v_i), \boldsymbol{\beta}_1(u_i, v_i), \boldsymbol{\beta}_2(u_i, v_i), \varphi_1, \varphi_2, \eta, i = 1, 2, ..., n\right\}$.

b. Menentukan fungsi likelihood untuk model penuh (*saturated*) pada himpunan parameter di bawah populasi yang melibatkan seluruh variabel prediktor

c. Menentukan fungsi likelihood pada himpunan parameter di bawah H_0 tanpa melibatkan variabel predictor.

d. Mendefinisikan $G^2 = -2\ln\left[\frac{L(\hat{\omega})}{L(\hat{\Omega})}\right]$

e. Menentukan daerah penolakan.

i.3) Pengujian serentak parameter $\boldsymbol{\beta}(u_i, v_i)$

Pada tahap ini, seluruh parameter $\boldsymbol{\beta}(u_i, v_i)$ diuji secara bersama-sama dengan hipotesis:

$H_0 : \beta_{l1}(u_i, v_i) = \beta_{l2}(u_i, v_i) = ... = \beta_{lq}(u_i, v_i) = 0, l = 1, 2; i = 1, 2, ..., n$

$H_1 :$ paling sedikit terdapat satu $\beta_{lr}(u_i, v_i) \neq 0, r = 1, 2, ..., q$

a. Menentukan himpunan parameter dibawah populasi (Ω) yaitu

$$\Omega = \left\{ \boldsymbol{\gamma}_1(u_i, v_i), \boldsymbol{\gamma}_2(u_i, v_i), \boldsymbol{\beta}_1(u_i, v_i), \boldsymbol{\beta}_2(u_i, v_i), \varphi_1, \varphi_2, \eta, i = 1, 2, ..., n \right\}$$

dan himpunan parameter di bawah H_0 yaitu

$$\omega = \left\{ \boldsymbol{\gamma}_1(u_i, v_i), \boldsymbol{\gamma}_2(u_i, v_i), \beta_{10}(u_i, v_i), \beta_{20}(u_i, v_i), \varphi_1, \varphi_2, \eta, i = 1, 2, ..., n \right\}.$$

b. Menentukan fungsi likelihood untuk model penuh (*saturated*) pada himpunan parameter di bawah populasi yang melibatkan seluruh variabel prediktor.

c. Menentukan fungsi likelihood pada himpunan parameter di bawah H_0 tanpa melibatkan variabel prediktor.

d. Mendefinisikan $G^2 = -2 \ln \left[\dfrac{L(\hat{\omega})}{L(\hat{\Omega})} \right]$

e. Menentukan daerah penolakan

3.2.3 Langkah-Langkah untuk Mendapatkan Faktor-Faktor yang Berpengaruh terhadap Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas

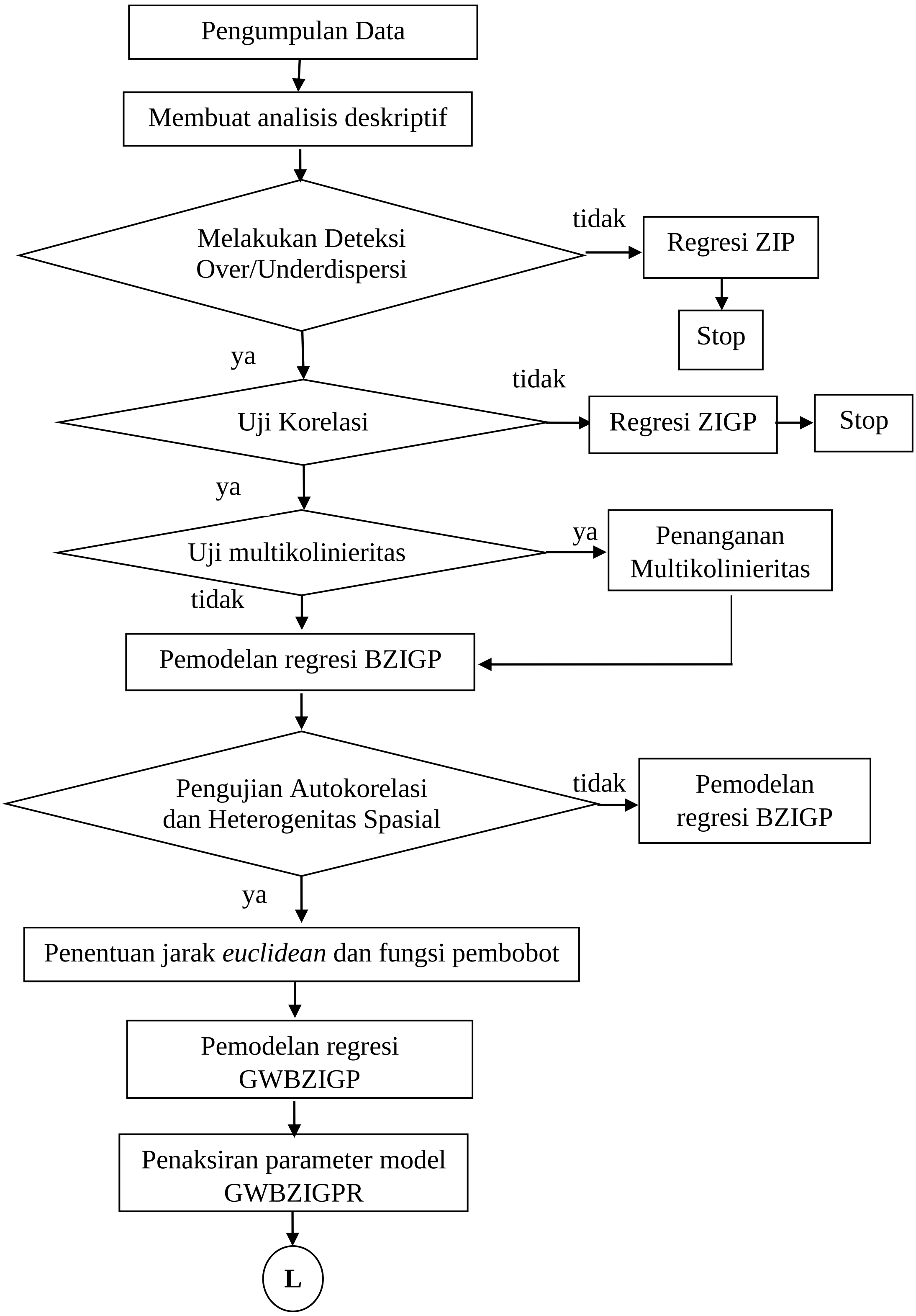
Untuk menentukan faktor-faktor yang mempengaruhi jumlah kematian ibu hamil dan jumlah kematian ibu nifas di 91 kecamatan yang berada di Karesidenan Pekalongan Provinsi Jawa Tengah Tahun 2017 dengan pendekatan model GWBZIGPR dilakukan langkah-langkah sebagai berikut:

1. Membuat statistik deskriptif mengenai variabel respon dan variabel prediktor
2. Melakukan deteksi overdispersi dengan *Varians Test* (VT) (Karlis dan Xekalaki, 2000):

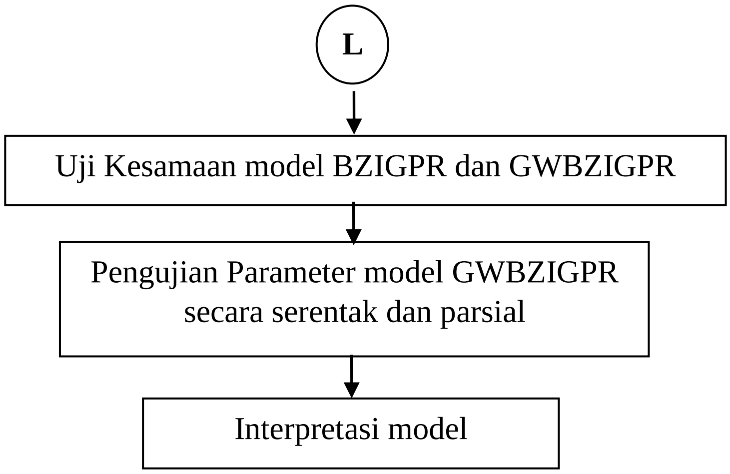
$$VT = \sum_{i=1}^{n} \frac{\left(y_i - \bar{y}\right)^2}{\bar{y}}$$

Data dikatakan mengalami underdispersi ketika VT < 1, overdispersi ketika VT > 1 dan equidispersi ketika VT = 1

3. Menguji keeratan hubungan antar respon menggunakan uji *Pearson Correlation.*
4. Melakukan pemeriksaan multikolinearitas antar variabel prediktor dengan nilai VIF. Apabila nilai VIF > 10 maka terdapat indikasi adanya multikolinearitas
5. Memodelkan data dengan model BZIGPR dengan langkah-langkah meliputi penaksiran parameter dan pengujian hipotesis seperti yang telah dijelaskan pada sub bab sebelumnya
6. Melakukan uji autokorelasi spasial dengan *Morans I* dan heterogenitas spasial untuk memeriksa adanya aspek spasial. Statistik uji yang digunakan adalah *Uji Glejser*.
7. Menghitung jarak *Euclidean* antar lokasi pengamatan berdasarkan letak geografis.
8. Menentukan fungsi pembobot spasial dengan 4 jenis pembobot fungsi kernel yaitu *fixed gaussian*, *adaptive gaussian*, *fixed bisquare*, dan *adaptive bisquare*. Pembobot yang dipilih adalah pembobot yang menghasilkan nilai GCV terkecil.
9. Mendapatkan model GWBZIGPR.
10. Melakukan penaksiran parameter pada model GWBZIGPR.
11. Melakukan pengujian kesamaan model BZIGPR dan GWBZIGPR dengan membandingkan nilai devians BZIGPR dan GWBZIGPR.
12. Pengujian hipotesis secara serentak dan parsial untuk model GWBZIGPR.
13. Melakukan interpretasi model yang didapatkan dan membentuk pengelompokkan variabel yang signifikan.



Gambar 3.2 Diagram Analisis Data dalam Menentukan Faktor-faktor yang Berpengaruh terhadap Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas



Gambar 3.2 Diagram Analsis Data dalam Menentukan Faktor-faktor yang Berpengaruh terhadap Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas

BAB 4
ANALISIS DAN PEMBAHASAN

Pada bab ini dibahas mengenai penaksiran parameter dengan *Maximum Likelihood Estimator* (MLE) dan pengujian hipotesis dengan *Maximum Likelihood Ratio Test* (MLRT) yang dilanjutkan dengan iterasi numerik menggunakan optimasi BHHH. Setelah diperoleh model BZIGPR dan GWBZIGPR kemudian dilakukan uji kesamaan model dan penghitungan ukuran kebaikan model dengan AICc. Model BZIGPR dan GWBZIGPR digunakan untuk memodelkan jumlah kematian ibu hamil dan jumlah kematian ibu nifas di Karesidenan Pekalongan Provinsi Jawa Tengah Tahun 2017 serta untuk mengetahui faktor-faktor yang mempengaruhinya.

4.1 Penaksiran Parameter Bivariate Zero Inflated Generalized Poisson Regression (BZIGPR)

Penaksiran parameter model BZIGPR dilakukan dengan menggunakan metode *Maximum Likelihood Estimation* (MLE). Misalkan $\left(Y_{1i},Y_{2i}\right)\sim ZIGP\left(\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\varphi_1,\varphi_2,\eta\right)$ untuk $i=1,...,n$ dan $Y_{obs}=\left\{\left(y_{1i},y_{2i}\right):i=1,...,n\right\}$. Didefinisikan

$$\mathbb{I}_1=\left\{i:y_{1i}=0,\ y_{2i}=0,\ i=1,2,...,n\right\},\quad n_1=\neq\left\{\mathbb{I}_1\right\},$$
$$\mathbb{I}_2=\left\{i:y_{1i}=0,\ y_{2i}>0,\ i=1,2,...,n\right\},\quad n_2=\neq\left\{\mathbb{I}_2\right\},$$
$$\mathbb{I}_3=\left\{i:y_{1i}>0,\ y_{2i}=0,\ i=1,2,...,n\right\},\quad n_3=\neq\left\{\mathbb{I}_3\right\},$$
$$\mathbb{I}_4=\left\{i:y_{1i}>0,\ y_{2i}>0,\ i=1,2,...,n\right\},\quad n_4=\neq\left\{\mathbb{I}_4\right\}=n-n_1-n_2-n_3$$

Model BZIGPR adalah

$$\mu_{li}=\exp\left(\mathbf{x}_i^T\boldsymbol{\beta}_l\right),\quad l=1,2;\quad i=1,2,...,n$$

$$p_{li}=\frac{\exp\left(\mathbf{x}_i^T\boldsymbol{\gamma}_l\right)}{1+\exp\left(\mathbf{x}_i^T\boldsymbol{\gamma}_l\right)}\text{ dan }\left(1-p_{li}\right)=\frac{1}{1+\exp\left(\mathbf{x}_i^T\boldsymbol{\gamma}_l\right)}\tag{4.1}$$

dengan mensubtitusikan persamaan (4.1) pada persamaan (2.71-2.73) sehingga diperoleh fungsi kepadatan peluang sebagai berikut

$$f\left(y_{1i},y_{2i}\right)=\begin{cases}A_i,\left(y_{1i}=0,y_{2i}=0\right)\\ B_i,\left(y_{1i}=0,y_{2i}>0\right)\\ C_i,\left(y_{1i}>0,y_{2i}=0\right)\\ D_i,\left(y_{1i}>0,y_{2i}>0\right)\end{cases} \tag{4.2}$$

dimana persamaan A_i, B_i, C_i, D_i dapat dilihat pada Lampiran 2. Tahap pertama dalam penaksiran parameter menggunakan MLE adalah membentuk fungsi *likelihood* yang didefinisikan sebagai berikut:

$$L(\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\varphi_1,\varphi_2,\eta)=\prod_{i=1}^{n} f\left(y_{1i,}y_{2i}\right)$$

$$L(\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\varphi_1,\varphi_2,\eta)=\prod_{i=1}^{n}\left(A_i\right)^{1-b_i-c_i-d_i}\left(B_i\right)^{b_i}\left(C_i\right)^{c_i}\left(D_i\right)^{d_i}$$

Tahap kedua adalah membentuk fungsi ln *likelihood* dari persamaan (4.2)

$$l=l(\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\varphi_1,\varphi_2,\eta)=\ln L(\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\varphi_1,\varphi_2,\eta)$$

$$=\ln\prod_{i=1}^{n}\left(A_i\right)^{1-b_i-c_i-d_i}\left(B_i\right)^{b_i}\left(C_i\right)^{c_i}\left(D_i\right)^{d_i}$$

$$l=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\ln A_i+\sum_{i=1}^{n}b_i\ln B_i+\sum_{i=1}^{n}c_i\ln C_i+\sum_{i=1}^{n}d_i\ln D_i \tag{4.3}$$

dimana $\ln A_i,\ln B_i,\ln C_i,\ln D_i$ dapat dilihat pada Lampiran 3. Selanjutnya untuk mendapatkan penaksir parameter maka fungsi $l(\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\varphi_1,\varphi_2,\eta)$ diturunkan terhadap parameter $\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\varphi_1,\varphi_2,\eta$ kemudian disamadengankan nol sehingga diperoleh turunan pertama pada Lampiran 4. Pada turunan pertama diperoleh persamaan yang tidak *close form*, sehingga untuk mendapatkan estimasi parameter BZIGPR dilakukan dengan cara menggunakan pendekatan numerik yaitu iterasi BHHH dengan langkah-langkah sebagai berikut: (Cameron dan Trivedi, 2005)

a) Menentukan nilai awal θ dan m=0 dengan nilai $\varepsilon>0$ untuk batas toleransi konvergensi. $\hat{\boldsymbol{\theta}}=\left[\hat{\boldsymbol{\gamma}}_{1(0)}^T\ \hat{\boldsymbol{\gamma}}_{2(0)}^T\ \hat{\boldsymbol{\beta}}_{1(0)}^T\ \hat{\boldsymbol{\beta}}_{2(0)}^T\ \hat{\varphi}_1\ \hat{\varphi}_2\ \hat{\eta}\right]^T$ dimana $\hat{\boldsymbol{\gamma}}_{1(0)}^T,\hat{\boldsymbol{\gamma}}_{2(0)}^T$, $\hat{\boldsymbol{\beta}}_{1(0)}^T$ dan $\hat{\boldsymbol{\beta}}_{2(0)}^T$ diperoleh dari taksiran ZIGPR univariat.

b) Menghitung vektor gradien

$$\mathbf{g}\left(\hat{\boldsymbol{\theta}}_m\right)=\left[\frac{\partial L(\bullet)}{\partial\hat{\boldsymbol{\gamma}}_{1(0)}^T}\ \frac{\partial L(\bullet)}{\partial\hat{\boldsymbol{\gamma}}_{2(0)}^T}\ \frac{\partial L(\bullet)}{\partial\hat{\boldsymbol{\beta}}_{1(0)}^T}\ \frac{\partial L(\bullet)}{\partial\hat{\boldsymbol{\beta}}_{2(0)}^T}\ \frac{\partial L(\bullet)}{\partial\hat{\varphi}_1}\ \frac{\partial L(\bullet)}{\partial\hat{\varphi}_2}\ \frac{\partial L(\bullet)}{\partial\hat{\eta}}\right]^T$$

dengan $L(\bullet)=\ln L(\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\varphi_1,\varphi_2,\eta)$.

c) Membentuk matriks Hessian

$$\mathbf{H}\left(\hat{\boldsymbol{\theta}}_m\right)=-\sum_{i=1}^{n}\mathbf{g}_i\left(\hat{\boldsymbol{\theta}}_m\right)\mathbf{g}_i\left(\hat{\boldsymbol{\theta}}_m\right)^T$$

d) Mensubtitusikan nilai $\hat{\boldsymbol{\theta}}_m$ pada elemen $\mathbf{g}\left(\hat{\boldsymbol{\theta}}_m\right)$ dan matriks Hessian $\mathbf{H}\left(\hat{\boldsymbol{\theta}}_m\right)$.

e) Lakukan iterasi mulai m=0 dengan persamaan $\hat{\boldsymbol{\theta}}_{m+1}=\hat{\boldsymbol{\theta}}_m-\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}_m\right)\mathbf{g}\left(\hat{\boldsymbol{\theta}}_m\right)$, iterasi akan berhenti jika $\left\|\hat{\boldsymbol{\theta}}_{m+1}-\hat{\boldsymbol{\theta}}_m\right\|\leq\varepsilon$ dimana ε adalah bilangan positif yang sangat kecil mendekati 0.

Ulangi langkah b dan seterusnya dengan m=m+1.

4.2 Pengujian Hipotesis BZIGPR

Fungsi likelihood yang berhubungan dengan model BZIGPR yaitu $L\left(\hat{\Omega}\right)$ dan $L(\hat{\omega})$. Nilai $L\left(\hat{\Omega}\right)$ adalah nilai *maximum likelihood* untuk model yang melibatkan variabel prediktor sedangkan nilai $L(\hat{\omega})$ adalah nilai *maximum likelihood* untuk model yang sederhana tanpa melibatkan variabel prediktor. Metode yang digunakan untuk pengujian serentak parameter pada model BZIGPR adalah *Maximum Likelihood Ratio Test* (MLRT). Pengujian serentak pada model BZIGPR bertujuan untuk mengetahui pengaruh variabel prediktor secara bersama-sama (simultan) terhadap variabel respon. Statistik uji yang digunakan adalah:

$$G_{BZIGPR}^2=-2\ln\left(\frac{L(\hat{\omega}_{BZIGPR})}{L\left(\hat{\Omega}_{BZIGPR}\right)}\right)=2\left[\ln L\left(\hat{\Omega}_{BZIGPR}\right)-\ln L(\hat{\omega}_{BZIGPR})\right]$$

G_{BZIGPR}^2 dapat didekati dengan distribusi χ^2 dengan derajat bebas v, di mana v adalah selisih jumlah parameter di bawah populasi dan jumlah parameter di bawah $H_0(\omega)$. Daerah penolakan H_0 jika $G^2_{BZIGPR} > \chi^2_{\alpha,v}$ dengan α adalah taraf signifikansi dan $v = n(\Omega_{BZIGPR}) - n(\omega_{BZIGPR})$

Pembuktian statistik uji G^2_{BZIGPR} mengikuti distribusi χ^2 adalah sebagai berikut:

$$G^2 = 2\left[L(\hat{\boldsymbol{\theta}}) - L(\hat{\boldsymbol{\theta}}_{\mathbf{0}})\right]$$

$$= \left(\hat{\boldsymbol{\theta}}_{\mathbf{1}} - \hat{\boldsymbol{\theta}}_{\mathbf{10}}\right)^T \left[\mathbf{I}^{11}(\hat{\boldsymbol{\theta}})\right]^{-1} \left(\hat{\boldsymbol{\theta}}_{\mathbf{1}} - \hat{\boldsymbol{\theta}}_{\mathbf{10}}\right)$$

$$= \hat{\boldsymbol{\beta}}^{*T} \left[\mathbf{I}^{11}(\hat{\boldsymbol{\theta}})\right]^{-1} \hat{\boldsymbol{\beta}}^{*}$$

$$= \left[\left[\mathbf{I}^{11}(\hat{\boldsymbol{\theta}})\right]^{-1/2} \hat{\boldsymbol{\beta}}^{*}\right]^T \left[\left[\mathbf{I}^{11}(\hat{\boldsymbol{\theta}})\right]^{-1/2} \hat{\boldsymbol{\beta}}^{*}\right]$$

$$= \hat{\boldsymbol{\beta}}^{*T} \left[\mathbf{I}^{11}(\hat{\boldsymbol{\theta}})\right]^{-1/2} \left[\mathbf{I}^{11}(\hat{\boldsymbol{\theta}})\right]^{-1/2} \hat{\boldsymbol{\beta}}^{*}$$

$$\mathbf{Z}^T\mathbf{Z} \xrightarrow{d} \chi^2_{mp}, n \to \infty$$

dimana:

$\left[\mathbf{I}^{11}(\hat{\boldsymbol{\theta}})\right] = \mathrm{cov}(\hat{\boldsymbol{\beta}}^{*})$ adalah matriks partisi dari matriks $\left[\mathbf{I}^{11}(\hat{\boldsymbol{\theta}})\right]^{-1}$, yakni

$\left[\mathbf{I}^{11}(\hat{\boldsymbol{\theta}})\right]^{-1} = \begin{bmatrix} \mathbf{I}^{11} & \mathbf{I}^{12} \\ \mathbf{I}^{21} & \mathbf{I}^{22} \end{bmatrix}$ dan $Z = \left[\mathbf{I}^{11}(\hat{\boldsymbol{\theta}})\right]^{-1/2} \hat{\boldsymbol{\beta}}^{*} \xrightarrow{d} N(\mathbf{0}, \mathbf{I}_{m,p})$, $n \to \infty$

mp adalah dimensi vektor $\boldsymbol{\beta}^{*}$ atau selisih antara banyaknya parameter pada himpunan Ω dan ω.

Berikut nilai devians dari model BZIGPR dari parameter di bawah populasi dan di bawah H_0. Himpunan parameter di bawah H_1 adalah $\Omega = \{\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \varphi_1, \varphi_2, \eta\}$ dengan fungsi *ln likelihood* sebagai berikut:

$$l^* = \ln L(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \varphi_1, \varphi_2, \eta) = \ln \prod_{i=1}^{n} \left(A_i^*\right)^{1-b_i-c_i-d_i} \left(B_i^*\right)^{b_i} \left(C_i^*\right)^{c_i} \left(D_i^*\right)^{d_i}$$

$$l^* = \sum_{i=1}^{n}(1-b_i-c_i-d_i)\ln A_i^* + \sum_{i=1}^{n} b_i \ln B_i^* + \sum_{i=1}^{n} c_i \ln C_i^* + \sum_{i=1}^{n} d_i \ln D_i^* \qquad (4.4)$$

dimana $\ln A_i^*, \ln B_i^*, \ln C_i^*, \ln D_i^*$ dapat dilihat pada Lampiran 5 dimana nilai $\hat{\Omega} = \left\{ \hat{\boldsymbol{\gamma}}_1, \hat{\boldsymbol{\gamma}}_2, \hat{\boldsymbol{\beta}}_1, \hat{\boldsymbol{\beta}}_2, \hat{\varphi}_1, \hat{\varphi}_2, \hat{\eta} \right\}$ diperoleh dari hasil penaksiran parameter pada subbab 4.1.

4.2.1 Pengujian serentak parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$

Pada tahap ini, seluruh parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$ diuji secara bersama-sama dengan hipotesis:

$H_0 : \beta_{11} = \beta_{12} = ... = \beta_{1q} = \beta_{21} = \beta_{22} = ... = \beta_{2q} = \gamma_{11} = \gamma_{12} = ... = \gamma_{1q} = \gamma_{21} = \gamma_{22} = ... = \gamma_{2q} = 0$

H_1 : Paling sedikit ada satu $\beta_{lr} \neq 0, \gamma_{lr} \neq 0, l = 1,2; r = 1,2,...,q$

Sedangkan fungsi *ln likelihood* d bawah H_0 $L(\omega)$ untuk

$\omega = \{\gamma_{10}, \gamma_{20}, \beta_{10}, \beta_{20}, \varphi_1, \varphi_2, \eta\}$ sebagai berikut

$$I = \ln L(\omega) = \sum_{i=1}^{n} (1 - b_i - c_i - d_i) \ln A_{1i} + \sum_{i=1}^{n} (b_i) \ln B_{1i} + \sum_{i=1}^{n} (c_i) \ln C_{1i} + \sum_{i=1}^{n} (d_i) \ln D_{1i} \quad (4.5)$$

dimana $\ln A_{1i}, \ln B_{1i}, \ln C_{1i}, \ln D_{1i}$ dapat dilihat pada Lampiran 6. Setelah fungsi *ln likelihood* di bawah H_0 terbentuk maka langkah selanjutnya adalah mencari turunan pertama dari masing-masing parameter yang berda di bawah H_0 yaitu $\hat{\beta}_{10}, \hat{\beta}_{20}, \hat{\gamma}_{10}, \hat{\gamma}_{20}, \varphi_1, \varphi_2, \eta$ untuk pengujian parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$. Hasil turunan pertama fungsi *ln likelihood* d bawah H_0 $L(\omega)$ dapat dilihat pada Lampiran 7.

Hasil turunan pertama masing-masing parameter tidak *close form* sehingga dilanjutkan dengan iterasi BHHH dengan langkah-langkah seperti yang dijelaskan pada subbab 4.1 sehingga dengan mensubtitusikan hasil estimasi parameter maka diperoleh fungsi *ln likelihood* d bawah H_0 $L(\hat{\omega})$ untuk parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$ sebagai berikut:

$$I^* = \ln L(\hat{\omega}) = \sum_{i=1}^{n} (1 - b_i - c_i - d_i) \ln A_{1i}^* + \sum_{i=1}^{n} (b_i) \ln B_{1i}^* + \sum_{i=1}^{n} (c_i) \ln C_{1i}^* + \sum_{i=1}^{n} (d_i) \ln D_{1i}^* \quad (4.6)$$

dimana $\ln A_{1i}^*, \ln B_{1i}^*, \ln C_{1i}^*, \ln D_{1i}^*$ dapat dilihat pada Lampiran 8. Sehingga nilai devians untuk pengujian parameter $\boldsymbol{\beta}$ sebagai berikut:

$$G^2 = 2\left(\ln L\left(\hat{\Omega}\right) - \ln L\left(\hat{\omega}\right)\right)$$

dengan $\ln L\left(\hat{\Omega}\right)$ yang telah didefinisikan pada persamaan (4.4) dan $\ln L\left(\hat{\omega}\right)$ pada persamaan (4.6). Daerah penolakan H_0 jika $\left|G^2\right| > \chi^2_{\alpha,v}$ dengan α adalah taraf signifikansi dan v adalah selisih jumlah parameter antara *ln likelihood* di bawah populasi dan ln likelihood di bawah H_0. Jika hasil dari pengujian serentak menghasilkan kesimpulan tolak H_0, maka pengujian dilanjutkan dengan uji secara parsial dengan statistik uji wald. Pengujian secara parsial ini bertujuan untuk mengetahui pengaruh variabel prediktor secara individual terhadap variabel respon. Hipotesis yang digunakan adalah sebagai berikut:

$i)\ H_0 : \beta_{lr} = 0$

$H_1 : \beta_{lr} \neq 0, l = 1,2; r = 1,2,...,q$

$ii)\ H_0 : \gamma_{lr} = 0$

$H_1 : \gamma_{lr} \neq 0, l = 1,2; r = 1,2,...,q$

Statistik uji yang digunakan:

$$Z_\beta = \frac{\hat{\beta}_{lr}}{se\left(\hat{\beta}_{lr}\right)} \text{ dimana } se\left(\hat{\beta}_{lr}\right) = \sqrt{\widehat{\text{var}}\left(\hat{\beta}_{lr}\right)}$$

$$Z_\gamma = \frac{\hat{\gamma}_{lr}}{se\left(\hat{\gamma}_{lr}\right)} \text{ dimana } se\left(\hat{\gamma}_{lr}\right) = \sqrt{\widehat{\text{var}}\left(\hat{\gamma}_{lr}\right)}$$

Nilai $\sqrt{\widehat{\text{var}}\,\hat{\beta}_{lr}}$ dan $\sqrt{\widehat{\text{var}}\,\hat{\gamma}_{lr}}$ diperoleh dari elemen diagonal utama yang bersesuaian dari matriks $-\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}\right)$, tolak H_0 jika $\left|Z_{hitung}\right| > Z_{\alpha/2}$.

4.2.2 Pengujian serentak parameter $\boldsymbol{\gamma}$

Hipotesis yang digunakan dalam pengujian ini adalah

$H_0 : \gamma_{11} = \gamma_{12} = ... = \gamma_{1q} = \gamma_{21} = \gamma_{22} = ... = \gamma_{2q} = 0$

$H_1 :$ paling sedikit terdapat satu $\gamma_{lr} \neq 0, l = 1,2; r = 1,2,...,q$

Fungsi *likelihood* d bawah H_0 dengan $\omega = \left\{\gamma_{10}, \gamma_{20}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \varphi_1, \varphi_2, \eta\right\}$ untuk parameter $\boldsymbol{\gamma}$ sebagai berikut:

$$J = \ln L(\omega) = \sum_{i=1}^{n}(1-b_i-c_i-d_i)\ln A_{2i} + \sum_{i=1}^{n}(b_i)\ln B_{2i} + \sum_{i=1}^{n}(c_i)\ln C_{2i} + \sum_{i=1}^{n}(d_i)\ln D_{2i} \tag{4.7}$$

dimana $\ln A_{2i}, \ln B_{2i}, \ln C_{2i}, \ln D_{2i}$ dapat dilihat pada Lampiran 9. Setelah fungsi likelihood di bawah H_0 terbentuk maka langkah selanjutnya adalah mencari turunan pertama dari masing-masing parameter yang berada di bawah H_0 yaitu $\hat{\boldsymbol{\beta}}_1, \hat{\boldsymbol{\beta}}_2, \hat{\gamma}_{10}, \hat{\gamma}_{20}, \varphi_1, \varphi_1, \eta$. Hasil turunan pertamanya dapat dilihat pada Lampiran 10 dan hasil turunan pertama masing-masing parameter tidak *close form* sehingga dilanjutkan dengan iterasi BHHH dengan langkah-langkah seperti yang dijelaskan pada subbab 4.1 sehingga fungsi *ln likelihood* d bawah H_0 $L(\hat{\omega})$ untuk parameter $\boldsymbol{\gamma}$ sebagai berikut:

$$J^* = \ln L(\hat{\omega}) = \sum_{i=1}^{n}(1-b_i-c_i-d_i)\ln A_{2i}^* + \sum_{i=1}^{n} b_i \ln B_{2i}^* + \sum_{i=1}^{n} c_i \ln C_{2i}^* + \sum_{i=1}^{n} d_i \ln D_{2i}^* \tag{4.8}$$

dimana $\ln A_{2i}, \ln B_{2i}, \ln C_{2i}, \ln D_{2i}$ dapat dilihat pada Lampiran 11.

Sehingga nilai devians sebagai berikut:

$$G^2 = 2\left(\ln L\left(\hat{\Omega}\right) - \ln L(\hat{\omega})\right)$$

Dengan $\ln L\left(\hat{\Omega}\right)$ yang telah didefinisikan pada persamaan (4.3) dan $\ln L(\hat{\omega})$ pada persamaan (4.8). Daerah penolakan H_0 jika $G^2 > \chi^2_{\alpha,v}$ dengan α adalah taraf signifikansi dan v adalah selisih jumlah parameter antara *ln likelihood* di bawah populasi dan ln likelihood di bawah H_0. Jika hasil dari pengujian serentak menghasilkan kesimpulan tolak H_0, maka pengujian dilanjutkan dengan uji secara parsial dengan statistik uji wald. Pengujian secara parsial ini bertujuan untuk mengetahui pengaruh variabel prediktor secara individual terhadap variabel respon.

Hipotesis yang digunakan adalah sebagai berikut:

$H_0 : \gamma_{lr} = 0$

$H_1 : \gamma_{lr} \neq 0, l = 1,2; r = 1,2,...,q$

Statistik uji yang digunakan:

$$Z = \frac{\hat{\gamma}_{lr}}{se(\hat{\gamma}_{lr})} \text{ dimana } se(\hat{\gamma}_{lr}) = \sqrt{\widehat{\text{var}}(\hat{\gamma}_{lr})}$$

Nilai $\sqrt{\text{var}(\hat{\gamma}_{lr})}$ diperoleh dari elemen diagonal utama dari matriks $-\mathbf{H}^{-1}(\hat{\boldsymbol{\theta}})$.

Tolak H_0 jika $|Z_{hitung}| > Z_{\alpha/2}$.

4.2.3 Pengujian serentak parameter $\boldsymbol{\beta}$

Pada tahap ini, seluruh parameter $\boldsymbol{\beta}$ diuji secara bersama-sama dengan hipotesis:

$H_0 : \beta_{11} = \beta_{12} = ... = \beta_{1q} = \beta_{21} = \beta_{22} = ... = \beta_{2q} = 0$

$H_1 :$ Paling sedikit ada satu $\beta_{lr} \neq 0, l = 1,2; r = 1,2,...,7$

Sedangkan fungsi *ln likelihood* d bawah H_0 $L(\omega)$ untuk

$\omega = \{\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \beta_{10}, \beta_{20}, \varphi_1, \varphi_2, \eta\}$ sebagai berikut

$$K = \ln L(\omega) = \sum_{i=1}^{n}(1 - b_i - c_i - d_i)\ln A_{3i} + \sum_{i=1}^{n} b_i \ln B_{3i} + \sum_{i=1}^{n} c_i \ln C_{3i} + \sum_{i=1}^{n} d_i \ln D_{3i} \qquad (4.5)$$

dimana $\ln A_{3i}, \ln B_{3i}, \ln C_{3i}, \ln D_{3i}$ dapat dilihat pada Lampiran 12.

Setelah fungsi *ln likelihood* di bawah H_0 terbentuk maka langkah selanjutnya adalah mencari turunan pertama dari masing-masing parameter yang berada di bawah H_0 yaitu $\hat{\beta}_{10}, \hat{\beta}_{20}, \hat{\boldsymbol{\gamma}}_1, \hat{\boldsymbol{\gamma}}_2, \hat{\varphi}_1, \hat{\varphi}_2, \hat{\eta}$ untuk pengujian parameter $\boldsymbol{\beta}$. Turunan pertama dari masing-masing parameter dapat dilihat pada Lampiran 13.

Hasil turunan pertama masing-masing parameter tidak *close form* sehingga dilanjutkan dengan iterasi BHHH dengan langkah-langkah seperti yang dijelaskan pada subbab 4.1 sehingga diperoleh fungsi *ln likelihood* d bawah H_0 $L(\hat{\omega})$ untuk parameter $\boldsymbol{\beta}$ sebagai berikut

$$K^{*}=\ln L(\hat{\omega})=\sum_{i=1}^{n}(1-b_i-c_i-d_i)\ln A_{3i}^{*}+\sum_{i=1}^{n}b_i\ln B_{3i}^{*}+\sum_{i=1}^{n}c_i\ln C_{3i}^{*}+\sum_{i=1}^{n}d_i\ln D_{3i}^{*} \tag{4.6}$$

dimana $\ln A_{3i}^{*},\ln B_{3i}^{*},\ln C_{3i}^{*},\ln D_{3i}^{*}$ dapat dilihat pada Lampiran 14 sehingga nilai devians untuk pengujian parameter $\boldsymbol{\beta}$ sebagai berikut:

$$G^2=2\left(\ln L\left(\hat{\Omega}\right)-\ln L(\hat{\omega})\right)$$

Dengan $\ln L\left(\hat{\Omega}\right)$ yang telah didefinisikan pada persamaan (4.4) dan $\ln L(\hat{\omega})$ pada persamaan (4.6). Daerah penolakan H_0 jika $G^2>\chi^2_{\alpha,v}$ dengan α adalah taraf signifikansi dan v adalah selisih jumlah parameter antara *ln likelihood* di bawah populasi dan ln likelihood di bawah H_0. Jika hasil dari pengujian serentak menghasilkan kesimpulan tolak H_0, maka pengujian dilanjutkan dengan uji secara parsial dengan statistik uji wald. Pengujian secara parsial ini bertujuan untuk mengetahui pengaruh variabel prediktor secara individual terhadap variabel respon. Hipotesis yang digunakan adalah sebagai berikut:

$H_0:\beta_{lr}=0$

$H_0:\beta_{lr}\neq 0, l=1,2; r=1,2,...,q$

Statistik uji yang digunakan:

$$Z=\frac{\hat{\beta}_{lr}}{se\left(\hat{\beta}_{lr}\right)} \text{ dimana } se\left(\hat{\beta}_{lr}\right)=\sqrt{\widehat{\text{var}}\left(\hat{\beta}_{lr}\right)}$$

Nilai $\sqrt{\widehat{\text{var}}\,\hat{\beta}_{lr}}$ diperoleh dari elemen diagonal utama yang bersesuaian dari matriks $-\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}\right)$, tolak H_0 jika $\left|Z_{hitung}\right|>Z_{\alpha/2}$.

4.3 Penaksiran Parameter Geographically Weighted Bivariate Zero Inflated Generalized Poisson Regression (GWBZIGPR)

Model GWBZIGPR merupakan pengembangan dari model BZIGPR dimana mempertimbangkan factor geografis berupa koordinat lintang dan koordinat bujur dari masing-masing pengamatan. Tentunya setiap daerah memiliki faktor geografis yang berbeda-beda, sehingga hal ini menunjukkan bahwa setiap daerah memiliki sifat lokal pada model GWBZIGPR. Jadi, bentuk fungsi ln *likelihood* dengan pembobot geografis $\left(w_{ii^*}\right)$ untuk masing-masing lokasi sebagai berikut:

Tahap pertama dalam penaksiran parameter menggunakan MLE adalah membentuk fungsi *likelihood* yang didefinisikan sebagai berikut:

$$L\left(\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right),\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right),\varphi_1,\varphi_2,\eta\right)=\prod_{i=1}^{n} f\left(y_{1i,}y_{2i}\right)^{w_{ii^*}}$$

$$L\left(\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right),\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right),\varphi_1,\varphi_2,\eta\right)$$
$$=\prod_{i=1}^{n}\left(\left(A_{4i}\right)^{1-b_i-c_i-d_i}\left(B_{4i}\right)^{b_i}\left(C_{4i}\right)^{c_i}\left(D_{4i}\right)^{d_i}\right)^{w_{ii^*}} \tag{4.7}$$

Tahap kedua adalah membentuk fungsi ln *likelihood* dari persamaan (4.7)

$$l\left(\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right),\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right),\varphi_1,\varphi_2,\eta\right)$$
$$=\ln L\left(\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right),\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right),\varphi_1,\varphi_2,\eta\right)$$
$$=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\ln A_{4i}+\sum_{i=1}^{n} w_{ii^*}b_i\ln B_{4i}+\sum_{i=1}^{n} w_{ii^*}c_i\ln C_{4i}+\sum_{i=1}^{n} w_{ii^*}d_i\ln D_{4i} \tag{4.8}$$

dimana $\ln A_{4i},\ln B_{4i},\ln C_{4i},\ln D_{4i}$ dapat dilihat pada Lampiran 15. Selanjutnya untuk mendapatkan penaksir parameter $\hat{\boldsymbol{\gamma}}_1\left(u_{i^*},v_{i^*}\right),\hat{\boldsymbol{\gamma}}_2\left(u_{i^*},v_{i^*}\right),\hat{\boldsymbol{\beta}}_1\left(u_{i^*},v_{i^*}\right),\hat{\boldsymbol{\beta}}_2\left(u_{i^*},v_{i^*}\right),\hat{\varphi}_1,\hat{\varphi}_2,\hat{\eta}$ maka fungsi $\ln L\left(\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right),\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right),\varphi_1,\varphi_2,\eta\right)$ diturunkan terhadap parameter $\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right),\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right),\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right),\varphi_1,\varphi_2,\eta$ kemudian disamadengankan nol sehingga diperoleh turunan pertama seperti pada Lampiran 16.

Pada turunan pertama diperoleh persamaan yang tidak *close form* maka diselesaikan menggunakan iterasi BHHH. Langkah-langkah untuk mendapatkan penaksir parameter GWBZIGP sebagai beikut:

a. Menentukan nilai awal $\hat{\boldsymbol{\theta}}$ dan m=0 dengan nilai $\varepsilon>0$ untuk batas toleransi konvergensi.

$\hat{\boldsymbol{\theta}} = \left[\hat{\boldsymbol{\gamma}}_{1(0)}^T(u_i,v_i) \; \hat{\boldsymbol{\gamma}}_{2(0)}^T(u_i,v_i) \; \hat{\boldsymbol{\beta}}_{1(0)}^T(u_i,v_i) \; \hat{\boldsymbol{\beta}}_{2(0)}^T(u_i,v_i) \; \hat{\varphi}_1 \;\; \hat{\varphi}_2 \; \hat{\eta} \right]^T$ dimana $\hat{\boldsymbol{\gamma}}_{1(0)}^T(u_i,v_i), \hat{\boldsymbol{\gamma}}_{2(0)}^T(u_i,v_i), \hat{\boldsymbol{\beta}}_{1(0)}^T(u_i,v_i), \hat{\boldsymbol{\beta}}_{2(0)}^T(u_i,v_i), \hat{\varphi}_1, \hat{\varphi}_2,$ dan $\hat{\eta}$ diperoleh dari taksiran BZIGPR univariat.

b. Menghitung vektor gradien $\mathbf{g}\left(\hat{\boldsymbol{\theta}}_m\right)$

$$\left[\frac{\partial l(\bullet)}{\partial \hat{\boldsymbol{\gamma}}_{1(0)}^T(u_i,v_i)} \; \frac{\partial l(\bullet)}{\partial \hat{\boldsymbol{\gamma}}_{2(0)}^T(u_i,v_i)} \; \frac{\partial l(\bullet)}{\partial \hat{\boldsymbol{\beta}}_{1(0)}^T(u_i,v_i)} \; \frac{\partial l(\bullet)}{\partial \hat{\boldsymbol{\beta}}_{2(0)}^T(u_i,v_i)} \; \frac{\partial l(\bullet)}{\partial \hat{\varphi}_1} \; \frac{\partial l(\bullet)}{\partial \hat{\varphi}_2} \; \frac{\partial l(\bullet)}{\partial \hat{\eta}} \right]^T$$

dengan $L(\bullet) = \ln L(\boldsymbol{\gamma}_1(u_i,v_i), \boldsymbol{\gamma}_2(u_i,v_i), \boldsymbol{\beta}_1(u_i,v_i), \boldsymbol{\beta}_2(u_i,v_i), \varphi_1, \varphi_2, \eta)$.

c. Membentuk matriks Hessian $\mathbf{H}\left(\hat{\boldsymbol{\theta}}_m\right) = -\sum_{i=1}^{n} \mathbf{g}_i\left(\hat{\boldsymbol{\theta}}_m\right) \mathbf{g}_i\left(\hat{\boldsymbol{\theta}}_m\right)^T$

d. Mensubtitusikan nilai $\hat{\boldsymbol{\theta}}_m$ pada elemen $\mathbf{g}\left(\hat{\boldsymbol{\theta}}_m\right)$ dan matriks Hessian.

e. Lakukan iterasi mulai m=0 dengan persamaan $\hat{\boldsymbol{\theta}}_{m+1} = \hat{\boldsymbol{\theta}}_m - \mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}_m\right)\mathbf{g}\left(\hat{\boldsymbol{\theta}}_m\right)$, iterasi akan berhenti jika $\left\| \hat{\boldsymbol{\theta}}_{m+1} - \hat{\boldsymbol{\theta}}_m \right\| \le \varepsilon$ dimana ε adalah bilangan positif yang sangat kecil mendekati 0.

f. Ulangi langkah b dan seterusnya dengan m=m+1.

4.4 Pengujian Hipotesis secara Serentak Parameter Model Geographically Weighted Bivariate Zero Inflated Generalized Poisson Regression

Statistik uji dalam pengujian parameter untuk model GWBZIGPR ditentukan oleh fungsi likelihood dari model tersebut yaitu $L\left(\hat{\Omega}\right)$ dan $L(\hat{\omega})$. $L\left(\hat{\Omega}\right)$ adalah nilai *maximum* likelihood untuk model dengan melibatkan variabel prediktor dan $L(\hat{\omega})$ adalah nilai maximum likelihood untuk model sederhana tanpa melibatkan variabel prediktor. Metode yang digunakan Pengujian serentak parameter pada model *Geographically Weighted Bivariate Generalized Poisson Regression* adalah *Maximum Likelihood Ratio Test* (MLRT).

$$G^2 = -2\ln\left(\frac{L(\hat{\omega})}{L(\hat{\Omega})}\right)$$
$$= 2\ln\left(L(\hat{\Omega}) - L(\hat{\omega})\right)$$

Pengujian serentak parameter model GWBZIGPR dilakukan untuk mengetahui signifikansi parameter $\boldsymbol{\beta}(u_i, v_i)$ dan $\boldsymbol{\gamma}(u_i, v_i)$ secara bersama-sama.

4.4.1 Pengujian Serentak Parameter $\boldsymbol{\beta}(u_i, v_i)$ dan $\boldsymbol{\gamma}(u_i, v_i)$

Pada tahap ini, seluruh parameter $\boldsymbol{\beta}(u_i, v_i)$ dan $\boldsymbol{\gamma}(u_i, v_i)$ diuji secara bersama-sama dengan hipotesis:

$H_0 : \beta_{11}(u_i, v_i) = ... = \beta_{1q}(u_i, v_i) = \beta_{21}(u_i, v_i) = ... = \beta_{2q}(u_i, v_i)$
$= \gamma_{11}(u_i, v_i) = ... = \gamma_{1q}(u_i, v_i) = \gamma_{21}(u_i, v_i) = ... = \gamma_{2q}(u_i, v_i) = 0, i = 1, 2, ..., n$

$H_1 :$ paling sedikit terdapat satu $\beta_{lr}(u_i, v_i) \neq 0, \gamma_{lr}(u_i, v_i) \neq 0; l = 1, 2; r = 1, 2, ..., q$

Dengan statistik uji yang digunakan:

$$G^2 = -2\ln\left(\frac{L(\hat{\omega})}{L(\hat{\Omega})}\right)$$
$$= 2\ln\left(L(\hat{\Omega}) - L(\hat{\omega})\right)$$

Fungsi likelihood di bawah populasi $L(\hat{\Omega})$ untuk

$\hat{\Omega} = \left\{\hat{\boldsymbol{\gamma}}_1(u_i, v_i), \hat{\boldsymbol{\gamma}}_2(u_i, v_i), \hat{\boldsymbol{\beta}}_1(u_i, v_i), \hat{\boldsymbol{\beta}}_2(u_i, v_i), \hat{\varphi}_1, \hat{\varphi}_2, \hat{\eta}, i = 1, 2, ..., n\right\}$ adalah

$$L(\hat{\Omega}) = \prod_{i=1}^{n} f(y_{1i,} y_{2i}) = \prod_{i=1}^{n}\left(A_{4i}^{*\ 1-b_i-c_i-d_i} B_{4i}^{*\ b_i} C_{4i}^{*\ c_i} D_{4i}^{*\ d_i}\right)$$

$$\ln L(\hat{\Omega}) = \ln \prod_{i=1}^{n}\left(A_{4i}^*\right)^{1-b_i-c_i-d_i}\left(B_{4i}^*\right)^{b_i}\left(C_{4i}^*\right)^{c_i}\left(D_{4i}^*\right)^{d_i}$$
$$= \sum_{i=1}^{n}(1 - b_i - c_i - d_i)\ln A_{4i}^* + \sum_{i=1}^{n} b_i \ln B_{4i}^* + \sum_{i=1}^{n} c_i \ln C_{4i}^* + \sum_{i=1}^{n} d_i \ln D_{4i}^* \quad (4.9)$$

dimana $\ln A_{4i}^*, \ln B_{4i}^*, \ln C_{4i}^*, \ln D_{4i}^*$ disajikan pada Lampiran 17.

Nilai $\hat{\boldsymbol{\beta}}_1(u_i,v_i),\hat{\boldsymbol{\beta}}_2(u_i,v_i),\hat{\boldsymbol{\gamma}}_1(u_i,v_i),\hat{\boldsymbol{\gamma}}_2(u_i,v_i)$ adalah nilai taksiran parameter yang diperoleh dari persamaan (4.8). Sedangkan fungsi likelihood d bawah H_0 $L(\omega)$ untuk $\omega=\{\gamma_{10}(u_i,v_i),\gamma_{20}(u_i,v_i),\beta_{10}(u_i,v_i),\beta_{20}(u_i,v_i),\varphi_1,\varphi_2,\eta,i=1,2,...,n\}$ sebagai berikut

$$R=\ln L(\omega)=\sum_{i=1}^{n} w_{ii^*}(1-b_i-c_i-d_i)\ln A_{5i}+\sum_{i=1}^{n} w_{ii^*}b_i\ln B_{5i}+\sum_{i=1}^{n} w_{ii^*}c_i\ln C_{5i}+\sum_{i=1}^{n} w_{ii^*}d_i\ln D_{5i} \quad (4.10)$$

dimana $\ln A_{5i},\ln B_{5i},\ln C_{5i},\ln D_{5i}$ dapat dilihat pada Lampiran 18.

Setelah fungsi likelihood di bawah H_0 terbentuk maka langkah selanjutnya adalah mencari turunan pertama dari masing-masing parameter yang berada di bawah H_0 $L(\omega)$ yaitu $\hat{\beta}_{10}(u_i,v_i),\hat{\beta}_{20}(u_i,v_i),\hat{\boldsymbol{\gamma}}_1(u_i,v_i),\hat{\boldsymbol{\gamma}}_2(u_i,v_i)$ untuk pengujian parameter $\boldsymbol{\beta}(u_i,v_i)$ dan $\boldsymbol{\gamma}(u_i,v_i)$. Hasil turunan pertama masing-masing parameter pada Lampiran 19 memberikan hasil yang tidak *close form* sehingga dilanjutkan dengan iterasi BHHH dengan langkah-langkah seperti yang dijelaskan pada subbab 4.1. Langkah selanjutnya adalah mensubtitusikan hasil turunan pertama yang diperoleh pada fungsi *ln likelihood* di bawah H_0 $L(\hat{\omega})$ untuk parameter $\boldsymbol{\beta}(u_i,v_i)$ dan $\boldsymbol{\gamma}(u_i,v_i)$ sebagai berikut:

$$R^*=\ln L(\hat{\omega})=\sum_{i=1}^{n}(1-b_i-c_i-d_i)\ln A_{5i}^*+\sum_{i=1}^{n} b_i\ln B_{5i}^*+\sum_{i=1}^{n} c_i\ln C_{5i}^*+\sum_{i=1}^{n} d_i\ln D_{5i}^* \quad (4.11)$$

dimana $\ln A_{5i}^*,\ln B_{5i}^*,\ln C_{5i}^*,\ln D_{5i}^*$ dapat dilihat pada Lampiran 20. Nilai devians untuk pengujian serentak parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$ menggunakan persamaan sebagai berikut:

$$G^2=2\left(\ln L(\hat{\Omega})-\ln L(\hat{\omega})\right)$$

Dengan $\ln L(\hat{\Omega})$ yang telah didefinisikan pada persamaan (4.9) dan $\ln L(\hat{\omega})$ pada persamaan (4.11). Daerah penolakan H_0 jika $G^2>\chi^2_{\alpha,v}$ dengan α adalah taraf signifikansi dan v adalah selisih jumlah parameter antara *ln likelihood* di bawah populasi dan ln likelihood di bawah H_0. Jika hasil dari pengujian serentak menghasilkan kesimpulan tolak H_0, maka pengujian dilanjutkan dengan uji secara parsial dengan statistik uji *wald*. Pengujian secara parsial ini bertujuan untuk mengetahui pengaruh variabel prediktor secara individual terhadap respon. Hipotesis yang digunakan adalah sebagai berikut:

$i)\ H_0 : \beta_{lr}\left(u_i, v_i\right) = 0$

$\quad H_1 : \beta_{lr}\left(u_i, v_i\right) \neq 0, l = 1,2; r = 1,2,...,q$

$ii)\ H_0 : \gamma_{lr}\left(u_i, v_i\right) = 0$

$\quad H_1 : \gamma_{lr}\left(u_i, v_i\right) \neq 0, l = 1,2; r = 1,2,...,q$

Statistik uji yang digunakan:

$$Z_\beta = \frac{\hat{\beta}_{lr}\left(u_i, v_i\right)}{se\left(\hat{\beta}_{lr}\left(u_i, v_i\right)\right)} \text{ dimana } se\left(\hat{\beta}_{lr}\left(u_i, v_i\right)\right) = \sqrt{\widehat{\text{var}}\left(\hat{\beta}_{lr}\left(u_i, v_i\right)\right)}$$

$$Z_\gamma = \frac{\hat{\gamma}_{lr}\left(u_i, v_i\right)}{se\left(\hat{\gamma}_{lr}\left(u_i, v_i\right)\right)} \text{ dimana } se\left(\hat{\gamma}_{lr}\left(u_i, v_i\right)\right) = \sqrt{\widehat{\text{var}}\left(\hat{\gamma}_{lr}\left(u_i, v_i\right)\right)}$$

Nilai $\sqrt{\text{var}\,\hat{\beta}_{lr}\left(u_i, v_i\right)}$ dan $\sqrt{\widehat{\text{var}}\left(\hat{\gamma}_{lr}\left(u_i, v_i\right)\right)}$ diperoleh dari diagonal utama dari matriks $-\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}\right)$, tolak H_0 jika $\left|Z_{hitung}\right| > Z_{\alpha/2}$.

4.4.2 Pengujian serentak parameter $\boldsymbol{\gamma}\left(u_i, v_i\right)$

Pada tahap ini, seluruh parameter $\boldsymbol{\gamma}\left(u_i, v_i\right)$ diuji secara bersama-sama dengan hipotesis:

$H_0 : \gamma_{11}\left(u_i, v_i\right) = \gamma_{12}\left(u_i, v_i\right) = ... = \gamma_{2q}\left(u_i, v_i\right) = 0, i = 1,2,...,n$

$H_1 :$ paling sedikit terdapat satu $\gamma_{lr}\left(u_i, v_i\right) \neq 0, l = 1,2; r = 1,2,...,q$

Fungsi *ln likelihood* d bawah H_0 $L\left(\omega\right)$ untuk $\omega = \left\{\gamma_{10}\left(u_i, v_i\right), \gamma_{20}\left(u_i, v_i\right), \boldsymbol{\beta}_1\left(u_i, v_i\right), \boldsymbol{\beta}_2\left(u_i, v_i\right), \varphi_1, \varphi_2, \eta, i = 1,2,...,n\right\}$ sebagai berikut

$$Q = \ln L\left(\omega\right) = \sum_{i=1}^{n} w_{ii^*}\left(1 - b_i - c_i - d_i\right)\ln A_{6i} + \sum_{i=1}^{n} w_{ii^*} b_i \ln B_{6i} + \sum_{i=1}^{n} w_{ii^*} c_i \ln C_{6i} + \sum_{i=1}^{n} w_{ii^*} d_i \ln D_{6i} \qquad (4.12)$$

dimana $\ln A_{6i}, \ln B_{6i}, \ln C_{6i}, \ln D_{6i}$ dapat dilihat pada Lampiran 21

Setelah fungsi likelihood di bawah H_0 terbentuk maka langkah selanjutnya adalah mencari turunan pertama dari masing-masing parameter yang berada di bawah H_0 yaitu $\hat{\boldsymbol{\beta}}_1(u_i,v_i), \hat{\boldsymbol{\beta}}_2(u_i,v_i), \hat{\boldsymbol{\gamma}}_{10}(u_i,v_i), \hat{\boldsymbol{\gamma}}_{20}(u_i,v_i), \hat{\varphi}_1, \hat{\varphi}_2, \hat{\eta}$ dan hasil turunan pertamanya dapat dilihat pada Lampiran 22. Hasil turunan pertama masing-masing parameter tidak *close form* sehingga dilanjutkan dengan iterasi BHHH dengan langkah-langkah seperti yang dijelaskan pada subbab 4.3 sehingga fungsi *ln likelihood* d bawah H_0 $L(\hat{\omega})$ untuk parameter $\boldsymbol{\gamma}$ sebagai berikut:

$$Q^* = \ln L(\hat{\omega}) = \sum_{i=1}^{n}(1-b_i-c_i-d_i)\ln A^*_{6i} + \sum_{i=1}^{n}(b_i)\ln B^*_{6i} + \sum_{i=1}^{n}(c_i)\ln C^*_{6i} + \sum_{i=1}^{n}(d_i)\ln D^*_{6i} \tag{4.13}$$

dimana $\ln A^*_{6i}, \ln B^*_{6i}, \ln C^*_{6i}, \ln D^*_{6i}$ dapat dilihat pada Lampiran 23. Sehingga nilai devians dapat dihitung sebagai berikut:

$$G^2 = 2\left(\ln L\left(\hat{\Omega}\right) - \ln L(\hat{\omega})\right)$$

Dengan $\ln L\left(\hat{\Omega}\right)$ yang telah didefinisikan pada persamaan (4.10) dan $\ln L(\hat{\omega})$ pada persamaan (4.13). Daerah penolakan H_0 jika $G^2 > \chi^2_{\alpha,v}$ dengan α adalah taraf signifikansi dan v adalah selisih jumlah parameter antara *ln likelihood* di bawah populasi dan ln likelihood di bawah H_0. Jika hasil dari pengujian serentak menghasilkan kesimpulan tolak H_0, maka pengujian dilanjutkan dengan uji secara parsial dengan statistik uji wald. Pengujian secara parsial ini bertujuan untuk mengetahui pengaruh variabel prediktor secara individual terhadap variabel respon. Hipotesis yang digunakan adalah sebagai berikut:

$H_0 : \gamma_{lr}(u_i,v_i) = 0$

$H_1 : \gamma_{lr}(u_i,v_i) \neq 0, l = 1,2; i = 1,2,...,n; r = 1,2,...,q$

Statistik uji yang digunakan:

$$Z = \frac{\hat{\gamma}_{lr}(u_i,v_i)}{se\left(\hat{\gamma}_{lr}(u_i,v_i)\right)} \text{ dimana } se(\hat{\gamma}_{lr}) = \sqrt{\hat{\text{var}}\left(\hat{\gamma}_{lr}(u_i,v_i)\right)}$$

Nilai $\sqrt{\widehat{\text{var}}\left(\hat{\gamma}_{lr}\left(u_i,v_i\right)\right)}$ diperoleh dari elemen diagonal utama dari matriks $-\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}\right)$. Tolak H_0 jika $\left|Z_{hitung}\right| > Z_{\alpha/2}$.

4.4.3 Pengujian Serentak Parameter $\boldsymbol{\beta}\left(u_i,v_i\right)$

Pada tahap ini, seluruh parameter $\boldsymbol{\beta}\left(u_i,v_i\right)$ diuji secara bersama-sama dengan hipotesis:

$H_0 : \beta_{11}\left(u_i,v_i\right) = \beta_{12}\left(u_i,v_i\right) = ... = \beta_{21}\left(u_i,v_i\right) = \beta_{22}\left(u_i,v_i\right) = ... = \beta_{2q}\left(u_i,v_i\right) = 0$

$H_1 :$ paling sedikit terdapat satu $\beta_{lr}\left(u_i,v_i\right) \neq 0; l = 1,2; r = 1,2,...,q$

Fungsi likelihood d bawah H_0 $L\left(\omega\right)$ untuk $\omega = \left\{\boldsymbol{\gamma}_1\left(u_i,v_i\right), \boldsymbol{\gamma}_2\left(u_i,v_i\right), \beta_{10}\left(u_i,v_i\right), \beta_{20}\left(u_i,v_i\right), \varphi_1, \varphi_2, \eta, i = 1,2,...,n\right\}$ sebagai berikut

$$R = \ln L\left(\omega\right) = \sum_{i=1}^{n} w_{ii^*}\left(1 - b_i - c_i - d_i\right)\ln A_{7i} + \sum_{i=1}^{n} w_{ii^*} b_i \ln B_{7i} w_{ii^*} + \sum_{i=1}^{n} w_{ii^*} c_i \ln C_{7i} w_{ii^*} + \sum_{i=1}^{n} w_{ii^*} d_i \ln D_{7i} \tag{4.14}$$

dimana $\ln A_{7i}, \ln B_{7i}, \ln C_{7i}, \ln D_{7i}$ dapat dilihat pada Lampiran 24. Setelah fungsi likelihood di bawah H_0 terbentuk maka langkah selanjutnya adalah mencari turunan pertama dari masing-masing parameter yang berda di bawah H_0 yaitu $\hat{\beta}_{10}\left(u_i,v_i\right), \hat{\beta}_{20}\left(u_i,v_i\right), \hat{\boldsymbol{\gamma}}_1\left(u_i,v_i\right), \hat{\boldsymbol{\gamma}}_2\left(u_i,v_i\right)$ untuk pengujian parameter $\boldsymbol{\beta}\left(u_i,v_i\right)$. Hasil turunan pertama masing-masing parameter pada Lampiran 25 tidak *close form* sehingga dilanjutkan dengan iterasi BHHH dengan langkah-langkah seperti yang dijelaskan pada subbab 4.3. selanjutnya mensubtitusikan hasil turunan parameter pada fungsi *ln likelihood* d bawah H_0 $L\left(\hat{\omega}\right)$ untuk parameter $\boldsymbol{\beta}$ sebagai berikut:

$$R^* = \ln L\left(\hat{\omega}\right) = \sum_{i=1}^{n}\left(1 - b_i - c_i - d_i\right)\ln A_{7i}^* + \sum_{i=1}^{n} b_i \ln B_{7i}^* + \sum_{i=1}^{n} c_i \ln C_{7i}^* + \sum_{i=1}^{n} d_i \ln D_{7i}^* \tag{4.15}$$

dimana $\ln A_{7i}^*, \ln B_{7i}^*, \ln C_{7i}^*, \ln D_{7i}^*$ dapat dilihat pada Lampiran 26.

Sehingga nilai devians untuk pengujian serentak parameter $\boldsymbol{\beta}$ sebagai berikut:

$$G^2 = 2\left(\ln L\left(\hat{\Omega}\right) - \ln L\left(\hat{\omega}\right)\right)$$

Dengan $\ln L\left(\hat{\Omega}\right)$ yang telah didefinisikan pada persamaan (4.10) dan $\ln L\left(\hat{\omega}\right)$ pada persamaan (4.12). Daerah penolakan H_0 jika $G^2 > \chi^2_{\alpha,v}$ dengan α adalah taraf signifikansi dan v adalah selisih jumlah parameter antara *ln likelihood* di bawah populasi dan ln likelihood di bawah H_0. Jika hasil dari pengujian serentak menghasilkan kesimpulan tolak H_0, maka pengujian dilanjutkan dengan uji secara parsial dengan statistik uji wald. Pengujian secara parsial ini bertujuan untuk mengetahui pengaruh variabel prediktor secara individual terhadap variabel respon. Hipotesis yang digunakan adalah sebagai berikut:

$H_0 : \beta_{lr}\left(u_i, v_i\right) = 0$

$H_1 : \beta_{lr}\left(u_i, v_i\right) \neq 0, l = 1,2; i = 1,2,...,n; r = 1,2,...,q$

Statistik uji yang digunakan:

$$Z = \frac{\hat{\beta}_{lr}\left(u_i, v_i\right)}{se\left(\hat{\beta}_{lr}\left(u_i, v_i\right)\right)} \text{ dimana } se\left(\hat{\beta}_{lr}\left(u_i, v_i\right)\right) = \sqrt{\text{vâr}\,\hat{\beta}_{lr}\left(u_i, v_i\right)}$$

Nilai $\sqrt{\text{vâr}\,\hat{\beta}_{lr}\left(u_i, v_i\right)}$ diperoleh dari diagonal utama dari matriks $-\mathbf{H}^{-1}\left(\hat{\boldsymbol{\theta}}\right)$, tolak H_0 jika $\left|Z_{hitung}\right| > Z_{\alpha/2}$.

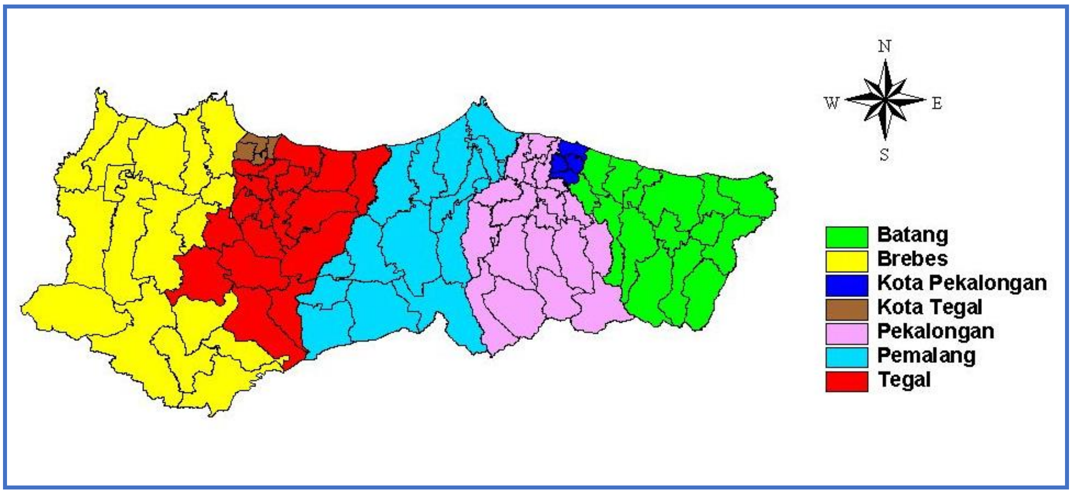
4.5 Pemodelan Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan dengan BZIGPR dan GWBZIGPR

Pada subbab ini akan dibahas mengenai pengaplikasian model BZIGPR dan GWBZIGPR sehingga dapat diketahui faktor-faktor apa saja yang mempengaruhi jumlah kematian ibu hamil dan jumlah kematian ibu nifas di Karesidenan Pekalongan Tahun 2017.

4.5.1 Gambaran Umum Mengenai Karesidenan Pekalongan

Karesidenan adalah pembagian admistratif dalam Provinsi pada zaman Hindia-Belanda hinga tahun 1950-an. Dalam Karesidenan terdiri dari beberapa

Kabupaten dan Kota. Tidak semua Provinsi di Indonesia terbentuk Karesidenan, hanya ada di pulau Jawa, Sumatra, Lombok, Kalimantan dan Sulawesi. Wilayah Karesidenan biasanya di bentuk di wilayah yang banyak penduduknya. Saat ini Karesidenan sudah tidak di gunakan lagi. Penggunaan Kata Karesidenan sudah tidak digunakan secara formal, saat ini penggunaan kata Karesidenan digunakan secara informal, Eks-Karesidenan hanya di gunakan dalam pengelompokan tanda kendaraan yang dilakukan berbagai wilayah di Indonesia salah satunya di pulau Jawa, baik Jawa tengah, Jawa Timur, Jawa Barat.



Gambar 4. 1 Peta Wilayah Karesidenan Pekalongan

Di Jawa Tengah sendiri Eks-Karesidenan terbagi terdapat 6 wilayah eks karesidenan, yaitu: Eks Karesidenan Banyumas, Eks Karesidenan Kedu, Eks Karesidenan Pati, Eks-Karesidenan Pekalongan, Eks Karesidenan Semarang, Eks Karesidenan Surakarta. Karesidenan Pekalongan Provinsi Jawa Tengah merupakan salah satu karesidenan yang berada di Provinsi Jawa Tengah tepatnya di bagian barat Provinsi Jawa Tengah dan terletak pada koordinat 6^0 – 7^0 23' Lintang Selatan dan 108^0 41' – 110^0 03' 06" Bujur Timur. Sebelah utara Karesidenan Pekalongan berbatasan dengan Laut Jawa, di sebelah timur dengan Kabupaten Kendal, di sebelah selatan berbatasan dengan Kabupaten Banyumas, Kabupaten Wonosobo, Kabupaten Banjarnegara, dan Kabupaten Purbalingga, Kabupaten Cilacap. Pada sebelah Barat berbatasan dengan Kabupaten Cirebon dan Kabupaten Kuningan Provinsi Jawa Barat. Luas wilayah Karesidenan Pekalongan adalah 5.366,75 km2 yang terdiri atas 2 Kota, 5 Kabupaten dan 91 kecamatan dengan wilayah terluas adalah Kabupaten Brebes dengan luas wilayah 1.662,96 Km^2. Dalam penelitian ini ruang lingkup wilayah yang digunakan adalah Eks-Karesidenan Pekalongan yang tebagi menjadi dari 5 kabupaten dan 2 kota wilyah admistratif yaitu:

1) Kabupaten Batang

Kabupaten Batang terletak antara 6° 51' 46'' - 7° 11'47 "lintang selatandan antara 109° 40' 19" dan 110^o 03'06" bujur timur Batas-batas Kabupaten Batang berbatasan dengan:

- Sebelah Barat: Kab.Pekalongan dan Kota Pekalongan
- Sebelah Selatan: Kab.Wonosobo dan Kab. Banjarnegara
- Sebelah Timur: Kab.Kendal I lahan
- Sebelah Utara: laut Jawa

Kabupaten Batang terbagi menjadi 15 Kecamatan dengan luas wilayah 7886416 Ha.

2) Kabupaten Pekalongan

Kabupaten Pekalongan terletak di antara 6° - 7° 23' lintang selatan dan antara 109° - 109° 78' bujur timur. Kabupaten Pekalongan berbatasan dengan:

- Sebelah Utara: Laut Jawa & Kota Pekalongan
- Sebelah Timur: Kabupaten Batang & Kota Pekalongan
- Sebelah Selatan: Kabupaten Bajarnegara
- Sebelah Barat: Kabupaten Pemalang

Luas wilayah Kabupaten Pekalongan adalah kurang lebih sebesar 836.13 km2. Terdiri dari 19 Kecamatan dan 285 Desa /Kelurahan 285. 11 desa pesisir pantai dan 274 tersebar 66 Desa/Kelurahan di wilayah dataran tinggi dan 219 tersebar berada di wilayah dataran rendah (Kabupeten Pekalongan dalam angka).

3) Kota Pekalongan

Kota Pekalongan terletak di dataran rendah pantai utara pulau jawa, dengan ketinggian 1 meter di atas permukaan laut dengan letak geografis 6^o 50'24" sampe dengan 6^o 55'44" lintang selatan dan 109^o 42'19" bujur timur serta koordinat fiktif 510.00-518.00 Km membujur dan 517.75-527.72 Km melintang. Batas wilayah secara admisratif sebagai berikut:

- Sebelah utara: Laut Jawa
- Sebelah Timur: Kab.Batang
- Sebelah Selatan: Kab.Pekalongan dan Kab Batang
- Sebelah Barat: Kab.Pekalongan

Kota Pekalongan di bagi menjadi 4 Kecamatan, Kecamatan Pekalongan barat sebesar 22 persen, Pekalongan Selatan sebesar 24 persen, Pekalongan Timur 21 persen dan Pekalongan Utara sebesar 33 persen. Dengan total luas Kota Pekalongan seluas 45,25 Km2 (Kota Pekalongan dalam angka).

4) Kabupaten Pemalang

Kabupaten Pemalang merupakan Kabupaten yang terletak diantara 109^{o} 17$^{'}$ 30” sampai 109^{o} 40$^{'}$ 30” Bujur Timur dan 8^{o} 52” 30” sampai 7^{0} 20’ 11” lintang selatan. Kabupaten Pemalang secara admistratif berbatasan dengan:

- Sebelah Utara: Laut Jawa
- Sebelah Selatan: Kabupaten Purbalingga
- Sebelah Barat: Kabupaten Tegal
- Sebelah Timur: Kabupaten Pekalongan

Luas Kabupaten Pemalang 1115,30 km2 yang terdiri dari 383,51 km2 luas lahan sawah dan 73179 km2 bukan sawah. Kabupaten Pemalang terdiri dari 14 Kecamatan.Wilayah yang tersebar di daerah dataran pantai antar 1-5 meter diatas permukaan air laut 18 desa dan 1 kelurahan diwilayah utara Kabupaten Pemalang, selanjutnya dataran rendah dengan ketiggian di antara 6-15 meter diatas permukaan laut. Daerah ini meliputi 98 desa dan 5 kelurahan terletak di bagian utara wilayah Kabupaten Pemalang serta di daerah dataran tinggi, dengan ketinggian sekitar 16-212 meter di atas permukaan laut meliputi 35 desa, terletak di bagian tengah dan selatan wilayah Kabupaten Pemalang. Wilayah terakhir merupakan wilayah pegunungan dengan ketinggian 213-924 meter diatas permukaan air laut daerah ini meliputi 55 Desa, terletak di bagian selatan wilayah Kabupaten

Pemalang. Ada 10 Desa yang merupakan Desa tertinggi dari Kabupaten Pemalang dengan ketinggian 925 meter diatas permukaan laut yang berbatsan langsung dengan kabupaten Purbalingga (Kabupaten Pemalang dalam angka).

5) Kabupaten Tegal

Kabupaten Tegal merupakan salah satu daerah Kabupaten di Eks Karesidenan Pekalongan dengan ibu Kota Slawi. Terletak di antara 108^o 57’ 6” sampai dengan 109^o 21’ 30” Bujur Timur dan 6^o 50’ 41” sampai dengan 7^o 15’ 30” Lintang Selatan. Adapun Batas-batas wilayah Kabupaten Tegal adalah:

- Sebelah Utara: Kota Tegal dan Laut Jawa
- Sebelah Selatan: Kabupaten Brebes dan Kab.Banyumas
- Sebelah Barat: Kabupaten Brebes
- Sebelah Timur: Kabupaten Pemalang

Secara Topografis Kab.Tegal terdiri dari 3 ketegori daerah: daerah pantai yang meliputi 3 Kecamatan yaitu Kecamatan Kramat, Kecamatan Suradadi dan Kecamatan Watureja, Sedangkan daerah dataran rendah melitupi 10 Kecamatan dan daerah dataran tinggi meliputi 5 Kecamatan. total Kecamatan yang ada di Kabupaten Tegal 18 Kecamatan. Dengan luas wilayah 87.879 Hektar.

6) Kota Tegal

Kota Tegal terletak di antara 109^o 8’ sampai 109^o 10’ garis Bujur Timur dan 6^o 5’ sampai dengan 6^o 54’ garis Lintang Selatan. Kota Tegal berbetasan dengan:

- Sebelah Utara: Laut Jawa
- Sebelah Timur: Kabupaten Tegal
- Sebelah Selatan: Kabupaten Tegal
- Sebelah Barat: Kabupaten Brebes

Luas wilayah Kota Tegal adalah 39,68 km2, secara administrasi Kota Tegal di bagi menjadi 4 Kecamatan dengan 27 Kelurahan, wilayah terluas adalah Kecamatan tegal barat sebesar 15,13 km2 di susul dengan

Kecamatan Margadana seluas 11,76 km2, lalu ada Kecamatan Tegal selatan 6,43 km2 dan Kecamatan Tegal timur seluas 6,36 km2.

7) Kabupaten Brebes

Kabupaten Brebes merupakan kabupaten yang terakhir yang berbetasan langsung dengan Provinsi Jawa Barat. Kabupaten Brebes berada di antara 6^{o} 44'- 7^{0} 21' Lintang Selatan dan antara 108^{o} 41'- 109^{o} 11'. Kabupaten Brebes berbatasan dengan:

- Sebelah Utara: Laut Jawa
- Sebelah Timur: Kab.Tegal dan Kota Tegal
- Sebelah Selatan: Karesidenan Banyumas
- Sebelah Barat: Provinsi Jawa barat

Kabuapten Brebes mempunyailuas wilayah seluas 1.662,96 km2, terdiri dari 17 Kecamatan dan 297 Desa/Kelurahan.

4.5.2 Gambaran Umum Variabel Penelitian

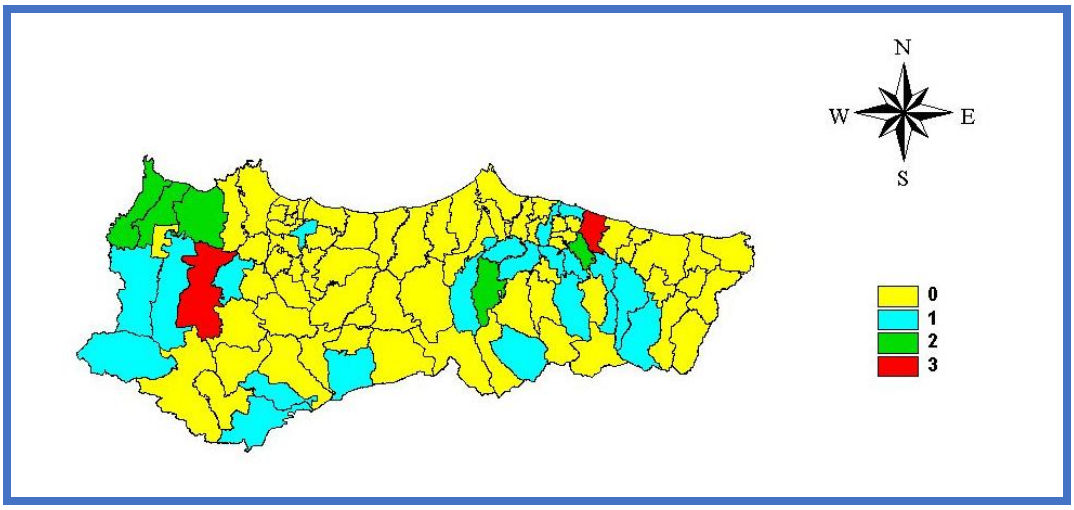
Tahapan awal dalam penelitian ini dilakukan dengan melakukan eksplorasi data yang meliputi respon dan variabel prediktor. Berdasarkan pengolahan data pada Lampiran 1 diperoleh output statistik deskriptif yang meliputi nilai minimum, nilai maksimum, mean dan standard deviasi dari setiap variabel tersebut yang disajikan dalam Tabel 4.1. Berdasarkan Tabel 4.1 menunjukkan bahwa jumlah kematian ibu baik saat hamil maupun nifas di Karesidenan Pekalongan paling banyak terdapat 3 kasus. Persentase imunisasi TT2+ pada ibu hamil (X_4) memiliki keragaman paling besar dibanding variabel prediktor lainnya yaitu 506,92. Persentase penanganan komplikasi kebidanan (X_6) memiliki rata-rata terkecil disbanding variable prediktor lainnya yaitu 30,17 dan persentase pemeriksaan kehamilan K1 (X_1) memiliki rata-rata yang paling besar dibanding variabel prediktor lainnya yaitu 97,89.

Tabel 4. 1 Nilai Statistik Deskriptif dari Variabel Penelitian

	Variabel	Min	Max	Mean	Varian
Y_1	Jumlah kematian ibu hamil	0,00	3,00	0,41	0,49
Y_2	Jumlah kematian ibu nifas	0,00	3,00	0,67	0,91
X_1	Persentase pemeriksaan kehamilan K1	46,60	100,00	97,89	42,63
X_2	Persentase pemeriksaan kehamilan K4	49,51	100,00	92,74	50,63
X_3	Persentase persalinan di tolong oleh tenaga kesehatan	79,59	100,00	97,75	16,11
X_4	Persentase imunisasi TT2+ pada ibu hamil	0,64	100,00	78,22	506,92
X_5	Persentase ibu hamil yang mendapatkan Fe3	61,17	100,00	93,18	50,80
X_6	Persentase penanganan komplikasi kebidanan	10,57	61,61	30,17	92,14
X_7	Rasio bidan per 100.000 penduduk	18,14	100,00	48,15	246,45

4.5.2.1 Jumlah Kematian Ibu Hamil di Karesidenan Pekalongan

Jumlah kematian ibu hamil merupakan jumlah kematian ibu akibat dari komplikasi kehamilan. Berdasarkan data dari Dinas Kesehatan Provinsi Jawa Tengan Tahun 2017 dapat diketahui bahwa jumlah kematian ibu hamil di Karesidenan Pekalongan tahun 2017 sebanyak 37 kematian dan Jumlah kematian ibu nifas sebanyak 61 kematian. Angka tersebut masih cukup jauh dibandingkan dengan target SDGs tahun 2030 yaitu sebesar 70/100.000 kelahiran hidup. Jumlah kematian ibu hamil tertinggi Tahun 2017 berada di Kecamatan Batang Kabupaten Batang dan Kecamatan Larangan Kabupaten Brebes sebesar 3 kematian. Jumlah kematian ibu hamil antar kecamatan cukup beragam dengan varians sebesar 0,49. Hal ini mengindikasikan bahwa program-program pengentasan jumlah kematian ibu hamil belum merata di setiap kecamatan di Karesidenan Pekalongan. Berdasarkan Gambar 4.2 terlihat hanya 2 kecamatan yang memiliki jumlah kematian ibu hamil terbanyak dibandingkan kecamatan lain yaitu Kecamatan Batang Kabupaten Batang dan Kecamatan Larangan Kabupaten Brebes. Sebagian besar kecamatan tidak memiliki kasus kematian ibu hamil selama Tahun 2017.

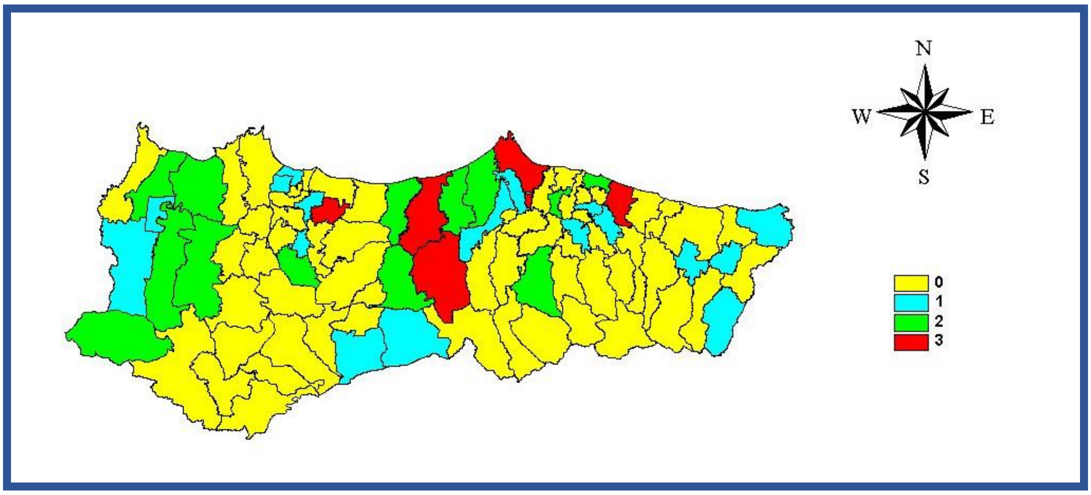


Gambar 4. 2 Persebaran Jumlah Kematian Ibu Hamil menurut Kecamatan di Karesidenan Pekalongan Tahun 2017

4.5.2.2 Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan

Jumlah kematian ibu nifas merupakan jumlah kematian ibu akibat dari komplikasi pada masa nifas. SDGs tahun 2030 menargetkan bahwa jumlah kematian ibu nifas di Indonesia sebesar 12/1000 kelahiran hidup. Berdasarkan Tabel 4.1 dapat diketahui bahwa jumlah kematian ibu nifas di Karesidenan Pekalongan sebesar 3 kematian namun angka ini belum dapat menggambarkan jumlah kematian ibu nifas yang sesungguhnya karena kasus – kasus kematian yang terlaporkan hanyalah kasus kematian yang terjadi di sarana pelayanan kesehatan, sedangkan kasus – kasus kematian yang terjadi di masyarakat belum seluruhnya terlaporkan. Jumlah kematian ibu nifas tertinggi berada di Kecamatan Batang Kabupaten Batang, Kecamatan Tarub Kabupaten Tegal, Kecamatan Losari Kabupaten Brebes, Kecamatan Bantarbolang, Pemalang dan Ulujami Kabupaten Pemalang.

Berdasarkan Gambar 4.3, kasus kematian ibu nifas menyebar di beberapa kecamatan dan terdapat 3 kecamatan di Kabupaten Pemalang dengan jumlah kematian ibu nifas terbanyak. Kasus kematian ibu nifas lebih banyak dibandingkan dengan kematian ibu hamil sehingga diharapkan program – program untuk menurunkan Jumlah Kematian Ibu Nifas yang dilakukan oleh instansi terkait lebih difokuskan pada kecamatan-kecamatan yang memiliki kasus kematian ibu nifas serta pencegahan untuk kecamatan yang tidak memiliki kasus kematian ibu nifas.



Gambar 4. 3 Persebaran Jumlah Kematian Ibu Nifas Menurut Kecamatan di Karesidenan Pekalongan Tahun 2017

4.5.3 Deteksi Over/Underdispersi

Deteksi under/overdispersi menggunakan *Variance Test* (VT) sebagai berikut:

$$VT = \sum_{i=1}^{91} \frac{\left(y_i - \bar{y}\right)^2}{\bar{y}}$$

Data dikatakan mengalami underdispersi ketika VT < 1, overdispersi ketika VT > 1 dan equidispersi ketika VT = 1. Hasil deteksi under/overdipersi untuk masing-masing respon memiliki nilai VT>1 sehingga dapat dikatakan terjadi overdispersi pada Data Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas. Untuk hasil penghitungannya dapat dilihat pada Lampiran 3.

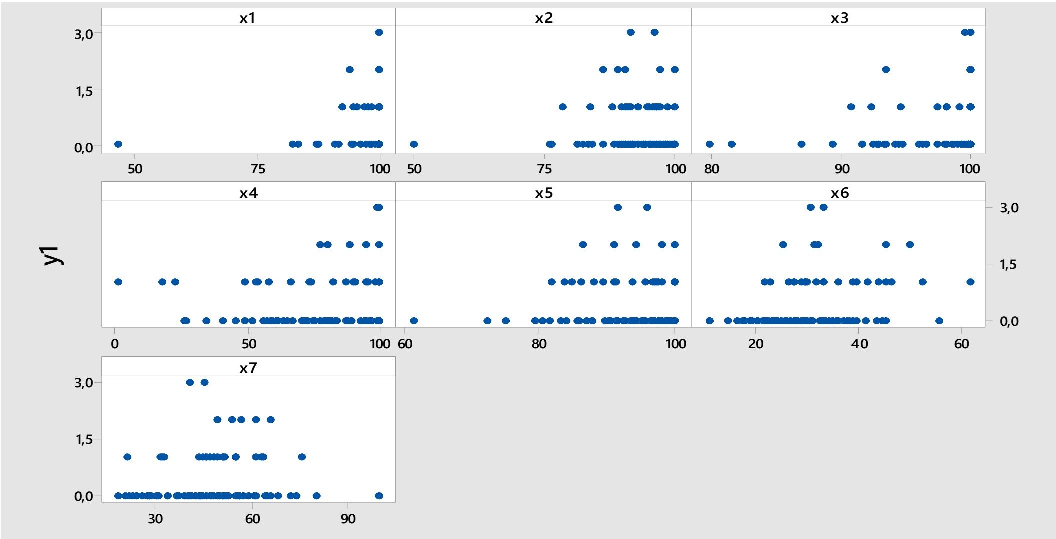
4.5.4 Pola Data antara Variabel Respon dan Variabel Prediktor

Tabel 4. 2 Koefisien Korelasi dan Signifikansi Variabel Penelitian

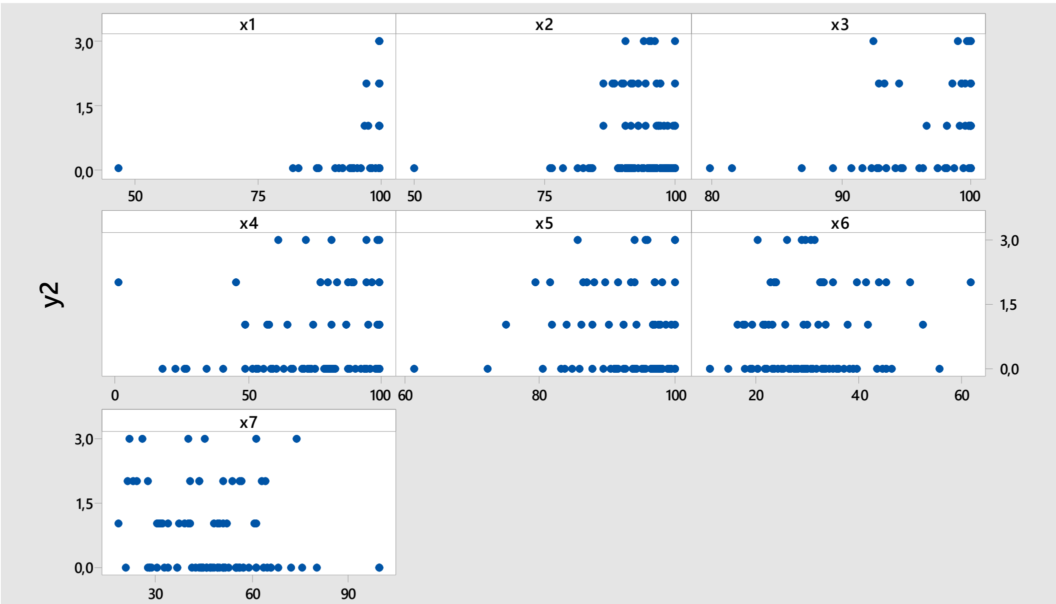
Variabel	Koefisien Korelasi (Signifikansi)						
	X_1	X_2	X_3	X_4	X_5	X_6	X_7
Y_1	0,08	0,02	0,14	0,07	0,05	0,30	0,08
	(0,45)	(0,86)	(0,17)	(0,54)	(0,63)	(0,00)	(0,46)
Y_2	0,21	0,11	0,16	0,11	-0,07	0,09	-0,21
	(0,05)	(0,28)	(0,14)	(0,32)	(0,52)	(0,39)	(0,04)

( ) = nilai *p-value*

Berdasarkan nilai koefisien korelasi pada Tabel 4.2, dapat diketahui bahwa terdapat hubungan antara variabel prediktor dengan respon. Variabel Persentase pemeriksaan kehamilan K1 (X_1), Persentase pemeriksaan kehamilan K4 (X_2), Persentase persalinan di tolong oleh tenaga kesehatan. (X_3) dan Persentase imunisasi TT2+ pada ibu hamil. (X_4) Persentase ibu hamil yang mendapatkan Fe3 (X_5), Persentase penanganan komplikasi kebidanan (X_6) dan Rasio bidan per 100.000 penduduk (X_7) mempunyai pola hubungan yang positif dengan jumlah kematian ibu hamil. Pola yang sama juga ditunjukkan pada variabel Persentase pemeriksaan kehamilan K1 (X_1), Persentase pemeriksaan kehamilan K4 (X_2), Persentase persalinan di tolong oleh tenaga kesehatan (X_3), Persentase imunisasi TT2+ pada ibu hamil. (X_4), dan Persentase penanganan komplikasi kebidanan (X_6) terhadap jumlah kematian ibu nifas. Akan tetapi, variabel Persentase ibu hamil yang mendapatkan Fe3 (X_5) dan Rasio bidan per 100.000 penduduk (X_7) memiliki pola yang negatif artinya setiap kenaikan variabel X_5 dan X_7 akan menurunkan jumlah kematian ibu nifas di Karesidenan Pekalongan. Analisis lebih lanjut menggunakan uji statistik perlu dilakukan untuk memastikan arah hubungan antara jumlah kematian ibu hamil dan jumlah kematian ibu nifas terhadap variabel yang memengaruhinya. Selain itu, besarnya pengaruh variabel-variabel tersebut terhadap jumlah kematian ibu hamil dan jumlah kematian ibu nifas di masing-masing kecamatan juga perlu dicari untuk menunjukan prioritas kebijakan di setiap wilayah.



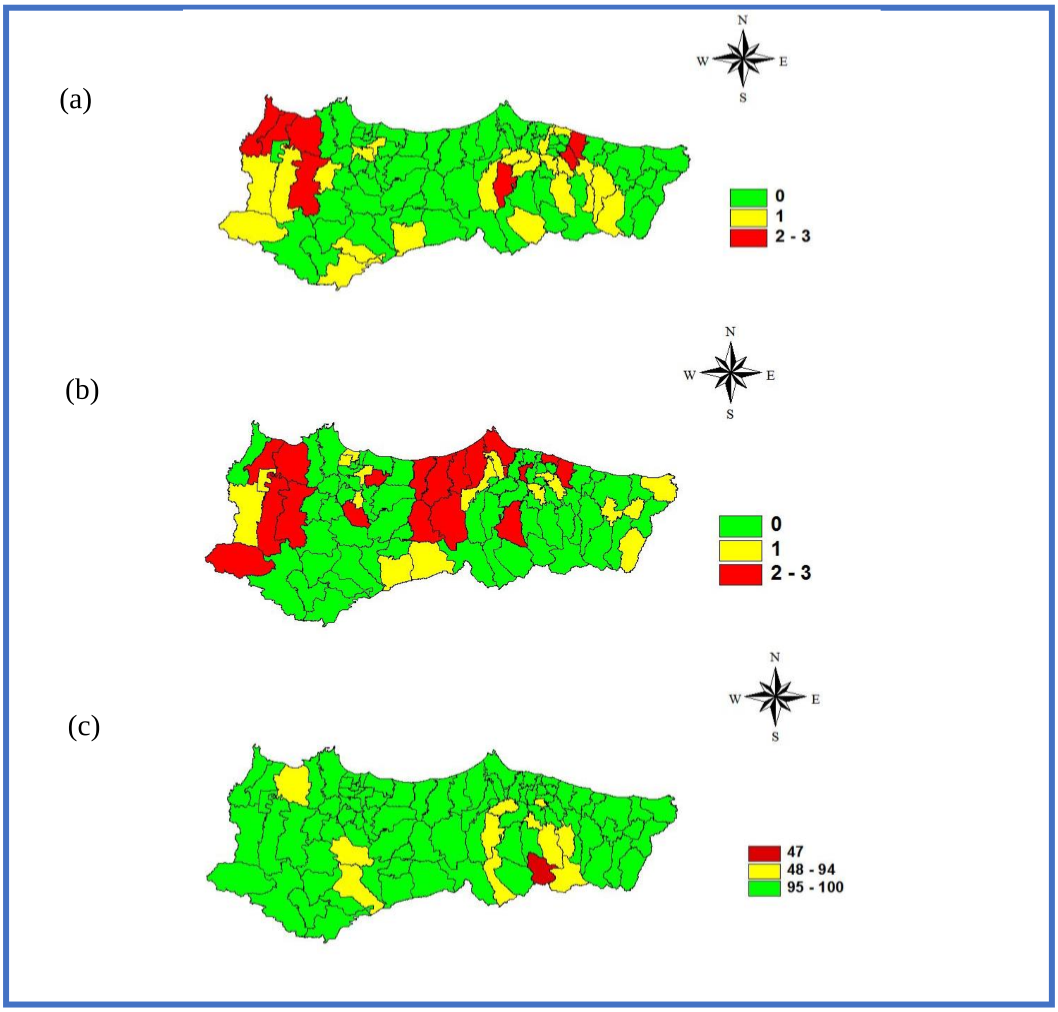
Gambar 4. 4 *Scatter Plot* Jumlah Kematian Ibu Hamil dengan Variabel Prediktornya



Gambar 4. 5 *Scatter Plot* Jumlah Kematian Ibu Nifas dengan Variabel Prediktornya

Berdasarkan Gambar 4.4 dan 4.5 sebagian besar variabel prediktor menunjukkan hubungan yang positip artinya setiap penambahan satuan pada variabel prediktor akan menaikkan Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas.

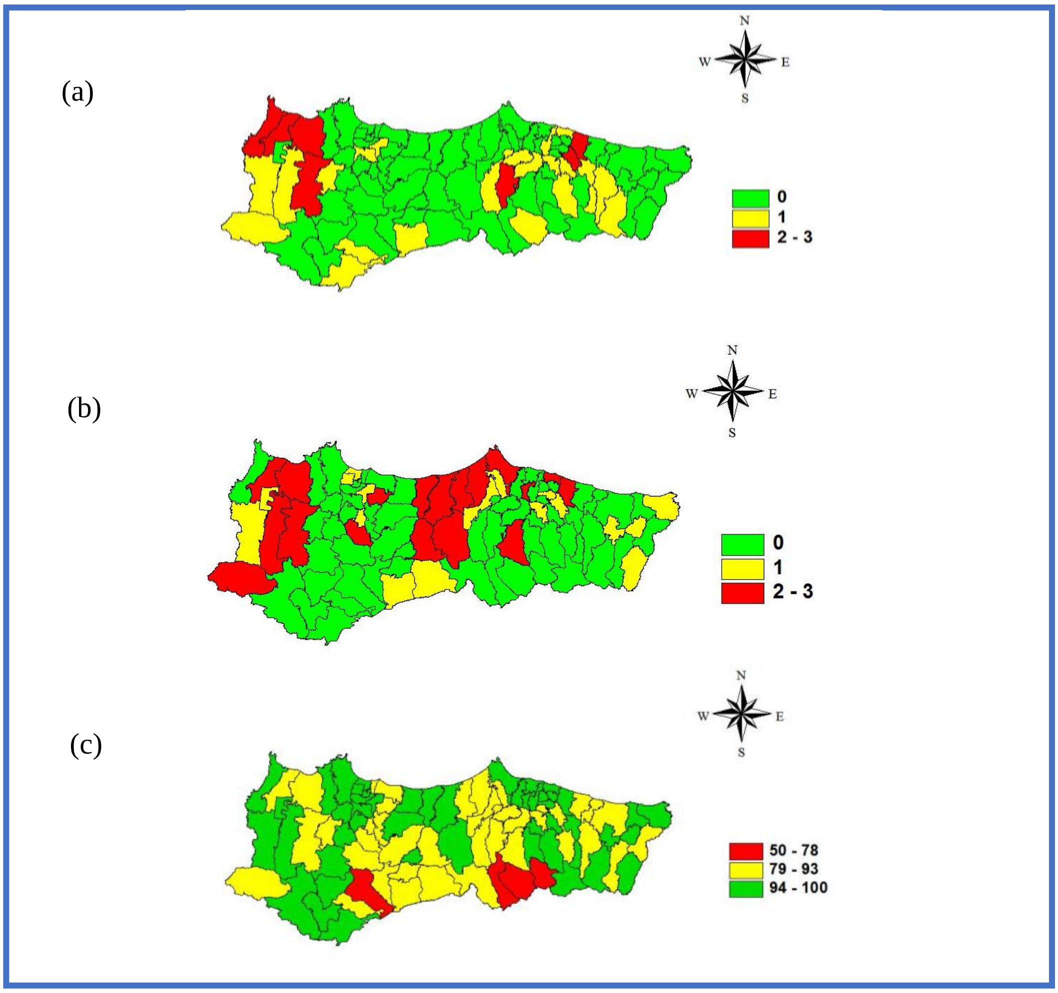
4.5.4.1 Jumlah Kematian Ibu Hamil dan Persentase Pemeriksaan Kehamilan K1



Gambar 4. 6 Pola Persebaran (a) Jumlah Kematian Ibu Hamil, (b) Jumlah Kematian Ibu Nifas dan (c) Persentase Kunjungan Kehamilan K1 di Karesidenan Pekalongan menurut Kecamatan

Pola persebaran persentase kunjungan kehamilan K1 pada sebagian besar kecamatan sudah di atas 95 persen namun hubungannya dengan jumlah kematian ibu hamil dan jumlah kematian ibu nifas menunjukkan pola yang berbeda-beda. Pada gambar 4.5 terlihat beberapa kecamatan di Kabupaten Brebes memiliki jumlah kematian ibu hamil tertinggi meskipun persentase kunjungan kehamilan K1 di atas 95 persen dan berkebalikan dengan salah satu kecamatan yang berada di Kota Pekalongan yang memiliki persentase kunjungan kehamilan K1 rendah namun jumlah kematian ibu hamilnya juga rendah. Hal yang sama juga terlihat pada beberapa kecamatan yang berada di Kabupaten Brebes dan Pemalang yang memiliki persentase kunjungan kehamilan K1 di atas 95 persen.

4.5.4.2 Jumlah Kematian Ibu Hamil dan Persentase Pemeriksaan Kehamilan K4

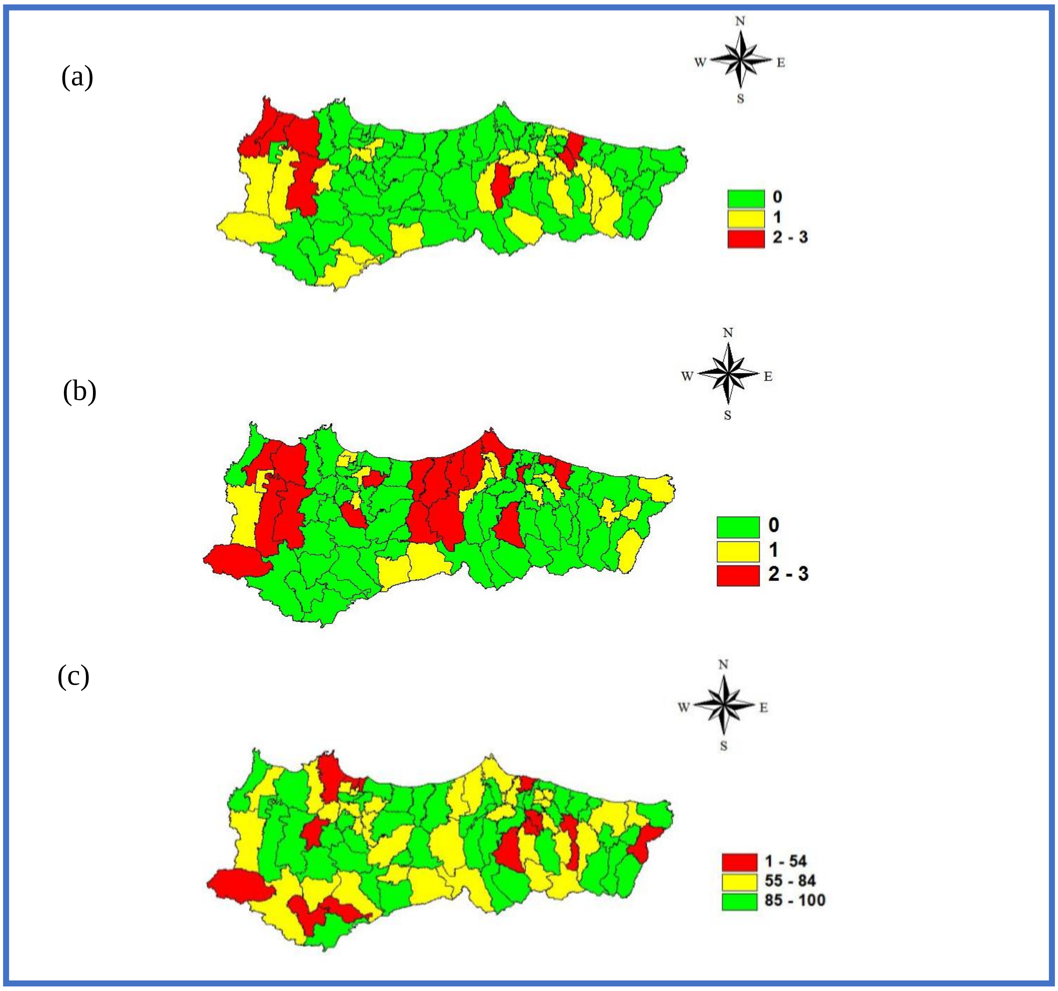


Gambar 4. 7 Pola Persebaran (a) Jumlah Kematian Ibu Hamil, (b) Jumlah Kematian Ibu Nifas dan (c) Persentase Kunjungan Kehamilan K4 di Karesidenan Pekalongan menurut Kecamatan

Persentase kunjungan kehamilan K4 berkisar antara 50-100 persen pada masing-masing kecamatan di Karesidenan Pekalongan dan menyebar pada masing-masing kecamatan. Terdapat pola yang negatip antara jumlah kematian ibu hamil dan jumlah kematan ibu nifas dengan persentase kunjungan ibu hamil K4 pada pada beberapa kecamatan antara lain di Kabupaten Tegal yang memiliki persentase kunjungan kehamilan K1 di atas 79 persen namun jumlah kematian ibu hamil lebih tinggi disbanding kecamatan lainnya, begitu juga pada beberpa kecamatan di Kabupaten Pemalang yang memiliki jumlah kematian ibu nifas tinggi dibanding kecamatan lain.

4.5.4.3 Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas dengan Persentase Imunisasi TT2+ pada Ibu Hamil

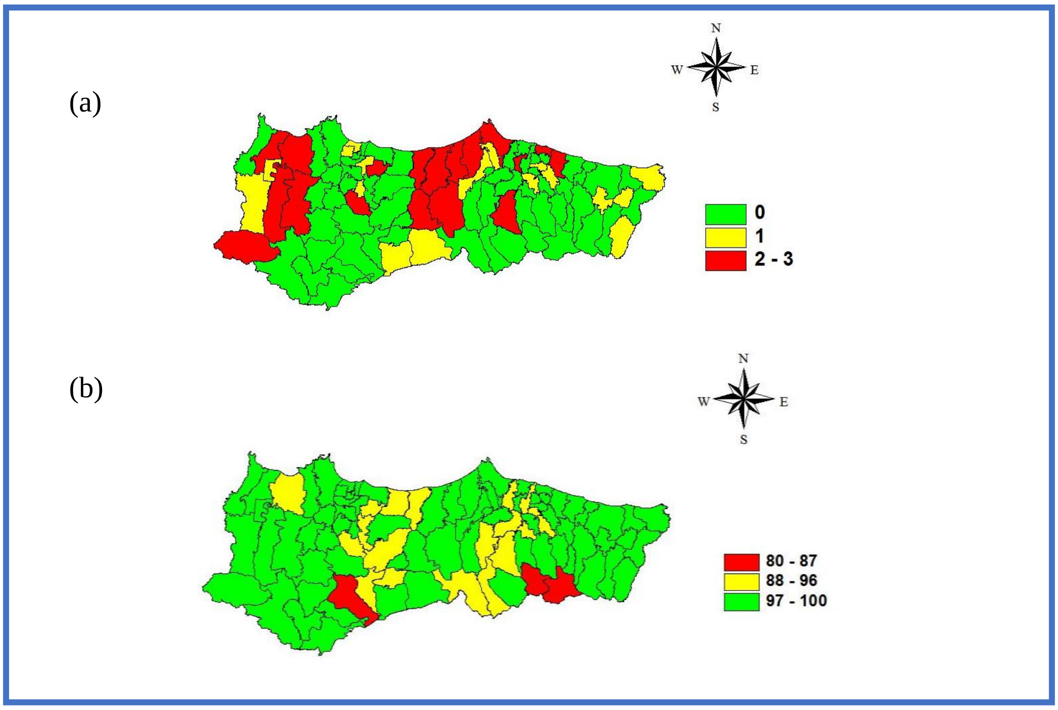
Persentase imunisasi TT2+ pada ibu hamil menyebar pada semua kecamatan dengan persentase 0,64 -100 persen dan masih terdapat ibu hamil yang kurang peduli dengan pentingnya imunisasi TT2+ selama masa hehamilan dan hal ini dapat terlihat pada nilai pesentasenya yang di bawah 55 persen. Pada gambar 4.6 menunjukkan bahwa sebagian besar Persentase imunisasi TT2+ pada ibu hamil pada setiap kecamatan menunjukkan pola yang negatif dengan jumlah kematian ibu hamil dan jumlah kematian ibu nifas yang berarti semakin banyak ibu hamil yang mendapat imunisasi TT2+ maka akan menurunkan jumlah kematian ibu hamil dan jumlah kematian ibu nifas di Karesidenan Pekalongan meskipun terdapat sebagian kecil kecamatan yang meiliki pola positip dan kecamatan seharusnya menjadi perhatian ke depannya oleh dinas kesehatan dalam meningkatan jumlah ibu hamil yang mendapat imunisasi TT2+.



Gambar 4. 8 Pola Persebaran (a) Jumlah Kematian Ibu Hamil, (b) Jumlah Kematian Ibu Nifas dan (c) Persentase Imunisasi TT2+ pada Ibu Hamil di Karesidenan Pekalongan menurut Kecamatan

4.5.4.4 Jumlah Kematian Ibu Nifas dan Persentase Persalinan Ditolong Tenaga Kesehatan

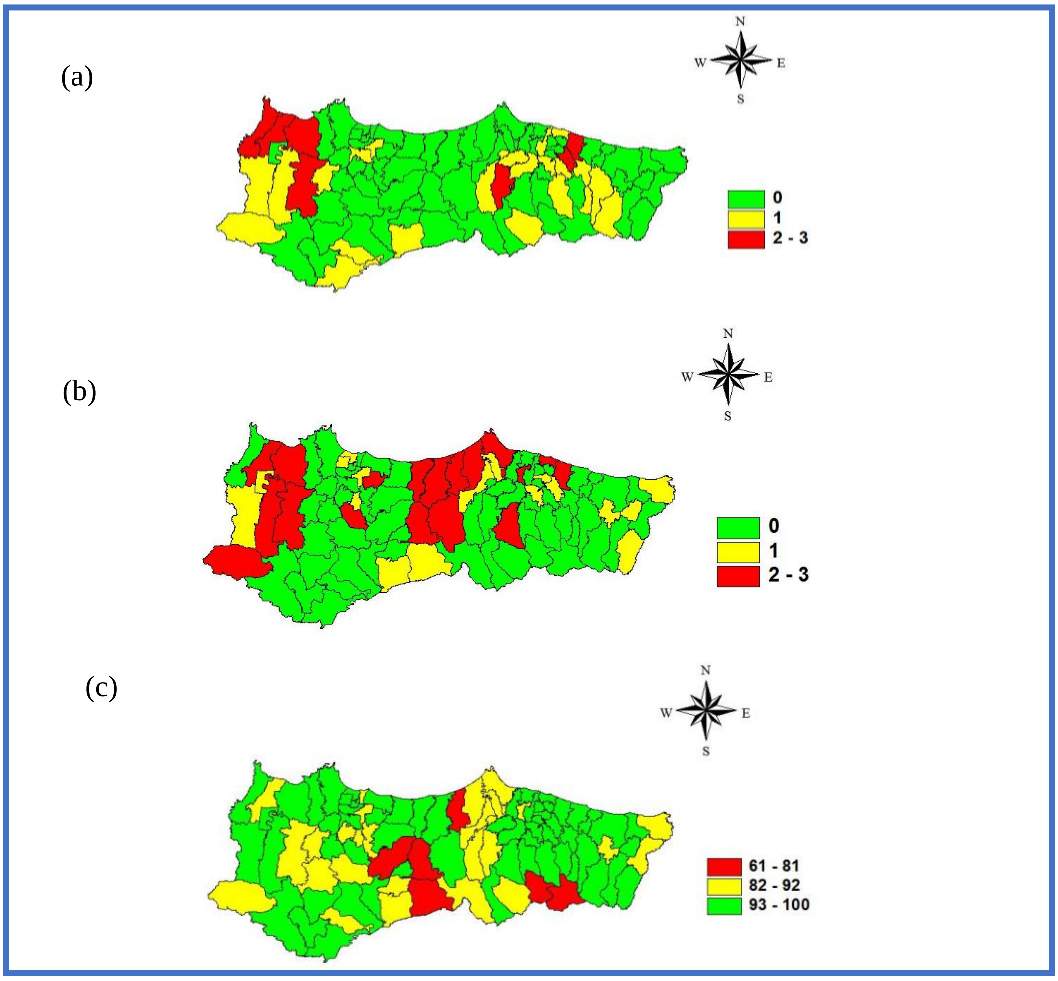
Persentase persalinan yang ditolong oleh tenaga kesehatan di Karesidenan cukup tinggi yaitu diatas 80 persen bahkan sebagian besar kecamatan memiliki persentase 100 persen, ini berarti kesadaran masyarakat akan pentingnya kelahiran ditolong oleh tenaga kesehatan sudah tinggi meskipun pada beberapa kecamatan di Kabupaten Brebes dan Kabupaten Pemalang menunjukkan hubungan yang positip yaitu masih terdapat jumlah kematian Ibu saat nifas.



Gambar 4. 9 Pola Persebaran (a) Jumlah Kematian Ibu Nifas dan (b) Persentase Persentase Persalinan Ditolong Tenaga Kesehatan di Karesidenan Pekalongan menurut Kecamatan

4.5.4.5 Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas dengan Persentase Ibu Hamil yang mendapatkan Fe3

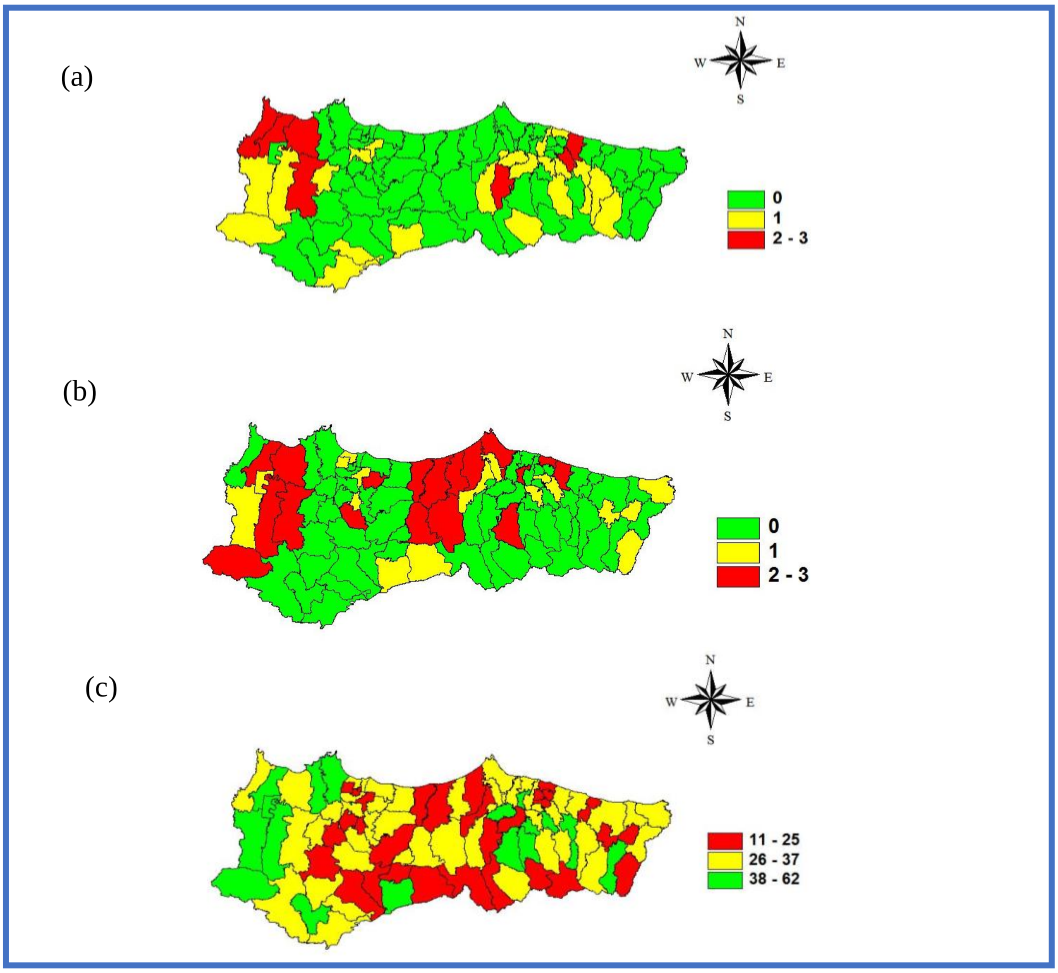
Pentingnya tablet Fe3 pada ibu hamil menjadi suatu keharusan bahwa seorang ibu hamil harus mengkonsumsi tablet tersebut hal ini dapat terlihat pada Gambar 4.8 dimana persentase ibu hamil yang mengkonsumsi tablet Fe3 sebagian besar diatas 92 persen meskipun masih ada kecamatan yang memiliki persentase di bawah 82 persen. Tingginya konsumsi tablet Fe3 dapat menyebabkan turunnya jumlah kematian ibu saat hamil meskipun pada beberapa kecamatan yang berada di Kabupaten Brebes dan Kabupaten Pemalang memiliki pola hubungan yang positip yaitu ketika persentase konsumsi Fe3 saat hamil menyebabkan meningkatnya jumlah kematian ibu saat nifas.



Gambar 4. 10 Pola Persebaran (a) Jumlah Kematian Ibu Hamil, (b) Jumlah Kematian Ibu Nifas dan (c) Persentase Ibu Hamil yang mendapatkan Fe di Karesidenan Pekalongan menurut Kecamatan

4.5.4.6 Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas dengan Persentase Penanganan Komplikasi Kebidanan

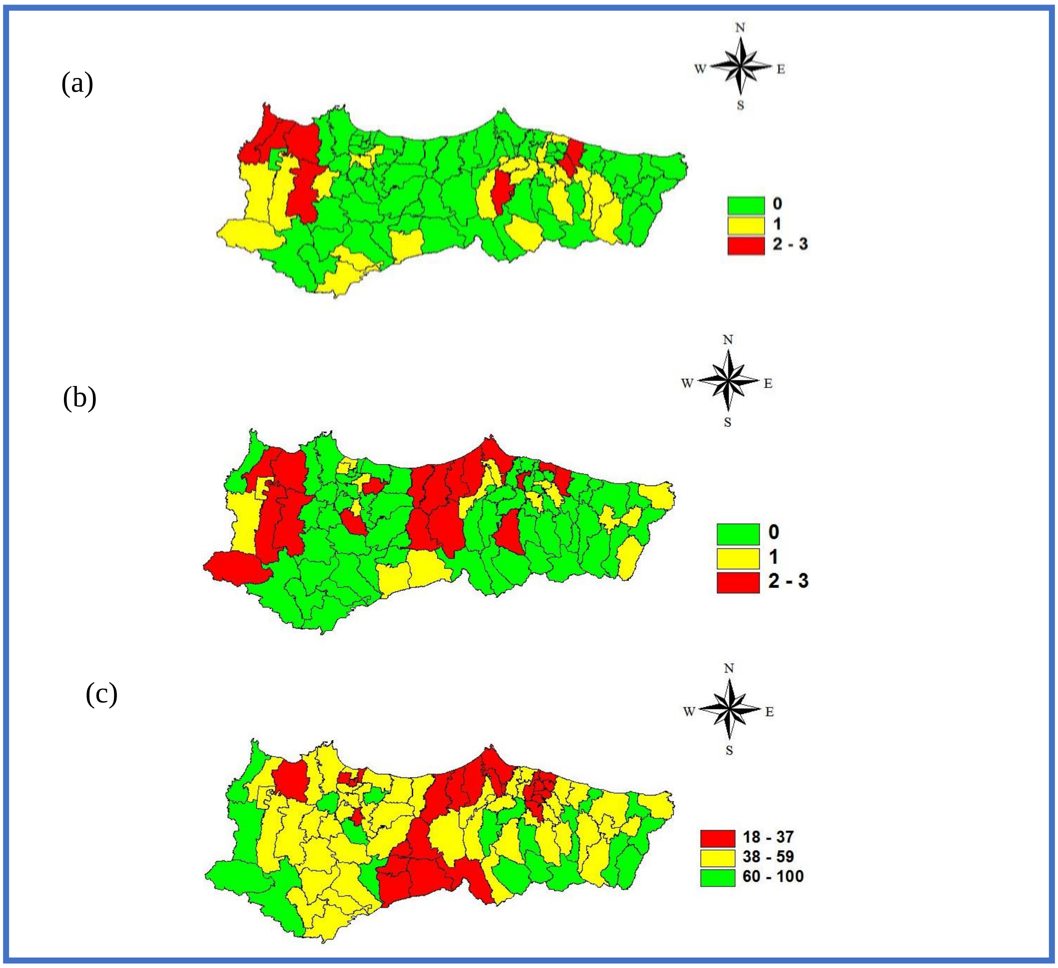
Persentase penanganan komplikasi kebidanan cukup beragam di Karesidenan Pekalongan dengan rentang 11-62 persen karena tidak semua komplikasi pada ibu hamil dan ibu nifas dilaporkan dan diperiksakann sehingga hal ini dapat memicu tingginya jumlah kematian ibu saat hamil maupun saat nifas.



Gambar 4. 11 Pola Persebaran (a) Jumlah Kematian Ibu Hamil, (b) Jumlah Kematian Ibu Nifas dan (c) Persentase Penanganan Komplikasi Kebidanan di Karesidenan Pekalongan menurut Kecamatan

4.5.4.7 Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas dengan Rasio Bidan per 100.000 Penduduk

Seperti diperlihatkan pada Tabel 4.1 bahwa rata-rata rasio bidan per 100.000 penduduk di Karesidenan Pekalongan adalah 48,15 yang dapat diartikan setiap 100.000 penduduk terdapat 48 bidan atau setiap 10.000 penduduk terdapat 5 bidan sehingga diharapkan dengan kehadiran bidan ditengah masyarakat khususnya ibu hamil dapat menjadikan jumlah kematian ibu semakin menurun meskipun terlihat di Gambar 4.12 pada beberapa kecamatan menunjukkan jumlah kematian ibu nifas yang tinggi.



Gambar 4. 12 Pola Persebaran (a) Jumlah Kematian Ibu Hamil, (b) Jumlah Kematian Ibu Nifas dan (c) Rasio Bidan per 100.000 penduduk di Karesidenan Pekalongan menurut Kecamatan

4.5.5 Uji Korelasi Respon

Analisis regresi *bivariate* menyaratkan bahwa kedua respon harus saling berkorelasi. Penelitian ini menggunakan data jumlah kematian ibu hamil (Y_1) dan jumlah kematian ibu nifas (Y_2) sebagai respon. Berdasarkan Lampiran 29 dapat diketahui bahwa korelasi antar variabel respon sebesar 0,269. Nilai tersebut menunjukkan adanya korelasi positif yang artinya semakin tinggi jumlah kematian ibu hamil, maka semakin tinggi pula jumlah kematian ibu nifas. Sebaliknya, semakin rendah jumlah kematian ibu hamil, maka semakin rendah pula jumlah kematian ibu nifas. Hipotesis untuk pengujian korelasi dari respon sebagai berikut:

$H_0 : \rho_{y_1,y_2} = 0$ (tidak terdapat hubungan antara Y_1 dan Y_2)

$H_1 : \rho_{y_1,y_2} \neq 0$ (terdapat hubungan antara Y_1 dan Y_2)

Statistik uji yang diperoleh adalah:

$$t = \frac{r_{y_1,y_2}\sqrt{n-2}}{\sqrt{1-\left(r_{y_1,y_2}\right)^2}} = \frac{0,2696\sqrt{91-2}}{\sqrt{1-0,2696^2}} = 2,6414$$

Nilai t yang diperoleh 2,6414 lebih besar dari $t_{(0.025;90)} = 1,98$ sehingga keputusan tolak H_0, atau terdapat hubungan antara variabel Y_1 dan Y_2. Berdasarkan hasil uji korelasi tersebut diperoleh kesimpulan bahwa terdapat hubungan antara jumlah kematian ibu hamil dan jumlah kematian ibu nifas di Karesidenan Pekalongan Tahun 2017, sehingga analisis dapat dilanjutkan ke analisis selanjutnya yaitu BZIGP.

4.5.6 Uji Multikolinearitas

Multikolinieritas merupakan kasus dimana terdapat hubungan antara variabel prediktor dalam model. Dari 7 variabel prediktor yang digunakan pada penelitian ini ada indikasi bahwa terdapat variabel yang saling berhubungan, sehingga diperlukan adanya uji multikolinieritas. Ada beberapa cara untuk mendeteksi kasus multikolinieritas yaitu dengan matrik kooefisien korelasi dan nilai Variance Inflation Factors (VIF) seperti pada persamaan (2.34). Menurut Gujarati (2003) apabila nilai VIF > 10, maka terjadi multikolinieritas. Hasil pengujian multikolinieritas antar variabel terdapat pada Lampiran 30.

Tabel 4. 3 Nilai VIF Variabel Prediktor

	X_1	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**	**X_7**
VIF	2,78	2,77	1,78	1,12	1,57	1,33	1,32

Berdasarkan Tabel 4.3 dapat dilihat bahwa nilai VIF setiap variabel prediktor bernilai kurang dari 10, maka dapat disimpulkan tidak ada kasus multikolinearitas pada variabel prediktor yang digunakan dalam penelitian ini.

4.5.7 Pemodelan Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan dengan BZIGPR

Pemodelan menggunakan BZIGPR menghasilkan parameter yang sama untuk setiap lokasi pengamatan, sehingga faktor – faktor yang memengaruhi jumlah kematian ibu hamil dan jumlah kematian ibu nifas di Karesidenan Pekalongan tahun 2017 dianggap sama untuk setiap kecamatan. Hasil pemodelan jumlah kematian ibu hamil dan jumlah kematian ibu nifas dengan BZIGPR dapat dilihat pada Lampiran 31.

Pada pengujian serentak parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$ dengan taraf signifikansi $\alpha = 5\%$ diperoleh nilai G^2 sebesar 3578,869 dan nilai $\chi^2_{(0,05;14)} = 23,685$. Pada pengujian serentak parameter $\boldsymbol{\beta}$ dengan taraf signifikansi $\alpha = 5\%$ diperoleh nilai G^2 sebesar 3576,365 dan nilai $\chi^2_{(0,05;14)} = 23,685$. Pada pengujian serentak parameter $\boldsymbol{\gamma}$ dengan taraf signifikansi $\alpha = 5\%$ diperoleh nilai G^2 sebesar 3531,279 dan nilai $\chi^2_{(0,05;14)} = 23,685$. Pada pengujian serentak baik pada parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$, $\boldsymbol{\beta}$, maupun $\boldsymbol{\gamma}$ menghasilkan nilai G^2 lebih besar dari χ^2_{tabel} sehingga dapat disimpulkan dengan keputusan tolak H_0 yang berarti bahwa minimal ada satu variabel yang berpengaruh dalam model.

Berdasarkan Tabel 4.4 dapat diketahui bahwa dengan tingkat signifikansi $\alpha = 5\%$ faktor – faktor yang signifikan berpengaruh terhadap jumlah kematian ibu hamil dan jumlah kematian ibu nifas di Karesidenan Pekalongan tahun 2017 pada parameter $\boldsymbol{\beta}$ adalah Variabel Persentase pemeriksaan kehamilan K1 (X_1), Persentase pemeriksaan kehamilan K4 (X_2), Persentase persalinan di tolong oleh tenaga kesehatan. (X_3) dan Persentase imunisasi TT2+ pada ibu hamil. (X_4) Persentase ibu hamil yang mendapatkan Fe3 (X_5), Persentase penanganan komplikasi kebidanan (X_6) dan Rasio bidan per 100.000 penduduk (X_7). Variabel yang signifikan pada parameter $\boldsymbol{\gamma}$ pada jumlah kematian ibu hamil adalah semua variabel prediktor namun pada jumlah kematian ibu nifas, variabel yang signifikan adalah Persentase pemeriksaan kehamilan K1 (X_1), Persentase pemeriksaan kehamilan K4 (X_2), Persentase persalinan di tolong oleh tenaga kesehatan (X_3), Persentase ibu hamil yang mendapatkan Fe3 (X_5) dan Persentase penanganan komplikasi kebidanan (X_6).

Tabel 4. 4 Nilai Taksiran Paramater Model BZIGPR

Parameter	**Nilai Taksiran**	**SE**	**Z Hitung**	**P-*Value***
γ_{10}	0,0100	0,0000	683,1607	0,0000
γ_{11}	0,0143	0,0013	11,1830	0,0000
γ_{12}	0,0139	0,0010	13,4849	0,0000
γ_{13}	0,0143	0,0012	11,3626	0,0000
γ_{14}	0,0124	0,0062	1,9886	0,0467
γ_{15}	0,0139	0,0011	12,1278	0,0000
γ_{16}	0,0120	0,0008	14,7161	0,0000
γ_{17}	0,0121	0,0014	8,8958	0,0000
γ_{20}	0,0120	0,0000	1495,4144	0,0000
γ_{21}	0,0109	0,0015	7,3192	0,0000
γ_{22}	0,0109	0,0014	7,7084	0,0000
γ_{23}	0,0109	0,0009	11,5491	0,0000
γ_{24}	0,0102	0,0055	1,8438	0,0652
γ_{25}	0,0105	0,0012	9,0523	0,0000
γ_{26}	0,0125	0,0021	5,8631	0,0000
γ_{27}	0,0106	0,0060	1,7603	0,0784
$\beta_{1.0}$	-10,4856	0,0000	-436572,2155	0,0000
β_{11}	0,0317	0,0012	26,0414	0,0000
β_{12}	-0,0412	0,0010	-39,6578	0,0000
β_{13}	0,0703	0,0012	57,8992	0,0000
β_{14}	0,0097	0,0019	5,2435	0,0000
β_{15}	0,0069	0,0012	5,9201	0,0000
β_{16}	0,0440	0,0006	78,8105	0,0000
β_{17}	0,0060	0,0011	5,3661	0,0000
β_{20}	-30,5023	0,0000	-1629897,9552	0,0000
β_{21}	0,3514	0,0008	415,7019	0,0000
β_{22}	0,0173	0,0007	25,7143	0,0000
β_{23}	-0,0375	0,0009	-43,0567	0,0000
β_{24}	0,0089	0,0028	3,1957	0,0014
β_{25}	-0,0268	0,0006	-47,4829	0,0000

Lanjutan Tabel 4.4

Parameter	Nilai Taksiran	SE	Z Hitung	P-*Value*
β_{26}	0,0254	0,0007	34,7496	0,0000
β_{27}	-0,0159	0,0008	-19,7507	0,0000
φ_1	1,0640	0,0008	1412,5907	0,0000
φ_2	1,2108	0,0008	1607,3857	0,0000
η	0,2690	0,0000	47773,0734	0,0000

Berdasarkan Tabel 4.4, maka diperoleh model BZIGPR untuk jumlah kematian ibu hamil $(\ln\hat{\mu}_1)$ dan jumlah kematian ibu nifas $(\ln\hat{\mu}_2)$ di Karesidenan Pekalongan Tahun 2017 adalah sebagai berikut:

a. Model regresi pada jumlah kematian ibu hamil

$$\ln\hat{\mu}_1 = -10,4856 + 0,0317x_{1i} - 0,0412x_{2i} + 0,0703x_{3i} + 0,0097x_{4i} + 0,0069x_{5i} + 0,0440x_{6i} + 0,0060x_{7i}$$

$$\text{logit } \hat{p}_1 = -0,01 + 0,0143x_{1i} + 0,0139x_{2i} + 0,0143x_{3i} + 0,0124x_{4i} + 0,0139x_{5i} + 0,0120x_{6i} + 0,0121x_{7i}$$

b. Model regresi pada jumlah kematian ibu nifas

$$\ln\hat{\mu}_2 = -30,5023 + 0,3514x_{1i} + 0,0173x_{2i} - 0,0375x_{3i} + 0,0089x_{4i} - 0,0268x_{5i} + 0,0254x_{6i} - 0,0159x_{7i}$$

$$\text{logit } \hat{p}_2 = 0,0120 + 0,0109x_{1i} + 0,0109x_{2i} + 0,0109x_{3i} + 0,0102x_{4i} + 0,0105x_{5i} + 0,0125x_{6i} + 0,0106x_{7i}$$

Perhitungan nilai taksiran jumlah kematian ibu hamil dan jumlah kematian ibu nifas untuk beberapa kecamatan disajikan dalam Tabel 4.5 berikut:

Tabel 4. 5 Nilai Taksiran Respon Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas

Kecamatan	Jumlah Kematian Ibu Hamil	Jumlah Kematian Ibu nifas	Kecamatan	Jumlah Kematian Ibu Hamil	Jumlah Kematian Ibu nifas
Wonotunggal	0,4278	1,3482	Banyuputih	0,4552	2,7675
Bandar	0,3148	1,5034	Subah	0,3802	1,7636
Blado	0,3747	2,1513	Pecalungan	0,1574	2,9073
Reban	1,4285	3,1228	Tulis	0,2551	1,1371
Bawang	0,2573	1,5591	Kandeman	0,3567	2,0062
Tersono	0,3529	1,1916	Batang	0,3689	2,4896

Untuk hasil lengkap nilai taksiran jumlah kematian ibu hamil dan jumlah kematian ibu nifas dapat dilihat pada Lampiran 33. Pemodelan Jumlah kematian ibu hamil dan jumlah kematian ibu nifas menggunakan BZIGPR menghasilkan nilai AICc sebesar 1098,138 serta MSE untuk jumlah kematian ibu hamil sebesar 0,4423004 dan MSE untuk jumlah kematian ibu nifas sebesar 2,853079.

4.5.8 Pengujian Heterogenitas Spasial Jumlah Kematian Ibu Hamil Dan Jumlah Kematian Ibu Nifas

Perbedaan karakteristik antar wilayah pengamatan menyebabkan adanya heterogenitas spasial. Untuk melihat ada tidaknya heterogenitas spasial dalam pemodelan jumlah kematian ibu hamil dan jumlah kematian ibu nifas ini, maka akan dilakukan pengujian heterogenitas spasial sesuai dengan persamaan (2.79) dengan hipotesis sebagai berikut:

H_0: $\Sigma_1 = \Sigma_2 = \ldots = \Sigma_n = \Sigma$

H_1: minimal ada satu $\Sigma_i \neq \Sigma$; $i = 1, 2, \ldots, 91$

Berdasarkan hasil pengolahan pada Lampiran 34, diketahui bahwa nilai statistik uji *glejser* sebesar 90,3217 lebih besar dari nilai $\chi^2_{(0,05;14)}$ sebesar 23,685, sehingga diperoleh keputusan tolak H_0. Hal ini berarti jumlah kematian ibu hamil dan jumlah kematian ibu nifas di Karesidenan Pekalongan Tahun 2017 mempunyai heterogenitas spasial antar wilayah, sehingga pemodelan dengan GWBZIGPR dapat dilakukan.

4.5.9 Penentuan Fungsi Pembobot Model GWBZIGPR pada Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan Tahun 2017

Berbeda dengan pemodelan menggunakan BZIGPR, pemodelan menggunakan GWBZIGPR menghasilkan penaksir parameter yang berbeda untuk setiap kecamatan. Hal ini merupakan pengaruh penambahan pembobot geografis yaitu koordinat lintang dan bujur pada setiap lokasi pengamatan. Informasi letak geografis tersebut kemudian digunakan untuk menghitung jarak euclid antar lokasi pengamatan. Hasil penghitungan jarak *euclid* terlampir pada Lampiran 35.

Penelitian ini menggunakan pembobot dengan fungsi *Kernel Gaussian* dan *Bisquare,* baik *fixed* maupun *adaptive* dengan rumus penghitungannya merujuk dengan subbab 2.15.2. Hal ini bertujuan untuk mengetahui bandwidth mana yang lebih sesuai untuk melakukan pembobotan spasial pada kasus Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan Tahun 2017. Pemilihan nilai bandwidth yang optimum dengan metode GCV. Nilai bandwidth yang optimum berdasarkan keempat fungsi kernel dapat dilihat pada Lampiran 36. Setelah diperoleh jarak *euclid* dan nilai bandwidth, maka selanjutnya dapat dilakukan pembobotan untuk masing – masing lokasi pengamatan. Pembobot spasial yang diperoleh untuk tiap – tiap lokasi tersebut kemudian digunakan untuk mengestimasi parameter model GWBGR sehingga setiap lokasi memiliki model yang berbeda.

Tabel 4. 6 Perbandingan AICc Model GWBGR berdasarkan Fungsi Pembobot

Fungsi Pembobot Kernel	**AICc**
Fixed Gaussian	871,2769
Adaptive Gaussian	872,0369
Fixed Bisquare	866,1279
Adaptive Bisquare	917,0569

Pembobot yang paling baik adalah fungsi pembobot dengan nilai AICc terkecil. Nilai AICc setiap fungsi pembobot yang diperoleh berdasarkan persamaan pada subbab 2.16. Berdasarkan nilai AICc pada Tabel 4.6, ternyata pemodelan GWBZIGPR dengan pembobot fungsi kernel *fixed bisquare* menghasilkan nilai AICc yang paling kecil yaitu sebesar 866,1279, sehingga dapat disimpulkan bahwa penggunaan pembobot fungsi kernel *fixed bisquare* menghasilkan pembobot yang lebih representatif dalam menggambarkan heterogenitas spasial Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas antar kecamatan di Karesidenan Pekalongan Tahun 2017. Pemodelan GWZIGPR dengan fungsi kernel *fixed bisquare* akan dijelaskan pada subbab selanjutnya. Nilai pembobot *fixed bisquare* terlampir pada Lampiran 37.

4.5.10 Pemodelan Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan Tahun 2017 dengan GWBZIGPR

Berdasarkan nilai AICc yang diperoleh pada subbab 4.5.9, maka fungsi kernel *fixed bisquare* dipilih sebagai fungsi pembobot model GWBZIGPR untuk memodelkan Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan Tahun 2017. Pemodelan dengan GWBZIGPR menghasilkan penaksir parameter yang berbeda untuk setiap kecamatan di Karesidenan Pekalongan. Perbedaan hasil taksiran parameter ini dipengaruhi oleh pembobot geografis yang digunakan, sehingga pada kasus Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas ini diperlukan pengujian kesamaan model antara model BZIGPR dengan model GWBZIGPR. Pengujian ini bertujuan untuk mengetahui signifikansi faktor geografis terhadap model. Hipotesis yang digunakan sebagai berikut:

$H_0 : \beta_{lr}\left(u_i, v_i\right) = \beta_{lr}, l = 1,2; r = 1,2,...,7; i = 1,2,...,91$

$H_1 :$ paling sedikit ada satu $\beta_{lr}\left(u_i, v_i\right) \neq \beta_{lr}$

Tabel 4. 7 Uji Kesamaan model BZIGP dan GWBZIGP Parameter $\boldsymbol{\beta}$

Model	**Devians**	**df**	**Fhit**	**Ftabel**
BZIGPR	3576,365	14	89,304	1,669
GWBZIGPR	3644,285	1274		

Pada taraf signifikasi $\alpha = 5\%$ diperoleh nilai $F_{hitung} = 89,304$ lebih besar dari $F_{0,05;14;1274} = 1,669$, sehingga keputusannya adalah tolak H_0 yang berarti terdapat perbedaan yang signifikan antara parameter model BZIGPR dengan model GWBZIGPR.

$H_0 : \gamma_{lr}\left(u_i, v_i\right) = \gamma_{lr}, l = 1,2; r = 1,2,...,7; i = 1,2,...,91$

$H_1 :$ paling sedikit ada satu $\gamma_{lr}\left(u_i, v_i\right) \neq \gamma_{lr}$

Tabel 4. 8 Uji Kesamaan model BZIGP dan GWBZIGP Parameter $\boldsymbol{\gamma}$

Model	Devians	df	Fhit	Ftabel
BZIGPR	3576,365	14	77,158	1,669
GWBZIGPR	4217,957	1274		

Pada taraf signifikasi $\alpha = 5\%$ diperoleh nilai $F_{hitung} = 77{,}158$ lebih besar dari $F_{0,05;14;1274} = 1{,}669$, sehingga keputusannya adalah tolak H_0 yang berarti terdapat perbedaan yang signifikan antara parameter model BZIGPR dengan model GWBZIGPR.

Selanjutnya, dilakukan uji serentak untuk mengetahui apakah minimal ada satu variabel prediktor yang berpengaruh terhadap Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan Tahun 2017. Hipotesis yang digunakan sebagai berikut:

$H_0 : \beta_{11}(u_i, v_i) = \cdots = \beta_{27}(u_i, v_i) = \gamma_{11}(u_i, v_i) = \cdots = \gamma_{27}(u_i, v_i) = 0, i = 1, 2, ..., 91$

$H_1 :$ paling sedikit ada satu $\beta_{lr}(u_i, v_i) \neq 0, \gamma_{lr}(u_i, v_i) \neq 0, l = 1, 2; r = 1, 2, ..., 7$

Berdasarkan hasil analisis uji serentak pada Lampiran 38 dapat diketahui nilai devians dari model GWBZIGPR *fixed bisquare* sesuai dengan persamaan (4.11) sebesar 4265,545 lebih besar dari nilai $\chi^2_{0,05;1274} = 1358{,}15$. Maka diperoleh kesimpulan tolak H_0 yang berarti minimal ada satu variabel prediktor yang berpengaruh signifikan dalam model. Pengujian serentak juga dilakukan pada parameter $\boldsymbol{\gamma}$ dengan hipotesis sebagai berikut:

$H_0 : \gamma_{11}(u_i, v_i) = \gamma_{12}(u_i, v_i) = \cdots \gamma_{27}(u_i, v_i) = 0, i = 1, 2, ..., 91$

$H_1 :$ paling sedikit ada satu $\gamma_{lr}(u_i, v_i) \neq 0, l = 1, 2; r = 1, 2, ..., 7$

Berdasarkan hasil analisis uji serentak pada Lampiran 38 dapat diketahui nilai devians dari model GWBZIGPR *fixed bisquare* sesuai dengan persamaan (4.13) sebesar 4217,957 lebih besar dari nilai $\chi^2_{0,05;1274} = 1358{,}15$. Maka diperoleh kesimpulan tolak H_0 yang berarti minimal ada satu variabel prediktor yang berpengaruh signifikan dalam model. Selanjutnya pengujian serentak pada parameter $\boldsymbol{\beta}$ dengan hipotesis sebagai berikut:

$H_0 : \beta_{11}(u_i, v_i) = \beta_{12}(u_i, v_i) = \cdots \beta_{27}(u_i, v_i) = 0, i = 1, 2, ..., 91$

H_1 : paling sedikit ada satu $\beta_{lr}(u_i, v_i) \neq 0, l = 1, 2; r = 1, 2, ..., 7$

Berdasarkan hasil analisis uji serentak pada Lampiran 38 dapat diketahui nilai devians dari model GWBZIGPR *fixed bisquare* sesuai dengan persamaan (4.15) sebesar 3644,285 lebih besar dari nilai $\chi^2_{0,05;1274} = 1358,15$. Maka diperoleh kesimpulan tolak H_0 yang berarti minimal ada satu variabel prediktor yang berpengaruh signifikan dalam model

Untuk mengetahui variabel mana yang berpengaruh terhadap model, maka dilakukan pengujian parsial di setiap kecamatan dengan hipotesis mengacu pada subbab 4.4 sebagai berikut:

$H_0 : \beta_{lr}(u_i, v_i) = 0$

$H_1 : \beta_{lr}(u_i, v_i) \neq 0, l = 1, 2; r = 1, 2, ..., 7; i = 1, 2, ..., 91$ dan

$H_0 : \gamma_{lr}(u_i, v_i) = 0$

$H_1 : \gamma_{lr}(u_i, v_i) \neq 0, l = 1, 2; r = 1, 2, ..., 7; i = 1, 2, ..., 91$

Hasil pengujian parsial variabel yang signifikan pada setiap lokasi dapat dilihat pada Lampiran 39-42.

Tabel 4. 9 Variabel Yang Signifikan pada Parameter $\boldsymbol{\gamma}$ Pada Model GWBZIGPR berdasarkan Kecamatan

No	Kecamatan	Respon	
		Jumlah Kematian Ibu Hamil	Jumlah Kematian Ibu Hamil
1	Wonotunggal	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
2	Bandar	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
3	Blado	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
4	Reban	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
5	Bawang	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
6	Tersono	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
7	Gringsing	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
8	Limpung	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
9	Banyuputih	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6

Lanjutan Tabel 4.9

No	Kecamatan	Respon	
		Jumlah Kematian Ibu Hamil	Jumlah Kematian Ibu Hamil
10	Subah	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
11	Pecalungan	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
12	Tulis	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
13	Kandeman	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
14	Batang	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
15	Warungasem	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
16	Kandangserang	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
17	Paninggaran	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
18	Lebakbarang	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
19	Petungkriyono	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
20	Talun	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
21	Doro	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
22	Karanganyar	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
23	Kajen	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
24	Kesesi	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
25	Sragi	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
26	Siwalan	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
27	Bojong	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
28	Wonopringgo	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
29	Kedungwuni	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
30	Karangdadap	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
31	Buaran	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
32	Tirto	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
33	Wiradesa	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
34	Wonokerto	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
35	Moga	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
36	Warungpring	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
37	Pulosari	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
38	Belik	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$

Lanjutan Tabel 4.9

No	Kecamatan	Respon	
		Jumlah Kematian Ibu Hamil	Jumlah Kematian Ibu Hamil
39	Watukumpul	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
40	Bodeh	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
41	Bantarbolang	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
42	Randudongkal	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
43	Pemalang	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
44	Taman	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
45	Petarukan	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
46	Ampelgading	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
47	Comal	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
48	Ulujami	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
49	Margasari	X_1, X_2, X_3, X_4, X_5	X_1, X_2, X_3, X_5
50	Bumijawa	$X_1, X_2, X_3, X_4, X_5, X_6$	X_1, X_2, X_3, X_5, X_6
51	Bojong	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
52	Balapulang	$X_1, X_2, X_3, X_4, X_5, X_6$	X_1, X_2, X_3, X_5, X_6
53	Pagerbarang	$X_1, X_2, X_3, X_4, X_5, X_6$	X_1, X_2, X_3, X_5, X_6
54	Lebaksiu	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
55	Jatinegara	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
56	Kedungbanteng	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
57	Pangkah	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
58	Slawi	$X_1, X_2, X_3, X_4, X_5, X_6$	X_1, X_2, X_3, X_5, X_6
59	Dukuhwaru	$X_1, X_2, X_3, X_4, X_5, X_6$	X_1, X_2, X_3, X_5, X_6
60	Adiwerna	$X_1, X_2, X_3, X_4, X_5, X_6$	X_1, X_2, X_3, X_5, X_6
61	Dukuhturi	$X_1, X_2, X_3, X_4, X_5, X_6$	X_1, X_2, X_3, X_5, X_6
62	Talang	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
63	Tarub	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
64	Kramat	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
65	Suradadi	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
66	Warureja	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_5, X_6, X_7$
67	Salem	X_4	X_4

Lanjutan Tabel 4.9

No	Kecamatan	Respon	
		Jumlah Kematian Ibu Hamil	Jumlah Kematian Ibu Hamil
68	Bantarkawung	X_1, X_3, X_4	X_1, X_5
69	Bumiayu	X_1, X_2, X_3, X_4, X_5	X_1, X_2, X_3, X_5
70	Paguyangan	X_1, X_2, X_3, X_4, X_5, X_6	X_1, X_2, X_3, X_5, X_6
71	Sirampog	X_1, X_2, X_3, X_4, X_5, X_6	X_1, X_2, X_3, X_5, X_6
72	Tonjong	X_1, X_2, X_3, X_4, X_5, X_6	X_1, X_2, X_3, X_5, X_6
73	Jatibarang	X_1, X_2, X_3, X_4, X_5	X_1, X_2, X_3, X_5
74	Larangan	X_1, X_2, X_3	X_1, X_2, X_3, X_5
75	Ketanggungan	X_1, X_3	X_1, X_5
76	Banjarharjo	X_4	-
77	Losari	-	-
78	Tanjung	X_1, X_3, X4	X_1, X_3, X_5
79	Kersana	X_1, X_2, X_3, X_4, X_5	X_1, X_3, X_5
80	Bulukamba	X_1, X_2, X_3, X_4, X_5	X_1, X_3, X_5
81	Wanasari	X_1, X_2, X_3, X_4, X_5	X_1, X_3, X_5
82	Brebes	X_1, X_2, X_3, X_4, X_5	X_1, X_2, X_3, X_5
83	Songgom	X_1, X_2, X_3, X_4, X_5	X_1, X_2, X_3, X_5
84	Pekalongan Barat	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_5, X_6
85	Pekalongan Timur	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_5, X_6
86	Pekalongan Utara	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_5, X_6
87	Pekalongan Selatan	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_5, X_6, X_7
88	Tegal Selatan	X_1, X_2, X_3, X_4, X_5, X_6	X_1, X_2, X_3, X_4, X_5, X_6
89	Tegal Timur	X_1, X_2, X_3, X_4, X_5, X_6	X_1, X_2, X_3, X_5, X_6
90	Tegal Barat	X_1, X_2, X_3, X_4, X_5, X_6	X_1, X_2, X_3, X_5, X_6
91	Margadana	X_1, X_2, X_3, X_4, X_5, X_6	X_1, X_2, X_3, X_5, X_6

Tabel 4. 10 Variabel yang Signifikanpada Parameter $\boldsymbol{\beta}$ Pada Model GWBZIGPR berdasarkan Kecamatan

No	Kecamatan	Respon	
		Jumlah Kematian Ibu Hamil	Jumlah Kematian Ibu Hamil
1	Wonotunggal	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
2	Bandar	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
3	Blado	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
4	Reban	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
5	Bawang	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
6	Tersono	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
7	Gringsing	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
8	Limpung	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
9	Banyuputih	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
10	Subah	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
11	Pecalungan	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
12	Tulis	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
13	Kandeman	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
14	Batang	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
15	Warungasem	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
16	Kandangserang	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
17	Paninggaran	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
18	Lebakbarang	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
19	Petungkriyono	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
20	Talun	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
21	Doro	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
22	Karanganyar	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
23	Kajen	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
24	Kesesi	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
25	Sragi	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
26	Siwalan	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
27	Bojong	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$

Lanjutan Tabel 4.10

No	Kecamatan	Respon	
		Jumlah Kematian Ibu Hamil	Jumlah Kematian Ibu Hamil
28	Wonopringgo	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
29	Kedungwuni	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
30	Karangdadap	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
31	Buaran	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
32	Tirto	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
33	Wiradesa	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
34	Wonokerto	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
35	Moga	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
36	Warungpring	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
37	Pulosari	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
38	Belik	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
39	Watukumpul	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
40	Bodeh	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
41	Bantarbolang	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
42	Randudongkal	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
43	Pemalang	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
44	Taman	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
45	Petarukan	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
46	Ampelgading	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
47	Comal	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
48	Ulujami	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
49	Margasari	$X_1, X_2, X_3, X_4, X_6, X_7$	X_1, X_2, X_3, X_5, X_6
50	Bumijawa	$X_1, X_2, X_3, X_4, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
51	Bojong	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
52	Balapulang	$X_1, X_2, X_3, X_4, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
53	Pagerbarang	$X_1, X_2, X_3, X_4, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6$
54	Lebaksiu	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
55	Jatinegara	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$
56	Kedungbanteng	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$	$X_1, X_2, X_3, X_4, X_5, X_6, X_7$

Lanjutan Tabel 4.10

No	Kecamatan	Respon	
		Jumlah Kematian Ibu Hamil	Jumlah Kematian Ibu Hamil
57	Pangkah	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6, X_7
58	Slawi	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6
59	Dukuhwaru	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6
60	Adiwerna	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6
61	Dukuhturi	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6
62	Talang	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6, X_7
63	Tarub	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6, X_7
64	Kramat	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6, X_7
65	Suradadi	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6, X_7
66	Warureja	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6, X_7
67	Salem	X_1, X_2, X_3, X_6	X_1, X_3
68	Bantarkawung	X_1, X_2, X_3, X_4, X_6	X_1, X_2, X_3
69	Bumiayu	X_1, X_2, X_3, X_4, X_6	X_1, X_2, X_3, X_5, X_6
70	Paguyangan	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6
71	Sirampog	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6
72	Tonjong	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6
73	Jatibarang	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_6
74	Larangan	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_6
75	Ketanggungan	X_1, X_2, X_3, X_4, X_6	X_1, X_2, X_3
76	Banjarharjo	X_1, X_2, X_3, X_4, X_6	X_1, X_3
77	Losari	X_1, X_2, X_3, X_4, X_6	X_1, X_3
78	Tanjung	X_1, X_2, X_4, X_6	X_1, X_2, X_3
79	Kersana	X_1, X_2, X_3, X_4, X_6	X_1, X_2, X_3, X_5, X_6
80	Bulukamba	X_1, X_2, X_3, X_4, X_6	X_1, X_2, X_3
81	Wanasari	X_1, X_2, X_3, X_4, X_6	X_1, X_2, X_3, X_5
82	Brebes	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_5, X_7
83	Songgom	X_1, X_2, X_3, X_5, X_6, X_7	X_1, X_2, X_3, X_5, X_7
84	Pekalongan Barat	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6, X_7
85	Pekalongan Timur	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6, X_7

Lanjutan Tabel 4.10

No	Kecamatan	Respon	
		Jumlah Kematian Ibu Hamil	Jumlah Kematian Ibu Hamil
86	Pekalongan Utara	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6, X_7
87	Pekalongan Selatan	X_1, X_2, X_3, X_4, X_5, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6, X_7
88	Tegal Selatan	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6
89	Tegal Timur	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6
90	Tegal Barat	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6
91	Margadana	X_1, X_2, X_3, X_4, X_6, X_7	X_1, X_2, X_3, X_4, X_5, X_6

Tabel 4. 11 Pengelompokkan Kecamatan berdasarkan Variabel Signifikan pada Parameter γ

Kecamatan		**Variabel**
Jumlah Kematian Ibu Hamil	**Jumlah Kematian Ibu Nifas**	
Salem, Banjarharjo	Salem	X_4
	Bantarkawung, Ketanggungan	X_1, X_5
Larangan		X_1, X_2, X_3
Bantarkawung, Ketanggungan, Tanjung		X_1, X_3, X_4
	Tanjung	X_1, X_3, X_5
	Margasari, Bumiayu, Jatibarang, Larangan, Kersana, Bulukamba, Wanasari, Brebes, Songgom	X_1, X_2, X_3, X_5
	Reban, Bawang, Tersono, Gringsing, Limpung, Banyuputih, Batang, Wonokerto, Pulosari, Bumijawa, Bojong, Balapulang, Pagerbarang, Lebaksiu, Jatinegara, Kedungbanteng, Pangkah, Slawi, Dukuhwaru, Adiwerna, Dukuhturi, Talang, Tarub, Suradadi, Paguyangan, Sirampog, Tonjong, Pekalongan Barat, Pekalongan Timur, Pekalongan Utara, Tegal Selatan, Tegal timur, Tegal Barat, Margadana	X_1, X_2, X_3, X_5, X_6
Margasari, Bumiayu, Jatibarang, Kersana, Bulukamba, Wanasari, Brebes, Songgom		X_1, X_2, X_3, X_4, X_5
Bumijawa, Balapulang, Pagerbarang, Slawi, Dukuhwaru, Adiwerna, Dukuhturi, Paguyangan, Sirampog, Tonjong, Tegal Selatan, Tegal Timur, Tegal Barat, Margadana	Wonotunggal, Bandar, Blado, Subah, Pencalungan, Tulis, Kandeman, Warungasem, Lebakbarang, Petungkriyono, Talun, Kramat	X_1, X_2, X_3, X_4, X_5, X_6
	Banjarharjo, Losari	Tidak ada yang signifikan

Lanjutan Tabel 4. 11

Kecamatan		Variabel
Jumlah Kematian Ibu Hamil	**Jumlah Kematian Ibu Nifas**	
	Kandangserang, Paninggaran, Doro, Karanganyar, Kajen, Kesesi, Sragi, Siwalan, Bojong, Wonopringgo, Kedungwuni, Karangdadap, Buaran, Tirto, Wiradesa, Moga, Warungpring, Belik, Watukumpul, Bodeh, Bantarbolang, Randudongkal, Pemalang, Taman, Petarukan, Ampelgading, Comal, Ulujami, Warureja, Pekalongan Selatan	X_1, X_2, X_3, X_5, X_6, X_7
Wonotunggal, Bandar, Blado, Reban, Bawang, Tersono, Gringsing, Limpung, Banyuputih, Subah, Pecalungan, Tulis, Kandeman, Batang, Warungasem, Kandangserang, Paninggaran, Lebakbarang, Petungkriyono, Talun, Doro, Karanganyar, Kajen, Kesesi, Sragi, Siwalan, Bojong, Wonopringgo, Kedungwuni, Karangdadap, Buaran, Tirto, Wiradesa, Wonokerto, Moga, Warungpring, Pulosari, Belik, Watukumpul, Bodeh, Bantarbolang, Randudongkal, Pemalang, Taman, Petarukan, Ampelgading, Comal, Ulujami, Bojong, Lebaksiu, Jatinegara, Kedungbanteng, Pangkah, Talang, Tarub, Kramat, Suradadi, Warureja, Pekalongan Barat, Pekalongan Timur, Pekalongan Utara, Pekalongan Selatan		X_1, X_2, X_3, X_4, X_5, X_6, X_7

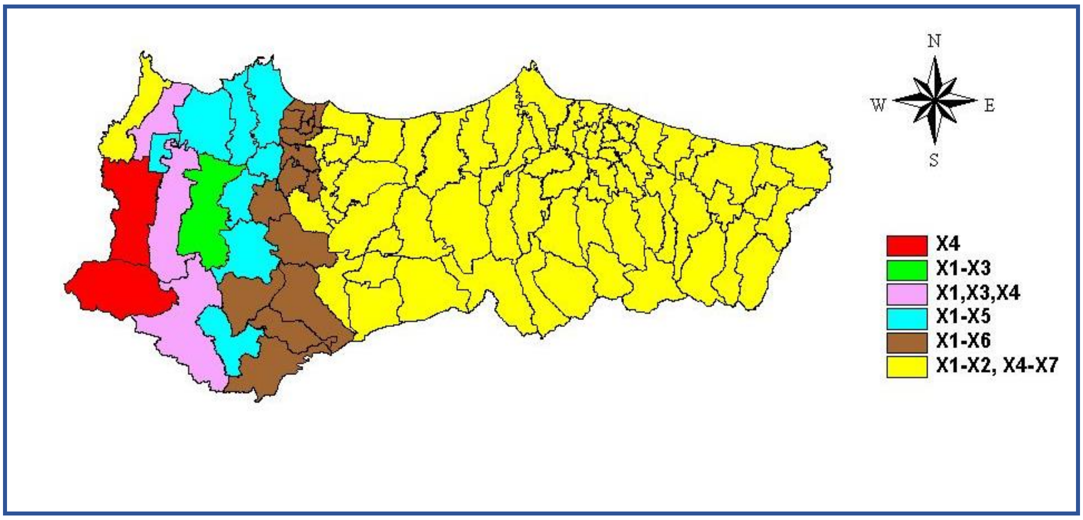
Tabel 4. 12 Pengelompokkan Kecamatan Berdasarkan Variabel Signifikan Pada Parameter **β**

Kecamatan		Variabel
Jumlah Kematian Ibu Hamil	**Jumlah Kematian Ibu Nifas**	
	Salem, Banjarharjo, Losari	X_1, X_3
	Bantarkawung, Ketanggungan, Tanjung, Bulukamba	X_1, X_2, X_3,
Losari		X_1, X_2, X_3, X_4
	Wanasari	X_1, X_2, X_3, X_5
Salem		X_1, X_2, X_3, X_6
Larangan, Ketanggungan, Banjarharjo, Tanjung, Kersana, Bulukamba, Wanasari		X_1, X_2, X_3, X_4, X_6
	Margasari, Bumiayu, Jatibarang, Larangan, Kersana, Brebes, Songgom	X_1, X_2, X_3, X_5, X_6
	Bumijawa, Balapulang, Pagerbarang, Slawi, Dukuhwaru, Adiwerna, Dukuhturi, Paguyangan, Sirampog, Tonjong, Tegal Selatan, tegal Timur, Tegal Barat, Margadana	X_1, X_2, X_3, X_4, $X_{5,}$, X_6
Margasari, Bumijawa, Balapulang, Pagebarang, Slawi, Dukuhwaru, Adiwerna, Dukuhturi, Bumiayu, Paguyangan, Sirampog, Tonjong, Jatibarang, Brebes, Tegal Selatan, Tegal Barat, Tegal Timur, margadana		X_1, X_2, X_3, X_4, $X_{6,}$, X_7
Songgom		X_1, X_2, X_3, X_5, X_6, X_7

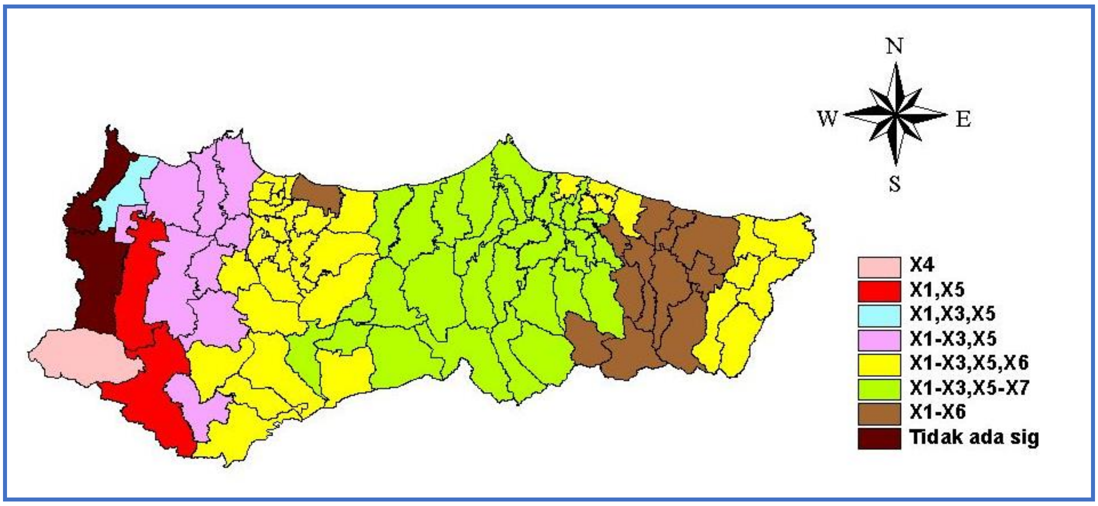
Lanjutan Tabel 4.12

Kecamatan		Variabel
Jumlah Kematian Ibu Hamil	**Jumlah Kematian Ibu Nifas**	
Wonotunggal, Bandar, Blado, Reban, Bawang, Tersono, Gringsing, Limpung, Banyuputih, Subah, Pecalungan, Tulis, Kandeman, Batang, Warungasem, Kandangserang, Paninggaran, Lebakbarang, Petungkriyono, Talun, Doro, Karanganyar, Kajen, Kesesi, Sragi, Siwalan, Bojong, Wonopringgo, Kedungwuni, Karangdadap, Buaran, Tirto, Wiradesa, Wonokerto, Moga, Warungpring, Pulosari, Belik, Watukumpul, Bodeh, Bantarbolang, Randudongkal, Pemalang, Taman, Petarukan, Ampelgading, Comal, Ulujami, Bojong, Lebaksiu, Jatinegara, Kedungbanteng, Pangkah, Talang, Tarub, Kramat, Suradadi, Warureja, Pekalongan Barat, Pekalongan Timur, Pekalongan Utara, Pekalongan Selatan	Wonotunggal, Bandar, Blado, Reban, Bawang, Tersono, Gringsing, Limpung, Banyuputih, Subah, Pecalungan, Tulis, Kandeman, Batang, Warungasem, Kandangserang, Paninggaran, Lebakbarang, Petungkriyono, Talun, Doro, Karanganyar, Kajen, Kesesi, Sragi, Siwalan, Bojong, Wonopringgo, Kedungwuni, Karangdadap, Buaran, Tirto, Wiradesa, Wonokerto, Moga, Warungpring, Pulosari, Belik, Watukumpul, Bodeh, Bantarbolang, Randudongkal, Pemalang, Taman, Petarukan, Ampelgading, Comal, Ulujami, Bojong, Lebaksiu, Jatinegara, Kedungbanteng, Pangkah, Talang, Tarub, Kramat, Suradadi, Warureja, Pekalongan Barat, Pekalongan Timur, Pekalongan Utara, Pekalongan Selatan	X_1, X_2, X_3, X_4, X_5, X_6, X_7

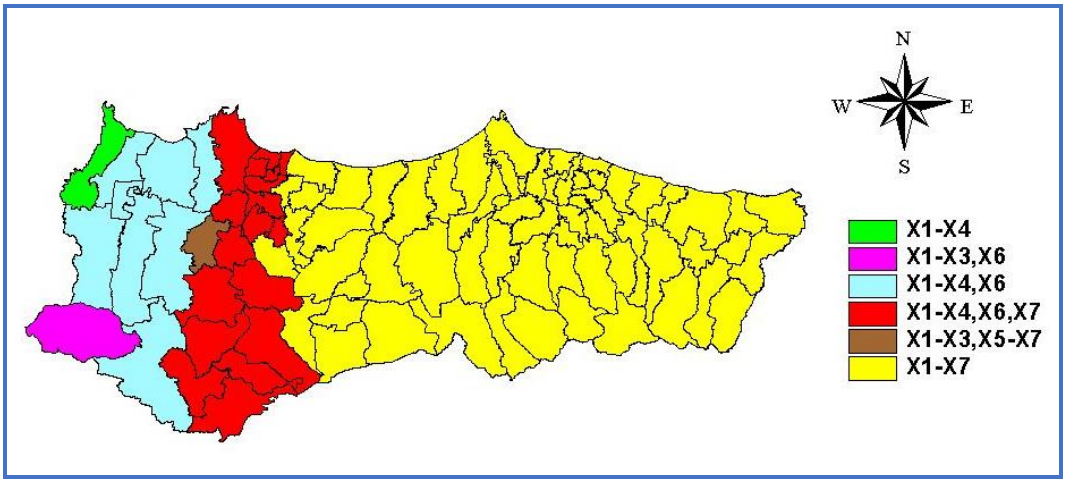
Apabila hasil pengelompokkan variabel yang signifikan berpengaruh terhadap Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas dipetakan maka akan terbentuk peta sebagai berikut:



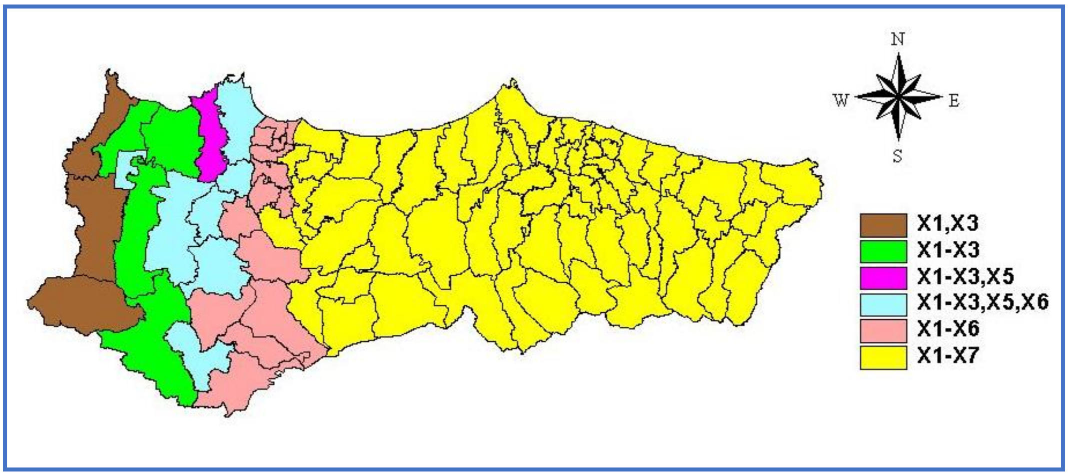
Gambar 4. 13 Pengelompokkan kecamatan di Karesidenan Pekalongan berdasarkan variabel yang signifikan pada model *zero state* pada Jumlah Kematian Ibu Hamil



Gambar 4. 14 Pengelompokkan kecamatan di Karesidenan Pekalongan berdasarkan variabel yang signifikan pada model *zero state* pada Jumlah Kematian Ibu Nifas



Gambar 4. 15 Pengelompokkan kecamatan di Karesidenan Pekalongan berdasarkan variabel yang signifikan pada model *poisson state* pada Jumlah Kematian Ibu Hamil



Gambar 4. 16 Pengelompokkan kecamatan di Karesidenan Pekalongan berdasarkan variabel yang signifikan pada model *poisson state* pada Jumlah Kematian Ibu Nifas

Contoh hasil pengujian parsial yang akan diintrepetasikan dipilih adalah Kecamatan Wanasari Kabupaten Brebes seperti tabel 4.13 berikut:

Tabel 4. 13 Uji Parsial parameter model GWBZIGPR di Kecamatan Wanasari

Parameter	**Nilai Taksiran**	**SE**	**Z Hitung**	**P-*Value***
γ_{10}	0,0100	0,0000	341,7580	0,0000
γ_{11}	0,0215	0,0042	5,0819	0,0000
γ_{12}	0,0167	0,0025	6,7874	0,0000
γ_{13}	0,0212	0,0044	4,8398	0,0000
γ_{14}	0,0284	0,0099	2,8765	0,0040
γ_{15}	0,0090	0,0035	2,5425	0,0110
γ_{16}	0,0048	0,0106	0,4470	0,6549
γ_{17}	-0,0138	0,0088	-1,5722	0,1159
γ_{20}	0,0120	0,0000	268,2501	0,0000
γ_{21}	0,0132	0,0046	2,8762	0,0040
γ_{22}	0,0102	0,0037	2,7582	0,0058
γ_{23}	0,0129	0,0039	3,3253	0,0009
γ_{24}	0,0017	0,0083	0,2042	0,8382
γ_{25}	0,0160	0,0060	2,6469	0,0081
γ_{26}	-0,0093	0,0153	-0,6090	0,5425
γ_{27}	0,0129	0,0102	1,2578	0,2085
$\beta_{1.0}$	-10,4858	0,0001	-92104,9306	0,0000
β_{11}	0,0254	0,0025	10,3476	0,0000
β_{12}	-0,0488	0,0024	-20,6242	0,0000
β_{13}	0.0644	0.0022	29.7909	0.0000
β_{14}	0,0456	0,0153	2,9766	0,0029
β_{15}	-0,0026	0,0047	-0,5635	0,5731
β_{16}	0,0372	0,0057	6,5463	0,0000
β_{17}	0,0001	0,0070	0,0092	0,9926
β_{20}	-30,5024	0,0001	-365651,0969	0,0000
β_{21}	0,3468	0,0044	79,5381	0,0000
β_{22}	0,0181	0,0041	4,4351	0,0000
β_{23}	-0,0428	0,0056	-7,6620	0,0000
β_{24}	0,0095	0,0149	0,6340	0,5261
β_{25}	-0,0138	0,0065	-2,1359	0,0327
β_{26}	0,0124	0,0086	1,4498	0,1471
β_{27}	0,0008	0,0137	0,0558	0,9555

4.5.11 Pemilihan Model Terbaik

Perbandingan nilai AICc antara BZIGPR dan BZIGPR disajikan dalam Tabel 4.14. Berdasarakan nilai AICc model BZIGPR dan GWBZIGPR dengan fungsi *kernel fix bisquare* maka dapat disimpulkan bahwa model GWBZIGPR *fix bisquare* lebih baik jika dibandingkan dengan model BZIGPR karena mempunyai nilai AICc yang lebih kecil.

Tabel 4. 14 Perbandingan AICc Model BZIGPR dan GWBZIGPR

Model	AICc
BZIGPR	1098,138
GWBZIGPR dengan *Fix Bisquare*	866,1279

Berdasarkan hasil tersebut maka akan dilanjutkan dengan interpretasi hasil model GWBZIGPR dengan fungsi *kernel fix bisquare*.

4.5.12 Interpretasi Model GWBZIGPR *Fix Bisquare*

Berdasarakan Tabel 4.12 maka model GWBZIGPR di Kecamatan Wanasari Kabupaten Brebes Tahun 2017 sebagaimana persamaan 2.75 sebagai berikut:

a. Model regresi *poisson state* untuk $\hat{\mu}_1$ pada Jumlah Kematian Ibu Hamil

$$\ln \hat{\mu}_{1i} = -10,4858 + 0,0254x_{1i} - 0,0488x_{2i} + 0,0212x_{3i} + 0,0456x_{4i} + m$$

dimana $m = -0,0026x_{5i} + 0,0372x_{6i} + 0,0001x_{7i}$

dengan variabel yang signifikan adalah Persentase pemeriksaan kehamilan K1 (X_1), Persentase pemeriksaan kehamilan K4 (X_2), Persentase pemeriksaan kehamilan K4 (X_2), Persentase penolong persalinan oleh tenaga kesehatan (X_3) dan Persentase penanganan komplikasi kebidanan (X_6) sehingga interpretasi modelnya sebagai berikut:

1. Setiap kenaikan 1 persen pemeriksaan kehamilan K1 maka akan menaikkan rata-rata jumlah kematian ibu hamil sebesar $e^{0,0254} = 1,026$ kali dengan asumsi variabel yang lain tetap.

2. Setiap kenaikan 1 persen pemeriksaan kehamilan K4 maka akan menurunkan rata-rata jumlah kematian ibu hamil sebesar $e^{-0,0488} = 0,952$ kali dengan asumsi variabel yang lain tetap.
3. Setiap kenaikan 1 persen imunisasi TT2+ pada ibu hamil maka akan menaikkan rata-rata jumlah kematian ibu hamil sebesar $e^{0,0456} = 1,047$ kali dengan asumsi variabel yang lain tetap.
4. Setiap kenaikan 1 persen penanganan komplikasi kebidanan maka akan menaikkan rata-rata jumlah kematian ibu hamil sebesar $e^{0,0372} = 1,038$ kali dengan asumsi variabel yang lain tetap.

b. Model regresi *poisson state* untuk $\hat{\mu}_2$ pada Jumlah Kematian Ibu Nifas

$$\ln \hat{\mu}_{2i} = -30,5024 + 0,3468x_{1i} + 0,0181x_{2i} - 0,0428x_{3i} + 0,0095x_{4i} + n$$

dimana $n = -0,0138x_{5i} + 0,0124x_{6i} + 0,0008x_{7i}$

dengan variabel yang signifikan adalah Persentase pemeriksaan kehamilan K1 (X_1), Persentase pemeriksaan kehamilan K4 (X_2), Persentase persalinan di tolong oleh tenaga kesehatan (X_3) dan Persentase ibu hamil yang mendapatkan Fe3 (X_5) sehingga interpretasi modelnya sebagai berikut:

1. Setiap kenaikan 1 persen pemeriksaan kehamilan K1 maka akan menaikkan rata-rata jumlah kematian ibu nifas sebesar $e^{0,3468} = 1,414$ kali dengan asumsi variabel yang lain tetap.
2. Setiap kenaikan 1 persen pemeriksaan kehamilan K4 maka akan menaikkan rata-rata jumlah kematian ibu nifas sebesar $e^{0,0181} = 1,018$ kali dengan asumsi variabel yang lain tetap.
3. Setiap kenaikan 1 persen persalinan di tolong oleh tenaga kesehatan (X_3) maka akan menurunkan rata-rata jumlah kematian ibu nifas sebesar $e^{-0,0428} = 0,958$ kali dengan asumsi variabel yang lain tetap.
4. Setiap kenaikan 1 persen ibu hamil yang mendapatkan Fe3 maka akan menurunkan rata-rata jumlah kematian ibu nifas sebesar $e^{-0,0138} = 0,986$ kali dengan asumsi variabel yang lain tetap.

c. Model regresi *zero state* untuk $\hat{p}_1$ pada Jumlah Kematian Ibu Hamil

$$\text{logit } \hat{p}_{1i} = 0,01 + 0,0215x_{1i} + 0,0167x_{2i} + 0,0212x_{3i} + 0,0284x_{4i} + o_1$$

dimana $o_1 = 0,0090x_{5i} + 0,0048x_{6i} - 0,0138x_{7i}$

berdasarkan model tersebut dapat diinterpretasikan bahwa peluang tidak terjadi kasus kematian ibu hamil di Kecamatan Wanasari dipengaruhi oleh Persentase pemeriksaan kehamilan K1 (X_1), Persentase pemeriksaan kehamilan K4 (X_2), Persentase penolong persalinan oleh tenaga kesehatan (X_3), Persentase imunisasi TT2+ pada ibu hamil (X_4) dan Persentase ibu hamil yang mendapatkan Fe3 (X_5) sehingga interpretasi modelnya sebagai berikut:

1. Setiap penambahan 1 persen pemeriksaan kehamilan K1 maka akan menaikkan resiko kematian ibu hamil sebesar $e^{0,0215} = 1,022$ kali.
2. Setiap penambahan 1 persen pemeriksaan kehamilan K4 maka akan menaikkan resiko kematian ibu hamil sebesar $e^{0,0167} = 1,017$ kali.
3. Setiap penambahan 1 persen ibu hamil yang mendapatkan TT2+ maka akan menaikkan resiko kematian ibu nifas sebesar $e^{0,0284} = 1,029$ kali.
4. Setiap penambahan 1 persen ibu hamil yang mendapatkan Fe3 maka akan menaikkan resiko kematian ibu nifas sebesar $e^{0,0090} = 1,009$ kali

d. Model regresi zero state untuk $\hat{p}_2$ pada Jumlah Kematian Ibu Nifas

$$\text{logit } \hat{p}_{2i} = 0,0120 + 0,0132x_{1i} + 0,0102x_{2i} + 0,0129x_{3i} - 0,0017x_{4i} + q_1$$

dimana $q_1 = 0,0160x_{5i} - 0,0093x_{6i} + 0,0129x_{7i}$

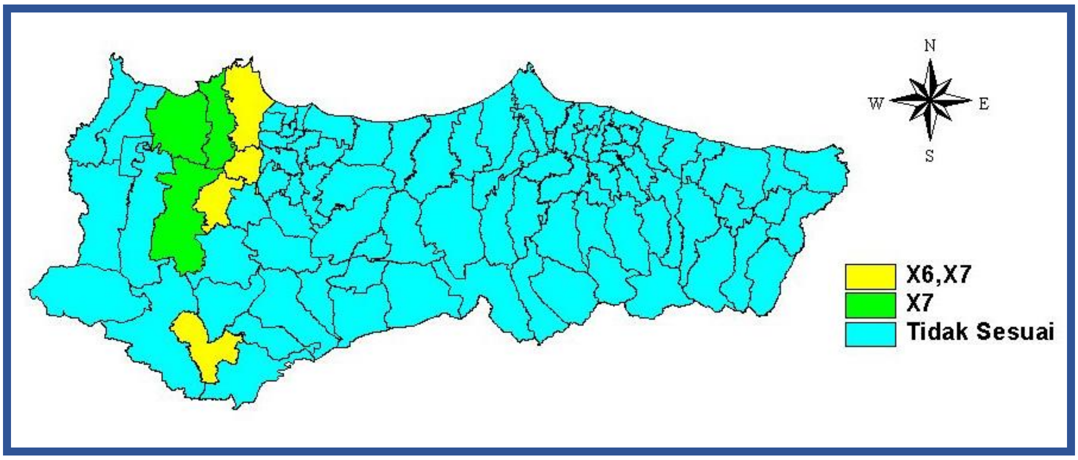
berdasarkan model tersebut dapat diinterpretasikan bahwa peluang tidak terjadi kasus kematian ibu nifas di Kecamatan Wanasari dipengaruhi oleh Persentase pemeriksaan kehamilan K1 (X_1), Persentase pemeriksaan kehamilan K4 (X_2), Persentase persalinan di tolong oleh tenaga kesehatan (X_3), dan Persentase ibu hamil yang mendapatkan Fe3 (X_5). sehingga interpretasi modelnya sebagai berikut:

1. Setiap penambahan 1 persen pemeriksaan kehamilan K1 maka akan menaikkan resiko kematian ibu hamil sebesar $e^{0,0132} = 1,0133$ kali.

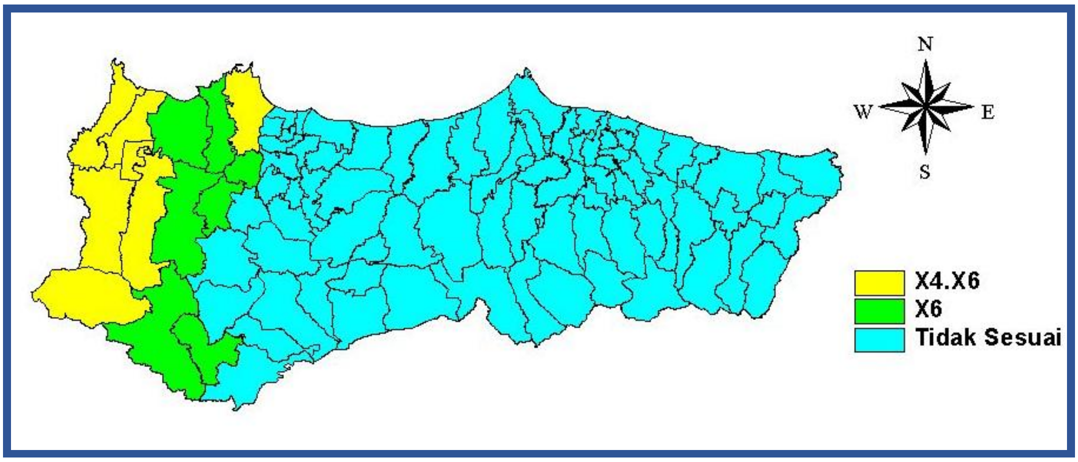
2. Setiap penambahan 1 persen pemeriksaan kehamilan K4 maka akan menaikkan resiko kematian ibu hamil sebesar $e^{0,0102} = 1,0102$ kali.
3. Setiap penambahan 1 persen persalinan di tolong oleh tenaga kesehatan maka akan menaikkan resiko kematian ibu hamil sebesar $e^{0,0129} = 1,0130$ kali.
4. Setiap penambahan 1 persen ibu hamil yang mendapatkan Fe3 maka akan menaikkan resiko kematian ibu nifas sebesar $e^{0.0160} = 1,0161$ kali.

Sebagian besar tanda koefisien parameter tidak sesuai tanda dan hal ini tidak sesuai dengan teori yang seharusnya hubungan variabel prediktor dan respon adalah negatif. Dengan adanya penambahan persentase pada variabel prediktor diharapan dapat mengurangi jumlah kematian ibu namun justru menambah jumlah ematian ibu, hal ini diduga disebabkan data yang digunakan pada penelitian ini adalah data *cross section* sehingga program kesehatan yang dilakukan pemerintah pada Tahun 2017 tidak berdampak langsung pada tahun tersebut melainkan berdampak pada tahun berikutnya. Berdasarkan *deep interview* dengan pihak Dinas Kesehatan Provinsi Jawa Tengah, seorang ibu hamil dihitung melakukan kunjungan K1 atau kunjungan K4 ketika ibu hamil tersebut berkunjung untuk memeriksakan kehamilannya pada suatu fasilitas kesehatan padahal di dalam kunjungan K1 maupun K4 terdapat 10 pemeriksaan yang harus dilakukan oleh tenaga kesehatan mulai dari pengukuran tinggi badan sampai tata laksana mendapatkan pengobatan sehingga ketika ibu hamil tersebut hanya dilakukan 1 pemeriksaan maupun 10 pemeriksaan tetap dihitung 1 kunjungan padahal dengan mengetahui riwayat pemeriksaan tersebut, penyebab kematian ibu dapat diantisipasi namun tolak ukur untuk menghitung kualitas kunjungan kehamilan berdasarakan 10 pemeriksaan tersebut belum ada. Selain itu, pada variabel persentase ibu hamil yang mendapatkan tablet Fe dalam hal ini ibu hamil tidak terpantau apakah tablet Fe yang diterima telah dikonsumsi atau tidak.

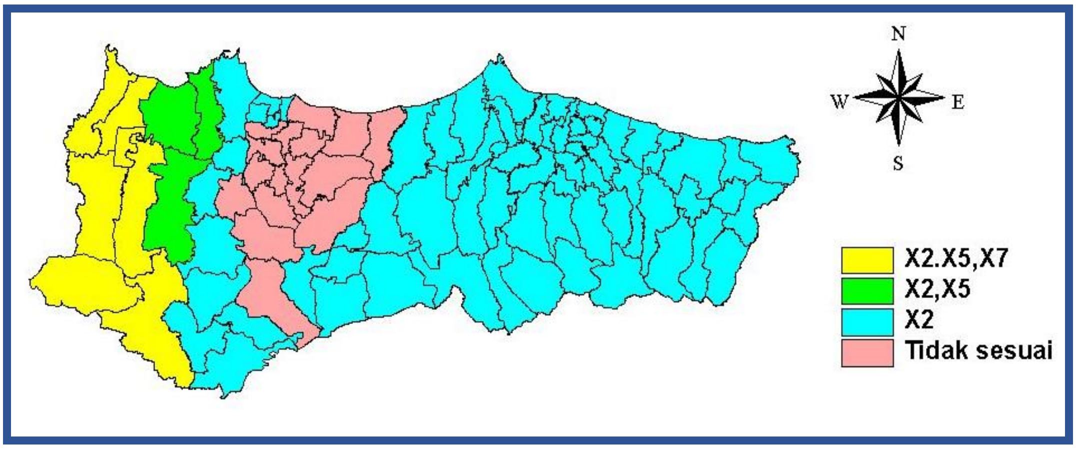
Sebaran variabel yang memiliki tanda koefisien sesuai dengan teori dapat dilihat pada gambar barikut.



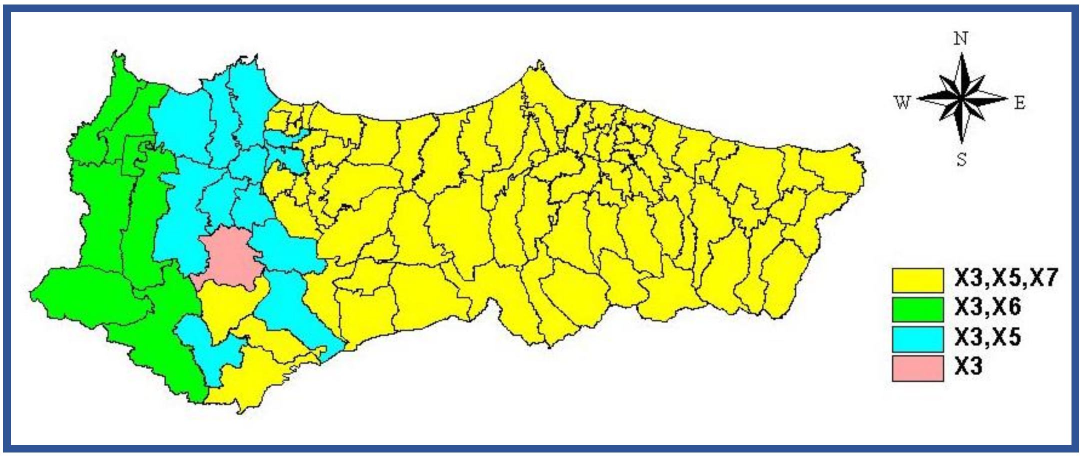
Gambar 4. 17 Pengelompokkan kecamatan di Karesidenan Pekalongan berdasarkan kesesuaian tanda pada model *zero state* pada Jumlah Kematian Ibu Hamil



Gambar 4. 18 Pengelompokkan kecamatan di Karesidenan Pekalongan berdasarkan kesesuaian tanda pada model *zero state* pada Jumlah Kematian Ibu Nifas



Gambar 4. 19 Pengelompokkan kecamatan di Karesidenan Pekalongan berdasarkan kesesuaian tanda pada model *poisson state* pada Jumlah Kematian Ibu Hamil



Gambar 4. 20 Pengelompokkan kecamatan di Karesidenan Pekalongan berdasarkan kesesuaian tanda pada model *poisson state* pada Jumlah Kematian Ibu Nifas

BAB 5
KESIMPULAN DAN SARAN

5.1 Kesimpulan

Berikut ini adalah kesimpulan yang diperoleh berdasarkan analisis Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas dengan menggunakan model BZIGPR dan GWBZIGPR

1. Penaksiran parameter menggunakan MLE dan menghasilkan bentuk yang tidak *close form* sehingga dilakukan iterasi numerik menggunakan metode BHHH. Adapun kelebihannya hanya memerlukan turunan pertama dari fungsi *ln likehood*.
2. Pemodelan menggunakan BZIGPR menghasilkan variabel yang signifikan mempengaruhi Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas yaitu Persentase pemeriksaan kehamilan K1, Persentase pemeriksaan kehamilan K4, Persentase persalinan di tolong oleh tenaga kesehatan, Persentase imunisasi TT2+ pada ibu hamil, Persentase ibu hamil yang mendapatkan Fe3, Persentase penanganan komplikasi kebidanan dan Rasio bidan per 100.000 penduduk .
3. Fungsi pembobot *kernel fix bisquare* menghasilkan nilai AICc paling kecil diantara pembobot kernel lainnya sehingga digunakan untuk model. GWBZIGPR.
4. Nilai AICc model GWBZIGPR lebih kecil dibanding model BZIGPR sehingga model GWBZIGPR lebih baik untuk memodelkan Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas di Karesidenan Pekalongan.
5. Pemodelan menggunakan GWBZIGPR pada model *zero state* menghasilkan 6 kelompok kecamatan berdasarkan kesamaan variabel yang signifikan terhadap Jumlah Kematian Ibu Hamil dan 8 kelompok kecamatan berdasarkan kesamaan variabel yang signifikan terhadap Jumlah Kematian Ibu Nifas. Sedangkan Pemodelan menggunakan GWBZIGPR pada model *poisson state* menghasilkan 6 kelompok

kecamatan berdasarkan kesamaan variabel yang signifikan terhadap Jumlah Kematian Ibu Hamil dan 6 kelompok kecamatan berdasarkan kesamaan variabel yang signifikan terhadap Jumlah Kematian Ibu Nifas.

5.2 Saran

Saran yang dapat diberikan untuk penelitian selanjutnya adalah:

1. Pada penelitian ini metode iterasi yang digunakan untuk menyelesaikan metode MLE yang yeng hasilkan turunan yang tidak *close form* adalah BHHH. Iterasi BHHH memerlukan inisial parameter awal yang diperoleh dari taksiran univariat, padahal metode yang diterapkan pada penelitian ini merupakan metode baru yang belum tersedia paket program untuk mendapatkan taksiran univariat nya sehingga harus melakukan *trial error* untuk mendapatkan taksiran yang memiliki *mean square error* terkecil sehingga penelitian selanjutnya disarankan menggunakan metode iterasi lainnya seperti interpolasi linear, metode scant dan sebagainya.
2. Penelitian selanjutnya disarankan menambahkan variabel lain dari kerangka kerja Mc Carty dan Maine yang belum ada pada penelitian ini dari sumber yang sama.
3. Saran untuk Dinas Kesehatan Provinsi Jawa Tengah khusunya Karesidenan Pekalongan agar lebih memperhatikan variabel prediktor yang mempunyai tanda koefisien parameter negatif dan berpengaruh signifikan terhadap Jumlah Kematian Ibu Hamil dan Jumlah Kematian Ibu Nifas.
4. Setiap prosedur dalam pemeriksaan kehamilan sampai penanganan persalinan hendaknya dilakukan pencatatan yang nantinya dapat dijadikan sebagai tolak ukur dalam penentuan seorang ibu hamil/nifas dikategorikan mendapatkan suatu layanan/tindakan kesehatan.

DAFTAR PUSTAKA

Agresti, A. (2002), “Categorical Data Analysis (Second Edition)”, John Wiley & Sons, Inc., New Jersey.

Aitken, A.C. (1936), “A Further Note on Multivariate Selection”, *Proceedings of the Edinburgh Mathematical Society*, Vol. 5, No. 1, hal 37-40.

Akaike, H. (1978), “A Bayesian Analysis of the minimum AIC Procedure”, Annals of the Institute of Statistical Mathematics, Part A Hal 914.

Almasi, A., Eshraghian, M.R., Moghimbeigi, A., Rahimi, A., Mohammad, K., dan Fallahigilan, S. (2016), “Multilevel Zero-Inflated Generalized Poisson Regression Modeling for Dispersed Correlated Count Data”, *Statistical Methodology*, Vol. 30, hal. 1-14.

Arnold, R. (2011), *Poisson Regression Modelling of the Effectiveness of the Meningococcal B Vaccine (MeNZB)*, School of Mathematics, Statistics and Operations Research, Victoria University of Wellington, Wellington.

ASEAN Secretariat (2017), *ASEAN Statistical Report on Millenium Development Goals 2017*, ASEAN Secretariat, Jakarta.

Badan Pusat Statistik (2014), *Kajian Indikator Sustainable Development Goals (SDGs)*, BPS RI, Jakarta.

Badan Pusat Statistik (2016), *Profil Penduduk Indonesia Hasil SUPAS 2015*, BPS RI, Jakarta.

Bedrick, E.J. dan Tsai, C.L (1994), “Model Selection for Multivariate Regression in Small Samples”, *Biometrics*, Vol. 50, No. 1, hal. 226-231.

Benassi, F. dan Naccarato, A. (2017), “Household in Potential Economic Distress. A Geographically Weighted Regression Model for Italy, 2001-2011”, *Spatial Statistics*, Vol. 21, hal. 362-376.

Best, D.J.(1999), *Test of fit and other nonparametric data analysis*, Univesity of Wollongong.

Brundson, C., Fotheringham, A.S., dan Charlton, M. (1998), “Geographically Weighted Regression-Modelling Spatial Non-Stationarity”, *The Statistician*, No. 47, Bagian 3, hal. 431-443.

Brunton, L.A., Alexander, N., Wint, W., Ashton, A, dan Broughan, J.M. (2017), “Using Geographically Weighted Regression to Explore the Spatially Heterogeneous Spread of Bovine Tuberculosis in England and Wales”, *Stochastic Environmental Research and Risk Assessment*, Vol. 31, No. 2, hal. 339-352.

Cameron, C., dan Ttrivedi, P. K. (2005). *Microeconometric Methods and applications*. UK: Cambridge University Press.

Campbell, J.T. (1934), “The Poisson Correlation Function”, *Proceedings of the Edinburgh Mathematical Society*, Vol. 4, No. 1, hal. 18-26.

Cheon, S., Song, S.H. dan Jung, B.C. (2009), "Tests for independence in a bivariate negative binomial model", *Journal of the Korean Statistical Society*, vol. 38, hal, 185-190.

Consul, P.C. dan Famoye, F. (1992), "Generalized Poisson Regression Model", *Communications in Statistics – Theory and Methods*, Vol. 21, No. 1, hal. 89-109.

Consul, P.C. dan Jain, G.C. (1973), "A Generalization of the Poisson Distribution", *Technometrics*, Vol. 15, No.4, hal. 791-799, doi: 10.2307/1267389.

Da Silva, A.R. dan Lima, A.O. (2017), "Geographically Weighted Beta Regression", *Spatial Statistics*, Vol. 21, hal. 279-303.

Da Silva, A.R. dan Mendes, F.F. (2018), "On Comparing Some Algorithms for Finding the Optimal Bandwith in Geographically Weighted Regression", *Applied Soft Computing Journal*, Vol. 73, hal. 943-957.

Destyanugraha, R. dan Kurniawan, R. (2017), "Pemodelan Angka Kematian Ibu di Indonesia Dengan Pendekatan Geographically Weighted Poisson Regression", *Jurnal Matematika, Sains dan Teknologi*, Vol. 18, No. 2, hal. 76-94.

Dinas Kesehatan (2018), "Profil Kesehatan Provinsi Jawa Tengah Tahun 2018", Dinkes Jawa Tengah, Semarang.

Emmanuel, C. (2015), *A Comparison of Poisson or Negative Binomial Regression and Lee-Carter Models of Forecasting Norwegian Male Mortality*, Thesis, University of Oslo, Oslo.

Famoye, F. (1993), "Restricted Generalized Poisson Regression Model", *Communications in Statistics – Theory and Methods*, Vol. 22, No. 5, hal. 1335-1354, doi: 10.1080/03610929308831089

Famoye, F. (2010), "A New Bivariate Generalized Poisson Distribution", *Statistica Neerlandica*, Vol 64, hal. 112-124, doi: 10.1111/j.1467-9574.2009.00446.x.

Famoye, F. (2015), "A Multivariate Generalized Poisson Regression Model", *Communications in Statistics – Theory and Methods*, Vol. 44, No. 3, hal. 497-511, doi: 10.1080/03610926.2012.743565.

Famoye, F. dan Consul, P.C. (1995), "Bivariate Generalized Poisson Distribution with Some Applications", *Metrika*, Vol. 42, hal. 127-138.

Famoye, F. dan Singh, K.P. (2006), "Zero-Inflated Generalized Poisson Regression Model with an Application to Domestic Violence Data", *Journal of Data Science 4*, hal. 117-130.

Famoye, F., Wulu, J.T., dan Singh, K.P. (2004), "On the Generalized Poisson Regression Model with an Application to Accident Data", *Journal of Data Science*, Vol 2, hal. 287-295.

Faroughi, P. dan Ismail, N. (2014), “A New Form of Bivariate Generalized Poisson Regression Model”, *AIP Conference Proceedings*, Vol. 1614, No. 1, hal. 923-928.

Fotheringham, A.S., Brundson, C., dan Charlton, M. (2002), *Geographically Weigted Regression: The Analysis of Spatially Varying Relationship*, John Wiley & Son Ltd, England.

Fransiska, R.D., Respati, S.H., dan Mudigdo, A. (2017), “Analysis of Maternal Mortality Determinants in Bondowoso Districts, East Java”, *Journal of Maternal and Child Health*, Vol. 2, No. 1, hal. 76-88.

Girum, T. dan Wasie, A. (2017), Correlates of Maternal Mortality in Developing Countries : An Ecological Study in 82 Countries, *Maternal Health, Neonatology and Perinatology*, Vol. 3.

Gupta, P.L., Gupta, R.C., dan Tripathi, R.C. (2004), “Score Test for Zero Inflated Generalized Poisson Regression Model”, *Communications in Statistics - Theory and Methods*, Vol. 33, No. 1, hal. 47-64.

Holgate, P. (1964), “Estimation for The Bivariate Poisson Distribution”, *Biometrika 51*, hal. 241-245.

Ismail, N. dan Faroughi, P. (2017), “Bivariate Zero-Inflated Generalized Poisson Regression Model with Flexible Covariance”, *Communications in Statistics – Theory and Methods*, Vol. 46, No. 15, hal. 7769-7785.

Ismail, N. dan Jemain, A.A. (2007), “ Handling Overdispersion with Negative Binomial and Generalized Poisson Regression Models”, *Casualty Actuarial Society Forum*, hal. 103-158.

Johnson, R. dan Wichern, D. (2007), *Applied Multivariate Statistical Analysis*, Harlow: Pearson Education Limited.

Jung, C.R. dan Winkelman, R (1993), Two aspect of labor mobility: A bivariate poisson regression approach. Journal empirical economics, vol 18, 543-556.

Karlis, D. dan Ntzoufras, I. (2005), “Bivariate Poisson and Diagonal Inflated Bivariate Poisson Regression Models in R”, *Journal of Statistical Software*, Vol. 14, No. 10, hal.1-36.

Karlis, D. dan Xekalaki, E. (2000), “A Simulation Comparison of Several Procedures for Testing the Poisson Assumption”, *The Statistician*, Vol. 49, part 3, hal.355-382.

Kawamura, K. (1973), “The Structure of Bivariate Poisson Distribution”, *Kodai Mathematical Journal*, Vol. 25, No. 2, hal. 246-256.

Kementerian Kesahatan Republik Indonesia. (2018). *Profil Kesehatan Indonesia 2017*. Kemenkes RI. Jakarta.

Lakshminarayana, J., Pandit, S.N.N., dan Rao, K.S. (1999), “On a Bivariate Poisson Distribution”, *Communications in Statistics – Theory and Methods*, Vol. 28, No. 2, hal. 267-276.

Lambert, D. (1992), "Zero-Inflated Poisson Regression With an Application to Defects in Manufacturing", *Technometrics*, Vol. 34, No. 1, hal. 1-14.

McCarthy, J., dan Maine, D. (1992), A Framework for Analizing The Determinants oMaternal Mortality. Studies in family planning, 23-33.

McCullagh, P. dan Nelder, J.A. (1989), *Generalized Linear Models*, 2nd edition, Chapman and Hall, London.

Novita, L., Salamah, M., dan Sutikno (2012), "Pemodelan Maternal Mortality di Jawa Timur dengan Pendekatan Geographically Weighted Poisson Regression (GWPR)", Diakses pada 5 Mei 2019, 19:12 melalui http://digilib.its.ac.id/public/ITS-Undergraduate-9311-Paper.pdf.

Ozmen, I. dan Famoye, F. (2007), "Count Regression Models with an Application to Zoological Data Containing Structural Zeros", *Journal of Data Science,* Vol. 5, No. 4, hal. 491-502.

Pangulimang, J., Purhadi, dan Sutikno (2016), "Parameter Estimation and Hypothesis Testing of Geographically Weighted Bivariate Zero-Inflated Poisson", *Proceeding of 3rd International Conference on Research, Implementation and Education of Mathematics and Science*, Universitas Negeri Yogyakarta, Yogyakarta, hal. 73-78.

Parodi, S. dan Bottarelli, E. (2006), "Poisson Regression Model in Epidemiology – An Introduction", *Annali della Facolta di Medicina Veterinaria - Universita di Parma,* Vol. XXVI, hal. 25-44.

Purhadi, Dewi, Y.S., dan Amaliana, L. (2015), "Zero Inflated Poisson and Geographically Weighted Zero-Inflated Poisson Regression Model: Application to Elephantiasis (*Filariasis*) Counts Data", *Journal of Mathematics and Statistics*, Vol. 11, No. 2, hal. 52-60.

Qomariyah, N., Purnami, S.W., dan Pramono, M.S. (2013), "Pemodelan Faktor-faktor yang Mempengaruhi Jumlah Kematian Ibu di Jatim dengan Pendekatan GWPR (Geographically Weighted Poisson Regression) Ditinjau dari Segi Fasilitas Kesehatan", *Jurnal Sains dan Seni POMITS*, Vol. 2, No. 2, hal. 311-316.

Ridout, M., Demetrio, C.G.B., dan Hinde, J. (1998), "Models for Count Data with Many Zeros", *Proceedings of the XIXth International Biometric Conference*, hal. 179-192.

Silva, A. R., dan Lima, A.D.(2017), Geographically Weighted Beta Regression, *Spatial statistics*.

Sugiarti, H., Purhadi, Sutikno, dan Purnami, S.W. (2016), "Parameter Estimation of Geographically Weighted Multivariate t Regression Model", *Journal of Theoretical and Applied Information Technology*, Vol. 92, No. 1, hal. 45-51.

Triyanto, Kuswardi, Y., dan Chrisnawati, H.E. (2018), "Modified Multivariate Poisson Regression to Modeling the Number of Mortality of Maternal, Early Neonatal and Post Neonatal in Central Java Province, Indonesia", *Applied Mathematical Sciences*, Vol. 12, No. 28, hal. 1373-1382.

Vernic, R. (1997), “On the Bivariate Generalized Poisson Distribution”, *Astin Bulletin*, Vol. 27, No. 1, hal. 23-32.

Wang, W. dan Famoye, F. (1997), “Modeling Household Fertility Decisions with Generalized Poisson Regression”, *Journal of Population Economics*, Vol. 10, No. 3, hal. 273-283.

Wardani, D.K. (2016), *Pendugaan Parameter dan Pengujian Hipotesis Biariate Generalized Poisson Regression*, Thesis, ITS, Surabaya.

World Health Organization (2015), *Trends in Maternal Mortality 1990 to 2015: Estimates by WHO, UNICEF, UNFPA, World Bank Group and The United Nations Population Division*, World Health Organization, Jenewa, Swiss.

Yuen, H.K., Chow, S.C., dan Tse, S.K. (2015), “On Statistical Tests for Homogeneity of Two Bivariate Zero-Inflated Poisson Populations”, *Journal of Biopharmaceutical Statistics*, Vol. 25, No. 1, hal. 44-53.

Zamani, H., Faroughi, P., dan Ismail, N. (2016), “Bivariate Generalized Poisson Regression Model: Applications on Health Care Data”, *Empirical Economics*, Vol. 51, No. 4, hal. 1607-1621.

Zhang, C., Tian, G. dan Huang, X. (2015), “Two New Bivariate Zero-Inflated Generalized Poisson Distribution With a Flexible Correlation Structure”, *Statistics, Optimization and Information Computing*, Vol. 3, hal. 105-137.

Zhang, H., Zhang, J., Lu, S., Cheng, S., dan Zhang, J. (2011), “Modeling Hotel Room Price with Geographically Weighted Regression”, *International Journal of Hospitality Management*, Vol. 30, hal. 1036-1043.

(Halaman ini sengaja dikosongkan)

LAMPIRAN

Lampiran 1: Data Penelitian

Kecamatan	Y_1	Y_2	X_1	X_2	X_3	X_4	X_5	X_6	X_7	u_i	v_i
Wonotunggal	1	0	100,00	94,49	100,00	53,20	100,00	39,34	63,68	-7,03	109,61
Bandar	1	0	100,00	91,41	100,00	66,64	93,65	28,72	54,81	-7,06	109,66
Blado	1	0	100,00	95,79	100,00	100,00	97,06	29,37	51,07	-7,11	109,75
Reban	0	0	100,00	91,30	100,00	94,15	92,56	55,22	68,32	-7,08	109,87
Bawang	0	1	100,00	100,00	100,00	100,00	100,00	22,98	60,74	-7,13	109,85
Tersono	0	0	100,00	83,15	100,00	40,63	83,15	29,61	64,98	-7,04	109,90
Gringsing	0	1	100,00	94,06	100,00	100,00	92,32	29,10	50,09	-6,96	109,93
Limpung	0	1	100,00	96,52	99,83	100,00	96,68	19,07	47,98	-7,02	109,85
Banyuputih	0	0	100,00	93,91	99,83	100,00	99,34	33,88	36,93	-6,96	109,88
Subah	0	0	100,00	91,94	99,87	80,30	94,12	30,53	55,43	-6,96	109,80
Pecalungan	0	1	100,00	100,00	99,81	100,00	83,85	17,43	40,39	-7,02	109,79
Tulis	0	0	100,00	90,51	100,00	99,27	96,93	14,45	61,45	-6,95	109,74
Kandeman	0	0	100,00	92,77	100,00	96,38	100,00	27,07	41,51	-6,93	109,70
Batang	3	3	100,00	95,99	100,00	98,91	95,82	30,43	45,27	-6,91	109,66
Warungasem	2	1	100,00	90,30	100,00	100,00	94,29	31,79	49,48	-6,95	109,67
Kandangserang	0	0	82,11	75,98	91,44	94,55	93,70	21,64	59,27	-7,15	109,40
Paninggaran	1	0	100,00	78,35	97,38	90,92	89,39	31,42	75,96	-7,15	109,51
Lebakbarang	0	0	46,60	49,51	79,59	81,55	61,17	10,68	100,00	-7,13	109,63
Petungkriyono	0	0	91,63	97,21	86,83	59,53	72,09	18,60	100,00	-7,15	109,67
Talun	0	0	83,30	81,10	99,42	79,08	96,70	29,17	54,91	-7,04	109,66
Doro	1	0	92,19	95,04	100,00	98,27	100,00	32,71	54,97	-7,06	109,62
Karanganyar	0	0	100,00	99,74	97,90	63,30	97,95	45,01	72,36	-7,05	109,56
Kajen	0	2	97,28	92,80	94,42	45,74	96,84	40,91	56,19	-7,06	109,50
Kesesi	2	0	93,72	88,75	93,33	88,35	90,79	25,10	66,18	-7,03	109,45
Sragi	1	0	92,43	90,21	100,00	74,04	98,04	46,13	46,16	-6,95	109,51
Siwalan	0	0	96,04	100,00	92,81	55,95	100,00	35,82	47,34	-6,89	109,52
Bojong	1	0	94,63	90,77	90,63	17,56	90,91	45,18	55,31	-6,97	109,55
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Tegal Selatan	0	0	100,00	93,53	100,00	70,80	93,53	19,95	28,54	-6,88	109,10
Tegal Timur	0	0	100,00	90,45	99,92	26,36	90,45	27,78	27,99	-6,87	109,11
Tegal Barat	0	1	100,00	97,75	100,00	48,83	97,75	25,54	40,67	-6,86	109,08
Margadana	0	1	100,00	96,89	100,00	57,04	96,89	17,08	30,45	-6,87	109,08

Lampiran 2: Fungsi kepadatan peluang BZIGPR

$$A_i = \frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}} + \frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}.\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right) + \frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1(u_i,v_i)e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right) + M_1$$

Dimana $M_1 = \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}} - \frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta(1-g_1)(1-g_2)\right)$

$$B_i = \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)^{y_{2i}}\frac{(1+\varphi_2 y_2)^{y_{2i}-1}}{y_2!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}(1+\varphi_2 y_2)}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}} + \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)\right)$$

$$C_i = \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)^{y_{1i}}\frac{(1+\varphi_1 y_1)^{y_{1i}-1}}{y_{1i}!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}(1+\varphi_1 y_{1i})}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}} + \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\exp\left(\frac{-e^{\mathbf{x}_i^{\mathbf{T}}\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^{\mathbf{T}}\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(e^{-y_{1i}}-g_1\right)(1-g_2)\right)\right)$$

$$D_i = \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_i,v_i)}}\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_i,v_i)}}\prod_{l=1}^{2}\left(\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}\right)^{y_{li}}\frac{(1+\varphi_l y_l)^{y_{li}-1}}{y_{li}!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}(1+\varphi_l y_{li})}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}\right)\right)\left(1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right)$$

Lampiran 3: Fungsi *ln likelihood* $L(\Omega)$ BZIGPR

$$\ln A_i = \ln\sum_{i=1}^{n_1}\left(\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}+\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)+\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+A_1\right)$$

$$=\left(\ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}+\ln\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)Q\right)\right)$$

dimana $A_1=\frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)Q$ dan $Q=\left(1+\eta(1-g_1)(1-g_2)\right)$

$$\ln B_i=\ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)^{y_{2i}}\frac{(1+\varphi_2 y_{2i})^{y_{2i}-1}}{y_2!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}+\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)\right)\right)$$

$$=\ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}+y_{2i}\mathbf{x}_i^T\boldsymbol{\beta}_2-y_{2i}\ln\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)+(y_{2i}-1)\ln(1+\varphi_2 y_{2i})-\ln y_{2i}!+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)+\ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}+\ln\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)P\right)$$

dimana $P=\left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)$

$$\ln C_i = \ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)^{y_{1i}}\frac{\left(1+\varphi_1 y_{1i}\right)^{y_{1i}-1}}{y_{1i}\,!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}+\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\right)$$

$$= \ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}+y_{1i}\ln e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-y_{1i}\ln\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}\,!+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+\ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}+\ln\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)R\right)$$

dimana $R=\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)$

$$\ln D_i = \ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\prod_{l=1}^{2}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}\right)^{y_{li}}\frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}\,!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}\left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$= \ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}+\ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}+y_{1i}\mathbf{x}_i^T\boldsymbol{\beta}_1-y_{1i}\ln\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}\,!+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+y_{2i}\mathbf{x}_i^T\boldsymbol{\beta}_2-y_{2i}\ln\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)+T$$

dimana $T=\left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right)-\ln y_{2i}\,!+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)+\ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$

Lampiran 4: Turunan Pertama fungsi $\ln L(\Omega)$ BZIGPR

Turunan pertama A terhadap $\boldsymbol{\gamma}_1$ terhadap adalah

$$\frac{\partial A}{\partial \boldsymbol{\gamma}_1} = -\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\,\mathbf{x}_i^T}{\left(1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)\left(1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\right)} + \frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\left(\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\right)\mathbf{x}_i^T}{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\left(\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)T}$$

dimana $\mathrm{T}=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama A terhadap $\boldsymbol{\gamma}_2$ adalah

$$\frac{\partial A}{\partial \boldsymbol{\gamma}_2} = -\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\right)\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)} + \frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\right)\mathbf{x}_i^T}{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)T}$$

Turunan pertama A terhadap $\boldsymbol{\beta}_1$ adalah

$$\frac{\partial A}{\partial \boldsymbol{\beta}_1} = \frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)T\,\frac{\partial g_1}{\partial \boldsymbol{\beta}_1}\right)$$

dimana $S = e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2\,\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)T$

Turunan pertama A terhadap $\boldsymbol{\beta}_2$ adalah

$$\frac{\partial A}{\partial \boldsymbol{\beta}_2}=\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2\varphi_2\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2\varphi_2\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}\right)T\frac{\partial g_2}{\partial \boldsymbol{\beta}_2}\right)$$

Turunan pertama A terhadap φ_1 adalah

$$\frac{\partial A}{\partial \varphi_1}=\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\frac{-\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}T\frac{\partial g_1}{\partial \varphi_1}\right)$$

Turunan pertama A terhadap φ_2 adalah

$$\frac{\partial A}{\partial \varphi_2}=\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\frac{-\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}T\frac{\partial g_2}{\partial \varphi_2}\right)$$

Turunan pertama A terhadap η adalah

$$\frac{\partial A}{\partial \eta}=\frac{1}{S}\left(\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)(1-g_1)(1-g_2)\right)$$

Turunan pertama B terhadap $\boldsymbol{\gamma}_1$ adalah

$$\frac{\partial B}{\partial \boldsymbol{\gamma}_1} = -\frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1}\right)^2} + \frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1}\mathbf{x}_i^T}{\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}$$

Turunan pertama B terhadap $\boldsymbol{\gamma}_2$ adalah

$$\frac{\partial B}{\partial \boldsymbol{\gamma}_2} = -\frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2}\right)^2}$$

Turunan pertama B terhadap $\boldsymbol{\beta}_1$ adalah

$$\frac{\partial B}{\partial \boldsymbol{\beta}_1} = \frac{\exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}} - \frac{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}\right)^2 \varphi_1 \mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}\right)^2}\right)}{\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)} \frac{\partial g_1}{\partial \boldsymbol{\beta}_1}$$

Turunan pertama B terhadap $\boldsymbol{\beta}_2$ adalah

$$\frac{\partial B}{\partial \boldsymbol{\beta}_2} = y_{2i}\mathbf{x}_i^T - \frac{y_{2i}\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)} + \left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\left(1+\varphi_2 y_{2i}\right)\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} - \frac{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2\left(1+\varphi_2 y_{2i}\right)\varphi_2 \mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2}\right) + \frac{1}{\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)} \frac{\partial g_2}{\partial \boldsymbol{\beta}_2}$$

Turunan pertama B terhadap φ_1 adalah

$$\frac{\partial B}{\partial \varphi_1}=\frac{-\exp\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\frac{\partial g_1}{\partial \varphi_1}$$

Turunan pertama B terhadap φ_2 adalah

$$\frac{\partial B}{\partial \varphi_2}=-y_{2i}\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)}+\frac{\left(y_{2i}-1\right)}{\left(1+\varphi_2 y_2\right)}y_{2i}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}y_{2i}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}+\frac{e^{2\mathbf{x}_i^T\boldsymbol{\beta}_2}\left(1+\varphi_2 y_2\right)}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)K}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)K\right)}\frac{\partial g_2}{\partial \varphi_2}$$

di mana $K=1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)$

Turunan pertama B terhadap η adalah

$$\frac{\partial B}{\partial \eta}=\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}$$

Turunan pertama C terhadap $\boldsymbol{\gamma}_1$ adalah

$$\frac{\partial C}{\partial \boldsymbol{\gamma}_1}=-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)}$$

Turunan pertama C terhadap $\boldsymbol{\gamma}_2$ adalah

$$\frac{\partial C}{\partial \boldsymbol{\gamma}_2}=-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\right)}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\mathbf{x}_i^T}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}$$

Turunan pertama C terhadap $\boldsymbol{\beta}_1$ adalah

$$\frac{\partial C}{\partial \boldsymbol{\beta}_1}=y_{1i}\left(\mathbf{x}_i^T-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)}\right)+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\left(1+\varphi_1 y_{1i}\right)\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\left(1+\varphi_1 y_{1i}\right)\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)+\frac{1}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_1}{\partial \boldsymbol{\beta}_1}$$

Turunan pertama C terhadap $\boldsymbol{\beta}_2$ adalah

$$\frac{\partial C}{\partial \boldsymbol{\beta}_2}=\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2\varphi_2\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_2}{\partial \boldsymbol{\beta}_2}$$

Turunan pertama C terhadap φ_1 adalah

$$\frac{\partial C}{\partial \varphi_1}=y_{1i}\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)}+\frac{\left(y_{1i}-1\right)}{\left(1+\varphi_1 y_1\right)}y_{1i}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}y_{1i}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\left(1+\varphi_1 y_1\right)}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_1}{\partial \varphi_1}$$

Turunan pertama C terhadap φ_2 adalah

$$\frac{\partial C}{\partial \varphi_2}=\frac{-\exp\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}\frac{\partial g_2}{\partial \varphi_2}$$

Turunan pertama C terhadap η adalah

$$\frac{\partial C}{\partial \eta}=\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}$$

Turunan pertama D terhadap $\boldsymbol{\gamma}_1$ adalah

$$\frac{\partial D}{\partial \boldsymbol{\gamma}_1}=-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)}$$

Turunan pertama D terhadap $\boldsymbol{\gamma}_2$ adalah

$$\frac{\partial D}{\partial \boldsymbol{\gamma}_2} = -\frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2} \mathbf{x}_i^T}{\left(1 + e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2}\right)}$$

Turunan pertama D terhadap $\boldsymbol{\beta}_1$ adalah

$$\frac{\partial D}{\partial \boldsymbol{\beta}_1} = y_{1i}\mathbf{x}_i^T - \frac{y_{1i}\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}\right)} + \left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}\left(1+\varphi_1 y_{1i}\right)\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}\left(1 - \frac{\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}\right)}\right)\right) + \frac{1}{\left(1+\eta\left(e^{-y_{1i}} - g_1\right)\left(e^{-y_{2i}} - g_2\right)\right)}\frac{\partial g_1}{\partial \boldsymbol{\beta}_1}$$

Turunan pertama D terhadap $\boldsymbol{\beta}_2$ adalah

$$\frac{\partial D}{\partial \boldsymbol{\beta}_2} = y_{2i}\mathbf{x}_i^T - \frac{y_{2i}\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)} + \left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\left(1+\varphi_2 y_{2i}\right)\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}\left(1 - \frac{\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)}\right)\right) + \frac{1}{\left(1+\eta\left(e^{-y_{1i}} - g_1\right)\left(e^{-y_{2i}} - g_2\right)\right)}\frac{\partial g_2}{\partial \boldsymbol{\beta}_2}$$

Turunan pertama D terhadap φ_1 adalah

$$\frac{\partial D}{\partial \varphi_1} = \frac{-y_{1i} e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}} + \frac{\left(y_{1i} - 1\right) y_{1i}}{\left(1+\varphi_1 y_{1i}\right)} + \left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}\left(y_{1i} - \frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}\left(1+\varphi_1 y_{1i}\right)}{\left(1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}\right)}\right)\right) + \frac{1}{\left(1+\eta\left(e^{-y_{1i}} - g_1\right)\left(e^{-y_{2i}} - g_2\right)\right)}\frac{\partial g_1}{\partial \varphi_1}$$

Turunan pertama D terhadap φ_2 adalah

$$\frac{\partial D}{\partial \varphi_2}=\frac{-y_{2i}e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}+\frac{(y_{2i}-1)y_{2i}}{(1+\varphi_2 y_{2i})}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\left(y_{2i}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}(1+\varphi_2 y_{2i})}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \varphi_2}$$

Turunan pertama D terhadap η adalah

$$\frac{\partial D}{\partial \eta}=\frac{\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}$$

Dimana $g_l=\exp\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}(t_l-1)\right)$, $\ln\ t_l-\left(\frac{\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}(t_l-1)}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}\right)+1=0$

$$\frac{\partial g_l}{\partial \beta_l}=(t_l-1)\left(1-\varphi_l t_l\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1\,\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)\right)^{-1}g_l$$

$$\frac{\partial g_l}{\partial \varphi_l}=\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}{\left(1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}\right)^2}\right)^2 t_l\,(t_l-1)\left(1-\varphi_l\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}{\left(1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}\right)^2}t_l\right)^{-1}g_l$$

Sehingga turunan pertama masing-masing parameter dari fungsi *ln likelihood* dari persamaan (4.3) sebagai berikut:

Turunan pertama fungsi l terhadap $\boldsymbol{\gamma}_1$

$$\frac{\partial l}{\partial \boldsymbol{\gamma}_1}=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(-\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\,\mathbf{x}_i^T}{\left(1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)\left(1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\right)}+\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\right)\mathbf{x}_i^T}{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\left(\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)T}\right)$$

$$+\sum_{i=1}^{n}\left(b_i\right)\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\mathbf{x}_i^T}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)+\sum_{i=1}^{n}\left(c_i\right)\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)}\right)+\sum_{i=1}^{n}\left(d_i\right)\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)}\right)$$

Turunan pertama fungsi l terhadap $\boldsymbol{\gamma}_2$

$$\frac{\partial l}{\partial \boldsymbol{\gamma}_2}=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\right)\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\left(e^{\left(\mathbf{x}_i^T\boldsymbol{\gamma}_1\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\right)\mathbf{x}_i^T}{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)T}\right)$$

$$+\sum_{i=1}^{n}\left(b_i\right)\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\right)}\right)+\sum_{i=1}^{n}\left(c_i\right)\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\right)}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\mathbf{x}_i^T}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n}\left(d_i\right)\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\right)}\right)$$

Turunan pertama fungsi l terhadap $\boldsymbol{\beta}_1$

$$\frac{\partial l}{\partial \boldsymbol{\beta}_1}=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)T\,\frac{\partial g_1}{\partial \boldsymbol{\beta}_1}\right)\right)$$

$$+\sum_{i=1}^{n}\left(b_i\right)\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\frac{\partial g_1}{\partial \boldsymbol{\beta}_1}\right)$$

$$+\sum_{i=1}^{n}\left(c_i\right)\left(y_{1i}\left(\mathbf{x}_i^T-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)}\right)+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\left(1+\varphi_1 y_{1i}\right)\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\left(1+\varphi_1 y_{1i}\right)\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)+\frac{1}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_1}{\partial \boldsymbol{\beta}_1}\right)$$

$$+\sum_{i=1}^{n}\left(d_i\right)\left(y_{1i}\mathbf{x}_i^T-\frac{y_{1i}\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\left(1+\varphi_1 y_{1i}\right)\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\left(1-\frac{\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \boldsymbol{\beta}_1}\right)$$

dimana $S=e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2\,\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)T$

$\mathrm{T}=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi l terhadap $\boldsymbol{\beta}_2$

$$\frac{\partial l}{\partial \boldsymbol{\beta}_2} = \sum_{i=1}^{n} \left(1 - b_i - c_i - d_i\right) \left(\frac{1}{S} \left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1} \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} \right) \left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} \mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} + \frac{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right) \varphi_2 \mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2} \right) + \exp\left(-\frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}} - \frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} \right) \left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} \mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} - \frac{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2 \varphi_2 \mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2} \right) T \frac{\partial g_2}{\partial \boldsymbol{\beta}_2} \right) \right)$$

$$+ \sum_{i=1}^{n} \left(b_i\right) \left(y_{2i} \mathbf{x}_i^T - \frac{y_{2i} \varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} \mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)} + \left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} \left(1+\varphi_2 y_{2i}\right) \mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} - \frac{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2 \left(1+\varphi_2 y_{2i}\right) \varphi_2 \mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2} \right) + \frac{1}{\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}} \right) \left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}} - g_2\right)\right) \right)} \frac{\partial g_2}{\partial \boldsymbol{\beta}_2} \right)$$

$$+ \sum_{i=1}^{n} \left(c_i\right) \left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} \right) \left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}} - g_1\right)\right) \left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} \mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} - \frac{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2 \varphi_2 \mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2} \right)}{\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} \right) \left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}} - g_1\right)\right) \right)} \frac{\partial g_2}{\partial \boldsymbol{\beta}_2} \right)$$

$$+ \sum_{i=1}^{n} \left(d_i\right) \left(y_{2i} \mathbf{x}_i^T - \frac{y_{2i} \varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} \mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)} + \left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} \left(1+\varphi_2 y_{2i}\right) \mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} \left(1 - \frac{\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)} \right) \right) + \frac{1}{\left(1+\eta\left(e^{-y_{1i}} - g_1\right)\left(e^{-y_{2i}} - g_2\right)\right)} \frac{\partial g_2}{\partial \boldsymbol{\beta}_2} \right)$$

Turunan pertama fungsi l terhadap φ_1

$$
\begin{aligned}
\frac{\partial l}{\partial \varphi_1} = & \sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\frac{-\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}T\frac{\partial g_1}{\partial \varphi_1}\right)\right) \\
& +\sum_{i=1}^{n}\left(b_i\right)\left(\frac{-\exp\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\frac{\partial g_1}{\partial \varphi_1}\right) \\
& +\sum_{i=1}^{n}\left(c_i\right)\left(y_{1i}\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)}+\frac{\left(y_{1i}-1\right)}{\left(1+\varphi_1 y_1\right)}y_{1i}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}y_{1i}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\left(1+\varphi_1 y_1\right)}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right) \\
& +\sum_{i=1}^{n}\left(d_i\right)\left(\frac{-y_{1i}e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}+\frac{\left(y_{1i}-1\right)y_{1i}}{\left(1+\varphi_1 y_{1i}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\left(y_{1i}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\left(1+\varphi_1 y_{1i}\right)}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right)
\end{aligned}
$$

Turunan pertama fungsi l terhadap φ_2

$$\frac{\partial l}{\partial \varphi_2}=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\frac{-\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}T\frac{\partial g_2}{\partial \varphi_2}\right)\right)$$

$$+\sum_{i=1}^{n}\left(b_i\right)\left(-y_{2i}\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)}+\frac{\left(y_{2i}-1\right)}{\left(1+\varphi_2 y_2\right)}y_{2i}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}y_{2i}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}+\frac{e^{2\mathbf{x}_i^T\boldsymbol{\beta}_2}\left(1+\varphi_2 y_2\right)}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \varphi_2}\right)$$

$$+\sum_{i=1}^{n}\left(c_i\right)\left(\frac{-\exp\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}\frac{\partial g_2}{\partial \varphi_2}\right)$$

$$+\sum_{i=1}^{n}\left(d_i\right)\left(\frac{-y_{2i}e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}+\frac{\left(y_{2i}-1\right)y_{2i}}{\left(1+\varphi_2 y_{2i}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\left(y_{2i}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\left(1+\varphi_2 y_{2i}\right)}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \varphi_2}\right)$$

Turunan pertama fungsi l terhadap η

$$\frac{\partial l}{\partial \eta}=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1-g_1\right)\left(1-g_2\right)\right)\right)+\sum_{i=1}^{n}\left(b_i\right)\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)$$

$$+\sum_{i=1}^{n}\left(c_i\right)\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n}\left(d_i\right)\left(\frac{\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}\right)$$

Lampiran 5: Fungsi *ln likelihood* di bawah populasi $L\left(\hat{\Omega}\right)$ BZIGPR

$$\ln A_i^* = \ln \sum_{i=1}^{n_1}\left(\frac{\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}\frac{\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}+\frac{\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}\frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}{1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)+\frac{\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}\frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right)+A_1\right)$$

$$=\left(\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}+\ln\left(e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}{1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}-\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}{1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)q\right)\right)$$

dimana $A_1=\frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}\frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}\exp\left(-\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}-\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}{1+\varphi_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)q$ dan $q=\left(1+\hat{\eta}\left(1-g_1\right)\left(1-g_2\right)\right)$

$$\ln B_i^*=\ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}{1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)^{y_{2i}}\frac{\left(1+\hat{\varphi}_2y_{2i}\right)^{y_{2i}-1}}{y_2!}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}\left(1+\hat{\varphi}_2y_{2i}\right)}{1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}+\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right)\left(1+\hat{\eta}\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\right)$$

$$=\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}+y_{2i}\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2-y_{2i}\ln\left(1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}\right)+\left(y_{2i}-1\right)\ln\left(1+\hat{\varphi}_2y_{2i}\right)-\ln y_{2i}!+\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}\left(1+\hat{\varphi}_2y_{2i}\right)}{1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)+\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}+\ln\left(e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}+\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right)P\right)$$

dimana $P=\left(1+\hat{\eta}\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$

$$\ln C_i^*=\ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right)^{y_{1i}}\frac{\left(1+\hat{\varphi}_1y_{1i}\right)^{y_{1i}-1}}{y_{1i}!}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}\left(1+\hat{\varphi}_1y_{1i}\right)}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right)\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}+\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}{1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\right)$$

$$=\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}+y_{1i}\ln e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}-y_{1i}\ln\left(1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}\right)+\left(y_{1i}-1\right)\ln\left(1+\hat{\varphi}_1y_{1i}\right)-\ln y_{1i}!+\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}\left(1+\hat{\varphi}_1y_{1i}\right)}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right)+\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}+\ln\left(e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}+\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}{1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)R\right)$$

Dimana $R=\left(1+\hat{\eta}\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)$

$$\ln D_i^* = \ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}\prod_{l=1}^{2}\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_l}}{1+\hat{\varphi}_l e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_l}}\right)^{y_{li}}\frac{\left(1+\hat{\varphi}_l y_{li}\right)^{y_{li}-1}}{y_{li}!}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_l}\left(1+\hat{\varphi}_l y_{li}\right)}{1+\hat{\varphi}_l e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_l}}\right)\left[1+\hat{\eta}\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$= \ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}} + \ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}} + y_{1i}\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1 - y_{1i}\ln\left(1+\hat{\varphi}_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}\right) + \left(y_{1i}-1\right)\ln\left(1+\hat{\varphi}_1 y_{1i}\right) - \ln y_{1i}! + \left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}\left(1+\hat{\varphi}_1 y_{1i}\right)}{1+\hat{\varphi}_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right) + y_{2i}\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2 - y_{2i}\ln\left(1+\hat{\varphi}_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}\right) + T$$

dimana $T = \left(y_{2i}-1\right)\ln\left(1+\hat{\varphi}_2 y_{2i}\right) - \ln y_{2i}! + \left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}\left(1+\hat{\varphi}_2 y_{2i}\right)}{1+\hat{\varphi}_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right) + \ln\left(1+\hat{\eta}\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$

Lampiran 6: Fungsi *ln likelihood* d bawah H_0 $L(\omega)$ parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$ BZIGPR

$$\ln A_{1i} = \ln\left(\frac{\mathrm{e}^{\gamma_{10}}}{1+\mathrm{e}^{\gamma_{10}}}\frac{\mathrm{e}^{\gamma_{20}}}{1+\mathrm{e}^{\gamma_{20}}}+\frac{\mathrm{e}^{\gamma_{10}}}{1+\mathrm{e}^{\gamma_{10}}}\frac{1}{1+\mathrm{e}^{\gamma_{20}}}\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)+A_1+A_2\right)$$

$$= \ln\frac{1}{1+e^{\gamma_{10}}}\frac{1}{1+e^{\gamma_{20}}}+\ln\left(e^{\gamma_{10}}e^{\gamma_{20}}+e^{\gamma_{10}}\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)+e^{\gamma_{20}}\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)+A_3\right)$$

dimana

$$A_1 = \frac{\mathrm{e}^{\gamma_{20}}}{1+\mathrm{e}^{\gamma_{20}}}\frac{1}{1+\mathrm{e}^{\gamma_{10}}}\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right),\ A_2 = \frac{1}{1+\mathrm{e}^{\gamma_{10}}}\frac{1}{1+\mathrm{e}^{\gamma_{20}}}\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta(1-g_1)(1-g_2)\right)$$

$$A_3 = \exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta(1-g_1)(1-g_2)\right)$$

$$\ln B_{1i} = \ln\left(\frac{1}{1+e^{\gamma_{20}}}\left(\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)^{y_{2i}}\frac{(1+\varphi_2 y_{2i})^{y_{2i}-1}}{y_2!}\exp\left(\frac{-e^{\beta_{20}}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\beta_{20}}}\right)\left(\frac{e^{\gamma_{10}}}{1+e^{\gamma_{10}}}+\frac{1}{1+e^{\gamma_{10}}}\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)\right)\right)$$

$$= \ln\frac{1}{1+e^{\gamma_{20}}}+y_{2i}\beta_{20}-y_{2i}\ln\left(1+\varphi_2 e^{\beta_{20}}\right)+(y_{2i}-1)\ln(1+\varphi_2 y_{2i})-\ln y_{2i}!+\left(\frac{-e^{\beta_{20}}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\beta_{20}}}\right)+\ln\frac{1}{1+e^{\gamma_{10}}}+\ln e^{\gamma_{10}}+\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)B_1$$

dimana $B_1 = \left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)$

$$\ln C_{1i} = \ln\left(\frac{1}{1+e^{\gamma_{10}}}\left(\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)^{y_{1i}} \frac{\left(1+\varphi_1 y_{1i}\right)^{y_{1i}-1}}{y_{1i}\,!} \exp\left(\frac{-e^{\beta_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}}}\right)\left(\frac{e^{\gamma_{20}}}{1+e^{\gamma_{20}}}+\frac{1}{1+e^{\gamma_{20}}}\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)C_1\right)\right)$$

$$= \ln\frac{1}{1+e^{\gamma_{10}}} + y_{1i}\ln e^{\beta_{10}} - y_{1i}\ln\left(1+\varphi_1 e^{\beta_{10}}\right) + \left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right) - \ln y_{1i}\,! + \left(\frac{-e^{\beta_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}}}\right) + \ln\frac{1}{1+e^{\gamma_{20}}} + \ln\left(e^{\gamma_{20}} + \exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)C_1\right)$$

dimana $C_1 = \left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)$

$$\ln D_{1i} = \ln\left(\frac{1}{1+e^{\gamma_{10}}}\frac{1}{1+e^{\gamma_{20}}}\prod_{l=1}^{2}\left(\frac{e^{\beta_{l0}}}{1+\varphi_l e^{\beta_{l0}}}\right)^{y_{li}} \frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}\,!}\exp\left(\frac{-e^{\beta_{l0}}\left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\beta_{l0}}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$= \ln\frac{1}{1+e^{\gamma_{10}}} + \ln\frac{1}{1+e^{\gamma_{20}}} + y_{1i}\beta_{10} - y_{1i}\ln\left(1+\varphi_1 e^{\beta_{10}}\right) + \left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right) - \ln y_{1i}\,! + \left(\frac{-e^{\beta_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}}}\right) + D_1$$

dimana $D_1 = y_{2i}\beta_{20} - y_{2i}\ln\left(1+\varphi_2 e^{\beta_{20}}\right) + \left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right) - \ln y_{2i}\,! + \left(\frac{-e^{\beta_{20}}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}}}\right) + \ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$

Lampiran 7: Turunan pertama fungsi *ln likelihood* d bawah H_0 $L(\omega)$ parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$ BZIGPR

Turunan pertama fungsi I terhadap γ_{10}

$$\frac{\partial I}{\partial \gamma_{10}}=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(-\frac{\mathrm{e}^{\gamma_{10}}}{\left(1+e^{\gamma_{10}}\right)\left(1+e^{\gamma_{20}}\right)}+N\right)+\sum_{i=1}^{n}\left(b_i\right)\left(-\frac{e^{\gamma_{10}}\mathbf{x}_i^T}{\left(1+e^{\gamma_{10}}\right)}+\frac{e^{\gamma_{10}}\mathbf{x}_i^T}{\left(e^{\gamma_{10}}+\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)$$

$$+\sum_{i=1}^{n}\left(c_i\right)\left(-\frac{e^{\gamma_{10}}}{\left(1+e^{\gamma_{10}}\right)}\right)+\sum_{i=1}^{n}\left(d_i\right)\left(-\frac{e^{\gamma_{10}}}{\left(1+e^{\gamma_{10}}\right)}\right)$$

dimana $$N=\frac{\mathrm{e}^{\gamma_{10}}\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\right)}{e^{\gamma_{10}}\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\right)+e^{\gamma_{20}}\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)+\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2\,\mathrm{e}^{\beta_{20}}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)}$$

Turunan pertama fungsi I terhadap γ_{20}

$$\frac{\partial I}{\partial \gamma_{20}} = \sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(-\frac{e^{\gamma_{20}}}{\left(1+e^{\gamma_{20}}\right)\left(1+e^{\gamma_{10}}\right)}+\frac{e^{\gamma_{20}}\left(e^{\gamma_{10}}+\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\right)}{e^{\gamma_{10}}\left(\mathrm{e}^{\gamma_{20}}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\right)+e^{\gamma_{20}}\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)+N}\right)+\sum_{i=1}^{n}\left(b_i\right)\left(-\frac{e^{\gamma_{20}}}{\left(1+e^{\gamma_{20}}\right)}\right)$$

$$+\sum_{i=1}^{n}\left(c_i\right)\left(-\frac{e^{\gamma_{20}}}{\left(1+e^{\gamma_{20}}\right)}+\frac{e^{\gamma_{20}}}{\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n}\left(d_i\right)\left(-\frac{e^{\gamma_{20}}}{\left(1+e^{\gamma_{20}}\right)}\right)$$

dimana $N = \exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2\,\mathrm{e}^{\beta_{20}}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi I terhadap β_{10}

$$\frac{\partial I}{\partial \beta_{10}}=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\gamma_{20}}\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}+\frac{\left(e^{\beta_{10}}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}}\right)^2}\right)+\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{\left(e^{\beta_{10}}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}}\right)^2}\right)T\frac{\partial g_1}{\partial \beta_{10}}\right)\right)$$

$$+\sum_{i=1}^{n}\left(b_i\right)\left(\frac{\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{\left(e^{\beta_{10}}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}}\right)^2}\right)}{\left(e^{\gamma_1}+\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\frac{\partial g_1}{\partial \beta_{10}}\right)$$

$$+\sum_{i=1}^{n}\left(c_i\right)\left(y_{1i}\left(1-\frac{e^{\mathbf{x}_i^T\beta_{10}}\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}}\right)}\right)+\left(\frac{-e^{\beta_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}}}-\frac{\left(e^{\beta_{10}}\right)^2\left(1+\varphi_1 y_{1i}\right)\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}}\right)^2}\right)+\frac{1}{\left(e^{\gamma_2}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_1}{\partial \beta_{10}}\right)$$

$$+\sum_{i=1}^{n}\left(d_i\right)\left(y_{1i}-\frac{y_{1i}\varphi_1 e^{\beta_{10}}}{\left(1+\varphi_1 e^{\beta_{10}}\right)}+\left(\frac{-e^{\beta_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}}}\left(1-\frac{\varphi_1 e^{\beta_{10}}}{\left(1+\varphi_1 e^{\beta_{10}}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \beta_{10}}\right)$$

dimana $S=e^{\gamma_{10}}\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\right)+e^{\gamma_{20}}\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)+\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2\,\mathrm{e}^{\beta_{20}}}\right)T$

$\mathrm{T}=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi I terhadap β_{20}

$$\frac{\partial I}{\partial \beta_{20}}=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\gamma_{10}}\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}+\frac{\left(e^{\beta_{20}}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2}\right)+U\frac{\partial g_2}{\partial \beta_{20}}\right)\right)$$

$$+\sum_{i=1}^{n}\left(b_i\right)\left(y_{2i}-\frac{y_{2i}\varphi_2 e^{\beta_{20}}}{\left(1+\varphi_2 e^{\beta_{20}}\right)}+\left(\frac{-e^{\beta_{20}}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}}}-\frac{\left(e^{\beta_{20}}\right)^2\left(1+\varphi_2 y_{2i}\right)\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2}\right)+\frac{1}{\left(e^{\gamma_1}+\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\frac{\partial g_2}{\partial \beta_{20}}\right)$$

$$+\sum_{i=1}^{n}\left(c_i\right)\left(\frac{\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}-\frac{\left(e^{\beta_{20}}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2}\right)}{\left(e^{\gamma_2}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_2}{\partial \beta_{20}}\right)$$

$$+\sum_{i=1}^{n}\left(d_i\right)\left(y_{2i}-\frac{y_{2i}\varphi_2 e^{\beta_{20}}}{\left(1+\varphi_2 e^{\beta_{20}}\right)}+\left(\frac{-e^{\beta_{20}}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}}}\left(1-\frac{\varphi_2 e^{\beta_{20}}}{\left(1+\varphi_2 e^{\beta_{20}}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \beta_{20}}\right)$$

dimana $U=\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}-\frac{\left(e^{\beta_{20}}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$

Turunan pertama fungsi I terhadap φ_1

$$\frac{\partial I}{\partial \varphi_1} = \sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\gamma_{20}}\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\frac{-\left(e^{\beta_{10}}\right)^2}{\left(1+\varphi_1 e^{\beta_{10}}\right)^2}+\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\frac{\left(e^{\beta_{10}}\right)^2}{\left(1+\varphi_1 e^{\beta_{10}}\right)^2}T\,\frac{\partial g_1}{\partial \varphi_1}\right)\right)$$

$$+\sum_{i=1}^{n} b_i\left(\frac{-\exp\left(\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(e^{\beta_{10}}\right)^2}{\left(e^{\gamma_1}+\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\left(1+\varphi_1 e^{\beta_{10}}\right)^2}\frac{\partial g_1}{\partial \varphi_1}\right)$$

$$+\sum_{i=1}^{n} c_i\left(y_{1i}\frac{e^{\beta_{10}}}{\left(1+\varphi_1 e^{\beta_{10}}\right)}+\frac{\left(y_{1i}-1\right)}{\left(1+\varphi_1 y_1\right)}y_{1i}+\left(\frac{-e^{\beta_{10}}y_{1i}}{1+\varphi_1 e^{\beta_{10}}}+\frac{\left(e^{\beta_{10}}\right)^2\left(1+\varphi_1 y_1\right)}{\left(1+\varphi_1 e^{\beta_{10}}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)}{\left(e^{\gamma_2}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right)$$

$$+\sum_{i=1}^{n} d_i\left(\frac{-y_{1i}e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}+\frac{\left(y_{1i}-1\right)y_{1i}}{\left(1+\varphi_1 y_{1i}\right)}+\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\left(y_{1i}-\frac{e^{\beta_{10}}\left(1+\varphi_1 y_{1i}\right)}{\left(1+\varphi_1 e^{\beta_{10}}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right)$$

Turunan pertama fungsi I terhadap φ_2

$$\frac{\partial I}{\partial \varphi_2} = \sum_{i=1}^{n} \left(1 - b_i - c_i - d_i\right) \left(\frac{1}{S} \left(e^{\gamma_{10}} \exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}} \right) \frac{-\left(e^{\beta_{20}}\right)^2}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2} + \exp\left(-\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}} - \frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}} \right) \frac{\left(e^{\beta_{20}}\right)^2}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2} T \frac{\partial g_2}{\partial \varphi_2} \right) \right)$$

$$+\sum_{i=1}^{n} b_i \left(-y_{2i} \frac{e^{\beta_{20}}}{\left(1+\varphi_2 e^{\beta_{20}}\right)} + \frac{\left(y_{2i}-1\right)}{\left(1+\varphi_2 y_2\right)} y_{2i} + \left(\frac{-e^{\beta_{20}} y_{2i}}{1+\varphi_2 e^{\beta_{20}}} + \frac{(e^{\beta_{20}})^2 \left(1+\varphi_2 y_2\right)}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2} \right) + \frac{\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}} \right) 1+\eta\left(1-g_1\right)\left(e^{-y_{2i}} - g_2\right)}{\left(e^{\gamma_1} + \exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}} \right) 1+\eta\left(1-g_1\right)\left(e^{-y_{2i}} - g_2\right) \right)} \frac{\partial g_2}{\partial \varphi_2} \right)$$

$$+\sum_{i=1}^{n} c_i \left(\frac{-\exp\left(\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}} \right) \left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}} - g_1\right)\right) \left(e^{\beta_{20}}\right)^2}{\left(e^{\gamma_2} + \exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}} \right) \left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}} - g_1\right)\right) \right) \left(1+\varphi_2 e^{\beta_{20}}\right)^2} \frac{\partial g_2}{\partial \varphi_2} \right)$$

$$+\sum_{i=1}^{n} d_i \left(\frac{-y_{2i} e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}} + \frac{\left(y_{2i}-1\right) y_{2i}}{\left(1+\varphi_2 y_{2i}\right)} + \left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}} \left(y_{2i} - \frac{e^{\beta_{20}} \left(1+\varphi_2 y_{2i}\right)}{\left(1+\varphi_2 e^{\beta_{20}}\right)} \right) \right) + \frac{1}{\left(1+\eta\left(e^{-y_{1i}} - g_1\right)\left(e^{-y_{2i}} - g_2\right)\right)} \frac{\partial g_2}{\partial \varphi_2} \right)$$

Turunan pertama fungsi I terhadap η

$$\frac{\partial I}{\partial \eta}=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1-g_1\right)\left(1-g_2\right)\right)\right)+\sum_{i=1}^{n}b_i\left(\frac{\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{\left(e^{\gamma_{10}}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_1 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)$$

$$+\sum_{i=1}^{n}c_i\left(\frac{\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)}{\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n}d_i\left(\frac{\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}\right)$$

Lampiran 8: Fungsi *ln likelihood* d bawah H_0 $L(\hat{\omega})$ parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$ BZIGPR

$$\ln A_{1i}^{*} = \ln\left(\frac{\mathrm{e}^{\hat{\gamma}_{10}}}{1+\mathrm{e}^{\hat{\gamma}_{10}}}\frac{\mathrm{e}^{\hat{\gamma}_{20}}}{1+\mathrm{e}^{\hat{\gamma}_{20}}}+\frac{\mathrm{e}^{\hat{\gamma}_{10}}}{1+\mathrm{e}^{\hat{\gamma}_{10}}}\frac{1}{1+\mathrm{e}^{\hat{\gamma}_{20}}}\exp\left(\frac{-e^{\hat{\beta}_{20}}}{1+\varphi_2 e^{\hat{\beta}_{20}}}\right)+A_1+A_2\right)$$

$$= \ln\frac{1}{1+e^{\hat{\gamma}_{10}}}\frac{1}{1+e^{\hat{\gamma}_{20}}}+\ln\left(e^{\hat{\gamma}_{10}}e^{\hat{\gamma}_{20}}+e^{\hat{\gamma}_{10}}\exp\left(\frac{-e^{\hat{\beta}_{20}}}{1+\varphi_2 e^{\hat{\beta}_{20}}}\right)+e^{\hat{\gamma}_{2}}\exp\left(\frac{-e^{\hat{\beta}_{10}}}{1+\varphi_1 e^{\hat{\beta}_{10}}}\right)+A_3\right)$$

$$\text{dimana } A_1 = \frac{\mathrm{e}^{\hat{\gamma}_{20}}}{1+\mathrm{e}^{\hat{\gamma}_{20}}}\frac{1}{1+\mathrm{e}^{\hat{\gamma}_{10}}}\exp\left(\frac{-e^{\hat{\beta}_{10}}}{1+\varphi_1 e^{\hat{\beta}_{10}}}\right),\; A_2 = \frac{1}{1+\mathrm{e}^{\hat{\gamma}_{10}}}\frac{1}{1+\mathrm{e}^{\hat{\gamma}_{20}}}\exp\left(-\frac{e^{\hat{\beta}_{10}}}{1+\varphi_1 e^{\hat{\beta}_{10}}}-\frac{e^{\hat{\beta}_{20}}}{1+\varphi_2 e^{\hat{\beta}_{20}}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

$$A_3 = \exp\left(-\frac{e^{\hat{\beta}_{10}}}{1+\hat{\varphi}_1 e^{\hat{\beta}_{10}}}-\frac{e^{\hat{\beta}_{20}}}{1+\hat{\varphi}_2 e^{\hat{\beta}_{20}}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

$$\ln B_{1i}^{*} = \ln\left(\frac{1}{1+e^{\hat{\gamma}_{20}}}\left(\frac{e^{\hat{\beta}_{20}}}{1+\varphi_2 e^{\hat{\beta}_{20}}}\right)^{y_{2i}}\frac{\left(1+\varphi_2 y_{2i}\right)^{y_{2i}-1}}{y_2\,!}\exp\left(\frac{-e^{\hat{\beta}_{20}}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\hat{\beta}_{20}}}\right)\left(\frac{e^{\hat{\gamma}_{10}}}{1+e^{\hat{\gamma}_{10}}}+\frac{1}{1+e^{\hat{\gamma}_{10}}}\exp\left(\frac{-e^{\hat{\beta}_{10}}}{1+\varphi_1 e^{\hat{\beta}_{10}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\right)$$

$$= \ln\frac{1}{1+e^{\hat{\gamma}_{20}}}+y_{2i}\hat{\beta}_{20}-y_{2i}\ln\left(1+\varphi_2 e^{\hat{\beta}_{20}}\right)+\left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right)-\ln y_{2i}\,!+\left(\frac{-e^{\hat{\beta}_{20}}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\hat{\beta}_{20}}}\right)+\ln\frac{1}{1+e^{\hat{\gamma}_{10}}}+\ln B_2$$

$$\text{dimana } B_1 = \left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right),\; B_2 = \left(e^{\hat{\gamma}_{10}}+\exp\left(\frac{-e^{\hat{\beta}_{10}}}{1+\varphi_1 e^{\hat{\beta}_{10}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)$$

$$\ln C_{1i}^{*}=\ln\left(\frac{1}{1+e^{\hat{\gamma}_{10}}}\left(\frac{e^{\hat{\beta}_{10}}}{1+\varphi_1 e^{\hat{\beta}_{10}}}\right)^{y_{1i}}\frac{\left(1+\varphi_1 y_{1i}\right)^{y_{1i}-1}}{y_{1i}\,!}\exp\left(\frac{-e^{\hat{\beta}_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\hat{\beta}_{10}}}\right)\left(\frac{e^{\hat{\gamma}_{20}}}{1+e^{\hat{\gamma}_{20}}}+\frac{1}{1+e^{\hat{\gamma}_{20}}}\exp\left(\frac{-e^{\hat{\beta}_{20}}}{1+\varphi_2 e^{\hat{\beta}_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\right)$$

$$=\ln\frac{1}{1+e^{\hat{\gamma}_{10}}}+y_{1i}\ln e^{\hat{\beta}_{10}}-y_{1i}\ln\left(1+\varphi_1 e^{\hat{\beta}_{10}}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}\,!+\left(\frac{-e^{\hat{\beta}_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\hat{\beta}_{10}}}\right)+\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\gamma}_{20}}}+\ln C_3$$

$$\text{dimana }\; C_3=\left(e^{\hat{\gamma}_{20}}+\exp\left(\frac{-e^{\hat{\beta}_{20}}}{1+\varphi_2 e^{\hat{\beta}_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)$$

$$\ln D_{1i}^{*}=\ln\left(\frac{1}{1+e^{\hat{\gamma}_{10}}}\frac{1}{1+e^{\hat{\gamma}_{20}}}\prod_{l=1}^{2}\left(\frac{e^{\hat{\beta}_{l0}}}{1+\varphi_l e^{\hat{\beta}_{l0}}}\right)^{y_{li}}\frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}\,!}\exp\left(\frac{-e^{\hat{\beta}_{l0}}\left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\hat{\beta}_{l0}}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$=\ln\frac{1}{1+e^{\hat{\gamma}_{10}}}+\ln\frac{1}{1+e^{\hat{\gamma}_{20}}}+y_{1i}\hat{\beta}_{10}-y_{1i}\ln\left(1+\varphi_1 e^{\hat{\beta}_{10}}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}\,!+\left(\frac{-e^{\hat{\beta}_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\hat{\beta}_{10}}}\right)+D_1$$

$$\text{dimana }\; D_1=y_{2i}\hat{\beta}_{20}-y_{2i}\ln\left(1+\varphi_2 e^{\hat{\beta}_{20}}\right)+\left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right)-\ln y_{2i}\,!+\left(\frac{-e^{\hat{\beta}_{20}}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\hat{\beta}_{20}}}\right)+\ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

Lampiran 9: Fungsi *ln likelihood* d bawah H_0 $L(\omega)$ parameter $\boldsymbol{\gamma}$ BZIGPR

$$\ln A_{2i}=\ln\left(\frac{\mathrm{e}^{\gamma_{10}}}{1+\mathrm{e}^{\gamma_{10}}}\frac{\mathrm{e}^{\gamma_{20}}}{1+\mathrm{e}^{\gamma_{20}}}+\frac{\mathrm{e}^{\gamma_{10}}}{1+\mathrm{e}^{\gamma_{10}}}\frac{1}{1+\mathrm{e}^{\gamma_{20}}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)+\frac{\mathrm{e}^{\gamma_{20}}}{1+\mathrm{e}^{\gamma_{20}}}\frac{1}{1+\mathrm{e}^{\gamma_{10}}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+A_2\right)$$

$$=\ln\frac{1}{1+e^{\gamma_{10}}}\frac{1}{1+e^{\gamma_{20}}}+\ln\left(e^{\gamma_{10}}e^{\gamma_{20}}+e^{\gamma_{10}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)+e^{\gamma_{20}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)\right)$$

$$\text{dimana } A_2=\frac{1}{1+\mathrm{e}^{\gamma_{10}}}\frac{1}{1+\mathrm{e}^{\gamma_{20}}}\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

$$\ln B_{2i}=\ln\left(\frac{1}{1+e^{\gamma_{20}}}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)^{y_{2i}}\frac{\left(1+\varphi_2y_{2i}\right)^{y_{2i}-1}}{y_2!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\left(1+\varphi_2y_{2i}\right)}{1+\varphi_2e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(\frac{e^{\gamma_{10}}}{1+e^{\gamma_{10}}}+\frac{1}{1+e^{\gamma_{10}}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\right)$$

$$=\ln\frac{1}{1+e^{\gamma_{20}}}+y_{2i}\boldsymbol{\beta}_2-y_{2i}\ln\left(1+\varphi_2e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)+\left(y_{2i}-1\right)\ln\left(1+\varphi_2y_{2i}\right)-\ln y_{2i}!+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\left(1+\varphi_2y_{2i}\right)}{1+\varphi_2e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)+\ln\frac{1}{1+e^{\gamma_{10}}}+\ln B_1$$

$$\text{dimana } B_1=\left(e^{\gamma_{10}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)$$

$$\ln C_{2i}=\ln\left(\frac{1}{1+e^{\gamma_{10}}}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)^{y_{1i}}\frac{\left(1+\varphi_1y_{1i}\right)^{y_{1i}-1}}{y_{1i}!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\left(1+\varphi_1y_{1i}\right)}{1+\varphi_1e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(\frac{e^{\gamma_{20}}}{1+e^{\gamma_{20}}}+\frac{1}{1+e^{\gamma_{20}}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\right)$$

$$=\ln\frac{1}{1+e^{\gamma_{10}}}+y_{1i}\ln e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}-y_{1i}\ln\left(1+\varphi_1e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1y_{1i}\right)-\ln y_{1i}!+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\left(1+\varphi_1y_{1i}\right)}{1+\varphi_1e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+\ln\frac{1}{1+e^{\gamma_{20}}}+\ln\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)C_1\right)$$

$$\text{dimana } C=\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

$$\ln D_{2i} = \ln\left(\frac{1}{1+e^{\gamma_{10}}}\frac{1}{1+e^{\gamma_{20}}}\prod_{l=1}^{2}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}\right)^{y_{li}}\frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}\,!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}\left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$= \ln\frac{1}{1+e^{\gamma_{10}}}+\ln\frac{1}{1+e^{\gamma_{20}}}+y_{1i}\mathbf{x}_i^T\boldsymbol{\beta}_1-y_{1i}\ln\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}\,!+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+D_1$$

$$\text{dimana } D_1 = y_{2i}\mathbf{x}_i^T\boldsymbol{\beta}_2-y_{2i}\ln\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)+\left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right)-\ln y_{2i}\,!+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)+\ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

Lampiran 10: Turunan pertama fungsi *ln likelihood* d bawah H_0 $L(\omega)$ parameter $\boldsymbol{\gamma}$ BZIGPR

Turunan pertama fungsi J terhadap γ_{10}

$$\frac{\partial J}{\partial \gamma_{10}} = \sum_{i=1}^{n} \left(1-b_i-c_i-d_i\right)\left(-\frac{e^{\gamma_{10}}}{\left(1+e^{\gamma_{10}}\right)\left(1+e^{\gamma_{20}}\right)}+N\right)$$

$$+\sum_{i=1}^{n} b_i \left(-\frac{e^{\gamma_{10}}}{\left(1+e^{\gamma_{10}}\right)}+\frac{e^{\gamma_{10}}}{\left(e^{\gamma_{10}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)M\right)}\right)+\sum_{i=1}^{n} c_i\left(-\frac{e^{\gamma_{10}}}{\left(1+e^{\gamma_{10}}\right)}\right)+\sum_{i=1}^{n} d_i\left(-\frac{e^{\gamma_{10}}}{\left(1+e^{\gamma_{10}}\right)}\right)$$

dimana $T=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$, $M=\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$

$$N=\frac{e^{\gamma_{10}}\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\right)\mathbf{x}_i^T}{e^{\gamma_{10}}\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\right)+e^{\gamma_{20}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)T}$$

Turunan pertama fungsi J terhadap γ_{20}

$$\frac{\partial J}{\partial \gamma_{20}} = \sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(-\frac{e^{\gamma_{20}}}{\left(1+e^{\gamma_{20}}\right)\left(1+e^{\gamma_{10}}\right)}+\frac{e^{\gamma_{20}}\left(e^{\gamma_{10}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\right)}{e^{\gamma_{10}}\left(\mathrm{e}^{\gamma_{20}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\right)+e^{\gamma_{20}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2\,\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)}\right)$$

$$+\sum_{i=1}^{n} b_i\left(-\frac{e^{\gamma_{20}}}{\left(1+e^{\gamma_{20}}\right)}\right)+\sum_{i=1}^{n} c_i\left(-\frac{e^{\gamma_{20}}}{\left(1+e^{\gamma_{20}}\right)}+\frac{e^{\gamma_{20}}}{\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n} d_i\left(-\frac{e^{\gamma_{20}}}{\left(1+e^{\gamma_{20}}\right)}\right)$$

Turunan pertama fungsi J terhadap $\boldsymbol{\beta}_2$

$$
\begin{aligned}
\frac{\partial J}{\partial \boldsymbol{\beta}_2} = {} & \sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\gamma_{10}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}\right)T\,\frac{\partial g_2}{\partial \boldsymbol{\beta}_2}\right)\right) \\
& +\sum_{i=1}^{n} b_i\left(y_{2i}-\frac{y_{2i}\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2\left(1+\varphi_2 y_{2i}\right)\varphi_2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}\right)+\frac{1}{\left(e^{\gamma_{10}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\frac{\partial g_2}{\partial \boldsymbol{\beta}_2}\right) \\
& +\sum_{i=1}^{n} c_i\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)^2}\right)}{\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_2}{\partial \boldsymbol{\beta}_2}\right) \\
& +\sum_{i=1}^{n} d_i\left(y_{2i}-\frac{y_{2i}\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\left(1-\frac{\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \beta_2}\right)
\end{aligned}
$$

Dimana $T=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi J terhadap $\boldsymbol{\beta}_1$

$$
\begin{aligned}
\frac{\partial J}{\partial \boldsymbol{\beta}_1} = {} & \sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\gamma_{20}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)T\,\frac{\partial g_1}{\partial \boldsymbol{\beta}_1}\right)\right) \\
& +\sum_{i=1}^{n} b_i\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)}{\left(e^{\gamma_{10}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\frac{\partial g_1}{\partial \boldsymbol{\beta}_1}\right) \\
& +\sum_{i=1}^{n} c_i\left(y_{1i}\left(1-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\varphi_1}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)}\right)+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\left(1+\varphi_1 y_{1i}\right)\varphi_1}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)+\frac{1}{\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_1}{\partial \boldsymbol{\beta}_1}\right) \\
& +\sum_{i=1}^{n} d_i\left(y_{1i}\mathbf{x}_i^T-\frac{y_{1i}\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\left(1-\frac{\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \boldsymbol{\beta}_1}\right)
\end{aligned}
$$

dimana

$$
S=e^{\gamma_{10}}\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\right)+e^{\gamma_{20}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2\,\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)
$$

Turunan pertama fungsi J terhadap φ_1

$$
\begin{aligned}
\frac{\partial J}{\partial \varphi_1} &= \sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\gamma_{20}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\frac{\left(-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)\frac{\partial g_1}{\partial \varphi_1}\right)\right) \\
&\quad +\sum_{i=1}^{n} b_i\left(\frac{-\exp\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}{\left(e^{\gamma_{10}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\frac{\partial g_1}{\partial \varphi_1}\right) \\
&\quad +\sum_{i=1}^{n} c_i\left(y_{1i}\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)}+\frac{\left(y_{1i}-1\right)}{\left(1+\varphi_1 y_1\right)}y_{1i}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}y_{1i}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2\left(1+\varphi_1 y_1\right)}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)}{\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right) \\
&\quad +\sum_{i=1}^{n} d_i\left(\frac{-y_{1i}e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}+\frac{\left(y_{1i}-1\right)y_{1i}}{\left(1+\varphi_1 y_{1i}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\left(y_{1i}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\left(1+\varphi_1 y_{1i}\right)}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right)
\end{aligned}
$$

Turunan pertama fungsi J terhadap φ_2

$$
\begin{aligned}
\frac{\partial J}{\partial \varphi_2} = & \sum_{i=1}^{n} \left(1 - b_i - c_i - d_i\right) \left(\frac{1}{S} \left(e^{\gamma_{10}} \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} \right) \frac{\left(-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2} + \exp\left(-\frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} - \frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}} \right) \frac{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2} \left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right) \frac{\partial g_2}{\partial \varphi_2} \right) \right) \\
& + \sum_{i=1}^{n} b_i \left(-y_{2i} \frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)} + \frac{\left(y_{2i}-1\right)}{\left(1+\varphi_2 y_2\right)} y_{2i} + \left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} y_{2i}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} + \frac{(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2})^2 \left(1+\varphi_2 y_2\right)}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2} \right) + \frac{\exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} \right) 1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{\left(e^{\gamma_1} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1}} \right) 1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right) \right)} \frac{\partial g_2}{\partial \varphi_2} \right) \\
& + \sum_{i=1}^{n} c_i \left(\frac{-\exp\left(\frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} \right) \left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right) \left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2}{\left(e^{\gamma_{20}} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} \right) \left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right) \right) \left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)^2} \frac{\partial g_2}{\partial \varphi_2} \right) \\
& + \sum_{i=1}^{n} d_i \left(\frac{-y_{2i} e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} + \frac{\left(y_{2i}-1\right) y_{2i}}{\left(1+\varphi_2 y_{2i}\right)} + \left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}} \left(y_{2i} - \frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_2} \left(1+\varphi_2 y_{2i}\right)}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2}\right)} \right) \right) + \frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)} \frac{\partial g_2}{\partial \varphi_2} \right)
\end{aligned}
$$

Turunan pertama fungsi J terhadap η

$$\frac{\partial J}{\partial \eta}=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1-g_1\right)\left(1-g_2\right)\right)\right)+\sum_{i=1}^{n}b_i\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{\left(e^{\gamma_{10}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)$$

$$+\sum_{i=1}^{n}c_i\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)}{\left(e^{\gamma_{20}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n}d_i\left(\frac{\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}\right)$$

Lampiran 11: Fungsi *ln likelihood* d bawah H_0 $L\left(\hat{\omega}\right)$ parameter $\boldsymbol{\gamma}$ BZIGPR

$$\ln A_{2i}^{*}=\ln\left(\frac{\mathrm{e}^{\hat{\gamma}_{10}}}{1+\mathrm{e}^{\hat{\gamma}_{10}}}\frac{\mathrm{e}^{\hat{\gamma}_{20}}}{1+\mathrm{e}^{\hat{\gamma}_{20}}}+\frac{\mathrm{e}^{\hat{\gamma}_{10}}}{1+\mathrm{e}^{\hat{\gamma}_{10}}}\frac{1}{1+\mathrm{e}^{\hat{\gamma}_{20}}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}{1+\varphi_2 e^{\hat{\boldsymbol{\beta}}_2}}\right)+\frac{\mathrm{e}^{\hat{\gamma}_{20}}}{1+\mathrm{e}^{\hat{\gamma}_{20}}}\frac{1}{1+\mathrm{e}^{\hat{\gamma}_{10}}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right)+\frac{1}{1+\mathrm{e}^{\hat{\gamma}_{10}}}\frac{1}{1+\mathrm{e}^{\hat{\gamma}_{20}}}\exp\left(-\frac{e^{\mathbf{x}_i\hat{\boldsymbol{\beta}}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}-\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)T\right)$$

$$=\ln\frac{1}{1+e^{\hat{\gamma}_{10}}}\frac{1}{1+e^{\hat{\gamma}_{20}}}+\ln\left(e^{\hat{\gamma}_{10}}e^{\hat{\gamma}_{20}}+e^{\hat{\gamma}_{10}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}{1+\varphi_2 e^{\hat{\boldsymbol{\beta}}_2}}\right)+e^{\hat{\gamma}_{20}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i\hat{\boldsymbol{\beta}}_1}}{1+\hat{\varphi}_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}-\frac{e^{\mathbf{x}_i\hat{\boldsymbol{\beta}}_2}}{1+\hat{\varphi}_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)\right)$$

$$\ln B_{2i}^{*}=\ln\left(\frac{1}{1+e^{\hat{\gamma}_{20}}}\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)^{y_{2i}}\frac{\left(1+\varphi_2 y_{2i}\right)^{y_{2i}-1}}{y_2\,!}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)\left(\frac{e^{\hat{\gamma}_{10}}}{1+e^{\hat{\gamma}_{10}}}+\frac{1}{1+e^{\hat{\gamma}_{10}}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\right)$$

$$=\ln\frac{1}{1+e^{\hat{\gamma}_{20}}}+y_{2i}\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2-y_{2i}\ln\left(1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}\right)+\left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right)-\ln y_{2i}\,!+\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)+\ln\frac{1}{1+e^{\hat{\gamma}_{10}}}+\ln\left(e^{\hat{\gamma}_{10}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right)B_1\right)$$

dimana $B_1=\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$

$$\ln C_{2i}^{*}=\ln\left(\frac{1}{1+e^{\hat{\gamma}_{10}}}\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right)^{y_{1i}}\frac{\left(1+\varphi_1 y_{1i}\right)^{y_{1i}-1}}{y_{1i}\,!}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right)\left(\frac{e^{\hat{\gamma}_{20}}}{1+e^{\hat{\gamma}_{20}}}+\frac{1}{1+e^{\hat{\gamma}_{20}}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\right)$$

$$=\ln\frac{1}{1+e^{\hat{\gamma}_{10}}}+y_{1i}\ln e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}-y_{1i}\ln\left(1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}\,!+\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1}}\right)+\ln\frac{1}{1+e^{\hat{\boldsymbol{\gamma}}_2}}+\ln C\left(e^{\hat{\gamma}_{20}}+\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}{1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)C_1\right)$$

dimana $C_1=\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)$

$$\ln D_{2i}^{*} = \ln\left(\frac{1}{1+e^{\hat{\gamma}_{10}}} \frac{1}{1+e^{\hat{\gamma}_{20}}} \prod_{l=1}^{2} \left(\frac{e^{\mathbf{x}_i^T \hat{\boldsymbol{\beta}}_l}}{1+\varphi_l e^{\mathbf{x}_i^T \hat{\boldsymbol{\beta}}_l}} \right)^{y_{li}} \frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}\,!} \exp\left(\frac{-e^{\mathbf{x}_i^T \hat{\boldsymbol{\beta}}_l} \left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\mathbf{x}_i^T \hat{\boldsymbol{\beta}}_l}} \right) \left[1+\eta \prod_{l=1}^{2} \left(e^{-y_{li}} - g_l \right) \right] \right)$$

$$= \ln \frac{1}{1+e^{\hat{\gamma}_{10}}} + \ln \frac{1}{1+e^{\hat{\gamma}_{20}}} + y_{1i} \mathbf{x}_i^T \hat{\boldsymbol{\beta}}_1 - y_{1i} \ln\left(1+\varphi_1 e^{\mathbf{x}_i^T \hat{\boldsymbol{\beta}}_1}\right) + \left(y_{1i}-1\right) \ln\left(1+\varphi_1 y_{1i}\right) - \ln y_{1i}\,! + \left(\frac{-e^{\mathbf{x}_i^T \hat{\boldsymbol{\beta}}_1} \left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T \hat{\boldsymbol{\beta}}_1}} \right) + D_1$$

$$\text{dimana } D_1 = y_{2i} \mathbf{x}_i^T \hat{\boldsymbol{\beta}}_2 - y_{2i} \ln\left(1+\varphi_2 e^{\mathbf{x}_i^T \hat{\boldsymbol{\beta}}_2}\right) + \left(y_{2i}-1\right) \ln\left(1+\varphi_2 y_{2i}\right) - \ln y_{2i}\,! + \left(\frac{-e^{\mathbf{x}_i^T \hat{\boldsymbol{\beta}}_2} \left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\mathbf{x}_i^T \hat{\boldsymbol{\beta}}_2}} \right) + \ln\left(1+\eta\left(e^{-y_{1i}} - g_1\right)\left(e^{-y_{2i}} - g_2\right)\right)$$

Lampiran 12: Fungsi *ln likelihood* d bawah H_0 $L(\omega)$ parameter $\boldsymbol{\beta}$ BZIGPR

$$\ln A_{3i} = \ln\left(\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}} \frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}} + \frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}} \frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}} \exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}} \right) + A_1 + A_2 \right)$$

$$= \ln \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}} \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}} + \ln\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1} e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2} + e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1} \exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}} \right) + e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2} \exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}} \right) + A_3 \right)$$

dimana

$$A_1 = \frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}} \frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}} \exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}} \right),\ A_2 = \frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}} \frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}} \exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}} - \frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}} \right) \left(1+\eta(1-g_1)(1-g_2)\right)$$

$$A_3 = \exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}} - \frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}} \right) \left(1+\eta(1-g_1)(1-g_2)\right)$$

$$\ln B_{3i} = \ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}} \left(\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}} \right)^{y_{2i}} \frac{(1+\varphi_2 y_{2i})^{y_{2i}-1}}{y_2 !} \exp\left(\frac{-e^{\beta_{20}}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\beta_{20}}} \right) \left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}} + \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}} \exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}} \right) \left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right) \right) \right)$$

$$= \ln \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}} + y_{2i}\beta_{20} - y_{2i}\ln\left(1+\varphi_2 e^{\beta_{20}}\right) + (y_{2i}-1)\ln(1+\varphi_2 y_{2i}) - \ln y_{2i}! + \left(\frac{-e^{\beta_{20}}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\beta_{20}}} \right) + \ln \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}} + \ln e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1} + \exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}} \right) B_1$$

dimana $B_1 = \left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)$

$$\ln C_{3i} = \ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\left(\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)^{y_{1i}}\frac{\left(1+\varphi_1 y_{1i}\right)^{y_{1i}-1}}{y_{1i}\,!}\exp\left(\frac{-e^{\beta_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}}}\right)\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}+\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)C_1\right)\right)$$

$$= \ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}} + y_{1i}\ln e^{\beta_{10}} - y_{1i}\ln\left(1+\varphi_1 e^{\beta_{10}}\right) + \left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right) - \ln y_{1i}\,! + \left(\frac{-e^{\beta_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}}}\right) + \ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}} + \ln\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2} + \exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)C_1\right)$$

$$\text{dimana}\, C_1 = \left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)$$

$$\ln D_{3i} = \ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}\prod_{l=1}^{2}\left(\frac{e^{\beta_{l0}}}{1+\varphi_l e^{\beta_{l0}}}\right)^{y_{li}}\frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}\,!}\exp\left(\frac{-e^{\beta_{l0}}\left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\beta_{l0}}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$= \ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}} + \ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}} + y_{1i}\beta_{10} - y_{1i}\ln\left(1+\varphi_1 e^{\beta_{10}}\right) + \left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right) - \ln y_{1i}\,! + \left(\frac{-e^{\beta_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}}}\right) + D_1$$

$$\text{dimana}\;\; D_1 = y_{2i}\beta_{20} - y_{2i}\ln\left(1+\varphi_2 e^{\beta_{20}}\right) + \left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right) - \ln y_{2i}\,! + \left(\frac{-e^{\beta_{20}}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}}}\right) + \ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

Lampiran 13: Turunan pertama fungsi *ln likelihood* d bawah H_0 $L(\omega)$ parameter $\boldsymbol{\beta}$ BZIGPR

Turunan pertama fungsi K terhadap $\boldsymbol{\gamma}_1$

$$\frac{\partial K}{\partial \boldsymbol{\gamma}_1}=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(-\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\right)}+N\right)+\sum_{i=1}^{n}b_i\left(-\frac{e^{\boldsymbol{\gamma}_1}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\mathbf{x}_i^T}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)$$

$$+\sum_{i=1}^{n}c_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)}\right)+\sum_{i=1}^{n}d_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)}\right)$$

dimana

$$N=\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\right)}{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)+\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2\,\mathrm{e}^{\beta_{20}}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)}$$

Turunan pertama fungsi K terhadap $\boldsymbol{\gamma}_2$

$$\frac{\partial K}{\partial \boldsymbol{\gamma}_2}=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\right)\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\right)}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\right)}{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\left(\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)+N}\right)$$

$$+\sum_{i=1}^{n}b_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\right)}\right)+\sum_{i=1}^{n}c_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\right)}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n}d_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\right)}\right)$$

dimana $N=\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2\,\mathrm{e}^{\beta_{20}}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi K terhadap β_{10}

$$\frac{\partial K}{\partial \beta_{10}}=\sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}+\frac{\left(e^{\beta_{10}}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}}\right)^2}\right)+\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{\left(e^{\beta_{10}}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}}\right)^2}\right)T\,\frac{\partial g_1}{\partial \beta_{10}}\right)\right)$$

$$+\sum_{i=1}^{n}b_i\left(\frac{\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{\left(e^{\beta_{10}}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}}\right)^2}\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\frac{\partial g_1}{\partial \beta_{10}}\right)$$

$$+\sum_{i=1}^{n}c_i\left(y_{1i}\left(1-\frac{e^{\beta_{10}}\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}}\right)}\right)+\left(\frac{-e^{\beta_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}}}-\frac{\left(e^{\beta_{10}}\right)^2\left(1+\varphi_1 y_{1i}\right)\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}}\right)^2}\right)+\frac{1}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_1}{\partial \beta_{10}}\right)$$

$$+\sum_{i=1}^{n}d_i\left(y_{1i}-\frac{y_{1i}\varphi_1 e^{\beta_{10}}}{\left(1+\varphi_1 e^{\beta_{10}}\right)}+\left(\frac{-e^{\beta_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}}}\left(1-\frac{\varphi_1 e^{\beta_{10}}}{\left(1+\varphi_1 e^{\beta_{10}}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \beta_{10}}\right)$$

dimana $S=e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)+\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2\,\mathrm{e}^{\beta_{20}}}\right)T$

$\mathrm{T}=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi K terhadap β_{20}

$$\frac{\partial K}{\partial \beta_{20}} = \sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}+\frac{\left(e^{\beta_{20}}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2}\right)+U\,\frac{\partial g_2}{\partial \beta_{20}}\right)\right)$$

$$+\sum_{i=1}^{n} b_i\left(y_{2i}-\frac{y_{2i}\varphi_2 e^{\beta_{20}}}{\left(1+\varphi_2 e^{\beta_{20}}\right)}+\left(\frac{-e^{\beta_{20}}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}}}-\frac{\left(e^{\beta_{20}}\right)^2\left(1+\varphi_2 y_{2i}\right)\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2}\right)+\frac{1}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \beta_{20}}\right)$$

$$+\sum_{i=1}^{n} c_i\left(\frac{\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}-\frac{\left(e^{\beta_{20}}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2}\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_2}{\partial \beta_{20}}\right)$$

$$+\sum_{i=1}^{n} d_i\left(y_{2i}-\frac{y_{2i}\varphi_2 e^{\beta_{20}}}{\left(1+\varphi_2 e^{\beta_{20}}\right)}+\left(\frac{-e^{\beta_{20}}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}}}\left(1-\frac{\varphi_2 e^{\beta_{20}}}{\left(1+\varphi_2 e^{\beta_{20}}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \beta_{20}}\right)$$

dimana $U=\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}-\frac{\left(e^{\beta_{20}}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$

Turunan pertama fungsi K terhadap φ_1

$$\frac{\partial K}{\partial \varphi_1} = \sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\frac{-\left(e^{\beta_{10}}\right)^2}{\left(1+\varphi_1 e^{\beta_{10}}\right)^2}+\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\frac{\left(e^{\beta_{10}}\right)^2}{\left(1+\varphi_1 e^{(\beta_{10})}\right)^2}T\,\frac{\partial g_1}{\partial \varphi_1}\right)\right)$$

$$+\sum_{i=1}^{n} b_i\left(\frac{-\exp\left(\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(e^{\beta_{10}}\right)^2}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\left(1+\varphi_1 e^{\beta_{10}}\right)^2}\frac{\partial g_1}{\partial \varphi_1}\right)$$

$$+\sum_{i=1}^{n} c_i\left(y_{1i}\frac{e^{\beta_{10}}}{\left(1+\varphi_1 e^{\beta_{10}}\right)}+\frac{\left(y_{1i}-1\right)}{\left(1+\varphi_1 y_1\right)}y_{1i}+\left(\frac{-e^{\beta_{10}}y_{1i}}{1+\varphi_1 e^{\beta_{10}}}+\frac{\left(e^{\beta_{10}}\right)^2\left(1+\varphi_1 y_1\right)}{\left(1+\varphi_1 e^{\beta_{10}}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right)$$

$$+\sum_{i=1}^{n} d_i\left(\frac{-y_{1i}e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}+\frac{\left(y_{1i}-1\right)y_{1i}}{\left(1+\varphi_1 y_{1i}\right)}+\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\left(y_{1i}-\frac{e^{\beta_{10}}\left(1+\varphi_1 y_{1i}\right)}{\left(1+\varphi_1 e^{\beta_{10}}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right)$$

Turunan pertama fungsi K terhadap φ_2

$$
\begin{aligned}
\frac{\partial K}{\partial \varphi_2} &= \sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\frac{-\left(e^{\beta_{20}}\right)^2}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2}+\exp\left(-\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\frac{\left(e^{\beta_{20}}\right)^2}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2}T\,\frac{\partial g_2}{\partial \varphi_2}\right)\right) \\
&+\sum_{i=1}^{n} b_i\left(-y_{2i}\frac{e^{\beta_{20}}}{\left(1+\varphi_2 e^{\beta_{20}}\right)}+\frac{\left(y_{2i}-1\right)}{\left(1+\varphi_2 y_2\right)}y_{2i}+\left(\frac{-e^{\beta_{20}}y_{2i}}{1+\varphi_2 e^{\beta_{20}}}+\frac{(e^{\beta_{20}})^2\left(1+\varphi_2 y_2\right)}{\left(1+\varphi_2 e^{\beta_{20}}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1}+\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \varphi_2}\right) \\
&+\sum_{i=1}^{n} c_i\left(\frac{-\exp\left(\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(e^{\beta_{20}}\right)^2}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\left(1+\varphi_2 e^{\beta_{20}}\right)^2}\frac{\partial g_2}{\partial \varphi_2}\right) \\
&+\sum_{i=1}^{n} d_i\left(\frac{-y_{2i}e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}+\frac{\left(y_{2i}-1\right)y_{2i}}{\left(1+\varphi_2 y_{2i}\right)}+\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\left(y_{2i}-\frac{e^{\beta_{20}}\left(1+\varphi_2 y_{2i}\right)}{\left(1+\varphi_2 e^{\beta_{20}}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \varphi_2}\right)
\end{aligned}
$$

Turunan pertama fungsi K terhadap η

$$\frac{\partial K}{\partial \eta} = \sum_{i=1}^{n}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(\exp\left(-\frac{e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}-\frac{e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1-g_1\right)\left(1-g_2\right)\right)\right)+\sum_{i=1}^{n} b_i\left(\frac{\exp\left(\frac{-e^{\beta_{10}}}{1+\varphi_1 e^{\beta_{10}}}\right)\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{\left(e^{\mathbf{x}_i^T\gamma_1}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_1 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)$$

$$+\sum_{i=1}^{n} c_i\left(\frac{\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)}{\left(e^{\mathbf{x}_i^T\gamma_2}+\exp\left(\frac{-e^{\beta_{20}}}{1+\varphi_2 e^{\beta_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n} d_i\left(\frac{\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}\right)$$

Lampiran 14: Fungsi *ln likelihood* d bawah H_0 $L(\hat{\omega})$ parameter $\boldsymbol{\beta}$ BZIGPR

$$\ln A_{3i}^{*} = \ln\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}} \frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}} + \frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}} \frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}} \exp\left(\frac{-e^{\hat{\beta}_{20}}}{1+\varphi_2 e^{\hat{\beta}_{20}}} \right) + A_1 + A_2 \right)$$

$$= \ln \frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}} \frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}} + \ln\left(e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1} e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2} + e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1} \exp\left(\frac{-e^{\hat{\beta}_{20}}}{1+\varphi_2 e^{\hat{\beta}_{20}}} \right) + e^{\hat{\boldsymbol{\gamma}}_2} \exp\left(\frac{-e^{\hat{\beta}_{10}}}{1+\varphi_1 e^{\hat{\beta}_{10}}} \right) + A_3 \right)$$

dimana $A_1 = \frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}} \frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}} \exp\left(\frac{-e^{\hat{\beta}_{10}}}{1+\varphi_1 e^{\hat{\beta}_{10}}} \right)$ $A_2 = \frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}} \frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}} \exp\left(-\frac{e^{\hat{\beta}_{10}}}{1+\varphi_1 e^{\hat{\beta}_{10}}} - \frac{e^{\hat{\beta}_{20}}}{1+\varphi_2 e^{\hat{\beta}_{20}}} \right) \left(1+\eta(1-g_1)(1-g_2)\right)$

$$A_3 = \exp\left(-\frac{e^{\hat{\beta}_{10}}}{1+\hat{\varphi}_1 e^{\hat{\beta}_{10}}} - \frac{e^{\hat{\beta}_{20}}}{1+\hat{\varphi}_2 e^{\hat{\beta}_{20}}} \right) \left(1+\eta(1-g_1)(1-g_2)\right)$$

$$\ln B_{3i}^{*} = \ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}} \left(\frac{e^{\hat{\beta}_{20}}}{1+\varphi_2 e^{\hat{\beta}_{20}}} \right)^{y_{2i}} \frac{(1+\varphi_2 y_{2i})^{y_{2i}-1}}{y_2\,!} \exp\left(\frac{-e^{\hat{\beta}_{20}}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\hat{\beta}_{20}}} \right) \left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}} + \frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}} \exp\left(\frac{-e^{\hat{\beta}_{10}}}{1+\varphi_1 e^{\hat{\beta}_{10}}} \right) \left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right) \right) \right)$$

$$= \ln \frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}} + y_{2i}\hat{\beta}_{20} - y_{2i} \ln\left(1+\varphi_2 e^{\hat{\beta}_{20}}\right) + (y_{2i}-1)\ln(1+\varphi_2 y_{2i}) - \ln y_{2i}\,! + \left(\frac{-e^{\hat{\beta}_{20}}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\hat{\beta}_{20}}} \right) + \ln \frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}} + \ln B_2$$

dimana $B_1 = \left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)$, $B_2 = \left(e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1} + \exp\left(\frac{-e^{\hat{\beta}_{10}}}{1+\varphi_1 e^{\hat{\beta}_{10}}} \right) \left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right) \right)$

$$\ln C_{3i}^{*}=\ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}\left(\frac{e^{\hat{\beta}_{10}}}{1+\varphi_1 e^{\hat{\beta}_{10}}}\right)^{y_{1i}}\frac{\left(1+\varphi_1 y_{1i}\right)^{y_{1i}-1}}{y_{1i}!}\exp\left(\frac{-e^{\hat{\beta}_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\hat{\beta}_{10}}}\right)\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}+\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}\exp\left(\frac{-e^{\hat{\beta}_{20}}}{1+\varphi_2 e^{\hat{\beta}_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\right)$$

$$=\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}+y_{1i}\ln e^{\hat{\beta}_{10}}-y_{1i}\ln\left(1+\varphi_1 e^{\hat{\beta}_{10}}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}!+\left(\frac{-e^{\hat{\beta}_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\hat{\beta}_{10}}}\right)+\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}+\ln C_3$$

dimana $C_3=\left(e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}+\exp\left(\frac{-e^{\hat{\beta}_{20}}}{1+\varphi_2 e^{\hat{\beta}_{20}}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)$

$$\ln D_{3i}^{*}=\ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}\prod_{l=1}^{2}\left(\frac{e^{\hat{\beta}_{l0}}}{1+\varphi_l e^{\hat{\beta}_{l0}}}\right)^{y_{li}}\frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}!}\exp\left(\frac{-e^{\hat{\beta}_{l0}}\left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\hat{\beta}_{l0}}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$=\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1}}+\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2}}+y_{1i}\hat{\beta}_{10}-y_{1i}\ln\left(1+\varphi_1 e^{\hat{\beta}_{10}}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}!+\left(\frac{-e^{\hat{\beta}_{10}}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\hat{\beta}_{10}}}\right)+D_1$$

dimana

$$D_1=y_{2i}\hat{\beta}_{20}-y_{2i}\ln\left(1+\varphi_2 e^{\hat{\beta}_{20}}\right)+\left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right)-\ln y_{2i}!+\left(\frac{-e^{\hat{\beta}_{20}}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\hat{\beta}_{20}}}\right)+\ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

Lampiran 15: Fungsi *ln likelihood* d bawah populasi $L(\Omega)$ GWBZIGPR

$$\ln A_i = \ln\left(\frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}} \frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}} + \frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}} \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}} \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}} \right) + A_1 + A_2 \right)$$

$$= \ln \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}} \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}} + \ln\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})} e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})} + e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})} \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}} \right) + e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})} \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}} \right) + A_3 \right)$$

$$\text{dimana } A_1 = \frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}} \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}} \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}} \right)$$

$$A_2 = \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}} \frac{1}{1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}} \exp\left(-\frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}} - \frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}} \right) \left(1+\eta(1-g_1)(1-g_2)\right)$$

$$\mathrm{A}_3 = \exp\left(-\frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}} - \frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}} \right) \left(1+\eta(1-g_1)(1-g_2)\right)$$

$$\ln B_i = \ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}} \left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)^{y_{2i}} \frac{\left(1+\varphi_2 y_{2i}\right)^{y_{2i}-1}}{y_2\,!} \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}} + \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}} \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)B_1\right)\right)$$

$$= \ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}} + y_{2i}\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right) - y_{2i}\ln\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right) + \left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right) - \ln y_{2i}\,! + \left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right) + \ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}} + \ln B_2$$

$$\text{dimana } B_2 = \left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)} + \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)B_1\right) \quad B_1 = \left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

$$\ln C_i = \ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}} \left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)^{y_{1i}} \frac{\left(1+\varphi_1 y_{1i}\right)^{y_{1i}-1}}{y_{1i}\,!} \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}} + \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}} \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)C_1\right)\right)$$

$$= \ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}} + y_{1i}\ln e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)} - y_{1i}\ln\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right) + \left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right) - \ln y_{1i}\,! + \left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right) + \ln C_2 + \ln C_3$$

$$\text{dimana } C_1 = \left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right), C_2 = \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}},$$

$$C_3 = \left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)} + \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)C_1\right)$$

$$\ln D_i = \ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}}\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}}\prod_{l=1}^{2}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}}\right)^{y_{li}}\frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}\,!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$= \ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}}+\ln\frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}}+y_{1i}\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)-y_{1i}\ln\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}\,!+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)+D_1$$

dimana

$$D_1 = y_{2i}\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)-y_{2i}\ln\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)+\left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right)-\ln y_{2i}\,!+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)+\ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

Lampiran 16: Turunan pertama fungsi *ln likelihood* d bawah populasi $L(\Omega)$ GWBZIGPR

Misalkan $\ln A_i = A$, $\ln B_i = B$, $\ln C_i = C$ dan $\ln D_i = D$

Turunan pertama A terhadap terhadap $\boldsymbol{\gamma}_1(u_i, v_i)$ adalah

$$\frac{\partial A}{\partial \boldsymbol{\gamma}_1(u_{i^*}, v_{i^*})} = -\frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\right)\left(1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}\right)} + \frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)\right)\mathbf{x}_i^T}{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)\right) + e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}\exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}\right) + E}$$

$$\text{dimana} \quad E = \exp\left(-\frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}} - \frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2\, e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)\left(1+\eta(1-g_1)(1-g_2)\right)$$

Turunan pertama A terhadap $\boldsymbol{\gamma}_2(u_i, v_i)$ adalah

$$\frac{\partial A}{\partial \boldsymbol{\gamma}_2(u_{i^*}, v_{i^*})} = -\frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}\right)\left(1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\right)} + \frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}\right)\right)\mathbf{x}_i^T}{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)\right) + e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}\exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}\right) + F}$$

$$\text{dimana} \quad F = \exp\left(-\frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}} - \frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2\, e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)\left(1+\eta(1-g_1)(1-g_2)\right)$$

Turunan pertama A terhadap $\boldsymbol{\beta}_1\left(u_i,v_i\right)$ adalah

$$\frac{\partial A}{\partial \boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}=\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)G\frac{\partial g_1}{\partial \boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)$$

dimana $T=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$, $G=\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)T$

$$S=e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2\,\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)T$$

Turunan pertama A terhadap $\boldsymbol{\beta}_2\left(u_i,v_i\right)$ adalah

$$\frac{\partial A}{\partial \boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}=\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_2\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)H\frac{\partial g_1}{\partial \boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)$$

$$\text{dimana } H=\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_2\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

Turunan pertama A terhadap φ_1 adalah

$$\frac{\partial A}{\partial\varphi_1}=\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\frac{-\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}T\frac{\partial g_1}{\partial\varphi_1}\right)$$

Turunan pertama A terhadap φ_2 adalah

$$\frac{\partial A}{\partial\varphi_2}=\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\frac{-\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}T\frac{\partial g_2}{\partial\varphi_2}\right)$$

Turunan pertama A terhadap η adalah

$$\frac{\partial A}{\partial\eta}=\frac{1}{S}\left(\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_1\right)\left(1-g_2\right)\right)$$

Turunan pertama B terhadap $\boldsymbol{\gamma}_1\left(u_i,v_i\right)$ adalah

$$\frac{\partial B}{\partial \boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}=-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}$$

Turunan pertama B terhadap $\boldsymbol{\gamma}_2\left(u_i,v_i\right)$ adalah

$$\frac{\partial B}{\partial \boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}=-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}$$

Turunan pertama B terhadap $\boldsymbol{\beta}_1\left(u_i,v_i\right)$ adalah

$$\frac{\partial B}{\partial \boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}=\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\frac{\partial g_1}{\partial \boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}$$

Turunan pertama B terhadap $\boldsymbol{\beta}_2$ adalah

$$\frac{\partial B}{\partial \boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)} = y_{2i}\mathbf{x}_i^T - \frac{y_{2i}\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)} + \left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}} - \frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2\left(1+\varphi_2 y_{2i}\right)\varphi_2\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\right) + J\,\frac{\partial g_2}{\partial \boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}$$

dimana $J = \dfrac{1}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)} + \exp\left(\dfrac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}$

Turunan pertama B terhadap φ_1 adalah

$$\frac{\partial B}{\partial \varphi_1} = \frac{-\exp\left(\dfrac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)} + \exp\left(\dfrac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}\,\frac{\partial g_1}{\partial \varphi_1}$$

Turunan pertama B terhadap φ_2 adalah

$$\frac{\partial B}{\partial \varphi_2} = -y_{2i}\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)} + \frac{\left(y_{2i}-1\right)}{\left(1+\varphi_2 y_2\right)}y_{2i} + \left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}y_{2i}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}} + \frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2\left(1+\varphi_2 y_2\right)}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\right) + \frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)K}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)} + \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)K\right)}\frac{\partial g_2}{\partial \varphi_2}$$

dimana $K = 1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)$

Turunan pertama B terhadap η adalah

$$\frac{\partial B}{\partial \eta} = \frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)} + \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}$$

Turunan pertama C terhadap $\boldsymbol{\gamma}_1\left(u_i,v_i\right)$ adalah

$$\frac{\partial C}{\partial \boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)} = -\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\right)}$$

Turunan pertama C terhadap $\boldsymbol{\gamma}_2(u_i, v_i)$ adalah

$$\frac{\partial C}{\partial \boldsymbol{\gamma}_2(u_{i^*}, v_{i^*})} = -\frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*}, v_{i^*})}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*}, v_{i^*})}\right)} + \frac{e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*}, v_{i^*})}\mathbf{x}_i^T}{\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*}, v_{i^*})} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}}\right)\left(1+\eta(1-g_2)\left(e^{-y_{1i}} - g_1\right)\right)\right)}$$

Turunan pertama C terhadap $\boldsymbol{\beta}_1(u_i, v_i)$ adalah

$$\frac{\partial C}{\partial \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})} = y_{1i}\left(\mathbf{x}_i^T - \frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})}\varphi_1 \mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})}\right)}\right) + \left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})}(1+\varphi_1 y_{1i})\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})}} - \frac{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})}\right)^2 (1+\varphi_1 y_{1i})\varphi_1 \mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})}\right)^2}\right) + L\frac{\partial g_1}{\partial \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})}$$

$$L = \frac{1}{\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*}, v_{i^*})} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}}\right)\left(1+\eta(1-g_2)\left(e^{-y_{1i}} - g_1\right)\right)\right)}$$

Turunan pertama C terhadap $\boldsymbol{\beta}_2(u_i, v_i)$ adalah

$$\frac{\partial C}{\partial \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})} = \frac{\exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}}\right)\left(1+\eta(1-g_2)\left(e^{-y_{1i}}-g_1\right)\right)\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}} - \frac{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}\right)^2 \varphi_2 \mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}\right)^2}\right)}{\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*}, v_{i^*})} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}}\right)\left(1+\eta(1-g_2)\left(e^{-y_{1i}}-g_1\right)\right)\right)} \frac{\partial g_2}{\partial \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}$$

Turunan pertama C terhadap $\varphi_1(u_i, v_i)$ adalah

$$\frac{\partial C}{\partial \varphi_1} = y_{1i} \frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})}\right)} + \frac{(y_{1i}-1)}{(1+\varphi_1 y_1)} y_{1i} + \left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})} y_{1i}}{1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})}} + \frac{\left(e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})}\right)^2 (1+\varphi_1 y_1)}{\left(1+\varphi_1 e^{\mathbf{x}_i^T \boldsymbol{\beta}_1(u_{i^*}, v_{i^*})}\right)^2}\right) + \frac{\exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}}\right)\left(1+\eta(1-g_2)\left(e^{-y_{1i}}-g_1\right)\right)}{M} \frac{\partial g_1}{\partial \varphi_1}$$

$$\text{dimana } M = \left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2(u_{i^*}, v_{i^*})} + \exp\left(\frac{-e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T \boldsymbol{\beta}_2(u_{i^*}, v_{i^*})}}\right)\left(1+\eta(1-g_2)\left(e^{-y_{1i}}-g_1\right)\right)\right)$$

Turunan pertama C terhadap $\varphi_2\left(u_i,v_i\right)$ adalah

$$\frac{\partial C}{\partial \varphi_2}=\frac{-\exp\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\frac{\partial g_2}{\partial \varphi_2}$$

Turunan pertama C terhadap η adalah

$$\frac{\partial C}{\partial \eta}=\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}$$

Turunan pertama D terhadap $\boldsymbol{\gamma}_1\left(u_i,v_i\right)$ adalah

$$\frac{\partial D}{\partial \boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}=-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\right)}$$

Turunan pertama D terhadap $\boldsymbol{\gamma}_2(u_i,v_i)$ adalah

$$\frac{\partial D}{\partial \boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}=-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\right)}$$

Turunan pertama D terhadap $\boldsymbol{\beta}_1(u_i,v_i)$ adalah

$$\frac{\partial D}{\partial \boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}=y_{1i}\mathbf{x}_i^T-\frac{y_{1i}\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\left(1-\frac{\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \beta_1\left(u_{i^*},v_{i^*}\right)}$$

Turunan pertama D terhadap $\boldsymbol{\beta}_2(u_i,v_i)$ adalah

$$\frac{\partial D}{\partial \boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}=y_{2i}\mathbf{x}_i^T-\frac{y_{2i}\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\left(1-\frac{\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \beta_2\left(u_{i^*},v_{i^*}\right)}$$

Turunan pertama D terhadap φ_1 adalah

$$\frac{\partial D}{\partial \varphi_1}=\frac{-y_{1i}e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(y_{1i}-1\right)y_{1i}}{\left(1+\varphi_1 y_{1i}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\left(y_{1i}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \varphi_1}$$

Turunan pertama D terhadap φ_2 adalah

$$\frac{\partial D}{\partial \varphi_2}=\frac{-y_{2i}e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(y_{2i}-1\right)y_{2i}}{\left(1+\varphi_2 y_{2i}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\left(y_{2i}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \varphi_2}$$

Turunan pertama D terhadap η adalah

$$\frac{\partial D}{\partial \eta}=\frac{\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}$$

Dimana $g_l=\exp\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}}\left(t_l-1\right)\right)$, $\ln\ t_l-\left(\frac{\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}\left(t_l-1\right)}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}}\right)+1=0$

$$\frac{\partial g_l}{\partial \beta_l\left(u_{i^*},v_{i^*}\right)}=\left(t_l-1\right)\left(1-\varphi_l t_l\left(\frac{-e^{\left(\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)\right)}\mathbf{x}_i^T}{1+\varphi_1 e^{\left(\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)\right)}}+\frac{e^{\left(\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)\right)^2}\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1\,\mathrm{e}^{\left(\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)\right)}\right)^2}\right)\right)^{-1}g_l$$

$$\frac{\partial g_l}{\partial \varphi_l}=\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)^2 t_l\left(t_l-1\right)\left(1-\varphi_l\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}\right)^2}t_l\right)^{-1}g_l$$

Sehingga turunan pertama masing-masing parameter dari fungsi *ln likelihood* dari persamaan (4.9) sebagai berikut:

Turunan pertama fungsi l terhadap $\boldsymbol{\gamma}_1(u_i, v_i)$

$$\frac{\partial l}{\partial \boldsymbol{\gamma}_1(u_{i^*}, v_{i^*})} = \sum_{i=1}^{n} w_{ii^*}(1-b_i-c_i-d_i)\left(-\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\right)\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}\right)}+N\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}b_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\right)}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\mathbf{x}_i^T}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}\right)\left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}c_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\right)}\right)+\sum_{i=1}^{n} w_{ii^*}d_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\right)}\right)$$

$$\text{dimana } N = \frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)\right)\mathbf{x}_i^T}{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2\,\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)T}$$

$$T = \left(1+\eta(1-g_1)(1-g_2)\right)$$

Turunan pertama fungsi l terhadap $\boldsymbol{\gamma}_2\left(u_i,v_i\right)$

$$\frac{\partial l}{\partial \boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\right)\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\right)\mathbf{x}_i^T}{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\left(\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)+N}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}b_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\right)}\right)+\sum_{i=1}^{n} w_{ii^*}c_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n} w_{ii^*}d_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\right)}\right)$$

dimana $N=\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2\,\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi l terhadap $\boldsymbol{\beta}_1(u_i,v_i)$

$$
\begin{aligned}
\frac{\partial l}{\partial \boldsymbol{\beta}_1(u_{i^*},v_{i^*})} = {} & \sum_{i=1}^{n} w_{ii^*}(1-b_i-c_i-d_i)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\right)^2\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\right)^2}\right)+OT\,\frac{\partial g_1}{\partial \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\right)\right) \\
& +\sum_{i=1}^{n} w_{ii^*}b_i\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}\right)\left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\right)^2\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\right)^2}\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}\right)\left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\frac{\partial g_1}{\partial \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\right) \\
& +\sum_{i=1}^{n} w_{ii^*}c_i\left(y_{1i}\left(\mathbf{x}_i^T-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\right)}\right)+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}(1+\varphi_1 y_{1i})\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\right)^2(1+\varphi_1 y_{1i})\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\right)^2}\right)+P\,\frac{\partial g_1}{\partial \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\right) \\
& +\sum_{i=1}^{n} w_{ii^*}d_i\left(y_{1i}\mathbf{x}_i^T-\frac{y_{1i}\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}(1+\varphi_1 y_{1i})\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}\left(1-\frac{\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\right)}\right)\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \boldsymbol{\beta}_1(u_{i^*},v_{i^*})}\right)
\end{aligned}
$$

dimana

$$S=e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)T$$

$$O=\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_1\mathbf{x}_i^T}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)$$

$$P=\frac{1}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}$$

$$\mathrm{T}=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

Turunan pertama fungsi l terhadap $\boldsymbol{\beta}_2\left(u_i,v_i\right)$

$$\frac{\partial l}{\partial \boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_2\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+U\right)\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}b_i\left(y_{2i}\mathbf{x}_i^T-\frac{y_{2i}\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2\left(1+\varphi_2 y_{2i}\right)\varphi_2\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+V\frac{\partial g_2}{\partial\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}c_i\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_2\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_2}{\partial\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}d_i\Bigg(y_{2i}\mathbf{x}_i^T-\frac{y_{2i}\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\left(1-\frac{\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)\Bigg)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial\beta_2\left(u_{i^*},v_{i^*}\right)}\Bigg)$$

dimana

$$U = \exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_2\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)\frac{\partial g_2}{\partial\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}$$

$$V = \frac{1}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}$$

Turunan pertama fungsi l terhadap φ_1

$$\frac{\partial l}{\partial \varphi_1}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\frac{-\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}T\frac{\partial g_1}{\partial \varphi_1}\right)\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}b_i\left(\frac{-\exp\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}\frac{\partial g_1}{\partial \varphi_1}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}c_i\left(y_{1i}\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{\left(y_{1i}-1\right)}{\left(1+\varphi_1 y_1\right)}y_{1i}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}y_{1i}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2\left(1+\varphi_1 y_1\right)}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}d_i\left(\frac{-y_{1i}e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(y_{1i}-1\right)y_{1i}}{\left(1+\varphi_1 y_{1i}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\left(y_{1i}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right)$$

dimana $T=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi l terhadap φ_2

$$
\begin{aligned}
\frac{\partial l}{\partial \varphi_2} = & \sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\frac{-\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}T\,\frac{\partial g_2}{\partial \varphi_2}\right)\right) \\
& +\sum_{i=1}^{n} w_{ii^*} b_i\left(-y_{2i}\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{\left(y_{2i}-1\right)}{\left(1+\varphi_2 y_2\right)}y_{2i}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}y_{2i}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2\left(1+\varphi_2 y_2\right)}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+\frac{\exp\left(\dfrac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)K}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}+\exp\left(\dfrac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)K\right)}\frac{\partial g_2}{\partial \varphi_2}\right) \\
& +\sum_{i=1}^{n} w_{ii^*} c_i\left(\frac{-\exp\left(\dfrac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\dfrac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\frac{\partial g_2}{\partial \varphi_2}\right) \\
& +\sum_{i=1}^{n} w_{ii^*} d_i\left(\frac{-y_{2i}e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(y_{2i}-1\right)y_{2i}}{\left(1+\varphi_2 y_{2i}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\left(y_{2i}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \varphi_2}\right)
\end{aligned}
$$

dimana $K=\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$

Turunan pertama fungsi l terhadap η

$$\frac{\partial l}{\partial \eta}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_1\right)\left(1-g_2\right)\right)\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} b_i\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} c_i\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n} w_{ii^*} d_i\left(\frac{\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}\right)$$

Lampiran 17: Fungsi *ln likelihood* d bawah populasi $L(\hat{\Omega})$ GWBZIGPR

$$\ln A_{4i}^{*} = \ln\left(\frac{\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}\frac{\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}+\frac{\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}\frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)+A_1+A_2\right)$$

$$= \ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}+\ln\left(e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)+A_3\right)$$

dimana $A_1 = \frac{\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}\frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right),$

$$A_2 = \frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1((u_i,v_i))}}\frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}\exp\left(-\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}-\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)\left(1+\eta(1-g_1)(1-g_2)\right)$$

$$A_3 = \exp\left(-\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}-\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)\left(1+\eta(1-g_1)(1-g_2)\right)$$

$$\ln B_{4i}^{*} = \ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)^{y_{2i}}\frac{(1+\hat{\varphi}_2 y_{2i})^{y_{2i}-1}}{y_2!}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}(1+\hat{\varphi}_2 y_{2i})}{1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}+\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\hat{\varphi}_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)B_1\right)\right)$$

$$= \ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}+y_{2i}\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)-y_{2i}\ln\left(1+\hat{\varphi}_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}\right)+(y_{2i}-1)\ln(1+\hat{\varphi}_2 y_{2i})-\ln y_{2i}!+\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}(1+\hat{\varphi}_2 y_{2i})}{1+\hat{\varphi}_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)+\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}+\ln B_2$$

dimana $B_1=\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$, $B_2=\left(e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)B_1\right)$

$$\ln C_{4i}^*=\ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)^{y_{1i}}\frac{\left(1+\hat{\varphi}_1y_{1i}\right)^{y_{1i}-1}}{y_{1i}!}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}\left(1+\hat{\varphi}_1y_{1i}\right)}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}+\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\varphi_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)C_1\right)\right)$$

$$=\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}+y_{1i}\ln e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}-y_{1i}\ln\left(1+\varphi_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}\right)+\left(y_{1i}-1\right)\ln\left(1+\hat{\varphi}_1y_{1i}\right)-\ln y_{1i}!+\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}\left(1+\hat{\varphi}_1y_{1i}\right)}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)+\ln C_2+\ln C_3$$

dimana $C_1=\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)$, $C_2=\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}$, $C_3=\left(e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)C_1\right)$

$$\ln D_{4i}^*=\ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}\prod_{l=1}^{2}\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_l(u_i,v_i)}}{1+\hat{\varphi}_le^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_l(u_i,v_i)}}\right)^{y_{li}}\frac{\left(1+\hat{\varphi}_ly_{li}\right)^{y_{li}-1}}{y_{li}!}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_l(u_i,v_i)}\left(1+\hat{\varphi}_ly_{li}\right)}{1+\hat{\varphi}_le^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_l(u_i,v_i)}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$=\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}+\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}+y_{1i}\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1\left(u_i,v_i\right)-y_{1i}\ln\left(1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}\right)+\left(y_{1i}-1\right)\ln\left(1+\hat{\varphi}_1y_{1i}\right)-\ln y_{1i}!+\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}\left(1+\hat{\varphi}_1y_{1i}\right)}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)+D_1$$

dimana

$$D_1=y_{2i}\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_i,v_i\right)-y_{2i}\ln\left(1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}\right)+\left(y_{2i}-1\right)\ln\left(1+\hat{\varphi}_2y_{2i}\right)-\ln y_{2i}!+\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}\left(1+\hat{\varphi}_2y_{2i}\right)}{1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)+\ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

Lampiran 18: Fungsi *ln likelihood* d bawah H_0 $L(\omega)$ parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$ GWBZIGPR

$$\ln A_{5i} = \ln\left(\frac{\mathrm{e}^{\gamma_{10}(u_{i^*},v_{i^*})}}{1+\mathrm{e}^{\gamma_{10}(u_{i^*},v_{i^*})}} \frac{\mathrm{e}^{\gamma_{20}(u_{i^*},v_{i^*})}}{1+\mathrm{e}^{\gamma_{20}(u_{i^*},v_{i^*})}} + \frac{\mathrm{e}^{\gamma_{10}(u_{i^*},v_{i^*})}}{1+\mathrm{e}^{\gamma_{10}(u_{i^*},v_{i^*})}} \frac{1}{1+\mathrm{e}^{\gamma_{20}(u_{i^*},v_{i^*})}} \exp\left(\frac{-e^{\beta_{20}(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\beta_{20}(u_{i^*},v_{i^*})}} \right) + A_1 + A_2 \right)$$

$$= \ln \frac{1}{1+e^{\gamma_{10}(u_{i^*},v_{i^*})}} \frac{1}{1+e^{\gamma_{20}(u_{i^*},v_{i^*})}} + \ln\left(e^{\gamma_{10}(u_{i^*},v_{i^*})} e^{\gamma_{20}(u_{i^*},v_{i^*})} + e^{\gamma_{10}(u_{i^*},v_{i^*})} \exp\left(\frac{-e^{\beta_{20}(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\beta_{20}(u_{i^*},v_{i^*})}} \right) + e^{\gamma_{20}(u_{i^*},v_{i^*})} \exp\left(\frac{-e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}} \right) + A_3 \right)$$

dimana

$$A_1 = \frac{\mathrm{e}^{\gamma_{20}(u_{i^*},v_{i^*})}}{1+\mathrm{e}^{\gamma_{20}(u_{i^*},v_{i^*})}} \frac{1}{1+\mathrm{e}^{\gamma_{10}(u_{i^*},v_{i^*})}} \exp\left(\frac{-e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}} \right),$$

$$A_2 = \frac{1}{1+\mathrm{e}^{\gamma_{10}(u_{i^*},v_{i^*})}} \frac{1}{1+\mathrm{e}^{\gamma_{20}(u_{i^*},v_{i^*})}} \exp\left(-\frac{e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}} - \frac{e^{\beta_{20}(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\beta_{20}(u_{i^*},v_{i^*})}} \right) \left(1+\eta(1-g_1)(1-g_2)\right)$$

$$A_3 = \exp\left(-\frac{e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}} - \frac{e^{\beta_{20}(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\beta_{20}(u_{i^*},v_{i^*})}} \right) \left(1+\eta(1-g_1)(1-g_2)\right)$$

$$\ln B_{5i} = \ln\left(\frac{1}{1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}} \left(\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}} \right)^{y_{2i}} \frac{\left(1+\varphi_2 y_{2i}\right)^{y_{2i}-1}}{y_2\,!} \exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}} \right) \left(\frac{e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}}{1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}} + \frac{1}{1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}} \exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}} \right) B_1 \right) \right)$$

$$= \ln \frac{1}{1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}} + y_{2i}\beta_{20}\left(u_{i^*},v_{i^*}\right) - y_{2i}\ln\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right) + \left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right) - \ln y_{2i}\,! + \left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}} \right) + \ln \frac{1}{1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}} + \ln B_2$$

$$\text{dimana } B_1 = \left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right),\ B_2 = \left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)} + \exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}} \right) B_1 \right)$$

$$\ln C_{5i} = \ln\left(\frac{1}{1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}} \left(\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}} \right)^{y_{1i}} \frac{\left(1+\varphi_1 y_{1i}\right)^{y_{1i}-1}}{y_{1i}\,!} \exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}} \right) \left(\frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}{1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}} + \frac{1}{1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}} \exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}} \right) C_1 \right) \right)$$

$$= \ln \frac{1}{1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}} + y_{1i}\ln e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)} - y_{1i}\ln\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right) + \left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right) - \ln y_{1i}\,! + \left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}} \right) + \ln C_2 + \ln C_3$$

$$\text{dimana } C_1 = \left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right), C_2 = \frac{1}{1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}, C_3 = \left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)} + \exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}} \right) C_1 \right)$$

$$\ln D_{5i} = \ln\left(\frac{1}{1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}}\frac{1}{1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}\prod_{l=1}^{2}\left(\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_l e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)^{y_{li}}\frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}!}\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$= \ln\frac{1}{1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}} + \ln\frac{1}{1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}} + y_{1i}\beta_{10}\left(u_{i^*},v_{i^*}\right) - y_{1i}\ln\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right) + \left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right) - \ln y_{1i}! + \left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right) + D_1$$

dimana

$$D_1 = y_{2i}\beta_{20}\left(u_{i^*},v_{i^*}\right) - y_{2i}\ln\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right) + \left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right) - \ln y_{2i}! + \left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right) + \ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

Lampiran 19: Turunan pertama fungsi *ln likelihood* d bawah H_0 $L(\omega)$ parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$ GWBZIGPR

Turunan pertama fungsi R terhadap $\boldsymbol{\gamma}_1(u_i,v_i)$

$$\frac{\partial R}{\partial\gamma_{10}\left(u_{i^*},v_{i^*}\right)}=\sum_{i=1}^{n}w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(-\frac{\mathrm{e}^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\right)\left(1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\right)}+N\right)$$

$$+\sum_{i=1}^{n}w_{ii^*}b_i\left(-\frac{e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}}{\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)+\sum_{i=1}^{n}w_{ii^*}c_i\left(-\frac{e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\right)}\right)$$

$$+\sum_{i=1}^{n}w_{ii^*}d_i\left(-\frac{e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\right)}\right)$$

$$\text{dimana } N=\frac{\mathrm{e}^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\right)}{e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\right)+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)+\exp\left(-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2\,\mathrm{e}^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)T}$$

$$T = \left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

Turunan pertama fungsi R terhadap $\gamma_{20}\left(u_i, v_i\right)$

$$\frac{\partial R}{\partial \gamma_{20}\left(u_{i^*}, v_{i^*}\right)} = \sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(-\frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\right)\left(1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\right)} + \frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)} + \exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\right)}{e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)} + \exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\right) + e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right) + N} \right)$$

$$+\sum_{i=1}^{n} w_{ii^*} b_i \left(-\frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\right)} \right) + \sum_{i=1}^{n} w_{ii^*} c_i \left(-\frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\right)} + \frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)} + \exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)} \right)$$

$$+\sum_{i=1}^{n} w_{ii^*} d_i \left(-\frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\right)} \right)$$

dimana

$$N = \exp\left(-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}} - \frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2\, \mathrm{e}^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}} \right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

Turunan pertama fungsi R terhadap $\beta_{10}\left(u_i,v_i\right)$

$$
\begin{aligned}
\frac{\partial R}{\partial \beta_{10}\left(u_{i^*},v_{i^*}\right)} &= \sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+OT\,\frac{\partial g_1}{\partial \beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)\right) \\
&+\sum_{i=1}^{n} w_{ii^*} b_i\left(\frac{\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)}{\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\frac{\partial g_1}{\partial \beta_{10}\left(u_{i^*},v_{i^*}\right)}\right) \\
&+\sum_{i=1}^{n} w_{ii^*} c_i\left(y_{1i}\left(1-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)}\right)+\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2\left(1+\varphi_1 y_{1i}\right)\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+P\,\frac{\partial g_1}{\partial \beta_{10}\left(u_{i^*},v_{i^*}\right)}\right) \\
&+\sum_{i=1}^{n} w_{ii^*} d_i\left(y_{1i}-\frac{y_{1i}\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)}+\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\left(1-\frac{\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)
\end{aligned}
$$

dimana

$$S = e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)} + \exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\right) + e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right) + \exp\left(-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}} - \frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2\, \mathrm{e}^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)T$$

$$O = \exp\left(-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}} - \frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}} - \frac{\left(e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2 \varphi_1}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right), \ \mathrm{T}=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

$$P = \frac{1}{\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)} + \exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}} - g_1\right)\right)\right)}$$

Turunan pertama fungsi R terhadap $\beta_{20}\left(u_i,v_i\right)$

$$\frac{\partial R}{\partial \beta_{20}\left(u_{i^*},v_{i^*}\right)}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+U_1\right)\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}b_i\left(y_{2i}-\frac{y_{2i}\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)}+\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2\left(1+\varphi_2 y_{2i}\right)\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+U_2\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}c_i\left(\frac{\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)}{\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_2}{\partial \beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}d_i\left(y_{2i}\mathbf{x}_i^T-\frac{y_{2i}\varphi_2 e^{\mathbf{x}_i^T\beta_{20}\left(u_i,v_i\right)}\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\beta_{20}\left(u_i,v_i\right)}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\beta_{20}\left(u_i,v_i\right)}\left(1+\varphi_2 y_{2i}\right)\mathbf{x}_i^T}{1+\varphi_2 e^{\mathbf{x}_i^T\beta_{20}\left(u_i,v_i\right)}}\left(1-\frac{\varphi_2 e^{\mathbf{x}_i^T\beta_{20}\left(u_i,v_i\right)}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\beta_{20}\left(u_i,v_i\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)$$

dimana

$$U_1 = \frac{\dfrac{\partial g_2}{\partial \beta_{20}\left(u_{i^*}, v_{i^*}\right)}}{\left(e^{\gamma_{10}\left(u_{i^*}, v_{i^*}\right)} + \exp\left(\dfrac{-e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}} - g_2\right)\right)\right)}$$

$$U_2 = \exp\left(-\frac{e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}} - \frac{e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}}\right)\left(\frac{-e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}} - \frac{\left(e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}\right)^2 \varphi_2}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}\right)^2}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}} - g_2\right)\right)\frac{\partial g_2}{\partial \beta_{20}\left(u_{i^*}, v_{i^*}\right)}$$

Turunan pertama fungsi R terhadap φ_1

$$\frac{\partial R}{\partial \varphi_1}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}+\exp\left(-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}T\,\frac{\partial g_1}{\partial \varphi_1}\right)\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} b_i\left(\frac{-\exp\left(\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}\frac{\partial g_1}{\partial \varphi_1}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} c_i\left(y_{1i}\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{\left(y_{1i}-1\right)}{\left(1+\varphi_1 y_1\right)}y_{1i}+\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}y_{1i}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2\left(1+\varphi_1 y_1\right)}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)}{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} d_i\left(\frac{-y_{1i}e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(y_{1i}-1\right)y_{1i}}{\left(1+\varphi_1 y_{1i}\right)}+\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\left(y_{1i}-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right)$$

dimana $T=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi R terhadap φ_2

$$\frac{\partial R}{\partial \varphi_2}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}+\exp\left(-\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}T\,\frac{\partial g_2}{\partial \varphi_2}\right)\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}b_i\left(-y_{2i}\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{\left(y_{2i}-1\right)}{\left(1+\varphi_2 y_2\right)}y_{2i}+\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}y_{2i}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}+\frac{(e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)})^2\left(1+\varphi_2 y_2\right)}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)K}{\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)K\right)}\frac{\partial g_2}{\partial \varphi_2}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}c_i\left(\frac{-\exp\left(\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}\frac{\partial g_2}{\partial \varphi_2}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}d_i\left(\frac{-y_{2i}e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(y_{2i}-1\right)y_{2i}}{\left(1+\varphi_2 y_{2i}\right)}+\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\left(y_{2i}-\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \varphi_2}\right)$$

dimana $K=1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)$, $\mathrm{T}=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi R terhadap η

$$\frac{\partial R}{\partial \eta}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(\exp\left(-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_1\right)\left(1-g_2\right)\right)\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} b_i\left(\frac{\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} c_i\left(\frac{\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)}{\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n} w_{ii^*} d_i\left(\frac{\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}\right)$$

Lampiran 20: Fungsi *ln likelihood* d bawah populasi $L\left(\hat{\omega}\right)$ parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$ GWBZIGPR

$$\ln A_{5i}^{*} = \ln\left(\frac{e^{\hat{\gamma}_{10}(u_i,v_i)}}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}} \frac{e^{\hat{\gamma}_{20}(u_i,v_i)}}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}} + \frac{e^{\hat{\gamma}_{10}(u_i,v_i)}}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}} \frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}} \exp\left(\frac{-e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}} \right) + \frac{e^{\hat{\gamma}_{20}(u_i,v_i)}}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}} \frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}} \exp\left(\frac{-e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}} \right) + A_1 \right)$$

$$= \ln \frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}} \frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}} + \ln\left(e^{\hat{\gamma}_{10}(u_i,v_i)} e^{\hat{\gamma}_{20}(u_i,v_i)} + e^{\hat{\gamma}_{10}(u_i,v_i)} \exp\left(\frac{-e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}} \right) + e^{\hat{\gamma}_{2}(u_i,v_i)} \exp\left(\frac{-e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}} \right) + A_2 \right)$$

dimana

$$A_1 = \frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}} \frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}} \exp\left(-\frac{e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}} - \frac{e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}} \right) \left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right),$$

$$A_2 = \exp\left(-\frac{e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}} - \frac{e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}} \right) \left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

$$\ln B_{5i}^{*} = \ln\left(\frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}} \left(\frac{e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}} \right)^{y_{2i}} \frac{\left(1+\varphi_2 y_{2i}\right)^{y_{2i}-1}}{y_2!} \exp\left(\frac{-e^{\hat{\beta}_{20}(u_i,v_i)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}} \right) \left(\frac{e^{\hat{\gamma}_{10}(u_i,v_i)}}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}} + \frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}} \exp\left(\frac{-e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}} \right) B_1 \right) \right)$$

$$= \ln \frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}} + y_{2i}\hat{\beta}_{20}\left(u_i,v_i\right) - y_{2i} \ln\left(1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}\right) + \left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right) - \ln y_{2i}! + \left(\frac{-e^{\hat{\beta}_{20}(u_i,v_i)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}} \right) + \ln \frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}} + \ln B_2$$

dimana $B_1 = \left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}} - g_2\right)\right)$, $B_2 = \left(e^{\hat{\gamma}_{10}(u_i,v_i)} + \exp\left(\frac{-e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}} \right) B_1 \right)$

$$\ln C_{5i}^{*}=\ln\left(\frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}}\left(\frac{e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\hat{\varphi}_1 e^{\hat{\beta}_{10}(u_i,v_i)}}\right)^{y_{1i}}\frac{\left(1+\hat{\varphi}_1 y_{1i}\right)^{y_{1i}-1}}{y_{1i}\,!}\exp\left(\frac{-e^{\hat{\beta}_{10}(u_i,v_i)}\left(1+\hat{\varphi}_1 y_{1i}\right)}{1+\hat{\varphi}_1 e^{\hat{\beta}_{10}(u_i,v_i)}}\right)\left(\frac{e^{\hat{\gamma}_{20}(u_i,v_i)}}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}}+\frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}}\exp\left(\frac{-e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\hat{\varphi}_2 e^{\hat{\beta}_{20}(u_i,v_i)}}\right)C_1\right)\right)$$

$$=\ln\frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}}+y_{1i}\ln e^{\hat{\beta}_{10}(u_i,v_i)}-y_{1i}\ln\left(1+\hat{\varphi}_1 e^{\hat{\beta}_{10}(u_i,v_i)}\right)+\left(y_{1i}-1\right)\ln\left(1+\hat{\varphi}_1 y_{1i}\right)-\ln y_{1i}\,!+\left(\frac{-e^{\mathbf{x}_i^T\hat{\beta}_{10}(u_i,v_i)}\left(1+\hat{\varphi}_1 y_{1i}\right)}{1+\hat{\varphi}_1 e^{\hat{\beta}_{10}(u_i,v_i)}}\right)+\ln C_2+\ln C_3$$

dimana $C_1=\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)$, $C_2=\dfrac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}}$, $C_3=\left(e^{\hat{\gamma}_{20}(u_i,v_i)}+\exp\left(\dfrac{-e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\hat{\varphi}_2 e^{\hat{\beta}_{20}(u_i,v_i)}}\right)C_1\right)$

$$\ln D_{5i}^{*}=\ln\left(\frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}}\frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}}\prod_{l=1}^{2}\left(\frac{e^{\hat{\beta}_{l0}(u_i,v_i)}}{1+\hat{\varphi}_l e^{\hat{\beta}_{l0}(u_i,v_i)}}\right)^{y_{li}}\frac{\left(1+\hat{\varphi}_l y_{li}\right)^{y_{li}-1}}{y_{li}\,!}\exp\left(\frac{-e^{\hat{\beta}_{l0}(u_i,v_i)}\left(1+\varphi_l y_{li}\right)}{1+\hat{\varphi}_l e^{\hat{\beta}_{l0}(u_i,v_i)}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$=\ln\frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}}+\ln\frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}}+y_{1i}\hat{\beta}_{10}\left(u_i,v_i\right)-y_{1i}\ln\left(1+\hat{\varphi}_1 e^{\hat{\beta}_{10}(u_i,v_i)}\right)+\left(y_{1i}-1\right)\ln\left(1+\hat{\varphi}_1 y_{1i}\right)-\ln y_{1i}\,!+\left(\frac{-e^{\hat{\beta}_{10}(u_i,v_i)}\left(1+\hat{\varphi}_1 y_{1i}\right)}{1+\hat{\varphi}_1 e^{\hat{\beta}_{10}(u_i,v_i)}}\right)+D_1$$

dimana

$$D_1=y_{2i}\hat{\beta}_{20}\left(u_i,v_i\right)-y_{2i}\ln\left(1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}\right)+\left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right)-\ln y_{2i}\,!+\left(\frac{-e^{\hat{\beta}_{20}}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\hat{\beta}_{20}}}\right)+\ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

Lampiran 21: Fungsi *ln likelihood* d bawah $H_0 \;\; L(\omega)$ parameter $\boldsymbol{\gamma}$ GWBZIGPR

$$\ln A_{6i} = \ln\left(\frac{\mathrm{e}^{\gamma_{10}(u_{i^*},v_{i^*})}}{1+\mathrm{e}^{\gamma_{10}(u_{i^*},v_{i^*})}} \frac{\mathrm{e}^{\gamma_{20}(u_{i^*},v_{i^*})}}{1+\mathrm{e}^{\gamma_{20}(u_{i^*},v_{i^*})}} + \frac{\mathrm{e}^{\gamma_{10}(u_{i^*},v_{i^*})}}{1+\mathrm{e}^{\gamma_{10}(u_{i^*},v_{i^*})}} \frac{1}{1+\mathrm{e}^{\gamma_{20}(u_{i^*},v_{i^*})}} \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}} \right) + \frac{\mathrm{e}^{\gamma_{20}(u_{i^*},v_{i^*})}}{1+\mathrm{e}^{\gamma_{20}(u_{i^*},v_{i^*})}} \frac{1}{1+\mathrm{e}^{\gamma_{10}(u_{i^*},v_{i^*})}} \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}} \right) + A_1 \right)$$

$$= \ln \frac{1}{1+e^{\gamma_{10}(u_{i^*},v_{i^*})}} \frac{1}{1+e^{\gamma_{20}(u_{i^*},v_{i^*})}} + \ln\left(e^{\gamma_{10}(u_{i^*},v_{i^*})} e^{\gamma_{20}(u_{i^*},v_{i^*})} + e^{\gamma_{10}(u_{i^*},v_{i^*})} \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}} \right) + e^{\gamma_{20}(u_{i^*},v_{i^*})} \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}} \right) + A_2 \right)$$

dimana $A_1 = \dfrac{1}{1+\mathrm{e}^{\gamma_{10}(u_{i^*},v_{i^*})}} \dfrac{1}{1+\mathrm{e}^{\gamma_{20}(u_{i^*},v_{i^*})}} \exp\left(-\dfrac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}} - \dfrac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}} \right) \left(1+\eta(1-g_1)(1-g_2)\right),$

$$A_2 = \exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}} - \frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}} \right) \left(1+\eta(1-g_1)(1-g_2)\right)$$

$$\ln B_{6i} = \ln\left(\frac{1}{1+e^{\gamma_{20}(u_{i^*},v_{i^*})}} \left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}} \right)^{y_{2i}} \frac{(1+\varphi_2 y_{2i})^{y_{2i}-1}}{y_2!} \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}} \right) \left(\frac{e^{\gamma_{10}(u_{i^*},v_{i^*})}}{1+e^{\gamma_{10}(u_{i^*},v_{i^*})}} + \frac{1}{1+e^{\gamma_{10}(u_{i^*},v_{i^*})}} \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}} \right) B_1 \right) \right)$$

$$= \ln \frac{1}{1+e^{\gamma_{20}(u_{i^*},v_{i^*})}} + y_{2i}\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right) - y_{2i}\ln\left(1+\varphi_2 e^{\boldsymbol{\beta}_2(u_i,v_i)}\right) + \left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right) - \ln y_{2i}! + \left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}} \right) + \ln \frac{1}{1+e^{\gamma_{10}(u_{i^*},v_{i^*})}} + \ln B_2$$

dimana $B_1 = \left(1+\eta(1-g_1)\left(e^{-y_{2i}} - g_2\right)\right),\; B_2 = \left(e^{\gamma_{10}(u_{i^*},v_{i^*})} + \exp\left(\dfrac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}} \right) B_1 \right)$

$$\ln C_{6i} = \ln\left(\frac{1}{1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)^{y_{1i}}\frac{\left(1+\varphi_1 y_{1i}\right)^{y_{1i}-1}}{y_{1i}\,!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}{1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}+\frac{1}{1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)C_1\right)\right)$$

$$= \ln\frac{1}{1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}} + y_{1i}\ln e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)} - y_{1i}\ln\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right) + \left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right) - \ln y_{1i}\,! + \left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right) + \ln C_2 + \ln C_3$$

$$\text{dimana } C_1 = \left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right),\, C_2 = \frac{1}{1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}},\, C_3 = \left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)} + \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)C_1\right)$$

$$\ln D_{6i} = \ln\left(\frac{1}{1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}}\frac{1}{1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}\prod_{l=1}^{2}\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}}\right)^{y_{li}}\frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}\,!}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\mathbf{x}_i^T\boldsymbol{\beta}_l\left(u_{i^*},v_{i^*}\right)}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$= \ln\frac{1}{1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}} + \ln\frac{1}{1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}} + y_{1i}\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right) - y_{1i}\ln\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right) + \left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right) - \ln y_{1i}\,! + \left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right) + D_1$$

dimana

$$D_1 = y_{2i}\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right) - y_{2i}\ln\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right) + \left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right) - \ln y_{2i}\,! + \left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right) + \ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

Lampiran 22: Turunan pertama fungsi *ln likelihood* d bawah H_0 $L(\omega)$ parameter $\boldsymbol{\gamma}$ GWBZIGPR

Turunan pertama fungsi Q terhadap $\gamma_{10}(u_i, v_i)$

$$\frac{\partial Q}{\partial \gamma_{10}(u_{i^*}, v_{i^*})} = \sum_{i=1}^{n} w_{ii^*}(1-b_i-c_i-d_i)\left(-\frac{e^{\gamma_{10}(u_{i^*},v_{i^*})}}{\left(1+e^{\gamma_{10}(u_{i^*},v_{i^*})}\right)\left(1+e^{\gamma_{10}(u_{i^*},v_{i^*})}\right)}+N\right)+\sum_{i=1}^{n} w_{ii^*}b_i\left(-\frac{e^{\gamma_{10}(u_{i^*},v_{i^*})}}{\left(1+e^{\gamma_{10}(u_{i^*},v_{i^*})}\right)}+\frac{e^{\gamma_{10}(u_{i^*},v_{i^*})}}{\left(e^{\gamma_{10}(u_{i^*},v_{i^*})}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}\right)M\right)}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}c_i\left(-\frac{e^{\gamma_{10}(u_{i^*},v_{i^*})}}{\left(1+e^{\gamma_{10}(u_{i^*},v_{i^*})}\right)}\right)+\sum_{i=1}^{n} w_{ii^*}d_i\left(-\frac{e^{\gamma_{10}(u_{i^*},v_{i^*})}}{\left(1+e^{\gamma_{10}(u_{i^*},v_{i^*})}\right)}\right)$$

dimana

$$N=\frac{e^{\gamma_{10}(u_{i^*},v_{i^*})}\left(e^{\gamma_{20}(u_{i^*},v_{i^*})}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)\right)}{e^{\gamma_{10}(u_{i^*},v_{i^*})}\left(e^{\gamma_{20}(u_{i^*},v_{i^*})}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)\right)+e^{\gamma_{20}(u_{i^*},v_{i^*})}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1(u_{i^*},v_{i^*})}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)T}$$

$T=\left(1+\eta(1-g_1)(1-g_2)\right)$, $M=\left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)$

Turunan pertama fungsi Q terhadap $\gamma_{20}\left(u_i, v_i\right)$

$$
\frac{\partial Q}{\partial \gamma_{20}\left(u_{i^*}, v_{i^*}\right)} = \sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(-\frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\right)\left(1+e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\right)} + \frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\right)\mathbf{x}_i^T}{e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\left(\mathrm{e}^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\right)+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)+N}\right)
$$

$$
+\sum_{i=1}^{n} w_{ii^*}b_i\left(-\frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\right)}\right)+\sum_{i=1}^{n} w_{ii^*}c_i\left(-\frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n} w_{ii^*}d_i\left(-\frac{e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\right)}\right)
$$

dimana $N = \exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2\,\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi Q terhadap $\boldsymbol{\beta}_1(u_i,v_i)$

$$\frac{\partial Q}{\partial \boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+OT\,\frac{\partial g_1}{\partial \boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} b_i\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)}{\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\frac{\partial g_1}{\partial \boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} c_i\left(y_{1i}\left(1-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\varphi_1}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)}\right)+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2\left(1+\varphi_1 y_{1i}\right)\varphi_1}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+P\,\frac{\partial g_1}{\partial \boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} d_i\left(\mathbf{x}_i^T-\frac{y_{1i}\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\left(1-\frac{\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)$$

dimana

$$S=e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\right)+e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2\,\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)T$$

$$O=\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}\right),\quad \mathrm{T}=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

$$P=\frac{1}{\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}$$

Turunan pertama fungsi Q terhadap $\boldsymbol{\beta}_2(u_i,v_i)$

$$
\begin{aligned}
\frac{\partial Q}{\partial \boldsymbol{\beta}_2(u_{i^*},v_{i^*})} = & \sum_{i=1}^{n} w_{ii^*}(1-b_i-c_i-d_i)\left(\frac{1}{S}\left(e^{\gamma_{10}(u_{i^*},v_{i^*})}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}\right)^2}\right)+U_1T\frac{\partial g_2}{\partial \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}\right)\right) \\
& +\sum_{i=1}^{n} w_{ii^*}b_i\left(y_{2i}-\frac{y_{2i}\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}\right)^2(1+\varphi_2 y_{2i})\varphi_2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}\right)^2}\right)+U_2\frac{\partial g_2}{\partial \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}\right) \\
& +\sum_{i=1}^{n} w_{ii^*}c_i\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)\left(1+\eta(1-g_2)\left(e^{-y_{1i}}-g_1\right)\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}-\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}\right)^2}\right)}{\left(e^{\gamma_{20}(u_{i^*},v_{i^*})}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2(u_{i^*},v_{i^*})}}\right)\left(1+\eta(1-g_2)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_2}{\partial \boldsymbol{\beta}_2(u_{i^*},v_{i^*})}\right) \\
& +\sum_{i=1}^{n} w_{ii^*}d_i\left(y_{2i}-\frac{y_{2i}\varphi_2 e^{\boldsymbol{\beta}_2(u_i,v_i)}}{\left(1+\varphi_2 e^{\boldsymbol{\beta}_2(u_i,v_i)}\right)}+\left(\frac{-e^{\boldsymbol{\beta}_2(u_i,v_i)}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\boldsymbol{\beta}_2(u_i,v_i)}}\left(1-\frac{\varphi_2 e^{\boldsymbol{\beta}_2(u_i,v_i)}}{\left(1+\varphi_2 e^{\boldsymbol{\beta}_2(u_i,v_i)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \boldsymbol{\beta}_2(u_i,v_i)}\right)
\end{aligned}
$$

dimana

$$U_1 = \frac{1}{\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)} + \exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}$$

$$U_2 = \exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}} - \frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}} - \frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)$$

$$\mathrm{T}=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

Turunan pertama fungsi Q terhadap φ_1

$$\frac{\partial Q}{\partial \varphi_1}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}T\,\frac{\partial g_1}{\partial \varphi_1}\right)\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}b_i\left(\frac{-\exp\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}\frac{\partial g_1}{\partial \varphi_1}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}c_i\left(y_{1i}\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{\left(y_{1i}-1\right)}{\left(1+\varphi_1 y_1\right)}y_{1i}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}y_{1i}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2\left(1+\varphi_1 y_1\right)}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)}{\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}d_i\left(\frac{-y_{1i}e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(y_{1i}-1\right)y_{1i}}{\left(1+\varphi_1 y_{1i}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\left(y_{1i}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{\left(1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right)$$

dimana $\mathrm{T}=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi Q terhadap φ_2

$$\frac{\partial Q}{\partial \varphi_2}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}+\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}T\,\frac{\partial g_2}{\partial \varphi_2}\right)\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}b_i\left(-y_{2i}\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{\left(y_{2i}-1\right)}{\left(1+\varphi_2 y_2\right)}y_{2i}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}y_{2i}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}+\frac{(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)})^2\left(1+\varphi_2 y_2\right)}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)K}{\left(e^{\gamma_1\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)K\right)}\frac{\partial g_2}{\partial \varphi_2}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}c_i\left(\frac{-\exp\left(\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)^2}\frac{\partial g_2}{\partial \varphi_2}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}d_i\left(\frac{-y_{2i}e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(y_{2i}-1\right)y_{2i}}{\left(1+\varphi_2 y_{2i}\right)}+\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\left(y_{2i}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{\left(1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \varphi_2}\right)$$

dimana $T=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$, $K=1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)$

Turunan pertama fungsi Q terhadap η

$$\frac{\partial Q}{\partial \eta}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(\exp\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_1\right)\left(1-g_2\right)\right)\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} b_i\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{\left(e^{\gamma_{10}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\mathbf{x}_i^T\boldsymbol{\beta}_1\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} c_i\left(\frac{\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)}{\left(e^{\gamma_{20}\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\mathbf{x}_i^T\boldsymbol{\beta}_2\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n} w_{ii^*} d_i\left(\frac{\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}\right)$$

Lampiran 23: Fungsi *ln likelihood* d bawah H_0 $L\left(\hat{\omega}\right)$ parameter $\boldsymbol{\gamma}$ GWBZIGPR

$$\ln A_{6i}^{*}=\ln\left(\frac{\mathrm{e}^{\hat{\gamma}_{10}(u_i,v_i)}}{1+\mathrm{e}^{\hat{\gamma}_{10}(u_i,v_i)}}\frac{\mathrm{e}^{\hat{\gamma}_{20}(u_i,v_i)}}{1+\mathrm{e}^{\hat{\gamma}_{20}(u_i,v_i)}}+\frac{\mathrm{e}^{\hat{\gamma}_{10}(u_i,v_i)}}{1+\mathrm{e}^{\hat{\gamma}_{10}(u_i,v_i)}}\frac{1}{1+\mathrm{e}^{\hat{\gamma}_{20}(u_i,v_i)}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)+A\frac{\mathrm{e}^{\hat{\gamma}_{20}(u_i,v_i)}}{1+\mathrm{e}^{\hat{\gamma}_{20}(u_i,v_i)}}\frac{1}{1+\mathrm{e}^{\hat{\gamma}_{10}(u_i,v_i)}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\varphi_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)+A_1\right)$$

$$=\ln\frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}}\frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}}+\ln\left(e^{\hat{\gamma}_{10}(u_i,v_i)}e^{\hat{\gamma}_{20}(u_i,v_i)}+e^{\hat{\gamma}_{10}(u_i,v_i)}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)+e^{\hat{\gamma}_{20}(u_i,v_i)}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)+A_2\right)$$

dimana

$$A_1=\frac{1}{1+\mathrm{e}^{\hat{\gamma}_{10}(u_i,v_i)}}\frac{1}{1+\mathrm{e}^{\hat{\gamma}_{20}(u_i,v_i)}}\exp\left(-\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\varphi_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}-\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\varphi_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)\left(1+\hat{\eta}\left(1-g_1\right)\left(1-g_2\right)\right)$$

$$A_3=\exp\left(-\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\hat{\varphi}_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}-\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\hat{\varphi}_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)\left(1+\hat{\eta}\left(1-g_1\right)\left(1-g_2\right)\right)$$

$$\ln B_{6i}^{*}=\ln\left(\frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}}\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\varphi_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)^{y_{2i}}\frac{\left(1+\varphi_2y_{2i}\right)^{y_{2i}-1}}{y_2!}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}\left(1+\varphi_2y_{2i}\right)}{1+\varphi_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)\left(\frac{e^{\hat{\gamma}_{10}(u_i,v_i)}}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}}+\frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\varphi_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)B_1\right)\right)$$

$$=\ln\frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}}+y_{2i}\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2\left(u_i,v_i\right)-y_{2i}\ln\left(1+\varphi_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}\right)+\left(y_{2i}-1\right)\ln\left(1+\varphi_2y_{2i}\right)-\ln y_{2i}!+\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}\left(1+\varphi_2y_{2i}\right)}{1+\varphi_2e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)+\ln\frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}}+\ln B_2$$

dimana $B_1=\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$, $B_2=\left(e^{\hat{\gamma}_{10}(u_i,v_i)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\varphi_1e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)B_1\right)$

$$\ln C_{6i}^{*}=\ln\left(\frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}}\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}{1+\hat{\varphi}_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)^{y_{1i}}\frac{\left(1+\varphi_1 y_{1i}\right)^{y_{1i}-1}}{y_{1i}!}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)\left(\frac{e^{\hat{\gamma}_{20}(u_i,v_i)}}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}}+\frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)C_1\right)\right)$$

$$=\ln\frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}}+y_{1i}\ln e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}-y_{1i}\ln\left(1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}!+\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)+\ln\frac{1}{1+e^{\hat{\gamma}_2(u_i,v_i)}}+\ln C_2$$

dimana $C_1=\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)$, $C_2=\left(e^{\hat{\gamma}_{20}(u_i,v_i)}+\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}{1+\hat{\varphi}_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}}\right)C_1\right)$

$$\ln D_{6i}^{*}=\ln\left(\frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}}\frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}}\prod_{l=1}^{2}\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_l(u_i,v_i)}}{1+\varphi_l e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_l(u_i,v_i)}}\right)^{y_{li}}\frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}!}\exp\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_l(u_i,v_i)}\left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_l(u_i,v_i)}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$=\ln\frac{1}{1+e^{\hat{\gamma}_{10}(u_i,v_i)}}+\ln\frac{1}{1+e^{\hat{\gamma}_{20}(u_i,v_i)}}+y_{1i}\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1\left(u_i,v_i\right)-y_{1i}\ln\left(1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}!+\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_1(u_i,v_i)}}\right)+D_1$$

dimana

$$D_1=y_{2i}\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2\left(u_i,v_i\right)-y_{2i}\ln\left(1+\varphi_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2(u_i,v_i)}\right)+\left(y_{2i}-1\right)\ln\left(1+\hat{\varphi}_2 y_{2i}\right)-\ln y_{2i}!+D_{11}$$

$$D_{11}=\left(\frac{-e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}\left(1+\hat{\varphi}_2 y_{2i}\right)}{1+\hat{\varphi}_2 e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}_2}}\right)+\ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

Lampiran 24: Fungsi *ln likelihood* d bawah H_0 $L(\omega)$ parameter $\boldsymbol{\beta}$ GWBZIGPR

$$\ln A_{7i} = \ln\left(\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}} \frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}}{1+\mathrm{e}^{\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}} + \frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}} \frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}} \exp\left(\frac{-e^{\beta_{20}(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\beta_{20}(u_{i^*},v_{i^*})}} \right) + A_1 + A_2 \right)$$

$$= \ln \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}} \frac{1}{1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}} + \ln\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})} e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})} + e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})} \exp\left(\frac{-e^{\beta_{20}(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\beta_{20}(u_{i^*},v_{i^*})}} \right) + e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})} \exp\left(\frac{-e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}} \right) + A_3 \right)$$

dimana

$$A_1 = \frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}} \frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}} \exp\left(\frac{-e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}} \right),$$

$$A_2 = \frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}} \frac{1}{1+\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}} \exp\left(-\frac{e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}} - \frac{e^{\beta_{20}(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\beta_{20}(u_{i^*},v_{i^*})}} \right) \left(1+\eta(1-g_1)(1-g_2)\right)$$

$$A_3 = \exp\left(-\frac{e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}} - \frac{e^{\beta_{20}(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\beta_{20}(u_{i^*},v_{i^*})}} \right) \left(1+\eta(1-g_1)(1-g_2)\right)$$

$$\ln B_{7i}=\ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\gamma_2\left(u_{i^*},v_{i^*}\right)}}\left(\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)^{y_{2i}}\frac{\left(1+\varphi_2 y_{2i}\right)^{y_{2i}-1}}{y_2!}\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{e^{\mathbf{x}_i^T\gamma_1\left(u_{i^*},v_{i^*}\right)}}{1+e^{\mathbf{x}_i^T\gamma_1\left(u_{i^*},v_{i^*}\right)}}+\frac{1}{1+e^{\gamma_1(u_i,v_i)}}\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)B_1\right)\right)$$

$$=\ln\frac{1}{1+e^{\mathbf{x}_i^T\gamma_2\left(u_{i^*},v_{i^*}\right)}}+y_{2i}\beta_{20}\left(u_{i^*},v_{i^*}\right)-y_{2i}\ln\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)+\left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right)-\ln y_{2i}!+\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)+\ln\frac{1}{1+e^{\mathbf{x}_i^T\gamma_1\left(u_{i^*},v_{i^*}\right)}}+\ln B_2$$

$$\text{dimana}\quad B_1=\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right),\ B_2=\left(e^{\mathbf{x}_i^T\gamma_1\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)B_1\right)$$

$$\ln C_{1i}=\ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\gamma_1\left(u_{i^*},v_{i^*}\right)}}\left(\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)^{y_{1i}}\frac{\left(1+\varphi_1 y_{1i}\right)^{y_{1i}-1}}{y_{1i}!}\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{e^{\gamma_2(u_i,v_i)}}{1+e^{\gamma_2(u_i,v_i)}}+\frac{1}{1+e^{\gamma_2(u_i,v_i)}}\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)C_1\right)\right)$$

$$=\ln\frac{1}{1+e^{\mathbf{x}_i^T\gamma_1\left(u_{i^*},v_{i^*}\right)}}+y_{1i}\ln e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}-y_{1i}\ln\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}!+\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)+\ln C_2+\ln C_3$$

$$\text{dimana}\, C_1=\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right),C_2=\frac{1}{1+e^{\mathbf{x}_i^T\gamma_2\left(u_{i^*},v_{i^*}\right)}},C_3=\left(e^{\mathbf{x}_i^T\gamma_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)C_1\right)$$

$$\ln D_{1i} = \ln\left(\frac{1}{1+e^{\mathbf{x}_l^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}} \frac{1}{1+e^{\mathbf{x}_l^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}} \prod_{l=1}^{2}\left(\frac{e^{\beta_{l0}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_l e^{\beta_{l0}\left(u_{i^*},v_{i^*}\right)}} \right)^{y_{li}} \frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}\,!} \exp\left(\frac{-e^{\beta_{l0}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\beta_{l0}\left(u_{i^*},v_{i^*}\right)}} \right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$= \ln\frac{1}{1+e^{\mathbf{x}_l^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}} + \ln\frac{1}{1+e^{\mathbf{x}_l^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}} + y_{1i}\beta_{10}\left(u_{i^*},v_{i^*}\right) - y_{1i}\ln\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right) + \left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right) - \ln y_{1i}\,! + \left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}} \right) + D_1$$

dimana

$$D_1 = y_{2i}\beta_{20}\left(u_{i^*},v_{i^*}\right) - y_{2i}\ln\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right) + \left(y_{2i}-1\right)\ln\left(1+\varphi_2 y_{2i}\right) - \ln y_{2i}\,! + \left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}} \right) + \ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

Lampiran 25: Turunan pertama fungsi *ln likelihood* d bawah H_0 $L(\omega)$ parameter $\boldsymbol{\beta}$ GWBZIGPR

Turunan pertama fungsi R terhadap $\boldsymbol{\gamma}_1(u_i,v_i)$

$$\frac{\partial R}{\partial \boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(-\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\right)\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}\right)}+N\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}b_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\right)}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}+\exp\left(\frac{-e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}}\right)\left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)+\sum_{i=1}^{n} w_{ii^*}c_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\right)}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}d_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\right)}\right)$$

dimana
$$N=\frac{\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}+\exp\left(\frac{-e^{\beta_{20}(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\beta_{20}(u_{i^*},v_{i^*})}}\right)\right)}{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1(u_{i^*},v_{i^*})}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}+\exp\left(\frac{-e^{\beta_{20}(u_{i^*},v_{i^*})}}{1+\varphi_2 e^{\beta_{20}(u_{i^*},v_{i^*})}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2(u_{i^*},v_{i^*})}\exp\left(\frac{-e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}}\right)+\exp\left(-\frac{e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}}-\frac{e^{\beta_{20}(u_{i^*},v_{i^*})}}{1+\varphi_2\,\mathrm{e}^{\beta_{20}(u_{i^*},v_{i^*})}}\right)T}$$

$$T=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

Turunan pertama fungsi R terhadap $\boldsymbol{\gamma}_2\left(u_i,v_i\right)$

$$\frac{\partial R}{\partial \boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\right)\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\right)}{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_1\left(u_{i^*},v_{i^*}\right)}\left(\mathrm{e}^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\right)+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)+N}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} b_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\right)}\right)+\sum_{i=1}^{n} w_{ii^*} c_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}}{\left(e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} d_i\left(-\frac{e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}}{\left(1+e^{\mathbf{x}_i^T\boldsymbol{\gamma}_2\left(u_{i^*},v_{i^*}\right)}\right)}\right)$$

dimana $N=\exp\left(-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2\,\mathrm{e}^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi R terhadap $\beta_{10}(u_i, v_i)$

$$\frac{\partial R}{\partial \beta_{10}(u_{i^*}, v_{i^*})} = \sum_{i=1}^{n} w_{ii^*}(1-b_i-c_i-d_i)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\gamma_2(u_{i^*},v_{i^*})}\exp\left(\frac{-e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}}\right)\left(\frac{-e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}}+\frac{\left(e^{\beta_{10}(u_{i^*},v_{i^*})}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}\right)^2}\right)+OT\frac{\partial g_1}{\partial \beta_{10}(u_{i^*},v_{i^*})}\right)\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}b_i\left(\frac{\exp\left(\frac{-e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}}\right)\left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)\left(\frac{-e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}}-\frac{\left(e^{\beta_{10}(u_{i^*},v_{i^*})}\right)^2\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}\right)^2}\right)}{\left(e^{\mathbf{x}_i^T\gamma_1(u_{i^*},v_{i^*})}+\exp\left(\frac{-e^{\beta_{10}(u_{i^*},v_{i^*})}}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}}\right)\left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\frac{\partial g_1}{\partial \beta_{10}(u_{i^*},v_{i^*})}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}c_i\left(y_{1i}\left(1-\frac{e^{\beta_{10}(u_{i^*},v_{i^*})}\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}\right)}\right)+\left(\frac{-e^{\beta_{10}(u_{i^*},v_{i^*})}(1+\varphi_1 y_{1i})}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}}-\frac{\left(e^{\beta_{10}(u_{i^*},v_{i^*})}\right)^2(1+\varphi_1 y_{1i})\varphi_1}{\left(1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}\right)^2}\right)+P\frac{\partial g_1}{\partial \beta_{10}(u_{i^*},v_{i^*})}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*}d_i\left(y_{1i}-\frac{y_{1i}\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}}{\left(1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}\right)}+\left(\frac{-e^{\beta_{10}(u_{i^*},v_{i^*})}(1+\varphi_1 y_{1i})}{1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}}\left(1-\frac{\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}}{\left(1+\varphi_1 e^{\beta_{10}(u_{i^*},v_{i^*})}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \beta_{10}(u_{i^*},v_{i^*})}\right)$$

dimana $S = e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1\left(u_{i^*}, v_{i^*}\right)} \left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2\left(u_{i^*}, v_{i^*}\right)} + \exp\left(\frac{-e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}} \right) \right) + e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2\left(u_{i^*}, v_{i^*}\right)} \exp\left(\frac{-e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}} \right) + \exp\left(-\frac{e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}} - \frac{e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_2 \mathrm{e}^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}} \right) T$

$$O = \exp\left(-\frac{e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}} - \frac{e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}} \right) \left(\frac{-e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}} - \frac{\left(e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)} \right)^2 \varphi_1}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)} \right)^2} \right), \ \mathrm{T} = \left(1+\eta\left(1-g_1\right)\left(1-g_2\right) \right)$$

$$P = \frac{1}{\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_2\left(u_{i^*}, v_{i^*}\right)} + \exp\left(\frac{-e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}} \right) \left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}} - g_1\right) \right) \right)}$$

Turunan pertama fungsi R terhadap $\beta_{20}(u_i,v_i)$

$$
\begin{aligned}
\frac{\partial R}{\partial \beta_{20}(u_i,v_i)} &= \sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\gamma_1\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+U_1\right)\right) \\
&+\sum_{i=1}^{n} w_{ii^*}b_i\left(y_{2i}-\frac{y_{2i}\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)}+\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2\left(1+\varphi_2 y_{2i}\right)\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+U_2\right) \\
&+\sum_{i=1}^{n} w_{ii^*}c_i\left(\frac{\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}-\frac{\left(e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2\varphi_2}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)}{\left(e^{\mathbf{x}_i^T\gamma_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\frac{\partial g_2}{\partial \beta_{20}\left(u_{i^*},v_{i^*}\right)}\right) \\
&+\sum_{i=1}^{n} w_{ii^*}d_i\left(y_{2i}\mathbf{x}_i^T-\frac{y_{2i}\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\mathbf{x}_i^T}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)}+\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)\mathbf{x}_i^T}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\left(1-\frac{\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)
\end{aligned}
$$

dimana

$$U_1 = \frac{\dfrac{\partial g_2}{\partial \beta_{20}\left(u_{i^*}, v_{i^*}\right)}}{\left(e^{\mathbf{x}_i^T \boldsymbol{\gamma}_1\left(u_{i^*}, v_{i^*}\right)} + \exp\left(\dfrac{-e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}} - g_2\right)\right)\right)}$$

$$U_2 = \exp\left(-\frac{e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*}, v_{i^*}\right)}} - \frac{e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}}\right)\left(\frac{-e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}} - \frac{\left(e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}\right)^2 \varphi_2}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*}, v_{i^*}\right)}\right)^2}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}} - g_2\right)\right)\frac{\partial g_2}{\partial \beta_{20}\left(u_{i^*}, v_{i^*}\right)}$$

Turunan pertama fungsi R terhadap φ_1

$$
\frac{\partial R}{\partial \varphi_1}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\gamma_2\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}+\exp\left(-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}T\,\frac{\partial g_1}{\partial \varphi_1}\right)\right)
$$

$$
+\sum_{i=1}^{n} w_{ii^*} b_i\left(\frac{-\exp\left(\dfrac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\left(e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(e^{\mathbf{x}_i^T\gamma_1\left(u_{i^*},v_{i^*}\right)}+\exp\left(\dfrac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}\,\frac{\partial g_1}{\partial \varphi_1}\right)
$$

$$
+\sum_{i=1}^{n} w_{ii^*} c_i\left(y_{1i}\,\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{\left(y_{1i}-1\right)}{\left(1+\varphi_1 y_1\right)}y_{1i}+\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}y_{1i}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2\left(1+\varphi_1 y_1\right)}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+H\,\frac{\partial g_1}{\partial \varphi_1}\right)
$$

$$
+\sum_{i=1}^{n} w_{ii^*} d_i\left(\frac{-y_{1i}e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(y_{1i}-1\right)y_{1i}}{\left(1+\varphi_1 y_{1i}\right)}+\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\left(y_{1i}-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_1 y_{1i}\right)}{\left(1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_1}{\partial \varphi_1}\right)
$$

dimana

$$H=\frac{\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)}{\left(e^{\mathbf{x}_i^T\gamma_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}$$

$$T=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$$

Turunan pertama fungsi R terhadap φ_2

$$
\begin{aligned}
\frac{\partial R}{\partial \varphi_2} &= \sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(e^{\mathbf{x}_i^T\gamma_1\left(u_{i^*},v_{i^*}\right)}\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}+\exp\left(-\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\frac{\left(e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}T\,\frac{\partial g_2}{\partial \varphi_2}\right)\right) \\
&+\sum_{i=1}^{n} w_{ii^*}b_i\left(-y_{2i}\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)}+\frac{\left(y_{2i}-1\right)}{\left(1+\varphi_2 y_2\right)}y_{2i}+\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}y_{2i}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2\left(1+\varphi_2 y_2\right)}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}\right)+\frac{\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)K}{\left(e^{\mathbf{x}_i^T\gamma_1\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)K\right)}\frac{\partial g_2}{\partial \varphi_2}\right) \\
&+\sum_{i=1}^{n} w_{ii^*}c_i\left(\frac{-\exp\left(\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\left(e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}{\left(e^{\mathbf{x}_i^T\gamma_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)^2}\frac{\partial g_2}{\partial \varphi_2}\right) \\
&+\sum_{i=1}^{n} w_{ii^*}d_i\left(\frac{-y_{2i}e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}+\frac{\left(y_{2i}-1\right)y_{2i}}{\left(1+\varphi_2 y_{2i}\right)}+\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\left(y_{2i}-\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\left(1+\varphi_2 y_{2i}\right)}{\left(1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}\right)}\right)\right)+\frac{1}{\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)}\frac{\partial g_2}{\partial \varphi_2}\right)
\end{aligned}
$$

dimana $K=1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)$, $\mathrm{T}=\left(1+\eta\left(1-g_1\right)\left(1-g_2\right)\right)$

Turunan pertama fungsi R terhadap η

$$\frac{\partial R}{\partial \eta}=\sum_{i=1}^{n} w_{ii^*}\left(1-b_i-c_i-d_i\right)\left(\frac{1}{S}\left(\exp\left(-\frac{e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}-\frac{e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_1\right)\left(1-g_2\right)\right)\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} b_i\left(\frac{\exp\left(\frac{-e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{10}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{\left(e^{\mathbf{x}_i^T\gamma_1\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_1 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)\right)}\right)$$

$$+\sum_{i=1}^{n} w_{ii^*} c_i\left(\frac{\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)}{\left(e^{\mathbf{x}_i^T\gamma_2\left(u_{i^*},v_{i^*}\right)}+\exp\left(\frac{-e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}{1+\varphi_2 e^{\beta_{20}\left(u_{i^*},v_{i^*}\right)}}\right)\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right)\right)}\right)+\sum_{i=1}^{n} w_{ii^*} d_i\left(\frac{\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}{1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)}\right)$$

Lampiran 26: Fungsi *ln likelihood* d bawah H_0 $L(\hat{\omega})$ parameter $\boldsymbol{\beta}$ GWBZIGPR

$$\ln A_{7i}^{*}=\ln\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}+\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}\exp\left(\frac{-e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}}\right)+\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}\exp\left(\frac{-e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}}\right)+A_1\right)$$

$$=\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}+\ln\left(e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}\exp\left(\frac{-e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}}\right)+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}\exp\left(\frac{-e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}}\right)+A_2\right)$$

dimana

$$A_1=\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}\exp\left(-\frac{e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}}-\frac{e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}}\right)\left(1+\eta(1-g_1)(1-g_2)\right),$$

$$A_2=\exp\left(-\frac{e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}}-\frac{e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}}\right)\left(1+\eta(1-g_1)(1-g_2)\right)$$

$$\ln B_{7i}^{*}=\ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}\left(\frac{e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}}\right)^{y_{2i}}\frac{(1+\varphi_2 y_{2i})^{y_{2i}-1}}{y_2!}\exp\left(\frac{-e^{\hat{\beta}_{20}(u_i,v_i)}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}}\right)\left(\frac{e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}+\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}\exp\left(\frac{-e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}}\right)B_1\right)\right)$$

$$=\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_2(u_i,v_i)}}+y_{2i}\hat{\beta}_{20}(u_i,v_i)-y_{2i}\ln\left(1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}\right)+(y_{2i}-1)\ln(1+\varphi_2 y_{2i})-\ln y_{2i}!+\left(\frac{-e^{\hat{\beta}_{20}(u_i,v_i)}(1+\varphi_2 y_{2i})}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}}\right)+\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}}+\ln B_2$$

dimana $B_1=\left(1+\eta(1-g_1)\left(e^{-y_{2i}}-g_2\right)\right)$, $B_2=\left(e^{\mathbf{x}_i^T\hat{\boldsymbol{\gamma}}_1(u_i,v_i)}+\exp\left(\frac{-e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}}\right)B_1\right)$

$$\ln C_{7i}^{*}=\ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\hat{\gamma}_1(u_i,v_i)}}\left(\frac{e^{\hat{\beta}_{10}(u_i,v_i)}}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}}\right)^{y_{1i}}\frac{\left(1+\varphi_1 y_{1i}\right)^{y_{1i}-1}}{y_{1i}!}\exp\left(\frac{-e^{\hat{\beta}_{10}(u_i,v_i)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}}\right)\left(\frac{e^{\mathbf{x}_i^T\hat{\gamma}_2(u_i,v_i)}}{1+e^{\mathbf{x}_i^T\hat{\gamma}_2(u_i,v_i)}}+\frac{1}{1+e^{\mathbf{x}_i^T\hat{\gamma}_2(u_i,v_i)}}\exp\left(\frac{-e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}}\right)C_1\right)\right)$$

$$=\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\gamma}_1(u_i,v_i)}}+y_{1i}\ln e^{\hat{\beta}_{10}(u_i,v_i)}-y_{1i}\ln\left(1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}!+\left(\frac{-e^{\hat{\beta}_{10}(u_i,v_i)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}}\right)+\ln C_2+\ln C_3$$

dimana $C_1=\left(1+\eta\left(1-g_2\right)\left(e^{-y_{1i}}-g_1\right)\right), C_2=\dfrac{1}{1+e^{\mathbf{x}_i^T\hat{\gamma}_2(u_i,v_i)}}, C_3=\left(e^{\mathbf{x}_i^T\hat{\gamma}_2(u_i,v_i)}+\exp\left(\dfrac{-e^{\hat{\beta}_{20}(u_i,v_i)}}{1+\varphi_2 e^{\hat{\beta}_{20}(u_i,v_i)}}\right)C_1\right)$

$$\ln D_{7i}^{*}=\ln\left(\frac{1}{1+e^{\mathbf{x}_i^T\hat{\gamma}_1(u_i,v_i)}}\frac{1}{1+e^{\mathbf{x}_i^T\hat{\gamma}_2(u_i,v_i)}}\prod_{l=1}^{2}\left(\frac{e^{\hat{\beta}_{l0}(u_i,v_i)}}{1+\varphi_l e^{\hat{\beta}_{l0}(u_i,v_i)}}\right)^{y_{li}}\frac{\left(1+\varphi_l y_{li}\right)^{y_{li}-1}}{y_{li}!}\exp\left(\frac{-e^{\hat{\beta}_{l0}(u_i,v_i)}\left(1+\varphi_l y_{li}\right)}{1+\varphi_l e^{\hat{\beta}_{l0}(u_i,v_i)}}\right)\left[1+\eta\prod_{l=1}^{2}\left(e^{-y_{li}}-g_l\right)\right]\right)$$

$$=\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\gamma}_1(u_i,v_i)}}+\ln\frac{1}{1+e^{\mathbf{x}_i^T\hat{\gamma}_2(u_i,v_i)}}+y_{1i}\hat{\beta}_{10}\left(u_i,v_i\right)-y_{1i}\ln\left(1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}\right)+\left(y_{1i}-1\right)\ln\left(1+\varphi_1 y_{1i}\right)-\ln y_{1i}!+\left(\frac{-e^{\hat{\beta}_{10}(u_i,v_i)}\left(1+\varphi_1 y_{1i}\right)}{1+\varphi_1 e^{\hat{\beta}_{10}(u_i,v_i)}}\right)+D_1$$

dimana

$$D_1=y_{2i}\hat{\beta}_{20}\left(u_i,v_i\right)-y_{2i}\ln\left(1+\hat{\varphi}_2 e^{\hat{\beta}_{20}(u_i,v_i)}\right)+\left(y_{2i}-1\right)\ln\left(1+\hat{\varphi}_2 y_{2i}\right)-\ln y_{2i}!+\left(\frac{-e^{\hat{\beta}_{20}}\left(1+\hat{\varphi}_2 y_{2i}\right)}{1+\hat{\varphi}_2 e^{\hat{\beta}_{20}}}\right)+\ln\left(1+\eta\left(e^{-y_{1i}}-g_1\right)\left(e^{-y_{2i}}-g_2\right)\right)$$

Lampiran 27: Statistik Deskriptif

Variabel	**Minimum**	**Maksimum**	**Mean**	**Varian**
Y_1	0,00	3,00	0,41	0,49
Y_2	0,00	3,00	0,67	0,91
X_1	46,60	100,00	97,89	42,63
X_2	49,51	100,00	92,74	50,63
X_3	79,59	100,00	97,75	16,11
X_4	0,64	100,00	78,22	506,92
X_5	61,17	100,00	93,18	50,80
X_6	10,57	61,61	30,17	92,14
X_7	18,14	100,00	48,15	246,45

Lampiran 28: Deteksi Under/Overdispersi

Kec	y1	y2	(y1-mean)^2	(y2-mean)^2	(y1-mean1) ^2/mean2	(y2-mean2) ^2/mean2
1	1	0	0,35213	0,44934	0,86605	0,67033
2	1	0	0,35213	0,44934	0,86605	0,67033
3	1	0	0,35213	0,44934	0,86605	0,67033
4	0	0	0,16532	0,44934	0,40659	0,67033
5	0	1	0,16532	0,10868	0,40659	0,16213
6	0	0	0,16532	0,44934	0,40659	0,67033
7	0	1	0,16532	0,10868	0,40659	0,16213
8	0	1	0,16532	0,10868	0,40659	0,16213
9	0	0	0,16532	0,44934	0,40659	0,67033
10	0	0	0,16532	0,44934	0,40659	0,67033
11	0	1	0,16532	0,10868	0,40659	0,16213
12	0	0	0,16532	0,44934	0,40659	0,67033
13	0	0	0,16532	0,44934	0,40659	0,67033
14	3	3	6,72576	5,42736	16,54173	8,09656
15	2	1	2,53894	0,10868	6,24443	0,16213
16	0	0	0,16532	0,44934	0,40659	0,67033
17	1	0	0,35213	0,44934	0,86605	0,67033
18	0	0	0,16532	0,44934	0,40659	0,67033
19	0	0	0,16532	0,44934	0,40659	0,67033
20	0	0	0,16532	0,44934	0,40659	0,67033
21	1	0	0,35213	0,44934	0,86605	0,67033
22	0	0	0,16532	0,44934	0,40659	0,67033
23	0	2	0,16532	1,76802	0,40659	2,63754
24	2	0	2,53894	0,44934	6,24443	0,67033
25	1	0	0,35213	0,44934	0,86605	0,67033
26	0	0	0,16532	0,44934	0,40659	0,67033
27	1	0	0,35213	0,44934	0,86605	0,67033
28	0	0	0,16532	0,44934	0,40659	0,67033
29	1	1	0,35213	0,10868	0,86605	0,16213
30	1	0	0,35213	0,44934	0,86605	0,67033
⋮	⋮	⋮	⋮	⋮	⋮	⋮
88	0	0	0,16532	0,44934	0,40659	0,67033
89	0	0	0,16532	0,44934	0,40659	0,67033
90	0	1	0,16532	0,10868	0,40659	0,16213
91	0	1	0,16532	0,10868	0,40659	0,16213
mean	0,4066	0,6703			**108,10811**	**122,492**

Lampiran 29: Uji Korelasi

Variabel	**Y_1**	**Y_2**	**X_1**	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**
Y_2	0,27							
P-value	0,01							
X_1	0,08	0,21						
P-value	0,45	0,05						
X_2	0,02	0,11	0,74					
P-value	0,86	0,28	0,00					
X_3	0,14	0,16	0,62	0,53				
P-value	0,17	0,14	0,00	0,00				
X_4	0,07	0,11	-0,02	0,03	0,04			
P-value	0,54	0,32	0,86	0,80	0,74			
X_5	0,05	-0,07	0,40	0,58	0,32	0,03		
P-value	0,63	0,52	0,00	0,00	0,00	0,77		
X_6	0,30	0,09	0,21	0,22	0,22	0,30	0,28	
P-value	0,00	0,39	0,05	0,04	0,04	0,01	0,01	
X_7	0,08	-0,21	0,39	0,28	0,37	-0,07	-0,12	0,15
P-value	0,46	0,04	0,00	0,01	0,00	0,53	0,24	0,15

```
Pearson's product-moment correlation

data:  y1 and y2
t = 2.6414, df = 89, p-value = 0.009752
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.06741667 0.45054575
sample estimates:
      cor
0.2696181
```

Lampiran 30: Uji Multikolinearitas

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	-2.360	2.088		-1.130	.262		
	x1	.009	.018	.085	.502	.617	.360	2.78
	x2	-.015	.017	-.154	-.906	.368	.360	2.77
	x3	.022	.024	.129	.951	.345	.563	1.78
	x4	.005	.003	.168	1.561	.122	.893	1.12
	x5	-.002	.013	-.024	-.189	.850	.639	1.57
	x6	.024	.009	.331	2.810	.006	.751	1.33
	x7	.003	.005	.075	.638	.525	.755	1.32

a. Dependent Variable: y1

Coefficientsa

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	.336	2.867		.117	.907		
	x1	.026	.025	.181	1.061	.292	.360	2.778
	x2	.003	.023	.021	.121	.904	.360	2.775
	x3	-.002	.032	-.008	-.058	.954	.563	1.775
	x4	.007	.005	.159	1.470	.145	.893	1.120
	x5	-.031	.017	-.231	-1.805	.075	.639	1.565
	x6	.019	.012	.191	1.614	.110	.751	1.332
	x7	-.011	.007	-.186	-1.578	.118	.755	1.324

a. Dependent Variable: y2

Lampiran 31: Uji Serentak Model BZIGPR

Parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$

```
==========================================================
     Uji Serentak parameter Beta&Gamma Model BZIGPR
==========================================================
G Hitung Daerah Kritis       Kesimpulan
3578,869   23,685         Tolak H0 dengan alpha 5%
==========================================================
```

Parameter $\boldsymbol{\beta}$

```
==========================================================
     Uji Serentak parameter Beta Model BZIGPR
==========================================================
G Hitung Daerah Kritis       Kesimpulan
3576,365   23,685         Tolak H0 dengan alpha 5%
==========================================================
```

Parameter $\boldsymbol{\gamma}$

```
==========================================================
     Uji Serentak parameter Beta Model BZIGPR
==========================================================
G Hitung Daerah Kritis       Kesimpulan
3531,279   23,685         Tolak H0 dengan alpha 5%
==========================================================
```

Lampiran 32: Penaksiran Parameter Model BZIGPR

```
==============================================================
              Estimasi Parameter BZIGPR di bawah H1
==============================================================
                  Estimate    Std Error       Z value P-value
gamma10        0,010020593 1,466799e-05      683,1607  0,0000
gamma11        0,014250720 1,274315e-03       11,1830  0,0000
gamma12        0,013853229 1,027317e-03       13,4849  0,0000
gamma13        0,014253116 1,254390e-03       11,3626  0,0000
gamma14        0,012357235 6,213937e-03        1,9886  0,0467
gamma15        0,013853962 1,142328e-03       12,1278  0,0000
gamma16        0,011984117 8,143543e-04       14,7161  0,0000
gamma17        0,012079313 1,357874e-03        8,8958  0,0000
gamma20        0,011986360 8,015410e-06     1495,4144  0,0000
gamma21        0,010927615 1,492999e-03        7,3192  0,0000
gamma22        0,010851384 1,407730e-03        7,7084  0,0000
gamma23        0,010852854 9,397170e-04       11,5491  0,0000
gamma24        0,010217308 5,541373e-03        1,8438  0,0652
gamma25        0,010507751 1,160783e-03        9,0523  0,0000
gamma26        0,012486124 2,129628e-03        5,8631  0,0000
gamma27        0,010633358 6,040555e-03        1,7603  0,0784
interceptb1 -10,485634770 2,401810e-05  -436572,2155  0,0000
b11            0,031749293 1,219183e-03       26,0414  0,0000
b12           -0,041172793 1,038201e-03      -39,6578  0,0000
b13            0,070282037 1,213869e-03       57,8992  0,0000
b14            0,009746819 1,858825e-03        5,2435  0,0000
b15            0,006904972 1,166365e-03        5,9201  0,0000
b16            0,044002777 5,583366e-04       78,8105  0,0000
b17            0,006024582 1,122713e-03        5,3661  0,0000
interceptb2 -30,502268086 1,871422e-05 -1629897,9552  0,0000
b21            0,351390939 8,452955e-04      415,7019  0,0000
b22            0,017260268 6,712323e-04       25,7143  0,0000
b23           -0,037471191 8,702753e-04      -43,0567  0,0000
b24            0,008943378 2,798546e-03        3,1957  0,0014
b25           -0,026837646 5,652060e-04      -47,4829  0,0000
b26            0,025369329 7,300605e-04       34,7496  0,0000
b27           -0,015890881 8,045720e-04      -19,7507  0,0000
psi1           1,064020770 7,532407e-04     1412,5907  0,0000
psi2           1,210820689 7,532857e-04     1607,3857  0,0000
eta            0,268991206 5,630603e-06    47773,0734  0,0000
==============================================================
```

Lampiran 33: Nilai Taksiran Respon Model BZIGPR

Kecamatan	Jumlah Kematian Ibu Hamil	Jumlah Kematian Ibu nifas	Kecamatan	Jumlah Kematian Ibu Hamil	Jumlah Kematian Ibu nifas
Wonotunggal	0,4278	1,3482	Kedungwuni	0,2008	0,8010
Bandar	0,3148	1,5034	Karangdadap	0,2609	0,4426
Blado	0,3747	2,1513	Buaran	0,1389	0,0322
Reban	1,4285	3,1228	Tirto	0,1611	2,1939
Bawang	0,2573	1,5591	Wiradesa	0,4268	3,5778
Tersono	0,3529	1,1916	Wonokerto	0,2013	0,7650
Gringsing	0,3826	2,3921	Moga	0,1694	2,9975
Limpung	0,2236	1,7915	Warungpring	0,2551	3,1577
Banyuputih	0,4552	2,7675	Pulosari	0,4564	4,8090
Subah	0,3802	1,7636	Belik	0,1774	4,3231
Pecalungan	0,1574	2,9073	Watukumpul	0,1460	2,8438
Tulis	0,2551	1,1371	Bodeh	0,4469	3,2819
Kandeman	0,3567	2,0062	Bantarbolang	0,2287	1,9503
Batang	0,3689	2,4896	Randudongkal	0,3719	4,9873
Warungasem	0,5078	2,2987	Pemalang	0,2141	2,4940
Kandangserang	0,1822	0,0029	Taman	0,3076	4,4771
Paninggaran	0,7055	1,4111	Petarukan	0,2456	2,7122
Lebakbarang	0,0518	0,0000	Ampelgading	0,2834	2,4779
Petungkriyono	0,0563	0,0696	Comal	0,3793	3,0773
Talun	0,3199	0,0038	Ulujami	0,1962	3,0962
Doro	0,3589	0,1271	Margasari	0,1805	2,3026
Karanganyar	0,4374	1,8577	Bumijawa	0,0991	0,0840
Kajen	0,2648	0,7407	Bojong	0,1399	0,7834
Kesesi	0,1992	0,2026	Balapulang	0,4370	0,0259
Sragi	0,5883	0,1746	Pagerbarang	0,2241	2,2302
Siwalan	0,1446	0,5867	Lebaksiu	0,3483	2,3456
Bojong	0,1775	0,3346	Jatinegara	0,1532	0,4599
Wonopringgo	0,1919	0,2583	Kedungbanteng	0,2845	1,9356
Pangkah	0,3816	1,7595	Ketanggungan	1,2690	5,4668
Slawi	0,1569	1,5650	Banjarharjo	0,6942	2,2857
Dukuhwaru	0,1576	1,5388	Losari	0,3510	1,8105
Adiwerna	0,2795	1,9175	Tanjung	1,0676	3,0099
Dukuhturi	0,3701	2,6674	Kersana	0,4744	2,5893
Talang	0,1764	0,9513	Bulukamba	0,6020	2,5498
Tarub	0,2606	1,5805	Wanasari	0,4426	2,0124
Kramat	0,4194	1,8295	Brebes	0,3253	1,6109
Suradadi	0,1822	3,3918	Songgom	0,3039	1,6148

Lanjutan Lampiran 33

Kecamatan	**Jumlah Kematian Ibu Hamil**	**Jumlah Kematian Ibu nifas**	**Kecamatan**	**Jumlah Kematian Ibu Hamil**	**Jumlah Kematian Ibu nifas**
Warureja	0,1719	2,1821	Pekalongan Barat	0,1747	2,1430
Salem	0,3697	1,1637	Pekalongan Timur	0,1872	1,7917
Bantarkawung	0,3747	1,6129	Pekalongan Utara	0,2012	2,7352
Bumiayu	0,2920	1,7237	Pekalongan Selatan	0,1710	1,9832
Paguyangan	0,2671	2,1446	Tegal Selatan	0,1742	1,9733
Sirampog	0,2652	1,4286	Tegal Timur	0,1756	1,6857
Tonjong	0,3277	1,7706	Tegal Barat	0,1674	1,4798
Jatibarang	0,3270	1,5230	Margadana	0,1210	1,5240
Larangan	0,4506	3,0399			

Lampiran 34: Uji Heterogenitas spasial

```
==========================================================
            Uji Heterogenitas Spasial
==========================================================
G Hitung     Daerah Kritis       Kesimpulan
90,32166   23,68479         Tolak H0 dengan alpha 5%
==========================================================
```

Lampiran 35: Jarak *Euclide*

Kec	1		2	3	4	5	6	7	8	9	…	91
1	0,000		0,058	0,162	0,269	0,262	0,291	0,329	0,245	0,278	…	0,553
2	0,058		0,000	0,105	0,213	0,204	0,239	0,284	0,194	0,237	…	0,611
3	0,162		0,105	0,000	0,124	0,102	0,166	0,233	0,134	0,197	…	0,713
4	0,269		0,213	0,124	0,000	0,052	0,055	0,137	0,064	0,126	…	0,822
5	0,262		0,204	0,102	0,052	0,000	0,107	0,189	0,109	0,175	…	0,815
6	0,291		0,239	0,166	0,055	0,107	0,000	0,083	0,049	0,082	…	0,837
7	0,329		0,284	0,233	0,137	0,189	0,083	0,000	0,099	0,052	…	0,854
8	0,245		0,194	0,134	0,064	0,109	0,049	0,099	0,000	0,069	…	0,788
9	0,278		0,237	0,197	0,126	0,175	0,082	0,052	0,069	0,000	…	0,802
10	0,206		0,169	0,156	0,140	0,176	0,123	0,127	0,079	0,075	…	0,728
11	0,184		0,135	0,099	0,103	0,126	0,109	0,150	0,061	0,105	…	0,728
12	0,155		0,131	0,159	0,187	0,211	0,180	0,189	0,133	0,137	…	0,666
13	0,144		0,137	0,190	0,231	0,252	0,224	0,227	0,177	0,175	…	0,628
14	0,131		0,144	0,218	0,273	0,291	0,270	0,273	0,222	0,221	…	0,583
15	0,104		0,108	0,181	0,244	0,258	0,246	0,260	0,198	0,208	…	0,595
16	0,242		0,280	0,355	0,479	0,453	0,514	0,564	0,472	0,516	…	0,427
17	0,155		0,179	0,244	0,368	0,341	0,406	0,461	0,366	0,415	…	0,516
18	0,103		0,082	0,121	0,246	0,219	0,285	0,344	0,247	0,301	…	0,612
19	0,129		0,090	0,089	0,213	0,183	0,255	0,320	0,221	0,280	…	0,651
20	0,050		0,014	0,115	0,219	0,213	0,242	0,284	0,196	0,235	…	0,604
21	0,028		0,046	0,145	0,258	0,246	0,285	0,329	0,240	0,280	…	0,569
22	0,053		0,102	0,200	0,315	0,302	0,341	0,381	0,295	0,331	…	0,513
23	0,109		0,160	0,255	0,372	0,358	0,398	0,438	0,353	0,388	…	0,461
24	0,159		0,214	0,312	0,426	0,414	0,450	0,485	0,403	0,433	…	0,403
25	0,129		0,185	0,289	0,384	0,385	0,396	0,415	0,347	0,363	…	0,441
26	0,173		0,223	0,324	0,407	0,415	0,411	0,418	0,362	0,367	…	0,438
27	0,090		0,146	0,250	0,347	0,347	0,361	0,384	0,312	0,332	…	0,476
28	0,046		0,092	0,195	0,289	0,289	0,303	0,330	0,255	0,278	…	0,534
29	0,062		0,099	0,197	0,285	0,289	0,296	0,318	0,247	0,266	…	0,542
30	0,076		0,080	0,162	0,238	0,246	0,247	0,270	0,199	0,218	…	0,590
31	0,100		0,120	0,204	0,273	0,285	0,276	0,288	0,228	0,236	…	0,566
32	0,144		0,186	0,282	0,358	0,368	0,360	0,366	0,311	0,314	…	0,491
33	0,126		0,166	0,261	0,336	0,347	0,339	0,347	0,291	0,296	…	0,508
34	0,169		0,205	0,294	0,357	0,373	0,353	0,351	0,306	0,300	…	0,513
35	0,451		0,500	0,585	0,707	0,685	0,738	0,780	0,694	0,730	…	0,269
36	0,387		0,438	0,528	0,648	0,629	0,676	0,714	0,631	0,663	…	0,251
…	…		…	…	…	…	…	…	…	…	…	…
91	0,553		0,611	0,713	0,822	0,815	0,837	0,854	0,788	0,802	…	0,000

Lampiran 36: Nilai Bandwith Optimum Fungsi Kernel

Kecamatan	***Fixed Gaussian***	***Adaptive Gaussian***	***Fixed Bisquare***	***Adaptive Bisquare***
Wonotunggal	1,007887	0,936782	1,401806	0,963488
Bandar	1,007887	0,990256	1,401806	1,013471
Blado	1,007887	1,082587	1,401806	1,106837
Reban	1,007887	1,202297	1,401806	1,221738
Bawang	1,007887	1,18407	1,401806	1,209023
Tersono	1,007887	1,227968	1,401806	1,252347
Gringsing	1,007887	1,256282	1,401806	1,291081
Limpung	1,007887	1,181103	1,401806	1,207221
Banyuputih	1,007887	1,204275	1,401806	1,239793
Subah	1,007887	1,130114	1,401806	1,164991
Pecalungan	1,007887	1,120071	1,401806	1,146897
Tulis	1,007887	1,067992	1,401806	1,106467
Kandeman	1,007887	1,029924	1,401806	1,076807
Batang	1,007887	0,984392	1,401806	1,036534
Warungasem	1,007887	0,997064	1,401806	1,037439
Kandangserang	1,007887	0,738231	1,401806	0,787043
Paninggaran	1,007887	0,848778	1,401806	0,890645
Lebakbarang	1,007887	0,965911	1,401806	0,998184
Petungkriyono	1,007887	1,004528	1,401806	1,037755
Talun	1,007887	0,986213	1,401806	1,011143
Doro	1,007887	0,944377	1,401806	0,967374
Karanganyar	1,007887	0,887871	1,401806	0,912151
Kajen	1,007887	0,830494	1,401806	0,854665
Kesesi	1,007887	0,777854	1,401806	0,806493
Sragi	1,007887	0,842621	1,401806	0,886968
Siwalan	1,007887	0,839793	1,401806	0,905476
Bojong	1,007887	0,87565	1,401806	0,912125
Wonopringgo	1,007887	0,929633	1,401806	0,962583
Kedungwuni	1,007887	0,941344	1,401806	0,976236
Karangdadap	1,007887	0,989167	1,401806	1,022549
Buaran	1,007887	0,967948	1,401806	1,012076
⋮	⋮	⋮	⋮	⋮
Tegal Selatan	1,007887	0,796611	1,401806	0,830579
Tegal Timur	1,007887	0,791982	1,401806	0,822596
Tegal Barat	1,007887	0,826498	1,401806	0,857562
Margadana	1,007887	0,821654	1,401806	0,854405

Lampiran 37: Pembobot fungsi *kernel fix bisquare*

lokasi	1	2	3	4	5	6	7	8	…	91
1	1,000	0,997	0,973	0,928	0,931	0,916	0,893	0,940	…	0,713
2	0,997	1,000	0,989	0,954	0,958	0,943	0,919	0,962	…	0,656
3	0,973	0,989	1,000	0,984	0,989	0,972	0,945	0,982	…	0,549
4	0,928	0,954	0,984	1,000	0,997	0,997	0,981	0,996	…	0,431
5	0,931	0,958	0,989	0,997	1,000	0,988	0,964	0,988	…	0,438
6	0,916	0,943	0,972	0,997	0,988	1,000	0,993	0,998	…	0,414
7	0,893	0,919	0,945	0,981	0,964	0,993	1,000	0,990	…	0,395
8	0,940	0,962	0,982	0,996	0,988	0,998	0,990	1,000	…	0,467
9	0,923	0,944	0,961	0,984	0,969	0,993	0,997	0,995	…	0,452
10	0,957	0,971	0,975	0,980	0,969	0,985	0,984	0,994	…	0,533
11	0,966	0,981	0,990	0,989	0,984	0,988	0,977	0,996	…	0,533
12	0,976	0,983	0,975	0,965	0,955	0,967	0,964	0,982	…	0,599
13	0,979	0,981	0,963	0,947	0,936	0,950	0,948	0,968	…	0,639
14	0,983	0,979	0,952	0,925	0,916	0,927	0,926	0,950	…	0,684
15	0,989	0,988	0,967	0,940	0,934	0,939	0,932	0,961	…	0,672
16	0,941	0,922	0,876	0,780	0,802	0,749	0,702	0,786	…	0,823
17	0,976	0,968	0,940	0,867	0,885	0,839	0,796	0,868	…	0,747
18	0,989	0,993	0,985	0,939	0,952	0,919	0,883	0,939	…	0,656
19	0,983	0,992	0,992	0,954	0,966	0,935	0,899	0,951	…	0,615
20	0,997	1,000	0,987	0,952	0,955	0,941	0,920	0,961	…	0,663
21	0,999	0,998	0,979	0,933	0,939	0,919	0,893	0,942	…	0,698
22	0,997	0,989	0,960	0,902	0,909	0,885	0,857	0,913	…	0,750
23	0,988	0,974	0,935	0,864	0,874	0,845	0,814	0,877	…	0,795
24	0,974	0,954	0,903	0,823	0,833	0,804	0,775	0,841	…	0,841
25	0,983	0,966	0,917	0,855	0,855	0,847	0,832	0,881	…	0,811
26	0,970	0,950	0,896	0,839	0,833	0,835	0,830	0,871	…	0,814
27	0,992	0,978	0,937	0,881	0,881	0,872	0,856	0,903	…	0,782
28	0,998	0,991	0,962	0,917	0,917	0,909	0,892	0,935	…	0,731
29	0,996	0,990	0,961	0,919	0,917	0,913	0,900	0,939	…	0,724
30	0,994	0,994	0,974	0,943	0,939	0,939	0,927	0,960	…	0,677
31	0,990	0,986	0,958	0,925	0,919	0,924	0,917	0,948	…	0,700
32	0,979	0,965	0,920	0,874	0,867	0,873	0,868	0,904	…	0,770
33	0,984	0,972	0,932	0,888	0,881	0,886	0,881	0,916	…	0,755
34	0,971	0,958	0,914	0,875	0,864	0,877	0,879	0,907	…	0,750
35	0,803	0,762	0,682	0,556	0,580	0,522	0,476	0,570	…	0,928
36	0,854	0,815	0,737	0,618	0,638	0,589	0,548	0,636	…	0,937
…	…	…	…	…	…	…	…	…	…	…
91	0,713	0,656	0,549	0,431	0,438	0,414	0,395	0,467	…	1,000

Lampiran 38: Uji Serentak Model GWBZIGPR

Parameter $\boldsymbol{\beta}$ dan $\boldsymbol{\gamma}$

```
==========================================================
     Uji Serentak parameter Gamma&Beta Model GWBZIGPR
==========================================================
G Hitung Daerah Kritis       Kesimpulan
4265,545   1358,15         Tolak H0 dengan alpha 5%
==========================================================
```

Parameter $\boldsymbol{\gamma}$

```
==========================================================
     Uji Serentak parameter Gamma Model BZIGPR
==========================================================
G Hitung Daerah Kritis       Kesimpulan
4217,957   1358,15         Tolak H0 dengan alpha 5%
==========================================================
```

Parameter $\boldsymbol{\beta}$

```
==========================================================
     Uji Serentak parameter Beta Model GWBZIGPR
==========================================================
G Hitung Daerah Kritis       Kesimpulan
3644,285   1358,15         Tolak H0 dengan alpha 5%
==========================================================
```

Lampiran 39: Penaksir parameter γ model GWBZIGPR

Kec	γ_{10}	γ_{11}	γ_{12}	γ_{13}	γ_{14}	γ_{15}	γ_{16}	γ_{17}
1	0,01003	0,01563	0,01503	0,01564	0,01237	0,01508	0,01242	0,01258
2	0,01003	0,01543	0,01488	0,01543	0,01227	0,01495	0,01228	0,01248
3	0,01002	0,01519	0,01471	0,01518	0,01228	0,01479	0,01208	0,01230
4	0,01002	0,01484	0,01443	0,01482	0,01231	0,01452	0,01182	0,01204
5	0,01002	0,01458	0,01421	0,01456	0,01258	0,01428	0,01166	0,01185
6	0,01002	0,01429	0,01397	0,01427	0,01256	0,01403	0,01149	0,01165
7	0,01001	0,01412	0,01382	0,01409	0,01258	0,01387	0,01137	0,01151
8	0,01001	0,01404	0,01375	0,01402	0,01280	0,01378	0,01135	0,01147
9	0,01001	0,01395	0,01367	0,01393	0,01277	0,01370	0,01129	0,01139
10	0,01001	0,01390	0,01362	0,01388	0,01291	0,01363	0,01128	0,01136
11	0,01001	0,01386	0,01358	0,01384	0,01304	0,01359	0,01126	0,01133
12	0,01001	0,01381	0,01354	0,01380	0,01305	0,01353	0,01124	0,01130
13	0,01001	0,01377	0,01351	0,01376	0,01310	0,01350	0,01124	0,01127
14	0,01000	0,01356	0,01283	0,01388	0,02724	0,01204	0,00939	0,00727
15	0,01001	0,01371	0,01345	0,01371	0,01321	0,01343	0,01121	0,01123
16	0,01000	0,01322	0,01251	0,01341	0,02711	0,01154	0,00992	0,00789
17	0,01000	0,01342	0,01259	0,01365	0,02781	0,01163	0,00967	0,00756
18	0,01001	0,01373	0,01346	0,01372	0,01367	0,01341	0,01120	0,01118
19	0,01001	0,01374	0,01348	0,01374	0,01369	0,01343	0,01119	0,01117
20	0,01001	0,01365	0,01340	0,01365	0,01343	0,01336	0,01117	0,01117
21	0,01000	0,01347	0,01264	0,01375	0,02802	0,01174	0,00940	0,00719
22	0,01000	0,01336	0,01258	0,01362	0,02763	0,01166	0,00953	0,00739
23	0,01000	0,01326	0,01251	0,01350	0,02733	0,01159	0,00966	0,00755
24	0,01000	0,01315	0,01246	0,01337	0,02694	0,01154	0,00977	0,00770
25	0,01000	0,01323	0,01255	0,01350	0,02704	0,01166	0,00963	0,00753
26	0,01000	0,01325	0,01260	0,01353	0,02702	0,01172	0,00962	0,00751
27	0,01000	0,01328	0,01256	0,01355	0,02734	0,01167	0,00954	0,00741
28	0,01000	0,01337	0,01260	0,01367	0,02779	0,01171	0,00940	0,00720
29	0,01000	0,01338	0,01262	0,01369	0,02783	0,01173	0,00937	0,00715
30	0,01000	0,01346	0,01266	0,01378	0,02822	0,01178	0,00924	0,00695
31	0,01000	0,01342	0,01266	0,01374	0,02800	0,01179	0,00929	0,00702
32	0,01000	0,01331	0,01262	0,01362	0,02747	0,01174	0,00947	0,00729
33	0,01000	0,01333	0,01262	0,01364	0,02761	0,01174	0,00942	0,00722
34	0,01000	0,01336	0,01267	0,01368	0,02764	0,01180	0,00940	0,00718
35	0,00999	0,01274	0,01226	0,01287	0,02679	0,01126	0,01025	0,00802
36	0,00999	0,01278	0,01227	0,01293	0,02657	0,01130	0,01016	0,00800
37	0,00999	0,01282	0,01229	0,01294	0,02745	0,01123	0,01023	0,00794
38	0,00999	0,01293	0,01231	0,01308	0,02739	0,01128	0,01004	0,00786
39	0,01000	0,01311	0,01237	0,01329	0,02787	0,01134	0,00984	0,00766

Lanjutan Lampiran 39

Kec	γ_{10}	γ_{11}	γ_{12}	γ_{13}	γ_{14}	γ_{15}	γ_{16}	γ_{17}
40	0,01	0,01294	0,0123	0,0131	0,02673	0,01139	0,00992	0,00783
41	0,00999	0,0128	0,0123	0,013	0,02653	0,01131	0,01011	0,00797
42	0,00999	0,0128	0,0123	0,013	0,02655	0,01131	0,0101	0,00797
43	0,00999	0,01275	0,0123	0,0129	0,02606	0,01139	0,01018	0,00806
44	0,01	0,01294	0,0124	0,0132	0,02649	0,01148	0,00993	0,00784
45	0,01	0,01303	0,0125	0,0133	0,02671	0,01154	0,00983	0,00773
46	0,01	0,01291	0,0124	0,0131	0,0265	0,01143	0,00995	0,00786
47	0,01	0,01311	0,0125	0,0134	0,02699	0,01158	0,0097	0,00758
48	0,01	0,01317	0,0126	0,0134	0,02712	0,01165	0,00966	0,00752
49	0,00998	0,02138	0,015	0,0213	0,02925	0,00811	-0,00114	-0,0069
50	0,00997	0,01371	0,0124	0,0137	0,02814	0,01062	0,01084	0,00158
51	0,01	0,01274	0,0115	0,0129	0,02953	0,00983	0,00968	0,00699
52	0,00997	0,01349	0,0123	0,0135	0,02758	0,0107	0,01093	0,00164
53	0,00997	0,0134	0,0123	0,0134	0,02743	0,01076	0,01105	0,00156
54	0,01	0,01256	0,0114	0,0128	0,02907	0,00983	0,00975	0,00677
55	0,01	0,0127	0,0114	0,0129	0,02916	0,00979	0,0096	0,00666
56	0,01	0,01266	0,0114	0,0129	0,02916	0,00979	0,0096	0,00651
57	0,01	0,01254	0,0114	0,0127	0,02908	0,00982	0,00975	0,00667
58	0,00997	0,01348	0,0124	0,0135	0,02755	0,0108	0,01065	0,00149
59	0,00997	0,01345	0,0124	0,0135	0,02755	0,01081	0,01073	0,00145
60	0,00997	0,01344	0,0124	0,0135	0,0276	0,01085	0,01073	0,00135
61	0,00997	0,01345	0,0124	0,0135	0,02769	0,01088	0,0107	0,00124
62	0,01	0,01253	0,0114	0,0128	0,02929	0,00981	0,00972	0,00642
63	0,01	0,01261	0,0114	0,0129	0,02931	0,00979	0,00963	0,00636
64	0,01	0,01261	0,0114	0,0129	0,02952	0,00979	0,00961	0,0062
65	0,01	0,01267	0,0114	0,0129	0,02945	0,00978	0,00955	0,00622
66	0,01	0,01277	0,0123	0,013	0,02624	0,01137	0,01012	0,00798
67	0,00921	0,01031	0,0124	0,0102	0,04272	0,00708	0,01018	0,02907
68	0,00945	0,01733	0,0077	0,0164	0,0398	0,00451	0,00832	0,01798
69	0,00999	0,02269	0,0152	0,0225	0,03021	0,00735	-0,00178	-0,0077
70	0,00998	0,01388	0,0125	0,0138	0,02907	0,0105	0,01119	0,00123
71	0,00997	0,01375	0,0124	0,0137	0,02852	0,01055	0,01104	0,00144
72	0,00998	0,01352	0,0124	0,0134	0,02803	0,01059	0,01142	0,00147
73	0,00997	0,02069	0,015	0,0207	0,02926	0,00866	-0,001	-0,007
74	0,01001	0,02271	0,0167	0,0225	0,02875	0,00783	0,00134	-0,0132
75	0,00949	0,0171	0,0076	0,0161	0,03889	0,00301	0,0059	0,02224
76	0,00922	0,00909	0,0135	0,0093	0,04226	0,00863	0,0119	0,02557
77	0,00915	0,00699	0,0156	0,0074	0,04403	0,01022	0,01348	0,02368
78	0,00947	0,0166	0,0078	0,016	0,03906	0,0041	0,0112	0,01413

Lanjutan Lampiran 39

Kec	γ_{10}	γ_{11}	γ_{12}	γ_{13}	γ_{14}	γ_{15}	γ_{16}	γ_{17}
79	0,00999	0,02199	0,01676	0,02175	0,02838	0,00857	0,00348	-0,01390
80	0,00998	0,02134	0,01662	0,02105	0,02825	0,00906	0,00476	-0,01359
81	0,00998	0,02146	0,01667	0,02117	0,02840	0,00900	0,00475	-0,01384
82	0,00997	0,02165	0,01520	0,02179	0,03108	0,00769	-0,00284	-0,00766
83	0,00998	0,02125	0,01496	0,02123	0,02957	0,00807	-0,00148	-0,01390
84	0,01000	0,01339	0,01264	0,01373	0,02828	0,01177	0,00921	0,00688
85	0,01000	0,01343	0,01267	0,01378	0,02848	0,01180	0,00913	0,00676
86	0,01000	0,01340	0,01267	0,01375	0,02831	0,01180	0,00919	0,00685
87	0,01000	0,01337	0,01261	0,01370	0,02825	0,01172	0,00924	0,00692
88	0,00997	0,01348	0,01241	0,01352	0,02788	0,01090	0,01056	0,00111
89	0,00997	0,01351	0,01243	0,01355	0,02798	0,01093	0,01049	0,00101
90	0,00997	0,01345	0,01241	0,01348	0,02793	0,01092	0,01071	0,00100
91	0,00997	0,01345	0,01240	0,01347	0,02788	0,01091	0,01070	0,00106

Lanjutan Lampiran 39

Kec	γ_{20}	γ_{21}	γ_{22}	γ_{23}	γ_{24}	γ_{25}	γ_{26}	γ_{27}
1	0,01199	0,01077	0,01065	0,01073	0,00997	0,01021	0,01227	0,01073
2	0,01198	0,01074	0,01062	0,01067	0,0103	0,0102	0,01212	0,01039
3	0,01198	0,01069	0,01058	0,01058	0,01067	0,01019	0,01193	0,00982
4	0,01198	0,01077	0,01063	0,01052	0,01079	0,01028	0,01182	0,00885
5	0,01198	0,0107	0,01057	0,01045	0,01126	0,01026	0,01171	0,00864
6	0,01198	0,01085	0,01069	0,0105	0,01109	0,01041	0,01175	0,00801
7	0,01198	0,01097	0,01079	0,01053	0,01087	0,01052	0,01179	0,00747
8	0,01198	0,01083	0,01068	0,01049	0,01144	0,01043	0,01168	0,00794
9	0,01198	0,01093	0,01076	0,01052	0,01122	0,01051	0,01172	0,00757
10	0,01198	0,01087	0,01072	0,01053	0,01151	0,01047	0,01168	0,00786
11	0,01198	0,0108	0,01066	0,01049	0,01176	0,01043	0,01163	0,00795
12	0,01198	0,01087	0,01072	0,01054	0,01167	0,01048	0,01167	0,00792
13	0,01198	0,01089	0,01074	0,01056	0,01165	0,01049	0,01169	0,00789
14	0,012	0,01191	0,01209	0,01148	0,00285	0,01037	0,01484	0,0132
15	0,01198	0,01087	0,01073	0,01056	0,01179	0,01048	0,01168	0,00794
16	0,01199	0,01155	0,01193	0,01133	0,00326	0,0104	0,01541	0,01354
17	0,01199	0,0117	0,01199	0,01138	0,00308	0,01029	0,0152	0,01354
18	0,01198	0,01067	0,01055	0,01042	0,01248	0,01032	0,01155	0,00812
19	0,01198	0,01063	0,01052	0,01039	0,01255	0,0103	0,01151	0,00808
20	0,01198	0,01078	0,01065	0,01049	0,01216	0,01042	0,01161	0,00801
21	0,012	0,01187	0,01207	0,01146	0,00287	0,0103	0,01499	0,01342

Lanjutan Lampiran 39

Kec	γ_{20}	γ_{21}	γ_{22}	γ_{23}	γ_{24}	γ_{25}	γ_{26}	γ_{27}
22	0,012	0,01179	0,01205	0,01144	0,003	0,01036	0,01509	0,01348
23	0,01199	0,01172	0,01202	0,01142	0,0031	0,0104	0,0152	0,01351
24	0,01199	0,01165	0,012	0,0114	0,00317	0,01047	0,0153	0,01352
25	0,012	0,01174	0,01205	0,01145	0,00299	0,01047	0,01518	0,01347
26	0,012	0,01175	0,01206	0,01145	0,00285	0,0105	0,0152	0,01347
27	0,012	0,01179	0,01207	0,01146	0,00294	0,01043	0,01513	0,01347
28	0,012	0,01187	0,01209	0,01148	0,00281	0,01038	0,01504	0,01343
29	0,012	0,01188	0,0121	0,01149	0,00276	0,01038	0,01502	0,01342
30	0,012	0,01196	0,01214	0,01151	0,00261	0,01034	0,01495	0,01335
31	0,012	0,01193	0,01213	0,01151	0,00262	0,01038	0,01499	0,01337
32	0,012	0,01183	0,0121	0,01148	0,00271	0,01045	0,01512	0,01345
33	0,012	0,01185	0,01211	0,01149	0,0027	0,01044	0,01509	0,01344
34	0,012	0,01186	0,01212	0,01149	0,00256	0,01045	0,01511	0,01343
35	0,01199	0,01128	0,01188	0,01124	0,00288	0,01065	0,01616	0,01368
36	0,01199	0,01137	0,01192	0,01129	0,00303	0,01064	0,01593	0,01362
37	0,01199	0,01125	0,01187	0,01122	0,00273	0,01058	0,01625	0,01374
38	0,01199	0,01141	0,01192	0,01129	0,00295	0,01052	0,01587	0,01368
39	0,01199	0,01155	0,01197	0,01134	0,00295	0,01041	0,01562	0,01368
40	0,01199	0,01154	0,01198	0,01137	0,0031	0,01058	0,01558	0,01358
41	0,01199	0,01141	0,01194	0,01131	0,00305	0,01064	0,01585	0,01361
42	0,01199	0,01141	0,01194	0,01131	0,00304	0,01064	0,01585	0,01361
43	0,01199	0,01139	0,01194	0,01131	0,00289	0,01073	0,01592	0,01361
44	0,01199	0,01155	0,012	0,01139	0,00293	0,01063	0,01558	0,01357
45	0,01199	0,01161	0,01203	0,01141	0,00285	0,0106	0,01549	0,01357
46	0,01199	0,01154	0,01199	0,01138	0,003	0,01063	0,01561	0,01358
47	0,01199	0,01168	0,01206	0,01144	0,00281	0,01055	0,01537	0,01355
48	0,012	0,01171	0,01207	0,01145	0,00266	0,01055	0,01536	0,01355
49	0,01204	0,01584	0,01262	0,01376	0,00093	0,01115	-0,00844	0,0124
50	0,01196	0,0109	0,01014	0,00818	0,00643	0,01299	0,02179	0,00954
51	0,01199	0,01122	0,01052	0,01146	0,00591	0,00962	0,01672	0,01246
52	0,01196	0,01104	0,01031	0,0083	0,00627	0,01303	0,02186	0,00917
53	0,01196	0,01111	0,01039	0,00833	0,00606	0,01309	0,02209	0,00887
54	0,01199	0,01128	0,01057	0,01141	0,00596	0,00976	0,01706	0,01221
55	0,01199	0,01132	0,01048	0,0114	0,00645	0,00972	0,01675	0,01206
56	0,01199	0,01135	0,01049	0,01138	0,00645	0,00976	0,0169	0,01193
57	0,01199	0,0113	0,01057	0,01139	0,00599	0,00978	0,01716	0,01213
58	0,01196	0,01115	0,01044	0,00834	0,0063	0,01306	0,02151	0,00914
59	0,01196	0,01116	0,01045	0,00834	0,0062	0,01309	0,02167	0,00901
60	0,01196	0,0112	0,01049	0,00833	0,00608	0,01312	0,02176	0,00888

Lanjutan Lampiran 39

Kec	γ_{20}	γ_{21}	γ_{22}	γ_{23}	γ_{24}	γ_{25}	γ_{26}	γ_{27}
61	0,01196	0,01122	0,01052	0,00831	0,00601	0,01315	0,02179	0,0088
62	0,01199	0,01134	0,01055	0,01135	0,00602	0,00981	0,01736	0,01193
63	0,01199	0,01137	0,0105	0,01135	0,00632	0,00978	0,01715	0,01184
64	0,01199	0,01139	0,01049	0,01133	0,00627	0,00979	0,0173	0,01175
65	0,01199	0,0114	0,01047	0,01134	0,00648	0,00977	0,01709	0,01174
66	0,01199	0,01142	0,01196	0,01133	0,00286	0,01072	0,01588	0,01362
67	0,01192	0,01781	0,01506	0,00142	-0,00334	0,02674	-0,02707	0,01022
68	0,01201	0,01483	0,00585	0,0058	-0,00251	0,03171	-0,02154	0,01213
69	0,01204	0,01635	0,01259	0,01382	0,00024	0,01103	-0,01013	0,01252
70	0,01196	0,01074	0,00991	0,00801	0,00607	0,013	0,02266	0,00944
71	0,01196	0,01084	0,01004	0,00812	0,00624	0,013	0,02224	0,00945
72	0,01196	0,01091	0,01014	0,00825	0,00591	0,013	0,02274	0,00899
73	0,01204	0,01554	0,01267	0,01375	0,00107	0,01124	-0,00763	0,01236
74	0,01205	0,01475	0,0112	0,0138	0,00057	0,01362	-0,01048	0,01314
75	0,01197	0,01385	0,00822	0,00409	-0,0007	0,03092	-0,01789	0,01024
76	0,01193	0,01787	0,01524	0,00109	-0,00304	0,02686	-0,02695	0,01024
77	0,01193	0,01664	0,01671	0,00138	-0,00368	0,02704	-0,0285	0,01003
78	0,01201	0,01544	0,00597	0,00921	-0,00123	0,02607	-0,01814	0,01152
79	0,01205	0,01378	0,01052	0,01327	0,00124	0,01515	-0,00986	0,01305
80	0,01204	0,01319	0,01022	0,01295	0,00183	0,01584	-0,00902	0,01282
81	0,01204	0,01323	0,01018	0,01294	0,0017	0,01596	-0,00934	0,01288
82	0,01204	0,01623	0,01295	0,01417	-0,00043	0,01105	-0,00963	0,01239
83	0,012	0,016	0,013	0,014	0,001	0,011	-0,009	0,012
84	0,012	0,01196	0,01217	0,01153	0,00238	0,0104	0,01505	0,01341
85	0,012	0,012	0,01219	0,01154	0,0023	0,01039	0,01501	0,01335
86	0,012	0,01197	0,01218	0,01153	0,0023	0,01041	0,01505	0,01339
87	0,012	0,01194	0,01216	0,01152	0,00246	0,0104	0,01506	0,01343
88	0,01196	0,01124	0,01054	0,00831	0,00601	0,01314	0,02168	0,00884
89	0,01196	0,01126	0,01055	0,0083	0,00598	0,01315	0,02165	0,00882
90	0,01196	0,01127	0,01055	0,00828	0,0058	0,01321	0,02202	0,00859
91	0,01196	0,01126	0,01054	0,00829	0,00586	0,0132	0,02196	0,00865

Lampiran 40: Penaksir parameter $\boldsymbol{\beta}$ model GWBZIGPR

Kec	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}	β_{17}
1	-10,48562	0,03205	-0,04091	0,07059	0,01042	0,00724	0,04416	0,00648
2	-10,48563	0,03174	-0,04119	0,07028	0,00988	0,00694	0,04411	0,00635
3	-10,48563	0,03128	-0,04161	0,06981	0,0092	0,00649	0,04401	0,00616
4	-10,48564	0,03075	-0,04211	0,06927	0,00858	0,00596	0,04388	0,00589
5	-10,48564	0,03055	-0,0423	0,06907	0,00828	0,00576	0,04382	0,00577
6	-10,48565	0,0303	-0,04253	0,06883	0,00809	0,00551	0,04375	0,00562
7	-10,48565	0,0301	-0,04273	0,06863	0,00795	0,0053	0,04368	0,0055
8	-10,48565	0,03025	-0,04259	0,06878	0,00797	0,00545	0,04374	0,00559
9	-10,48565	0,03012	-0,04271	0,06865	0,0079	0,00532	0,04369	0,00551
10	-10,48565	0,03026	-0,04258	0,0688	0,00796	0,00546	0,04374	0,00559
11	-10,48565	0,03026	-0,0426	0,06879	0,00791	0,00546	0,04374	0,00559
12	-10,48565	0,03031	-0,04254	0,06885	0,00797	0,00551	0,04375	0,00561
13	-10,48565	0,03033	-0,04253	0,06887	0,00798	0,00553	0,04376	0,00561
14	-10,4856	0,03133	-0,04226	0,06998	0,0112	0,00607	0,04408	0,00698
15	-10,48565	0,03035	-0,04251	0,06889	0,00797	0,00555	0,04376	0,00562
16	-10,48561	0,03118	-0,04253	0,06978	0,01142	0,0058	0,04369	0,00637
17	-10,4856	0,03124	-0,0425	0,06986	0,01174	0,00585	0,04379	0,00665
18	-10,48565	0,03034	-0,04254	0,06888	0,00785	0,00554	0,04376	0,00564
19	-10,48565	0,03029	-0,04259	0,06883	0,0078	0,00549	0,04375	0,00562
20	-10,48565	0,03033	-0,04255	0,06886	0,00788	0,00552	0,04376	0,00561
21	-10,4856	0,03128	-0,04242	0,06992	0,01174	0,00593	0,04392	0,00687
22	-10,4856	0,03127	-0,04242	0,0699	0,01161	0,00593	0,04388	0,00673
23	-10,4856	0,03125	-0,04244	0,06987	0,0115	0,0059	0,04382	0,00658
24	-10,48561	0,03122	-0,04244	0,06984	0,01132	0,00589	0,04379	0,00644
25	-10,4856	0,03126	-0,04237	0,06989	0,01124	0,00597	0,0439	0,00659
26	-10,4856	0,03126	-0,04235	0,06989	0,01109	0,00598	0,04393	0,00659
27	-10,4856	0,03127	-0,04238	0,0699	0,01138	0,00596	0,04391	0,00667
28	-10,4856	0,03127	-0,04239	0,06992	0,01155	0,00595	0,04394	0,00681
29	-10,4856	0,03127	-0,04239	0,06992	0,01153	0,00596	0,04396	0,00684
30	-10,4856	0,03127	-0,0424	0,06992	0,01163	0,00594	0,044	0,00696
31	-10,4856	0,03127	-0,04238	0,06992	0,01149	0,00597	0,04401	0,00691
32	-10,4856	0,03126	-0,04236	0,06991	0,01125	0,00598	0,04397	0,00672
33	-10,4856	0,03127	-0,04236	0,06991	0,01132	0,00598	0,04397	0,00676
34	-10,4856	0,03126	-0,04235	0,06991	0,01121	0,00599	0,044	0,00678
35	-10,48562	0,03087	-0,04278	0,06945	0,01086	0,00547	0,04343	0,00579
36	-10,48561	0,03097	-0,04267	0,06956	0,0109	0,0056	0,04352	0,00591
37	-10,48561	0,03084	-0,04287	0,06941	0,01107	0,00539	0,04338	0,00581
38	-10,48561	0,031	-0,04272	0,06959	0,0113	0,00558	0,04352	0,00604
39	-10,48561	0,0311	-0,04266	0,0697	0,01163	0,00567	0,04361	0,00629

Lanjutan Lampiran 40

Kec	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}	β_{17}
40	-10,48561	0,03112	-0,04252	0,06973	0,01111	0,00578	0,04368	0,00619
41	-10,48561	0,03101	-0,04262	0,0696	0,01091	0,00565	0,04357	0,00596
42	-10,48561	0,03101	-0,04263	0,0696	0,01092	0,00565	0,04357	0,00596
43	-10,48562	0,03097	-0,04259	0,06957	0,01053	0,00566	0,04359	0,00587
44	-10,48561	0,03112	-0,04247	0,06974	0,01083	0,00583	0,04374	0,00618
45	-10,48561	0,03116	-0,04243	0,06978	0,01089	0,00588	0,0438	0,00629
46	-10,48561	0,03111	-0,04249	0,06972	0,01089	0,0058	0,04371	0,00615
47	-10,48561	0,0312	-0,0424	0,06983	0,01104	0,00592	0,04385	0,00643
48	-10,48561	0,0312	-0,04239	0,06984	0,01097	0,00594	0,04389	0,00648
49	-10,48564	0,02693	-0,04806	0,06541	0,03373	0,00097	0,04195	0,00733
50	-10,48562	0,02875	-0,04451	0,06746	0,02086	0,00262	0,04086	0,00463
51	-10,4856	0,03001	-0,04409	0,06856	0,01772	0,00398	0,04212	0,00427
52	-10,48563	0,02881	-0,04434	0,06754	0,02002	0,00278	0,04099	0,00474
53	-10,48563	0,02879	-0,04429	0,06752	0,01957	0,00281	0,04101	0,00478
54	-10,4856	0,03001	-0,04396	0,06858	0,01745	0,00406	0,04215	0,00418
55	-10,4856	0,03004	-0,04397	0,06863	0,01818	0,00406	0,04213	0,00427
56	-10,4856	0,03003	-0,04394	0,06863	0,01818	0,00408	0,04214	0,00424
57	-10,4856	0,03	-0,04395	0,06858	0,01751	0,00406	0,04215	0,00416
58	-10,48562	0,02893	-0,04427	0,06769	0,02015	0,00288	0,04118	0,00493
59	-10,48563	0,0289	-0,04426	0,06766	0,01998	0,00287	0,04116	0,00492
60	-10,48563	0,02889	-0,04426	0,06765	0,01989	0,00286	0,04117	0,00496
61	-10,48563	0,02889	-0,04426	0,06766	0,01991	0,00285	0,04119	0,005
62	-10,4856	0,02997	-0,04394	0,06857	0,01772	0,00404	0,04213	0,00412
63	-10,4856	0,02999	-0,04394	0,06861	0,01813	0,00406	0,04213	0,00418
64	-10,4856	0,02997	-0,04394	0,06859	0,01823	0,00404	0,04212	0,00414
65	-10,4856	0,02999	-0,04394	0,06862	0,01846	0,00406	0,04212	0,0042
66	-10,48562	0,03099	-0,04258	0,0696	0,01061	0,00569	0,04361	0,00592
67	-10,48559	0,02971	-0,05376	0,067	0,05739	-0,00711	0,03482	-0,02234
68	-10,48561	0,02775	-0,05057	0,06532	0,04331	-0,00289	0,03551	-0,00375
69	-10,48565	0,0265	-0,04879	0,06488	0,0362	0,00036	0,04129	0,00712
70	-10,48563	0,02841	-0,0448	0,06709	0,02122	0,00224	0,04046	0,00432
71	-10,48563	0,02859	-0,04463	0,06728	0,02091	0,00245	0,04067	0,00448
72	-10,48563	0,02848	-0,04455	0,06718	0,01985	0,00249	0,04062	0,00442
73	-10,48563	0,0271	-0,04775	0,06566	0,03272	0,00119	0,04237	0,00749
74	-10,48572	0,0257	-0,04859	0,06449	0,04179	-0,00114	0,03893	0,00239
75	-10,48566	0,02832	-0,05216	0,06559	0,04942	-0,00439	0,03497	-0,00926
76	-10,48558	0,0296	-0,05313	0,06727	0,05542	-0,0065	0,03536	-0,02122
77	-10,48557	0,02951	-0,05371	0,06738	0,05753	-0,00707	0,03522	-0,02371
78	-10,48563	0,02709	-0,04908	0,06514	0,04288	-0,00198	0,03472	-0,00552

Lanjutan Lampiran 40

Kec	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}	β_{17}
79	-10,48574	0,02547	-0,04867	0,06443	0,04422	-0,00209	0,03782	0,00082
80	-10,48574	0,02543	-0,04873	0,06448	0,04523	-0,00253	0,03736	0,00029
81	-10,48575	0,02536	-0,04879	0,06442	0,04562	-0,00263	0,03722	0,00006
82	-10,48563	0,02686	-0,04805	0,06539	0,03311	0,00144	0,04273	0,00716
83	-10,48564	0,02698	-0,04796	0,06548	0,03327	0,00117	0,04222	0,00731
84	-10,4856	0,03124	-0,0424	0,0699	0,01143	0,00594	0,04402	0,00691
85	-10,4856	0,03124	-0,0424	0,0699	0,01147	0,00594	0,04405	0,00698
86	-10,4856	0,03124	-0,04239	0,0699	0,01137	0,00595	0,04404	0,00693
87	-10,4856	0,03124	-0,04241	0,0699	0,01149	0,00594	0,044	0,00689
88	-10,48562	0,0289	-0,04428	0,06769	0,02015	0,00284	0,04123	0,00504
89	-10,48562	0,02891	-0,04428	0,0677	0,02024	0,00283	0,04125	0,00508
90	-10,48563	0,02883	-0,04429	0,06762	0,01987	0,0028	0,04117	0,00502
91	-10,48563	0,02885	-0,04428	0,06763	0,01991	0,00281	0,04118	0,00502

Lanjutan Lampiran 40

Kec	β_{20}	β_{21}	β_{22}	β_{23}	β_{24}	β_{25}	β_{26}	β_{27}
1	-30,50226	0,35157	0,01744	-0,03731	0,00959	-0,02674	0,0255	-0,01585
2	-30,50226	0,35165	0,01752	-0,03723	0,00948	-0,02665	0,02556	-0,01576
3	-30,50226	0,35184	0,01769	-0,03704	0,0094	-0,02648	0,02565	-0,01561
4	-30,50226	0,3521	0,01793	-0,03678	0,00941	-0,02625	0,02574	-0,01547
5	-30,50226	0,35209	0,01791	-0,03679	0,00925	-0,02624	0,02574	-0,01542
6	-30,50226	0,35215	0,01796	-0,03672	0,00923	-0,02619	0,02575	-0,01542
7	-30,50226	0,35222	0,01803	-0,03665	0,00925	-0,02612	0,02576	-0,01543
8	-30,50226	0,35203	0,01784	-0,03684	0,00901	-0,02628	0,02571	-0,01544
9	-30,50226	0,35208	0,01789	-0,03678	0,00905	-0,02624	0,02572	-0,01545
10	-30,50226	0,35191	0,01773	-0,03695	0,00885	-0,02638	0,02568	-0,01549
11	-30,50226	0,3519	0,01771	-0,03697	0,0088	-0,02638	0,02568	-0,01546
12	-30,50226	0,35181	0,01762	-0,03705	0,0087	-0,02646	0,02564	-0,01553
13	-30,50226	0,35176	0,01758	-0,03709	0,00864	-0,0265	0,02563	-0,01557
14	-30,50223	0,35242	0,01807	-0,03644	0,0075	-0,02645	0,02645	-0,01584
15	-30,50227	0,35171	0,01752	-0,03714	0,00856	-0,02654	0,02561	-0,01558
16	-30,50223	0,35224	0,01792	-0,03661	0,00755	-0,02656	0,02613	-0,01578
17	-30,50223	0,35229	0,01795	-0,03656	0,00772	-0,02656	0,02623	-0,01587
18	-30,50226	0,35173	0,01753	-0,03713	0,00845	-0,02652	0,02564	-0,01547
19	-30,50226	0,35178	0,01758	-0,03708	0,00849	-0,02647	0,02567	-0,01542
20	-30,50227	0,35171	0,01751	-0,03715	0,00848	-0,02654	0,02562	-0,01552
21	-30,50223	0,35235	0,018	-0,03651	0,00769	-0,02654	0,02635	-0,01589

Lanjutan Lampiran 40

Kec	β_{20}	β_{21}	β_{22}	β_{23}	β_{24}	β_{25}	β_{26}	β_{27}
22	-30,50223	0,35228	0,01795	-0,03657	0,00759	-0,02656	0,02628	-0,01587
23	-30,50223	0,35224	0,01792	-0,03661	0,00751	-0,02657	0,02622	-0,01584
24	-30,50223	0,35221	0,0179	-0,03664	0,00739	-0,02657	0,02618	-0,0158
25	-30,50223	0,35226	0,01795	-0,03658	0,00734	-0,02653	0,02627	-0,01582
26	-30,50223	0,3523	0,01799	-0,03654	0,00724	-0,02649	0,0263	-0,0158
27	-30,50223	0,35228	0,01795	-0,03657	0,00742	-0,02654	0,02629	-0,01584
28	-30,50223	0,35233	0,01799	-0,03653	0,00754	-0,02653	0,02635	-0,01587
29	-30,50223	0,35235	0,018	-0,03651	0,00753	-0,02652	0,02637	-0,01587
30	-30,50223	0,35241	0,01805	-0,03646	0,00762	-0,02649	0,02644	-0,01587
31	-30,50223	0,3524	0,01805	-0,03646	0,00752	-0,02648	0,02643	-0,01586
32	-30,50223	0,35234	0,01801	-0,03651	0,00734	-0,02649	0,02636	-0,01583
33	-30,50223	0,35235	0,01801	-0,0365	0,00739	-0,02649	0,02637	-0,01584
34	-30,50223	0,35239	0,01806	-0,03646	0,00732	-0,02645	0,02641	-0,01582
35	-30,50224	0,35228	0,01801	-0,03655	0,00721	-0,02639	0,02604	-0,01537
36	-30,50224	0,35223	0,01796	-0,0366	0,00717	-0,02646	0,02605	-0,01549
37	-30,50223	0,35233	0,01805	-0,0365	0,00736	-0,02638	0,02605	-0,01539
38	-30,50223	0,35225	0,01796	-0,03658	0,00741	-0,02649	0,02607	-0,01561
39	-30,50223	0,35225	0,01794	-0,03659	0,00756	-0,02655	0,02613	-0,01577
40	-30,50224	0,3522	0,01791	-0,03664	0,00722	-0,02653	0,02611	-0,01569
41	-30,50224	0,35222	0,01794	-0,03662	0,00714	-0,02647	0,02607	-0,01554
42	-30,50224	0,35222	0,01794	-0,03662	0,00715	-0,02648	0,02607	-0,01554
43	-30,50224	0,35227	0,01801	-0,03656	0,00688	-0,02639	0,0261	-0,01543
44	-30,50224	0,35225	0,01796	-0,03658	0,00701	-0,02647	0,02616	-0,01565
45	-30,50223	0,35227	0,01798	-0,03656	0,00704	-0,02647	0,02621	-0,0157
46	-30,50224	0,35222	0,01794	-0,03661	0,00706	-0,02649	0,02614	-0,01564
47	-30,50223	0,35228	0,01798	-0,03656	0,00714	-0,02649	0,02625	-0,01576
48	-30,50223	0,35233	0,01802	-0,03651	0,0071	-0,02645	0,0263	-0,01575
49	-30,5023	0,34545	0,01524	-0,04488	0,01773	-0,0194	0,02093	0,00162
50	-30,50229	0,34653	0,01453	-0,04327	0,02229	-0,02671	0,02576	-0,00472
51	-30,50223	0,35089	0,01627	-0,03803	0,01791	-0,02919	0,02407	-0,02062
52	-30,50229	0,34662	0,01456	-0,04316	0,02175	-0,02675	0,02577	-0,00479
53	-30,50229	0,34666	0,0146	-0,04313	0,02154	-0,0267	0,02576	-0,00464
54	-30,50223	0,35074	0,01618	-0,03818	0,01796	-0,02922	0,02415	-0,0203
55	-30,50223	0,35049	0,01595	-0,03846	0,01878	-0,02942	0,02401	-0,02036
56	-30,50223	0,35041	0,01589	-0,03854	0,01898	-0,02945	0,02404	-0,02026
57	-30,50223	0,35067	0,01613	-0,03825	0,01814	-0,02925	0,02416	-0,02025
58	-30,50229	0,34656	0,01443	-0,04326	0,02186	-0,02698	0,02596	-0,00464
59	-30,50229	0,34657	0,01446	-0,04325	0,0218	-0,02693	0,02594	-0,00456
60	-30,50229	0,34656	0,01445	-0,04327	0,02183	-0,02693	0,02597	-0,00441

Lanjutan Lampiran 40

Kec	β_{20}	β_{21}	β_{22}	β_{23}	β_{24}	β_{25}	β_{26}	β_{27}
61	-30,50229	0,34654	0,01442	-0,04331	0,02192	-0,02695	0,02602	-0,00427
62	-30,50223	0,35054	0,01602	-0,0384	0,01865	-0,02935	0,02416	-0,02016
63	-30,50223	0,35039	0,01588	-0,03856	0,01911	-0,02946	0,02408	-0,02018
64	-30,50223	0,35034	0,01584	-0,03862	0,0194	-0,02951	0,0241	-0,02014
65	-30,50223	0,35027	0,01577	-0,0387	0,01957	-0,02956	0,02404	-0,02016
66	-30,50224	0,35226	0,018	-0,03656	0,00689	-0,02641	0,02611	-0,01548
67	-30,50216	0,34489	0,01673	-0,04902	0,00907	0,004	-0,00422	0,01225
68	-30,50227	0,34038	0,01826	-0,05038	0,01427	0,00778	-0,01025	0,00367
69	-30,5023	0,34534	0,01562	-0,04494	0,0171	-0,01808	0,02048	0,00255
70	-30,50229	0,34649	0,01467	-0,04329	0,02291	-0,02633	0,02553	-0,00448
71	-30,50229	0,34653	0,0146	-0,04326	0,02251	-0,02653	0,02563	-0,00465
72	-30,50229	0,34671	0,0148	-0,04303	0,02188	-0,02633	0,02545	-0,0048
73	-30,50229	0,34546	0,01496	-0,04493	0,01858	-0,02023	0,02117	0,00134
74	-30,50235	0,34652	0,01735	-0,04319	0,01154	-0,01535	0,01582	0,00107
75	-30,50229	0,34143	0,01822	-0,04955	0,01108	0,00518	-0,00395	0,00499
76	-30,50216	0,3455	0,01592	-0,04881	0,01097	0,001	-0,00352	0,01187
77	-30,50214	0,34564	0,01527	-0,04925	0,01235	0,00116	-0,00475	0,01323
78	-30,50228	0,34074	0,01802	-0,05038	0,01481	0,00576	-0,01176	0,00445
79	-30,50236	0,34674	0,01789	-0,04282	0,01001	-0,01433	0,01357	0,00077
80	-30,50236	0,34675	0,01803	-0,04276	0,00959	-0,01409	0,01276	0,00062
81	-30,50236	0,34675	0,01811	-0,04277	0,00945	-0,0138	0,0124	0,00077
82	-30,50229	0,34497	0,01467	-0,04571	0,01976	-0,01912	0,02089	0,00307
83	-30,5023	0,34535	0,01505	-0,04507	0,01829	-0,01953	0,02096	0,00177
84	-30,50223	0,35243	0,01808	-0,03642	0,00744	-0,02645	0,02647	-0,01585
85	-30,50222	0,35247	0,01811	-0,03639	0,00749	-0,02643	0,02651	-0,01585
86	-30,50222	0,35246	0,01811	-0,03639	0,00742	-0,02643	0,02649	-0,01584
87	-30,50223	0,35241	0,01806	-0,03645	0,00747	-0,02647	0,02644	-0,01586
88	-30,50229	0,34649	0,01436	-0,04338	0,02214	-0,02705	0,02609	-0,0042
89	-30,50229	0,34646	0,01433	-0,04342	0,02226	-0,02709	0,02614	-0,0041
90	-30,50229	0,34649	0,0144	-0,04338	0,02209	-0,02693	0,02607	-0,00398
91	-30,50229	0,3465	0,0144	-0,04337	0,02206	-0,02695	0,02605	-0,00406

Lampiran 41: *P-value* Parameter $\boldsymbol{\gamma}$ model GWBZIGPR

Kec	γ_{10}	γ_{11}	γ_{12}	γ_{13}	γ_{14}	γ_{15}	γ_{16}	γ_{17}
1	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
2	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
3	0,0000	0,0000	0,0000	0,0000	0,0002	0,0000	0,0000	0,0000
4	0,0000	0,0000	0,0000	0,0000	0,0005	0,0000	0,0000	0,0000
5	0,0000	0,0000	0,0000	0,0000	0,0006	0,0000	0,0000	0,0000
6	0,0000	0,0000	0,0000	0,0000	0,0006	0,0000	0,0000	0,0000
7	0,0000	0,0000	0,0000	0,0000	0,0006	0,0000	0,0000	0,0000
8	0,0000	0,0000	0,0000	0,0000	0,0004	0,0000	0,0000	0,0000
9	0,0000	0,0000	0,0000	0,0000	0,0004	0,0000	0,0000	0,0000
10	0,0000	0,0000	0,0000	0,0000	0,0002	0,0000	0,0000	0,0000
11	0,0000	0,0000	0,0000	0,0000	0,0002	0,0000	0,0000	0,0000
12	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
13	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
14	0,0000	0,0000	0,0000	0,0000	0,0002	0,0000	0,0000	0,0012
15	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
16	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
17	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0001
18	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
19	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
20	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
21	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0006
22	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0002
23	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0001
24	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0001
25	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0002
26	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0002
27	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0003
28	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0006
29	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0007
30	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0015
31	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0013
32	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0005
33	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0007
34	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0009
35	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
36	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
37	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
38	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
39	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

Lanjutan Lampiran 41

Kec	γ_{10}	γ_{11}	γ_{12}	γ_{13}	γ_{14}	γ_{15}	γ_{16}	γ_{17}
40	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
41	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
42	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
43	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
44	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
45	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0001
46	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
47	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0001
48	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0002
49	0,0000	0,0000	0,0000	0,0000	0,0004	0,0005	0,8413	0,2404
50	0,0000	0,0000	0,0000	0,0000	0,0003	0,0000	0,0000	0,7168
51	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0007
52	0,0000	0,0000	0,0000	0,0000	0,0003	0,0000	0,0000	0,7036
53	0,0000	0,0000	0,0000	0,0000	0,0004	0,0000	0,0000	0,7205
54	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0019
55	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0017
56	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0029
57	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0026
58	0,0000	0,0000	0,0000	0,0000	0,0003	0,0000	0,0000	0,7264
59	0,0000	0,0000	0,0000	0,0000	0,0003	0,0000	0,0000	0,7351
60	0,0000	0,0000	0,0000	0,0000	0,0003	0,0000	0,0000	0,7558
61	0,0000	0,0000	0,0000	0,0000	0,0003	0,0000	0,0000	0,7765
62	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0054
63	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0050
64	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0079
65	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0064
66	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
67	0,0000	0,5667	0,5254	0,5107	0,0042	0,6738	0,6901	0,3688
68	0,0000	0,0084	0,2279	0,0151	0,0013	0,3658	0,5024	0,4101
69	0,0000	0,0000	0,0000	0,0000	0,0005	0,0038	0,7689	0,2184
70	0,0000	0,0000	0,0000	0,0000	0,0004	0,0000	0,0000	0,7934
71	0,0000	0,0000	0,0000	0,0000	0,0003	0,0000	0,0000	0,7494
72	0,0000	0,0000	0,0000	0,0000	0,0006	0,0000	0,0000	0,7495
73	0,0000	0,0000	0,0000	0,0000	0,0003	0,0001	0,8580	0,2290
74	0,0000	0,0000	0,0000	0,0000	0,0051	0,0559	0,8975	0,2476
75	0,0000	0,0070	0,2251	0,0135	0,0018	0,6165	0,7020	0,3922
76	0,0000	0,6040	0,4764	0,5437	0,0038	0,6099	0,6279	0,4090
77	0,0000	0,7077	0,4402	0,6544	0,0053	0,5763	0,5935	0,4654
78	0,0000	0,0044	0,2018	0,0083	0,0015	0,4040	0,3558	0,4863

Lanjutan Lampiran 41

Kec	γ_{10}	γ_{11}	γ_{12}	γ_{13}	γ_{14}	γ_{15}	γ_{16}	γ_{17}
79	0,0000	0,0000	0,0000	0,0000	0,0047	0,0241	0,7425	0,1540
80	0,0000	0,0000	0,0000	0,0000	0,0037	0,0089	0,6508	0,1190
84	0,0000	0,0000	0,0000	0,0000	0,0040	0,0110	0,6549	0,1159
85	0,0000	0,0000	0,0000	0,0000	0,0006	0,0030	0,6567	0,2102
86	0,0000	0,0000	0,0000	0,0000	0,0004	0,0006	0,7989	0,2333
87	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0020
88	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0030
89	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0024
90	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0016
91	0,0000	0,0000	0,0000	0,0000	0,0003	0,0000	0,0000	0,8003

Lanjutan Lampiran 41

Kec	γ_{20}	γ_{21}	γ_{22}	γ_{23}	γ_{24}	γ_{25}	γ_{26}	γ_{27}
1	0,0000	0,0000	0,0000	0,0000	0,0394	0,0000	0,0000	0,0888
2	0,0000	0,0000	0,0000	0,0000	0,0392	0,0000	0,0000	0,1045
3	0,0000	0,0000	0,0000	0,0000	0,0478	0,0000	0,0000	0,1410
4	0,0000	0,0000	0,0000	0,0000	0,0744	0,0000	0,0000	0,2197
5	0,0000	0,0000	0,0000	0,0000	0,0582	0,0000	0,0000	0,2239
6	0,0000	0,0000	0,0000	0,0000	0,0719	0,0000	0,0000	0,2756
7	0,0000	0,0000	0,0000	0,0000	0,0886	0,0000	0,0000	0,3276
8	0,0000	0,0000	0,0000	0,0000	0,0484	0,0000	0,0000	0,2611
9	0,0000	0,0000	0,0000	0,0000	0,0599	0,0000	0,0000	0,2978
10	0,0000	0,0000	0,0000	0,0000	0,0355	0,0000	0,0000	0,2518
11	0,0000	0,0000	0,0000	0,0000	0,0298	0,0000	0,0000	0,2391
12	0,0000	0,0000	0,0000	0,0000	0,0240	0,0000	0,0000	0,2309
13	0,0000	0,0000	0,0000	0,0000	0,0207	0,0000	0,0000	0,2263
14	0,0000	0,0000	0,0000	0,0000	0,6567	0,0000	0,0000	0,0549
15	0,0000	0,0000	0,0000	0,0000	0,0158	0,0000	0,0000	0,2129
16	0,0000	0,0000	0,0000	0,0000	0,5857	0,0000	0,0000	0,0374
17	0,0000	0,0000	0,0000	0,0000	0,6149	0,0000	0,0000	0,0371
18	0,0000	0,0000	0,0000	0,0000	0,0095	0,0000	0,0000	0,1923
19	0,0000	0,0000	0,0000	0,0000	0,0113	0,0000	0,0000	0,2021
20	0,0000	0,0000	0,0000	0,0000	0,0117	0,0000	0,0000	0,2007
21	0,0000	0,0000	0,0000	0,0000	0,6447	0,0000	0,0000	0,0415
22	0,0000	0,0000	0,0000	0,0000	0,6208	0,0000	0,0000	0,0382
23	0,0000	0,0000	0,0000	0,0000	0,6027	0,0000	0,0000	0,0363

Lanjutan lampiran 41

Kec	γ_{20}	γ_{21}	γ_{22}	γ_{23}	γ_{24}	γ_{25}	γ_{26}	γ_{27}
24	0,0000	0,0000	0,0000	0,0000	0,0394	0,0000	0,0000	0,0888
25	0,0000	0,0000	0,0000	0,0000	0,0392	0,0000	0,0000	0,1045
26	0,0000	0,0000	0,0000	0,0000	0,0478	0,0000	0,0000	0,1410
27	0,0000	0,0000	0,0000	0,0000	0,0744	0,0000	0,0000	0,2197
28	0,0000	0,0000	0,0000	0,0000	0,0582	0,0000	0,0000	0,2239
29	0,0000	0,0000	0,0000	0,0000	0,0719	0,0000	0,0000	0,2756
30	0,0000	0,0000	0,0000	0,0000	0,0886	0,0000	0,0000	0,3276
31	0,0000	0,0000	0,0000	0,0000	0,0484	0,0000	0,0000	0,2611
32	0,0000	0,0000	0,0000	0,0000	0,0599	0,0000	0,0000	0,2978
33	0,0000	0,0000	0,0000	0,0000	0,0355	0,0000	0,0000	0,2518
34	0,0000	0,0000	0,0000	0,0000	0,0298	0,0000	0,0000	0,2391
35	0,0000	0,0000	0,0000	0,0000	0,0240	0,0000	0,0000	0,2309
36	0,0000	0,0000	0,0000	0,0000	0,0207	0,0000	0,0000	0,2263
37	0,0000	0,0000	0,0000	0,0000	0,6567	0,0000	0,0000	0,0549
38	0,0000	0,0000	0,0000	0,0000	0,0158	0,0000	0,0000	0,2129
39	0,0000	0,0000	0,0000	0,0000	0,5857	0,0000	0,0000	0,0374
40	0,0000	0,0000	0,0000	0,0000	0,6149	0,0000	0,0000	0,0371
41	0,0000	0,0000	0,0000	0,0000	0,0095	0,0000	0,0000	0,1923
42	0,0000	0,0000	0,0000	0,0000	0,0113	0,0000	0,0000	0,2021
43	0,0000	0,0000	0,0000	0,0000	0,0117	0,0000	0,0000	0,2007
44	0,0000	0,0000	0,0000	0,0000	0,6447	0,0000	0,0000	0,0415
45	0,0000	0,0000	0,0000	0,0000	0,6208	0,0000	0,0000	0,0382
46	0,0000	0,0000	0,0000	0,0000	0,6027	0,0000	0,0000	0,0363
47	0,0000	0,0000	0,0000	0,0000	0,5897	0,0000	0,0000	0,0362
48	0,0000	0,0000	0,0000	0,0000	0,6171	0,0000	0,0000	0,0405
49	0,0000	0,0000	0,0000	0,0000	0,6380	0,0000	0,0000	0,0446
50	0,0000	0,0000	0,0000	0,0000	0,6259	0,0000	0,0000	0,0402
51	0,0000	0,0000	0,0000	0,0000	0,6473	0,0000	0,0000	0,0422
52	0,0000	0,0000	0,0000	0,0000	0,6553	0,0000	0,0000	0,0437
53	0,0000	0,0000	0,0000	0,0000	0,6795	0,0000	0,0000	0,0478
54	0,0000	0,0000	0,0000	0,0000	0,6776	0,0000	0,0000	0,0483
55	0,0000	0,0000	0,0000	0,0000	0,6595	0,0000	0,0000	0,0459
56	0,0000	0,0000	0,0000	0,0000	0,6618	0,0000	0,0000	0,0457
57	0,0000	0,0000	0,0000	0,0000	0,6829	0,0000	0,0000	0,0503
58	0,0000	0,0000	0,0000	0,0000	0,6319	0,0000	0,0000	0,0489
59	0,0000	0,0000	0,0000	0,0000	0,6070	0,0000	0,0000	0,0434
60	0,0000	0,0000	0,0000	0,0000	0,6560	0,0000	0,0000	0,0505

Lannjutan Lampiran 41

Kec	γ_{20}	γ_{21}	γ_{22}	γ_{23}	γ_{24}	γ_{25}	γ_{26}	γ_{27}
61	0,0000	0,0000	0,0000	0,0007	0,3683	0,0000	0,0000	0,2649
62	0,0000	0,0000	0,0000	0,0000	0,3612	0,0000	0,0000	0,1065
63	0,0000	0,0000	0,0000	0,0000	0,3330	0,0000	0,0000	0,1013
64	0,0000	0,0000	0,0000	0,0000	0,3400	0,0000	0,0000	0,1087
65	0,0000	0,0000	0,0000	0,0000	0,3197	0,0000	0,0000	0,1019
66	0,0000	0,0000	0,0000	0,0000	0,6272	0,0000	0,0000	0,0460
67	0,0000	0,4127	0,5814	0,9492	0,7700	0,4301	0,3285	0,4842
68	0,0000	0,0096	0,2674	0,3543	0,7849	0,0117	0,2229	0,2913
69	0,0000	0,0000	0,0000	0,0002	0,9759	0,0005	0,5093	0,1816
70	0,0000	0,0000	0,0000	0,0038	0,4153	0,0000	0,0000	0,2722
71	0,0000	0,0000	0,0000	0,0023	0,3820	0,0000	0,0000	0,2495
72	0,0000	0,0000	0,0000	0,0018	0,4128	0,0000	0,0000	0,2996
73	0,0000	0,0000	0,0000	0,0001	0,8815	0,0002	0,5876	0,1550
74	0,0000	0,0028	0,0025	0,0003	0,9462	0,0184	0,5018	0,2052
75	0,0000	0,0220	0,2709	0,6764	0,9398	0,0083	0,3068	0,3988
76	0,0000	0,3570	0,5597	0,9579	0,7808	0,4111	0,3104	0,4680
77	0,0000	0,3843	0,5478	0,9454	0,7429	0,4288	0,2972	0,5090
78	0,0000	0,0018	0,2003	0,0500	0,8888	0,0034	0,2716	0,3069
79	0,0000	0,0036	0,0046	0,0006	0,8815	0,0117	0,5223	0,2062
80	0,0000	0,0040	0,0052	0,0008	0,8244	0,0078	0,5536	0,2037
81	0,0000	0,0040	0,0058	0,0009	0,8382	0,0081	0,5425	0,2085
82	0,0000	0,0000	0,0000	0,0001	0,9554	0,0006	0,5195	0,2046
83	0,0000	0,0000	0,0000	0,0001	0,9231	0,0002	0,5516	0,1653
84	0,0000	0,0000	0,0000	0,0000	0,7074	0,0000	0,0000	0,0511
85	0,0000	0,0000	0,0000	0,0000	0,7213	0,0000	0,0000	0,0545
86	0,0000	0,0000	0,0000	0,0000	0,7197	0,0000	0,0000	0,0542
87	0,0000	0,0000	0,0000	0,0000	0,6956	0,0000	0,0000	0,0481
88	0,0000	0,0000	0,0000	0,0007	0,3670	0,0000	0,0000	0,2592
89	0,0000	0,0000	0,0000	0,0007	0,3709	0,0000	0,0000	0,2609
90	0,0000	0,0000	0,0000	0,0008	0,3912	0,0000	0,0000	0,2872
91	0,0000	0,0000	0,0000	0,0008	0,3845	0,0000	0,0000	0,2803

Lampiran 42: *P-value* Parameter $\boldsymbol{\beta}$ model GWBZIGPR

Kec	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}	β_{17}
1	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
2	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
3	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
4	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
5	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
6	0,0000	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000
7	0,0000	0,0000	0,0000	0,0000	0,0000	0,0002	0,0000	0,0000
8	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
9	0,0000	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000
10	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
11	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
12	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
13	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
14	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
15	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
16	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
17	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
18	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
19	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
20	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
21	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
22	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
23	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
24	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
25	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
26	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
27	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
28	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
29	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
30	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
31	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
32	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
33	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
34	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
35	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
36	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
37	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
38	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
39	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

Lanjutan Lampiran 42

Kec	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}	β_{17}
40	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
41	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
42	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
43	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
44	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
45	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
46	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
47	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
48	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
49	0,0000	0,0000	0,0000	0,0000	0,0003	0,6431	0,0000	0,0006
50	0,0000	0,0000	0,0000	0,0000	0,0017	0,1729	0,0000	0,0046
51	0,0000	0,0000	0,0000	0,0000	0,0032	0,0154	0,0000	0,0062
52	0,0000	0,0000	0,0000	0,0000	0,0017	0,1359	0,0000	0,0039
53	0,0000	0,0000	0,0000	0,0000	0,0019	0,1320	0,0000	0,0044
54	0,0000	0,0000	0,0000	0,0000	0,0028	0,0117	0,0000	0,0065
55	0,0000	0,0000	0,0000	0,0000	0,0016	0,0102	0,0000	0,0044
56	0,0000	0,0000	0,0000	0,0000	0,0015	0,0098	0,0000	0,0046
57	0,0000	0,0000	0,0000	0,0000	0,0026	0,0116	0,0000	0,0066
58	0,0000	0,0000	0,0000	0,0000	0,0010	0,1122	0,0000	0,0026
59	0,0000	0,0000	0,0000	0,0000	0,0011	0,1152	0,0000	0,0029
60	0,0000	0,0000	0,0000	0,0000	0,0012	0,1168	0,0000	0,0031
61	0,0000	0,0000	0,0000	0,0000	0,0011	0,1183	0,0000	0,0032
62	0,0000	0,0000	0,0000	0,0000	0,0022	0,0121	0,0000	0,0072
63	0,0000	0,0000	0,0000	0,0000	0,0016	0,0107	0,0000	0,0056
64	0,0000	0,0000	0,0000	0,0000	0,0016	0,0117	0,0000	0,0062
65	0,0000	0,0000	0,0000	0,0000	0,0013	0,0105	0,0000	0,0051
66	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
67	0,0000	0,0000	0,0000	0,0000	0,0341	0,5212	0,0131	0,4381
68	0,0000	0,0000	0,0000	0,0000	0,0071	0,5590	0,0000	0,6563
69	0,0000	0,0000	0,0000	0,0000	0,0004	0,8758	0,0000	0,0016
70	0,0000	0,0000	0,0000	0,0000	0,0038	0,2842	0,0000	0,0137
71	0,0000	0,0000	0,0000	0,0000	0,0025	0,2181	0,0000	0,0079
72	0,0000	0,0000	0,0000	0,0000	0,0044	0,2173	0,0000	0,0120
73	0,0000	0,0000	0,0000	0,0000	0,0002	0,5572	0,0000	0,0005
74	0,0000	0,0000	0,0000	0,0000	0,0060	0,8051	0,0000	0,7251
75	0,0000	0,0000	0,0000	0,0000	0,0245	0,5168	0,0000	0,4664
76	0,0000	0,0000	0,0000	0,0000	0,0320	0,5476	0,0082	0,4384
77	0,0000	0,0000	0,0000	0,0000	0,0312	0,5279	0,0095	0,3706
78	0,0000	0,0000	0,0000	0,0000	0,0082	0,6975	0,0000	0,4722

Lanjutan Lampiran 42

Kec	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}	β_{17}
79	0,0000	0,0000	0,0000	0,0000	0,0038	0,6558	0,0000	0,9057
80	0,0000	0,0000	0,0000	0,0000	0,0029	0,5835	0,0000	0,9668
84	0,0000	0,0000	0,0000	0,0000	0,0029	0,5731	0,0000	0,9926
85	0,0000	0,0000	0,0000	0,0000	0,0003	0,4908	0,0000	0,0012
86	0,0000	0,0000	0,0000	0,0000	0,0003	0,5692	0,0000	0,0006
87	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
88	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
89	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
90	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
91	0,0000	0,0000	0,0000	0,0000	0,0009	0,1188	0,0000	0,0030

Lanjutan Lampiran 42

Kec	β_{20}	β_{21}	β_{22}	β_{23}	β_{24}	β_{25}	β_{26}	β_{27}
1	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
2	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
3	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
4	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
5	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
6	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
7	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
8	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
9	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
10	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
11	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
12	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
13	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
14	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
15	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
16	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
17	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
18	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
19	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
20	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
21	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
22	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
23	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
24	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

Lanjutan Lampiran 42

Kec	β_{20}	β_{21}	β_{22}	β_{23}	β_{24}	β_{25}	β_{26}	β_{27}
25	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
26	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
27	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
28	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
29	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
30	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
31	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
32	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
33	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
34	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
35	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
36	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
37	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
38	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
39	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
40	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
41	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
42	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
43	0,0000	0,0000	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
44	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
45	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
46	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
47	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
48	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
49	0,0000	0,0000	0,0000	0,0000	0,0868	0,0000	0,0000	0,8975
50	0,0000	0,0000	0,0000	0,0000	0,0228	0,0000	0,0000	0,6969
51	0,0000	0,0000	0,0000	0,0000	0,0462	0,0000	0,0000	0,0000
52	0,0000	0,0000	0,0000	0,0000	0,0235	0,0000	0,0000	0,6867
53	0,0000	0,0000	0,0000	0,0000	0,0256	0,0000	0,0000	0,6964
54	0,0000	0,0000	0,0000	0,0000	0,0434	0,0000	0,0000	0,0000
55	0,0000	0,0000	0,0000	0,0000	0,0318	0,0000	0,0000	0,0000
56	0,0000	0,0000	0,0000	0,0000	0,0304	0,0000	0,0000	0,0000
57	0,0000	0,0000	0,0000	0,0000	0,0416	0,0000	0,0000	0,0000
58	0,0000	0,0000	0,0000	0,0000	0,0190	0,0000	0,0000	0,6927
59	0,0000	0,0000	0,0000	0,0000	0,0203	0,0000	0,0000	0,6987
60	0,0000	0,0000	0,0000	0,0000	0,0206	0,0000	0,0000	0,7095
61	0,0000	0,0000	0,0000	0,0000	0,0204	0,0000	0,0000	0,7195
62	0,0000	0,0000	0,0000	0,0000	0,0378	0,0000	0,0000	0,0000
63	0,0000	0,0000	0,0000	0,0000	0,0312	0,0000	0,0000	0,0000

Lanjutan Lampiran 42

Kec	β_{20}	β_{21}	β_{22}	β_{23}	β_{24}	β_{25}	β_{26}	β_{27}
64	0,0000	0,0000	0,0000	0,0000	0,0303	0,0000	0,0000	0,0000
65	0,0000	0,0000	0,0000	0,0000	0,0270	0,0000	0,0000	0,0000
66	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
67	0,0000	0,0000	0,0586	0,0000	0,7025	0,9006	0,8762	0,6089
68	0,0000	0,0000	0,0001	0,0000	0,4109	0,5730	0,5453	0,8134
69	0,0000	0,0000	0,0000	0,0000	0,1292	0,0002	0,0000	0,8493
70	0,0000	0,0000	0,0000	0,0000	0,0316	0,0000	0,0000	0,7278
71	0,0000	0,0000	0,0000	0,0000	0,0269	0,0000	0,0000	0,7082
72	0,0000	0,0000	0,0000	0,0000	0,0353	0,0000	0,0000	0,7004
73	0,0000	0,0000	0,0000	0,0000	0,0635	0,0000	0,0000	0,9141
74	0,0000	0,0000	0,0000	0,0000	0,4569	0,0091	0,0350	0,9392
75	0,0000	0,0000	0,0001	0,0000	0,5428	0,6984	0,8648	0,7424
76	0,0000	0,0000	0,0569	0,0000	0,6213	0,9735	0,8915	0,6011
77	0,0000	0,0000	0,0766	0,0000	0,5827	0,9701	0,8590	0,5701
78	0,0000	0,0000	0,0000	0,0000	0,3642	0,6550	0,4797	0,7601
79	0,0000	0,0000	0,0000	0,0000	0,5094	0,0219	0,1000	0,9558
80	0,0000	0,0000	0,0000	0,0000	0,5162	0,0274	0,1288	0,9636
81	0,0000	0,0000	0,0000	0,0000	0,5261	0,0327	0,1471	0,9555
82	0,0000	0,0000	0,0000	0,0000	0,0645	0,0000	0,0000	0,8152
83	0,0000	0,0000	0,0000	0,0000	0,0746	0,0000	0,0000	0,8877
84	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
85	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
86	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
87	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
88	0,0000	0,0000	0,0000	0,0000	0,0186	0,0000	0,0000	0,7245
89	0,0000	0,0000	0,0000	0,0000	0,0181	0,0000	0,0000	0,7316
90	0,0000	0,0000	0,0000	0,0000	0,0211	0,0000	0,0000	0,7413
91	0,0000	0,0000	0,0000	0,0000	0,0207	0,0000	0,0000	0,7349

Lampiran 43: Syntax BZIGPR

```
data=read.csv(file.choose(),header=T,sep=";")
library(gamlss)
library(MASS)
library(hypergeo)
library(goft)
library(maxLik)
maxit=1000
y=as.matrix(data[,1:2])
y1=as.matrix(data[,1])
y2=as.matrix(data[,2])
n=nrow(data)
x=as.matrix(cbind(rep(1,n),data[,-c(1,2)]))
p=ncol(x)

#### dummy
b=matrix(0,n,1)
c=matrix(0,n,1)
d=matrix(0,n,1)
e=matrix(0,n,1)
a=matrix(0,n,4)
for(i in 1:n)
{
  if ((y[i,1]==0)&(y[i,2]!=0))
  {
    b[i]=1
    c[i]=0
    d[i]=0
    e[i]=0
  }
  if ((y[i,1]!=0)&(y[i,2]==0))
  {
    b[i]=0
    c[i]=1
    d[i]=0
    e[i]=0
  }
  if ((y[i,1]!=0)&(y[i,2]!=0))
  {
    b[i]=0
    c[i]=0
    d[i]=1
    e[i]=0
  }
  e[i]=1-b[i]-c[i]-d[i]
  a[i,]=cbind(b[i],c[i],d[i],e[i])
  }
b=a[,1]
c=a[,2]
d=a[,3]
e=a[,4]
#mencari parameter awal
library(AER)
p1 <- glm(y1 ~x1+x2+x3+x4+x5+x6+x7 , family = poisson, data =
data)
p2 <- glm(y2 ~x1+x2+x3+x4+x5+x6+x7 , family = poisson, data =
data)
```

```
p1$coefficients
p2$coefficients
beta1=as.matrix(p1$coefficients)
beta2=as.matrix(p2$coefficients)
gamma1=as.matrix(c(rep(0.01,1,8)))
gamma2=as.matrix(c(rep(0.012,1,8)))
psi1=1.065181
psi2=1.209661
eta=0.269
##  iterasi
set.seed(100)
eps=100
iterasi=0
while (eps > 0.0000000001 && iterasi<100)
{
 ### turunan pertama ################################

 tetaawal<-  as.matrix(c(gamma1,gamma2,beta1,beta2,psi1,psi2,eta))
  g1=matrix(0,n,1)
  g2=matrix(0,n,1)
  dg1beta1=matrix(0,n,p)
  dg2beta2=matrix(0,n,p)
  dg1psi1=matrix(0,n,1)
  dg2psi2=matrix(0,n,1)
  t=exp(-1)
  b1x<-x%*%beta1
  b2x<-x%*%beta2
  eb1=exp(b1x)
  eb2=exp(b2x)
  gamma1x<-x%*%gamma1
  gamma2x<-x%*%gamma2
  egam1=exp(gamma1x)
  egam2=exp(gamma2x)
  lambda1=eb1/(1+(psi1*eb1))
  lambda2=eb2/(1+(psi2*eb2))
  pi1=egam1/(1+egam1)
  pi2=egam2/(1+egam2)
  pi10=1/(1+egam1)
  pi20=1/(1+egam2)
for(i in 1:n)
{
#turunan g1
  g1[i]=exp(lambda1[i]*(t-1))
  g2[i]=exp(lambda2[i]*(t-1))
dg1beta1[i,]=(t-1)*1/(1-(psi1*t*(-
lambda1[i]*x[i,])*(1+(psi1*lambda1[i]))))*g1[i]
dg1psi1[i]=(-lambda1[i]^2)*t*(t-1)*1/(1-(psi1*t*(-
lambda1[i])))*g1[i]
dg2beta2[i,]=(t-1)*1/(1-(psi2*t*(-
lambda2[i]*x[i,])*(1+(psi2*lambda2[i]))))*g2[i]
dg2psi2[i]=(-lambda2[i]^2)*t*(t-1)*1/(1-(psi2*t*(-
lambda2[i])))*g2[i]
  }
## turunan gamma1 #########################################
  A11=matrix(0,n,p)
  A211=matrix(0,n,p)
  A2211=matrix(0,n,p)
  A2221=matrix(0,n,p)
```

```
A2231=matrix(0,n,p)
A221=matrix(0,n,p)
A21=matrix(0,n,p)
A1=matrix(0,n,p)
B11=matrix(0,n,p)
B211=matrix(0,n,p)
B221=matrix(0,n,p)
B21=matrix(0,n,p)
B1=matrix(0,n,p)
C1=matrix(0,n,p)
D1=matrix(0,n,p)
pers.11=matrix(0,n,p)
pers.21=matrix(0,n,p)
pers.31=matrix(0,n,p)
pers.41=matrix(0,n,p)
dgamma1=matrix(0,n,p)
sumdgamma1=matrix(0,p)
dg=matrix(0,n,1)
dh=matrix(0,n,1)
z=matrix(0,n,1)
u=matrix(0,n,1)
for (j in 1:p)
{
  for (i in 1:n)
  {
    #persamaan 1
    dg[i]=1+(eta*(1-g1[i])*(1-g2[i]))
    A11[i,]=egam1[i]*x[i,]*pi10[i]*pi20[i]
    A211[i,]=egam1[i]*(egam1[i]+exp(-lambda2[i]))
    A2211[i,]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
    A2221[i,]=egam2[i]*(exp(-lambda1[i]))
    A2231[i,]=exp(-lambda1[i]-lambda2[i])*dg[i]
    A221[i,]=A2211[i]+A2221[i]+A2231[i]
    A21[i,]=A211[i,]/A221[i,]
    A1[i,]=A11[i,]+A21[i,]
    pers.11[i,]=(1-b[i]-c[i]-d[i])*A1[i,]

    #persamaman2
    u[i]=1+(eta*(1-g1[i])*((exp(-y2[i]))-g2[i]))
    B11[i,]=-pi1[i]*x[i,]
    B211[i,]=egam1[i]*x[i,]
    B221[i,]=egam1[i]+((exp(-lambda1[i]))*u[i])
    B21[i,]=B211[i,]/B221[i,]
    B1[i,]=B11[i,]+B21[i,]
    pers.21[i,]=(b[i])*B1[i,]

    #persamaan3
    C1[i,]=-pi1[i]*x[i,]
    pers.31[i,]=(c[i])*C1[i,]

    #persamaan4
    D1[i,]=-pi1[i]*x[i,]
    pers.41[i,]=(d[i])*D1[i,]
    }
}
dgamma1=pers.11+pers.21+pers.31+pers.41
dgamma=ifelse(is.nan(dgamma1),0,dgamma1)
dgamma1
```

```
#####  turunan gamma2 #####################################
  A12=matrix(0,n,p)
  A212=matrix(0,n,p)
  A2212=matrix(0,n,p)
  A2222=matrix(0,n,p)
  A2232=matrix(0,n,p)
  A222=matrix(0,n,p)
  A22=matrix(0,n,p)
  A2=matrix(0,n,p)
  C12=matrix(0,n,p)
  C212=matrix(0,n,p)
  C222=matrix(0,n,p)
  C22=matrix(0,n,p)
  B2=matrix(0,n,p)
  C12=matrix(0,n,p)
  C2=matrix(0,n,p)
  D2=matrix(0,n,p)
  pers.12=matrix(0,n,p)
  pers.22=matrix(0,n,p)
  pers.32=matrix(0,n,p)
  pers.42=matrix(0,n,p)
  dgamma2=matrix(0,n,p)
  sumdgamma2=matrix(0,p)
  dg=matrix(0,n,1)
  dh=matrix(0,n,1)
  z=matrix(0,n,1)
  u=matrix(0,n,1)
  for (j in 1:p)
  {
    for (i in 1:n)
    {
      #persamaan 1
      dg[i]=1+(eta*(1-g1[i])*(1-g2[i]))
      A12[i,]=-pi2[i]*x[i,]*pi20[i]
      A212[i,]=egam2[i]*(egam1[i]+exp(-lambda1[i]))*x[i,]
      A2212[i,]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
      A2222[i,]=egam2[i]*exp(-lambda1[i])
      A2232[i,]=exp(-lambda1[i]-lambda2[i])*dg[i]
      A222[i,]=A2212[i,]+A2222[i,]+A2232[i,]
      A22[i,]=A212[i,]/A222[i,]
      A2[i,]=A12[i,]+A22[i,]
      pers.12[i,]=(1-b[i]-c[i]-d[i])*A2[i,]

      #persamaman2
      B2[i,]=-pi2[i]*x[i,]
      pers.22[i,]=(b[i])*B2[i,]

      #persamaan3
      C12[i,]=-pi2[i]*x[i,]
      C212[i,]=egam2[i]*x[i,]
      C222[i,]=egam2[i]+((exp(-lambda2[i]))*(1+(eta*(1-
g1[i])*(exp(-y2[i])-g2[i]))))
      C22[i,]=C212[i,]/C222[i,]
      C2[i,]=C12[i,]+C22[i,]
      pers.32[i,]=(c[i])*C2[i,]

      #persamaan4
      D2[i,]=-pi2[i]*x[i,]
```

```
      pers.42[i,]=(d[i])*D2[i,]
      }
  }
  dgamma2=pers.12+pers.22+pers.32+pers.42
  dgamma=ifelse(is.nan(dgamma2),0,dgamma2)
  dgamma2

  ##### turunan beta1 #######################################
  s11=matrix(0,n,1)
  s12=matrix(0,n,1)
  s13=matrix(0,n,1)
  s=matrix(0,n,1)
  A13=matrix(0,n,p)
  A23=matrix(0,n,p)
  A3=matrix(0,n,p)
  B13=matrix(0,n,p)
  B23=matrix(0,n,p)
  B3=matrix(0,n,p)
  C13=matrix(0,n,p)
  C23=matrix(0,n,p)
  C330=matrix(0,n,p)
  C33=matrix(0,n,p)
  C3=matrix(0,n,p)
  D13=matrix(0,n,p)
  D23=matrix(0,n,p)
  D33=matrix(0,n,p)
  D43=matrix(0,n,p)
  D3=matrix(0,n,p)
  dg=matrix(0,n,1)
  dh=matrix(0,n,1)
  z=matrix(0,n,1)
  u=matrix(0,n,1)
  pers.13=matrix(0,n,p)
  pers.23=matrix(0,n,p)
  pers.33=matrix(0,n,p)
  pers.43=matrix(0,n,p)
  dbeta1=matrix(0,n,p)
  sumdbeta1=matrix(0,p)
  for (i in 1:n)
  {
    for (j in 1:p)
    {
      # Persamaan1
      dg[i]=1+(eta*(1-g1[i])*(1-g2[i]))
      s11[i]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
      s12[i]=egam2[i]*exp(-lambda1[i])
      s13[i]=(exp(-lambda1[i]-lambda2[i]))*dg[i]
      s[i]=s11[i]+s12[i]+s13[i]
      A13[i,]=egam2[i]*exp(-lambda1[i])*((-
lambda1[i]*x[i,])+((lambda1[i])^2*psi1*x[i,]))
      A23[i,]=exp(-lambda1[i]-lambda2[i])*((-lambda1[i]*x[i,])-
((lambda1[i])^2*psi1*x[i,]))*t*dg1beta1[i]
      A3[i,]=1/s[i]*(A13[i,]+A23[i,])
      pers.13[i,]=(1-b[i]-c[i]-d[i])*A3[i,]

      # Persamaan2
      u[i]=1+(eta*(1-g1[i])*((exp(-y2[i]))-g2[i]))
```

```
      B13[i,]=exp(-lambda1[i])*u[i]*((-lambda1[i]*x[i,])-
((lambda1[i])^2*x[i,]*psi1))
      B23[i,]=egam1[i]+(exp(-lambda1[i])*u[i])
      B3[i,]=B13[i,]/B23[i,]*dg1beta1[i]
      pers.23[i,]=(b[i])*B3[i,]

      # Persamaan3
      C13[i,]=y1[i]*(x[i,]-(lambda1[i]*psi1*x[i,]))
      C23[i,]=-
(lambda1[i])*x[i,]*(1+(psi1*y1[i]))*(1+(lambda1[i]*psi1))
      C330[i,]=egam2[i]+(exp(-lambda2[i])*(1+(eta*(1-g2[i])*(exp(-
y1[i])-g1[i]))))
      C33[i,]=1/C330[i,]*dg1beta1[i]
      C3[i,]=C13[i,]+C23[i,]+C33[i,]
      pers.33[i,]=(c[i])*C3[i,]

      #persamaan4
      D13[i,]=y1[i]*x[i,]
      D23[i,]=-(lambda1[i])*y1[i]*psi1*x[i,]
      D33[i,]=(-(lambda1[i]*(1+(y1[i]*psi1)*x[i,])))*(1-
(lambda1[i]*psi1))
      z[i]=1+(eta*(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i]))
      D43[i,]=1/z[i]*dg1beta1[i]
      D3[i,]=D13[i,]+D23[i,]+D33[i,]+D43[i,]
      pers.43[i,]=(d[i])*D3[i,]
      }
  }
  dbeta1=pers.13+pers.23+pers.33+pers.43
  dbeta1=ifelse(is.nan(dbeta1),0,dbeta1)
  dbeta1

  ##### turunan beta2########################################
  A14=matrix(0,n,p)
  A24=matrix(0,n,p)
  A4=matrix(0,n,p)
  B14=matrix(0,n,p)
  B24=matrix(0,n,p)
  B34=matrix(0,n,p)
  B440=matrix(0,n,p)
  B44=matrix(0,n,p)
  B4=matrix(0,n,p)
  C14=matrix(0,n,p)
  C24=matrix(0,n,p)
  C4=matrix(0,n,p)
  D14=matrix(0,n,p)
  D24=matrix(0,n,p)
  D34=matrix(0,n,p)
  D44=matrix(0,n,p)
  D4=matrix(0,n,p)
  dg=matrix(0,n,1)
  dh=matrix(0,n,1)
  z=matrix(0,n,1)
  u=matrix(0,n,1)
  pers.14=matrix(0,n,p)
  pers.24=matrix(0,n,p)
  pers.34=matrix(0,n,p)
  pers.44=matrix(0,n,p)
  dbeta2=matrix(0,n,p)
```

```
  sumdbeta2=matrix(0,p)
  for (i in 1:n)
  {
    for (j in 1:p)
    {
      # Persamaan1
      dg[i]=1+(eta*(1-g1[i])*(1-g2[i]))
      s11[i]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
      s12[i]=egam2[i]*exp(-lambda1[i])
      s13[i]=(exp(-lambda1[i]-lambda2[i]))*dg[i]
      s[i]=s11[i]+s12[i]+s13[i]
      A14[i,]=egam1[i]*exp(-lambda2[i])*((-
lambda2[i]*x[i,])+((lambda2[i])^2*psi2*x[i,]))
      A24[i,]=exp(-lambda2[i]-lambda1[i])*((-lambda2[i]*x[i,])-
((lambda2[i])^2*psi2*x[i,]))*t*dg2beta2[i]
      A4[i,]=1/s[i]*(A14[i,]+A24[i,])
      pers.14[i,]=(1-b[i]-c[i]-d[i])*A4[i,]

      # persamaan2
      u[i]=1+(eta*(1-g1[i])*((exp(-y2[i]))-g2[i]))
      B14[i,]=y2[i]*(x[i,]-(lambda2[i]*psi2*x[i,]))
      B24[i,]=-
(lambda2[i])*x[i,]*(1+(psi2*y2[i]))*(1+(lambda2[i]*psi2))
      B440[i,]=egam1[i]+(exp(-lambda1[i])*u[i])
      B44[i,]=1/B440[i,]*dg2beta2[i]
      B4[i,]=B14[i,]+B24[i,]+B34[i,]+B44[i,]
      pers.24[i,]=(b[i])*B4[i,]

      #persamaan3
      dh[i]=1+(eta*(1-g2[i])*((exp(-y1[i]))-g1[i]))
      C14[i,]=exp(-lambda2[i])*u[i]*((-lambda2[i]*x[i,])-
((lambda2[i])^2*x[i,]*psi2))
      C24[i,]=egam2[i]+(exp(-lambda2[i])*dh[i])
      C4[i,]=C14[i,]/C24[i,]*dg2beta2[i]
      pers.34[i,]=(c[i]*C4[i,])

      #persamaan4
      z[i]=1+(eta*(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i]))
      D14[i,]=(y2[i]*x[i,])
      D24[i,]=-lambda2[i]*y2[i]*psi2*x[i,]
      D34[i,]=(-(lambda2[i]*(1+(y2[i]*psi2)*x[i,])))*(1-
(lambda2[i]*psi2))
      D44[i,]=(1/z[i]*dg2beta2[i])
      D4[i,]=D14[i,]+D24[i,]+D34[i,]+D44[i,]
      pers.44[i,]=(d[i])*D4[i,]
      }
  }
  dbeta2=pers.14+pers.24+pers.34+pers.44
  dbeta2=ifelse(is.nan(dbeta2),0,dbeta2)
  dbeta2

  #### turunan psi1 ######################################
  A15=matrix(0,n,1)
  A25=matrix(0,n,1)
  A35=matrix(0,n,1)
  A5=matrix(0,n,1)
  B15=matrix(0,n,1)
  B25=matrix(0,n,1)
```

```
B5=matrix(0,n,1)
C15=matrix(0,n,1)
C25=matrix(0,n,1)
C35=matrix(0,n,1)
C415=matrix(0,n,1)
C425=matrix(0,n,1)
C45=matrix(0,n,1)
C5=matrix(0,n,1)
D15=matrix(0,n,1)
D25=matrix(0,n,1)
D35=matrix(0,n,1)
D45=matrix(0,n,1)
D5=matrix(0,n,1)
dg=matrix(0,n,1)
dh=matrix(0,n,1)
z=matrix(0,n,1)
u=matrix(0,n,1)
pers.15=matrix(0,n,1)
pers.25=matrix(0,n,1)
pers.35=matrix(0,n,1)
pers.45=matrix(0,n,1)
dpsi1=matrix(0,n,1)
sumdpsi1=matrix(0,1)
for (i in 1:n)
{
  # Persamaan1
  dg[i]=1+(eta*(1-g1[i])*(1-g2[i]))
  z[i]=1+(eta*(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i]))
  s11[i]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
  s12[i]=egam2[i]*exp(-lambda1[i])
  s13[i]=(exp(-lambda1[i]-lambda2[i]))*dg[i]
  s[i]=s11[i]+s12[i]+s13[i]
  A15[i]=1/s[i]
  A25[i]=egam2[i]*(-lambda1[i])*(-lambda2[i]^2)
  A35[i]=exp(-lambda1[i]-
lambda2[i])*dg[i]*lambda1[i]^2*dg1psi1[i]
  A5[i]=(A25[i,]*A35[i,])/A15[i]
  pers.15[i]=(1-b[i]-c[i]-d[i])*A5[i]

  # Persamaan2
  u[i]=1+(eta*(1-g1[i])*((exp(-y2[i]))-g2[i]))
  B15[i]=exp(-lambda1[i])*u[i]*(eb1[i]^2)
  B25[i]=egam1[i]+(exp(-lambda1[i])*u[i]*(1+(psi1*eb1[i])^2))
  B5[i]=B15[i]/B25[i]*dg1psi1[i]
  pers.25[i]=(b[i])*B5[i]

  # Persamaan3
  dh[i]=1+(eta*(1-g2[i])*((exp(-y1[i]))-g1[i]))
  C15[i]=(y1[i]*lambda1[i])
  C25[i]=((y1[i]-1)/(1+(psi1*y1[i]))*y1[i])
  C35[i]=-lambda1[i]*(y1[i]+(lambda1[i]*(1+(psi1*y1[i]))))
  C415[i]=egam2[i]+(exp(-lambda2[i])*dh[i])
  C425[i]=exp(-lambda2[i])*dh[i]
  C45[i]=C425[i]/ C415[i]*dg1psi1[i]
  C5[i]=C15[i]+C25[i]+C35[i]+C45[i]
  pers.35[i]=(c[i])*C5[i]
```

```
    # Persamaan4
    z[i]=1+(eta*(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i]))
    D15[i]=-lambda1[i]*y1[i]
    D25[i]=(y1[i]-1)*pi10[i]*y1[i]
    D35[i]=-lambda1[i]*(y1[i]-(lambda1[i]*(1+(psi1*y1[i]))))
    D45[i]=1/z[i]*dg1psi1[i]
    D5[i]=D15[i]+D25[i]+D35[i]+D45[i]
    pers.45[i]=(d[i])*D5[i]
  }
  dpsi1=pers.15+pers.25+pers.35+pers.45
  dpsi1=ifelse(is.nan(dpsi1),0,dpsi1)
  sumdpsi1<-sum(dpsi1)
  dpsi1
  sumdpsi1

  #### turunan psi2 #######################################
  A16=matrix(0,n,1)
  A26=matrix(0,n,1)
  A36=matrix(0,n,1)
  A6=matrix(0,n,1)
  B16=matrix(0,n,1)
  B26=matrix(0,n,1)
  B36=matrix(0,n,1)
  B46=matrix(0,n,1)
  B6=matrix(0,n,1)
  C16=matrix(0,n,1)
  C26=matrix(0,n,1)
  C6=matrix(0,n,1)
  D16=matrix(0,n,1)
  D26=matrix(0,n,1)
  D36=matrix(0,n,1)
  D46=matrix(0,n,1)
  D6=matrix(0,n,1)
  dg=matrix(0,n,1)
  dh=matrix(0,n,1)
  z=matrix(0,n,1)
  u=matrix(0,n,1)
  pers.16=matrix(0,n,1)
  pers.26=matrix(0,n,1)
  pers.36=matrix(0,n,1)
  pers.46=matrix(0,n,1)
  dpsi2=matrix(0,n,1)
  sumdpsi2=matrix(0,1)
  for (i in 1:n)
  {
    # Persamaan1
    dg[i]=1+(eta*(1-g1[i])*(1-g2[i]))
    s11[i]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
    s12[i]=egam2[i]*exp(-lambda1[i])
    s13[i]=(exp(-lambda1[i]-lambda2[i]))*dg[i]
    s[i]=s11[i]+s12[i]+s13[i]
    A16[i]=1/s[i]
    A26[i]=egam1[i]*exp(-lambda2[i])*-(lambda2[i]^2)
    A36[i]=exp(-lambda1[i]-
lambda2[i])*dg[i]*(lambda2[i]^2)*dg2psi2[i]
    A6[i]=(A26[i]+A36[i])/A16[i]
    pers.16[i]=(1-b[i]-c[i]-d[i])*A6[i]
```

```
  # Persamaan2
  u[i]=1+(eta*(1-g1[i])*((exp(-y2[i]))-g2[i]))
  B16[i]=-y2[i]*lambda2[i]
  B26[i]=(y2[i]-1)/(1+(psi2*y2[i]))*y2[i]
  B36[i]=-lambda2[i]*(y2[i]-(lambda2[i]*(1+(psi2*y2[i]))))
  B46[i]=exp(-lambda1[i])*u[i]*dg1psi1[i]
  B6[i]=B16[i]+B26[i]+B36[i]+B46[i]
  pers.26[i]=(b[i])*B6[i]

  # Persamaan3
  dh[i]=1+(eta*(1-g2[i])*((exp(-y1[i]))-g1[i]))
  C16[i]=exp(-lambda2[i])*dh[i]*eb2[i]^2
  C26[i]=egam2[i]+(exp(-lambda2[i])*dh[i]*(1+(psi2*eb2[i])^2))
  C6[i]=C16[i]/C26[i]*dg2psi2[i]
  pers.36[i]=(c[i])*C26[i]

  # Persamaan4
  z[i]=1+(eta*(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i]))
  D16[i]=-lambda2[i]*y2[i]
  D26[i]=(y2[i]-1)*y2[i]/(1+(psi2*y2[i]))
  D36[i]=-lambda2[i]*(y2[i]-(lambda2[i]*(1+(psi2*y2[i]))))
  D46[i]=1/z[i]*dg2psi2[i]
  D6[i]=D16[i]+D26[i]+D36[i]+D46[i]
  pers.46[i]=(d[i])*D6[i]

}
dpsi2=pers.15+pers.26+pers.36+pers.46
dpsi2=ifelse(is.nan(dpsi2),0,dpsi2)
sumdpsi2<-sum(dpsi2)
dpsi2
sumdpsi2

### turunan eta ########################################
A17=matrix(0,n,1)
A27=matrix(0,n,1)
A7=matrix(0,n,1)
B17=matrix(0,n,1)
B27=matrix(0,n,1)
B7=matrix(0,n,1)
C17=matrix(0,n,1)
C27=matrix(0,n,1)
C7=matrix(0,n,1)
D7=matrix(0,n,1)
dg=matrix(0,n,1)
dh=matrix(0,n,1)
z=matrix(0,n,1)
u=matrix(0,n,1)
s=matrix(0,n,1)
pers.17=matrix(0,n,1)
pers.27=matrix(0,n,1)
pers.37=matrix(0,n,1)
pers.47=matrix(0,n,1)
deta=matrix(0,n,1)
sumdeta=matrix(0,1)
for (i in 1:n)
{
  # Persamaan1
  dg[i]=1+(eta*(1-g1[i])*(1-g2[i]))
```

```
  s11[i]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
  s12[i]=egam2[i]*exp(-lambda1[i])
  s13[i]=(exp(-lambda1[i]-lambda2[i]))*dg[i]
  s[i]=s11[i]+s12[i]+s13[i]
  A17[i]=1/s[i]
  A27[i]=exp(-lambda1[i]-lambda2[i])*dg[i]
  A7[i]=A17[i]*A27[i]
  pers.17[i]=(1-b[i]-c[i]-d[i])*A7[i]

  # Persamaan2
  u[i]=1+(eta*(1-g1[i])*((exp(-y2[i]))-g2[i]))
  B17[i]=exp(-lambda1[i])*(1-g1[i])*(exp(-y2[i])-g2[i])
  B27[i]=egam1[i]+(exp(-lambda1[i])*u[i])
  B7[i]=B17[i]/B27[i]
  pers.27[i]=(b[i])*B7[i]

  # Persamaan3
  dh[i]=1+(eta*(1-g2[i])*((exp(-y1[i]))-g1[i]))
  C17[i]=exp(-lambda2[i])*(1-g2[i])*(exp(-y1[i])-g1[i])
  C27[i]=egam2[i]+(exp(-lambda2[i])*dh[i])
  C7[i]=C17[i]/C27[i]
  pers.37[i]=(c[i])*C7[i]

  # Persamaan4
  z[i]=1+(eta*(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i]))
  D7[i]=(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i])/z[i]
  pers.47[i]=(d[i])*D7[i]

}
deta=pers.17+pers.27+pers.37+pers.47
deta=ifelse(is.nan(deta),0,deta)
sumdeta<-sum(deta)
deta
sumdeta

b111=rep(NA,p)
for (l in 1:p)
  for(i in 1:n)
  {
    b111[l]=sum(dbeta1[,l])
  }
b112=rep(NA,p)
for (l in 1:p)
  for(i in 1:n)
  {
    b112[l]=sum(dbeta2[,l])
  }
g111=rep(NA,p)
for (l in 1:p)
  for(i in 1:n)
  {
    g111[l]=sum(dgamma1[,l])
  }
g112=rep(NA,p)
for (l in 1:p)
  for(i in 1:n)
  {
    g112[l]=sum(dgamma2[,l])
```

```
    }
  #parameter
  I<-cbind(dgamma1,dgamma2,dbeta1,dbeta2,dpsi1,dpsi2,deta)
  g<-as.matrix(c(g111,g112,b111,b112,sumdpsi1,sumdpsi2,sumdeta))
  Hinv<-ginv(t(I)%*%(I))
  teta1<-tetaawal-(Hinv%*%g)

  eps<-(norm(teta1-tetaawal,type=c("2")))
  pgamma1<-teta1[1:8]
  pgamma2<-teta1[9:16]
  #gama1<-ifelse(gama1>0,gama1,0.1)
  pbeta1<-teta1[17:24]
  pbeta2<-teta1[25:32]
  ppsi1<-teta1[33]
  ppsi2<-teta1[34]
  peta=teta1[35]

teta1=as.matrix(c(pgamma1,pgamma2,pbeta1,pbeta2,ppsi1,ppsi2,peta))
  error=abs(teta1-tetaawal)
  tetaawal<-teta1
  eps=sqrt(sum(error^2))
  iterasi<-iterasi+1
  print(teta1)
  print(eps)
  cat(paste('BGR (Iterasi ke : ',iterasi,', epsilon :
',eps,')\n',sep=""))
}

teta1
se<-sqrt(diag(Hinv))
Z<-teta1/se
pval=2*pnorm(abs(Z), lower.tail = FALSE)
{{
cat("==========================================================","
\n")
  cat(" Estimasi Parameter BZIGPR di bawah H1","\n")
cat("==========================================================","
\n")
}
table=cbind(teta1=round(teta1,9),se,Z=round(Z,4),pval=round(pval,4
))
  table2=data.frame(table,stringsAsFactors=FALSE)

rownames(table2)=c("gamma1.0","gamma1.1","gamma1.2","gamma1.3","ga
mma1.4","gamma1.5","gamma1.6","gamma1.7","gamma2.0","gamma2.1","ga
mma2.2","gamma2.3","gamma2.4","gamma2.5","gamma2.6","gamma2.7","in
tercept.b1","b1.1","b1.2","b1.3","b1.4","b1.5","b1.6","b1.7","inte
rcept.b2","b2.1","b2.2","b2.3","b2.4","b2.5","b2.6","b2.7","psi.1"
,"psi.2","eta")
  colnames(table2)=c("Estimate","Std Error","Z value","P-value")
  write.table(table2,"D:/BZIGPR.txt",sep="\t")
  print(table2)

{cat("==========================================================",
"\n")
  }}
pb1x=x%*%pbeta1
pb2x=x%*%pbeta2
```

```
y1had=exp(pb1x)
y2had=exp(pb2x)
MSE1=mean((y1had-y1)^2)
MSE2=mean((y2had-y2)^2)
MSE1
MSE2

################ uji serentak BZIGPR ######################
LOMEGA
LH0_Beta
LH0_Gamma
p=7
n=91
df=2*p
csqtbl=qchisq(0.05,df)
Gsquare_Beta=2*(LOMEGA-LLH0_BETA)
Gsquare_Gamma=2*(LOMEGA-LLH0_GAMMA)

AIC=(-2*LOMEGA)+(2*df)
AICc=AIC+((2*df*(df+1))/(n-df-1))
```

Lampiran 44: Syntax Uji Heterogenitas

```
data=read.csv(file.choose(),header=T,sep=";")
e1=as.matrix((data[,1]))
e2=as.matrix((data[,2]))
x=as.matrix((data[,-c(1,2)]))
n=nrow(data)
k=ncol(x)
e11=e1^2
e22=e2^2
E=cbind(e11,e22)
G=lm(E~x[,1]+x[,2]+x[,3]+x[,4]+x[,5]+x[,6]+x[,7])
g=G$fit
h=G$coef
covar1=(t(E-g)%*%(E-g))/n
det1=det(covar1)
#g0=cbind(e1-mean(e1),e2-mean(e2))
g0=cbind(e1-h[1,1],e2-h[1,2])
covar0=(t(g0)%*%g0)/n
det0=det(covar0)
#Gvalue=-(n-2-1-0.5*3)*log(det1/det0)
Gvalue=-(n-k-1-0.5*(2-k+1))*log(det1/det0)
chitab=qchisq(0.95,(2*k),lower.tail=TRUE)
```

Lampiran 45: Syntax Pembobot

```
data=read.csv(file.choose(),header=T,sep=";")
library(MASS)
library(spgwr)
maxit=1000
y=as.matrix(data[,1:2])
y1=as.matrix(data[,1])
y2=as.matrix(data[,2])
n=nrow(data)
x=as.matrix(cbind(rep(1,n),data[,3:9]))
p=ncol(x)
u=data$u
v=data$v

#################  bandwith #############################
p<-ncol(x)
coord  = as.matrix(cbind(data[c(10,11)]))
u      = as.matrix(coord[,1])
v      = as.matrix(coord[,2])
ban.y1.fg<-ggwr.sel(y1~x1+x2+x3+x4+x5+x6+x7,data = data, coords =
coord,adapt = FALSE, gweight = gwr.Gauss,verbose = FALSE)
b.y1.fg<-ggwr(y1~x1+x2+x3+x4+x5+x6+x7,data = data, coords = coord,
adapt = ban.y1.fg)
g.fg1<-as.matrix(b.y1.fg$bandwidth)
ban.y2.fg<-ggwr.sel(y2~x1+x2+x3+x4+x5+x6+x7,data = data, coords =
coord,adapt = FALSE, gweight = gwr.Gauss,verbose = FALSE)
b.y2.fg<-ggwr(y2~x1+x2+x3+x4+x5+x6+x7,data = data, coords = coord,
adapt = ban.y2.fg)
g.fg2<-as.matrix(b.y2.fg$bandwidth)
ban.y1.ab<-ggwr.sel(y1~x1+x2+x3+x4+x5+x6+x7,data = data, coords =
coord,adapt = TRUE, gweight = gwr.bisquare,verbose = FALSE)
b.y1.ab<-ggwr(y1~x1+x2+x3+x4+x5+x6+x7,data = data, coords = coord,
adapt = ban.y1.ab)
g.ab1<-as.matrix(b.y1.ab$bandwidth)
ban.y2.ab<-ggwr.sel(y2~x1+x2+x3+x4+x5+x6+x7,data = data, coords =
coord,adapt = TRUE, gweight = gwr.bisquare,verbose = FALSE)
b.y2.ab<-ggwr(y2~x1+x2+x3+x4+x5+x6+x7,data = data, coords =
coord,adapt = ban.y2.ab)
g.ab2<-as.matrix(b.y2.ab$bandwidth)
ban.y1.ag<-ggwr.sel(y1~x1+x2+x3+x4+x5+x6+x7,data = data, coords =
coord,adapt = TRUE, gweight = gwr.Gauss,verbose = FALSE)
b.y1.ag<-ggwr(y1~x1+x2+x3+x4+x5+x6+x7,data = data, coords =
coord,adapt = ban.y1.ag)
g.ag1<-as.matrix(b.y1.ag$bandwidth)

ban.y2.ag<-ggwr.sel(y2~x1+x2+x3+x4+x5+x6+x7,data = data, coords =
coord,adapt = TRUE, gweight = gwr.Gauss,verbose = FALSE)
b.y2.ag<-ggwr(y2~x1+x2+x3+x4+x5+x6+x7,data = data, coords =
coord,adapt = ban.y2.ag)
g.ag2<-as.matrix(b.y2.ag$bandwidth)
ban.y1.fb<-ggwr.sel(y1~x1+x2+x3+x4+x5+x6+x7,data = data, coords =
coord,adapt = FALSE, gweight = gwr.bisquare,verbose = FALSE)
b.y1.fb<-ggwr(y1~x1+x2+x3+x4+x5+x6+x7,data = data, coords = coord,
adapt = ban.y1.fb)
g.fb1<-as.matrix(b.y1.fb$bandwidth)
ban.y2.fb<-ggwr.sel(y2~x1+x2+x3+x4+x5+x6+x7,data = data, coords =
coord,adapt = FALSE, gweight = gwr.bisquare,verbose = FALSE)
```

```
b.y2.fb<-ggwr(y2~x1+x2+x3+x4+x5+x6+x7,data = data, coords = coord,
adapt = ban.y2.fb)
g.fb2<-as.matrix(b.y2.fb$bandwidth)
cbind(g.fg1,g.fg2,g.ab1,g.ab2,g.ag1,g.ag2,g.fb1,g.fb2)

#### menghitung jarak euclidian ########################
d=matrix(0,n,n)
for (i in 1:n)
{
  for (j in 1:n)
  {
    d[i,j]=sqrt((u[i]-u[j])^2+(v[i]-v[j])^2)
  }}
write.table(d,file = "D:/euclidian.txt",sep="\t")

###### fixed gaussian
#Rumus bandwidth fixed gaussian Y1
g=matrix(c(g.fg1),nrow = n,ncol=n)
wfg1=exp(-0.5*((d/g)^2))
beta.gcv1<-matrix(0,n,p)
Y1hat.fg<-matrix(0,n,1)
W=wfg1
GCV1=matrix(0,91,1)
for (i in 1:n)
{
  W1=matrix(W[,i]) ; #W1 = (W1[-i]);
beta.gcv1[i,]=ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])%*%
y1
  Y1hat.fg[i]=(x[i,])%*%(beta.gcv1[i,])
  trc<-x%*%ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])
  GCV1[i]=sum(n*(((y1-Y1hat.fg)*(y1-Y1hat.fg))/(n-
sum(diag(trc)))^2))
}
g=g.fg1
b.GCV1=cbind(g,GCV1)
b.GCV1=b.GCV1[order(b.GCV1[,2]),]

opt.b.fg1=b.GCV1[1,1]
sum(GCV1)

#Rumus bandwidth fixed gaussian Y2
g=matrix(c(g.fg2),nrow = n,ncol=n)
wfg2=matrix(0,n,n)
wfg2=exp(-0.5*((d/g)^2))
beta.gcv2<-matrix(0,n,p)
Y2hat.fg<-matrix(0,n,1)
W=wfg2
GCV2=matrix(0,91,1)
for (i in 1:n)
  #for ( j in 1:n){
{

beta.gcv2[i,]=ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])%*%
y2
  Y2hat.fg[i]=(x[i,])%*%(beta.gcv2[i,])
  trc<-x%*%ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])
  GCV2[i]=sum(n*(((y2-Y2hat.fg)*(y2-Y2hat.fg))/(n-
sum(diag(trc)))^2))
```

```
}
g=g.fg2
b.GCV2=cbind(g,GCV2)
b.GCV2=b.GCV2[order(b.GCV2[,2]),]
opt.b.fg2=b.GCV2[1,1]

######## Adaptive bisquare
#menghitung GCV untuk bandwith Adaptive bisquare (Y1)
g=g.ab1
wab1=matrix(0,n,n)

for (ii in 1:n)
{ for (jj in 1:n)
{ #Rumus bandwidth bisquare
  if (d[ii,jj]<=g[ii])
  { wab1[ii,jj]=(1-(d[ii,jj]/g[ii,])^2)^2}  else {
    wab1[ii,jj]=0 }}}
beta.gcv<-matrix(0,n,p)
Y1hat.ab<-matrix(0,n,1)
W=wab1
GCV3=matrix(0,91,1)
for (i in 1:n)
  #for ( j in 1:n){
{
beta.gcv1[i,]=ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])%*%
y1
  Y1hat.ab[i]=(x[i,])%*%(beta.gcv1[i,])
  trc<-x%*%ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])
  GCV3[i]=sum(n*(((y1-Y1hat.ab)*(y1-Y1hat.ab))/(n-
sum(diag(trc)))^2))
}
g=g.ab1
b.GCV3=cbind(g,GCV3)
b.GCV3=b.GCV3[order(b.GCV3[,2]),]
#opt.b.ab1=b.GCV3[1,1]

#Rumus bandwidth adapt bisquare  Y2
g=g.ab2
wab2=matrix(0,n,n)
for (ii in 1:n)
{ for (jj in 1:n)
{ #Rumus bandwidth bisquare
  if (d[ii,jj]<=g[ii])
  { wab2[ii,jj]=(1-(d[ii,jj]/g[ii,])^2)^2}  else {
    wab2[ii,jj]=0 }}}
beta.gcv<-matrix(0,n,p)
Y2hat.ab<-matrix(0,n,1)
W=wab2
GCV4=matrix(0,91,1)
for (i in 1:n)
  #for ( j in 1:n){
{
beta.gcv1[i,]=ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])%*%
y2
  Y2hat.ab[i]=(x[i,])%*%(beta.gcv1[i,])
  trc<-x%*%ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])
  GCV4[i]=sum(n*(((y2-Y2hat.ab)*(y2-Y2hat.ab))/(n-
sum(diag(trc)))^2))}
```

```
g=g.ab2
b.GCV4=cbind(g,GCV4)
b.GCV4=b.GCV4[order(b.GCV4[,2]),]

########  Adaptive gaussian
g=g.ag1
wag1=matrix(0,n,n)
#Rumus bandwidth adaptive gaussian y1
for (ii in 1:n)
{ for (jj in 1:n)
  #Rumus bandwidth adaptive gaussian

{ wag1[ii,jj]=exp(-0.5*(d[ii,jj]/g[ii,])^2)}}
beta.gcv<-matrix(0,n,p)
Y1hat.ag<-matrix(0,n,1)
W=wag1
GCV5=matrix(0,91,1)
for (i in 1:n)
{
beta.gcv1[i,]=ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])%*%
y1
  Y1hat.ag[i]=(x[i,])%*%(beta.gcv1[i,])
  trc<-x%*%ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])
  GCV5[i]=sum(n*(((y1-Y1hat.ag)*(y1-Y1hat.ag))/(n-
sum(diag(trc)))^2))}
g=g.ag1
b.GCV5=cbind(g,GCV5)
b.GCV5=b.GCV5[order(b.GCV5[,2]),]
g=g.ag2
wag2=matrix(0,n,n)

#Rumus bandwidth adaptive gaussian y2
for (ii in 1:n)
{ for (jj in 1:n)
  #Rumus adaptive gaussian

{ wag2[ii,jj]=exp(-0.5*(d[ii,jj]/g[ii,])^2)}}
beta.gcv<-matrix(0,n,p)
Y2hat.ag<-matrix(0,n,1)
W=wag2
GCV6=matrix(0,91,1)
for (i in 1:n)
  {
beta.gcv1[i,]=ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])%*%
y2
  Y2hat.ag[i]=(x[i,])%*%(beta.gcv1[i,])
  trc<-x%*%ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])
  GCV6[i]=sum(n*(((y2-Y2hat.ag)*(y2-Y2hat.ag))/(n-
sum(diag(trc)))^2))
}
g=g.ag2
b.GCV6=cbind(g,GCV6)
b.GCV6=b.GCV6[order(b.GCV6[,2]),]
#opt.b.ag6=b.GCV6[1,1]

############ fix bisquare
g=g.fb1
wfb1=matrix(0,n,n)
```

```
for (ii in 1:n)
{ for (jj in 1:n)
{ #Rumus bandwidth fix bisquare
  if (d[ii,jj]<=g[ii])
  { wfb1[ii,jj]=(1-(d[ii,jj]/g[ii,])^2)^2}  else {
    wfb1[ii,jj]=0 }}}
beta.gcv<-matrix(0,n,p)
Y1hat.fb<-matrix(0,n,1)
W=wfb1
GCV7=matrix(0,91,1)
for (i in 1:n)
 {
beta.gcv1[i,]=ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])%*%
y1
  Y1hat.fb[i]=(x[i,])%*%(beta.gcv1[i,])
  trc<-x%*%ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])
  GCV7[i]=sum(n*(((y1-Y1hat.fb)*(y1-Y1hat.fb))/(n-
sum(diag(trc)))^2))
}
g=g.fb1
b.GCV7=cbind(g,GCV7)
b.GCV7=b.GCV7[order(b.GCV7[,2]),]
opt.b.fb1=b.GCV7[1,1]
g=g.fb2
wfb2=matrix(0,n,n)

for (ii in 1:n)
{ for (jj in 1:n)
{ #Rumus bandwidth bisquare
  if (d[ii,jj]<=g[ii])
  { wfb2[ii,jj]=(1-(d[ii,jj]/g[ii,])^2)^2}  else {
    wfb2[ii,jj]=0 }}}
beta.gcv<-matrix(0,n,p)
Y2hat.fb<-matrix(0,n,1)
W=wfb2
GCV8=matrix(0,91,1)
for (i in 1:n)
{
beta.gcv1[i,]=ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])%*%
y2
  Y2hat.fb[i]=(x[i,])%*%(beta.gcv1[i,])
  trc<-x%*%ginv(t(x)%*%diag(W[,i])%*%x)%*%t(x)%*%diag(W[,i])
  GCV8[i]=sum(n*(((y2-Y2hat.fb)*(y2-Y2hat.fb))/(n-
sum(diag(trc)))^2))}
g=g.fb2
b.GCV8=cbind(g,GCV8)
b.GCV8=b.GCV8[order(b.GCV8[,2]),]
opt.b.fb2=b.GCV8[1,1]


############# pembobot   ################################

band=rbind(b.GCV1[1,],b.GCV2[1,])
j=band[order(band[,2],decreasing=FALSE),]

GCVgab=rbind(cbind(b.GCV1,b.GCV3,b.GCV5,b.GCV7)[1,c(2,4,6,8)],(cbi
nd(b.GCV2,b.GCV4,b.GCV6,b.GCV8))[1,c(2,4,6,8)])
print(GCVgab[1,])
```

```
#Rumus adaptif bisquare
g.opt.ab=matrix(g.ab1)
w.ab=matrix(0,n,n)
k=min(ii,jj)
for (ii in 1:n)
{ for (jj in 1:n)
{
  if (d[ii,jj]<=g.opt.ab[k])
  { w.ab[ii,jj]=(1-(d[ii,jj]/g.opt.ab[k])^2)^2}  else {
    w.ab[ii,jj]=0 }}}
write.table(g.ab1,file = "D:/bandwith.ab.txt")
write.table(w.ab,file = "D:/pembobot.ab.txt",sep="\t")
write.csv(w.ab,file = "D:/pembobot.ab.csv")
w=w.ab

#Rumus fix bisquare
g.opt.fb=opt.b.fb1
w.fb=matrix(0,n,n)
for (ii in 1:n)
{ for (jj in 1:n)
{ #Rumus bandwidth fix bisquare
  if (d[ii,jj]<=g.opt.fb)
  { w.fb[ii,jj]=(1-(d[ii,jj]/g.opt.fb)^2)^2}  else {
    w.fb[ii,jj]=0 }}}
bdwt.fb=as.matrix(c(rep(g.opt.fb,1,n)))
write.table(bdwt.fb,file = "D:/bandwith.fb.txt")
write.table(w.fb,file = "D:/pembobot.fb.txt",sep="\t")
write.csv(w.fb,file = "D:/pembobot.fb.csv")
w.fb

#Rumus adaptif gaussian
g.opt.ag=matrix(g.ag1)
w.ag=matrix(0,n,n)
k=min(ii,jj)
for (ii in 1:n)
{ for (jj in 1:n)
{
  w.ag[ii,jj]=exp(-0.5*(d[ii,jj]/g.opt.ag[k])^2)}}

write.table(g.ag1,file = "D:/bandwith.ag.txt")
write.table(w.ag,file = "D:/pembobot.ag.txt",sep="\t")
write.csv(w.ag,file = "D:/pembobot.ag.csv")
w.ag

#Rumus fix gaussian
g.opt.fg=opt.b.fg1
w.fg=matrix(0,n,n)
for (ii in 1:n)
{ for (jj in 1:n)
{
  w.fg[ii,jj]=exp(-0.5*(d[ii,jj]/g.opt.fg)^2)}}

bdwt.fg=as.matrix(c(rep(g.opt.fg,1,n)))
write.table(bdwt.fg,file = "D:/bandwith.fg.txt")
write.table(w.fg,file = "D:/pembobot.fg.txt",sep="\t")
write.csv(w.fg,file = "D:/pembobot.fg.csv")
w.fg
```

Lampiran 46: Syntax Estimasi Parameter GWBZIGPR *fix bisquare*

```
data=read.csv(file.choose(),header=T,sep=";")
library(gamlss)
library(MASS)
library(hypergeo)
library(goft)
y=as.matrix(data[,1:2])
y1=as.matrix(data[,1])
y2=as.matrix(data[,2])
n=nrow(data)
x=as.matrix(cbind(rep(1,n),data[,-c(1,2)]))
p=ncol(x)

#### dummy #################################################
b=matrix(0,n,1)
c=matrix(0,n,1)
d=matrix(0,n,1)
e=matrix(0,n,1)
a=matrix(0,n,4)
for(i in 1:n)
{
  if ((y[i,1]==0)&(y[i,2]!=0))
  {
    b[i]=1
    c[i]=0
    d[i]=0
    e[i]=0
  }
  if ((y[i,1]!=0)&(y[i,2]==0))
  {
    b[i]=0
    c[i]=1
    d[i]=0
    e[i]=0
  }
  if ((y[i,1]!=0)&(y[i,2]!=0))
  {
    b[i]=0
    c[i]=0
    d[i]=1
    e[i]=0
  }
  e[i]=1-b[i]-c[i]-d[i]
  a[i,]=cbind(b[i],c[i],d[i],e[i])

}
b=a[,1]
c=a[,2]
d=a[,3]
e=a[,4]

#mencari parameter awal

library(AER)
p1 <- glm(y1 ~x1+x2+x3+x4+x5+x6+x7 , family = poisson, data =
data)
```

```
p2 <- glm(y2 ~x1+x2+x3+x4+x5+x6+x7 , family = poisson, data =
data)
p1$coefficients
p2$coefficients
beta1=as.matrix(p1$coefficients)
beta2=as.matrix(p2$coefficients)
gamma1=as.matrix(c(rep(0.01,1,8)))
gamma2=as.matrix(c(rep(0.012,1,8)))
psi1=1.065181
psi2=1.209661
eta=0.269
w=read.csv(file = "D:/pembobot.fb.csv")

##  iterasi
pb1x=matrix(0,n,1)
pb2x=matrix(0,n,1)
ppgamma1x=matrix(0,n,1)
ppgamma2x=matrix(0,n,1)
set.seed(100)
for (gw in 1:n)
{
eps=100
iterasi=0
while (eps > 0.00000001 && iterasi<100)
{
 tetaawal<-as.matrix(c(gamma1,gamma2,beta1,beta2,psi1,psi2,eta))
  g1=matrix(0,n,1)
  g2=matrix(0,n,1)
  dg1beta1=matrix(0,n,p)
  dg2beta2=matrix(0,n,p)
  dg1psi1=matrix(0,n,1)
  dg2psi2=matrix(0,n,1)
  t=exp(-1)
  b1x<-x%*%beta1
  b2x<-x%*%beta2
  eb1=exp(b1x)
  eb2=exp(b2x)
  gamma1x<-x%*%gamma1
  gamma2x<-x%*%gamma2
  egam1=exp(gamma1x)
  egam2=exp(gamma2x)
  lambda1=eb1/(1+(psi1*eb1))
  lambda2=eb2/(1+(psi2*eb2))
  pi1=egam1/(1+egam1)
  pi2=egam2/(1+egam2)
  pi10=1/(1+egam1)
  pi20=1/(1+egam2)

  for(i in 1:n)
  {
    #turunan g1
    g1[i]=exp(lambda1[i]*(t-1))
    g2[i]=exp(lambda2[i]*(t-1))
    dg1beta1[i,]=(t-1)*1/(1-(psi1*t*(-
lambda1[i]*x[i,])*(1+(psi1*lambda1[i]))))*g1[i]
    dg1psi1[i]=(-lambda1[i]^2)*t*(t-1)*1/(1-(psi1*t*(-
lambda1[i])))*g1[i]
```

```
    dg2beta2[i,]=(t-1)*1/(1-(psi2*t*(-
lambda2[i]*x[i,])*(1+(psi2*lambda2[i]))))*g2[i]
    dg2psi2[i]=(-lambda2[i]^2)*t*(t-1)*1/(1-(psi2*t*(-
lambda2[i])))*g2[i]
  }
  ##### turunan gamma1 ##############################
  A11=matrix(0,n,p)
  A211=matrix(0,n,p)
  A2211=matrix(0,n,p)
  A2221=matrix(0,n,p)
  A2231=matrix(0,n,p)
  A221=matrix(0,n,p)
  A21=matrix(0,n,p)
  A1=matrix(0,n,p)
  B11=matrix(0,n,p)
  B211=matrix(0,n,p)
  B221=matrix(0,n,p)
  B21=matrix(0,n,p)
  B1=matrix(0,n,p)
  C1=matrix(0,n,p)
  D1=matrix(0,n,p)
  pers.11=matrix(0,n,p)
  pers.21=matrix(0,n,p)
  pers.31=matrix(0,n,p)
  pers.41=matrix(0,n,p)
  dgamma1=matrix(0,n,p)
  sumdgamma1=matrix(0,p)
  dg=matrix(0,n,1)
  dh=matrix(0,n,1)
  z=matrix(0,n,1)
  u=matrix(0,n,1)
  for (j in 1:p)
    for (i in 1:n)
    {{
      #persamaan 1
      dg[i]=(1+(eta*(1-g1[i])*(1-g2[i])))
      A11[i,]=egam1[i]*x[i,]*pi10[i]*pi20[i]*w[gw,i]
      A211[i,]=egam1[i]*(egam1[i]+exp(-lambda2[i]))
      A2211[i,]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
      A2221[i,]=egam2[i]*(exp(-lambda1[i]))
      A2231[i,]=(exp(-lambda1[i]-lambda2[i])*dg[i])
      A221[i,]=A2211[i]+A2221[i]+A2231[i]
      A21[i,]=A211[i,]/A221[i,]*w[gw,i]
      A1[i,]=A11[i,]+A21[i,]
      pers.11[i,]=(1-b[i]-c[i]-d[i])*A1[i,]

      #persamaman2
      u[i]=1+(eta*(1-g1[i])*((exp(-y2[i]))-g2[i]))
      B11[i,]=-pi1[i]*x[i,]*w[gw,i]
      B211[i,]=egam1[i]*x[i,]
      B221[i,]=(egam1[i]+((exp(-lambda1[i]))*u[i]))
      B21[i,]=B211[i,]/B221[i,]*w[gw,i]
      B1[i,]=B11[i,]+B21[i,]
      pers.21[i,]=(b[i])*B1[i,]

      #persamaan3
      C1[i,]=-pi1[i]*x[i,]*w[gw,i]
      pers.31[i,]=(c[i])*C1[i,]
```

```
    #persamaan4
    D1[i,]=-pi1[i]*x[i,]*w[gw,i]
    pers.41[i,]=(d[i])*D1[i,]

  }
  }
dgamma1=pers.11+pers.21+pers.31+pers.41
dgamma=ifelse(is.nan(dgamma1),0,dgamma1)
dgamma1

#####  turunan gamma2 ##################################
A12=matrix(0,n,p)
A212=matrix(0,n,p)
A2212=matrix(0,n,p)
A2222=matrix(0,n,p)
A2232=matrix(0,n,p)
A222=matrix(0,n,p)
A22=matrix(0,n,p)
A2=matrix(0,n,p)
C12=matrix(0,n,p)
C212=matrix(0,n,p)
C222=matrix(0,n,p)
C22=matrix(0,n,p)
B2=matrix(0,n,p)
C12=matrix(0,n,p)
C2=matrix(0,n,p)
D2=matrix(0,n,p)
pers.12=matrix(0,n,p)
pers.22=matrix(0,n,p)
pers.32=matrix(0,n,p)
pers.42=matrix(0,n,p)
dgamma2=matrix(0,n,p)
sumdgamma2=matrix(0,p)
dg=matrix(0,n,1)
dh=matrix(0,n,1)
z=matrix(0,n,1)
u=matrix(0,n,1)
for (j in 1:p)
  for (i in 1:n)
  {{
    #persamaan 1
    dg[i]=1+(eta*(1-g1[i])*(1-g2[i]))
    A12[i,]=-pi2[i]*x[i,]*pi20[i]*w[gw,i]
    A212[i,]=egam2[i]*(egam1[i]+exp(-lambda1[i]))*x[i,]
    A2212[i,]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
    A2222[i,]=egam2[i]*exp(-lambda1[i])
    A2232[i,]=exp(-lambda1[i]-lambda2[i])*dg[i]
    A222[i,]=A2212[i,]+A2222[i,]+A2232[i,]
    A22[i,]=A212[i,]/A222[i,]*w[gw,i]
    A2[i,]=A12[i,]+A22[i,]
    pers.12[i,]=(1-b[i]-c[i]-d[i])*A2[i,]

    #persamaman2
    B2[i,]=-pi2[i]*x[i,]*w[gw,i]
    pers.22[i,]=(b[i])*B2[i,]

    #persamaan3
```

```
      C12[i,]=-pi2[i]*x[i,]*w[gw,i]
      C212[i,]=egam2[i]*x[i,]
      C222[i,]=(egam2[i]+((exp(-lambda2[i]))*(1+(eta*(1-
g1[i])*(exp(-y2[i])-g2[i])))))
      C22[i,]=C212[i,]/C222[i,]*w[gw,i]
      C2[i,]=C12[i,]+C22[i,]
      pers.32[i,]=(c[i])*C2[i,]

      #persamaan4
      D2[i,]=-pi2[i]*x[i,]*w[gw,i]
      pers.42[i,]=(d[i])*D2[i,]

    }
    }
  dgamma2=pers.12+pers.22+pers.32+pers.42
  dgamma=ifelse(is.nan(dgamma2),0,dgamma2)
  dgamma2

  ##### turunan beta1 #################################
  s11=matrix(0,n,1)
  s12=matrix(0,n,1)
  s13=matrix(0,n,1)
  s=matrix(0,n,1)
  A13=matrix(0,n,p)
  A23=matrix(0,n,p)
  A3=matrix(0,n,p)
  B13=matrix(0,n,p)
  B23=matrix(0,n,p)
  B3=matrix(0,n,p)
  C13=matrix(0,n,p)
  C23=matrix(0,n,p)
  C330=matrix(0,n,p)
  C33=matrix(0,n,p)
  C3=matrix(0,n,p)
  D13=matrix(0,n,p)
  D23=matrix(0,n,p)
  D33=matrix(0,n,p)
  D43=matrix(0,n,p)
  D3=matrix(0,n,p)
  dg=matrix(0,n,1)
  dh=matrix(0,n,1)
  z=matrix(0,n,1)
  u=matrix(0,n,1)
  pers.13=matrix(0,n,p)
  pers.23=matrix(0,n,p)
  pers.33=matrix(0,n,p)
  pers.43=matrix(0,n,p)
  dbeta1=matrix(0,n,p)
  sumdbeta1=matrix(0,p)
  for (i in 1:n)
    for (j in 1:p)
    {{
      # Persamaan1
      dg[i]=1+(eta*(1-g1[i])*(1-g2[i]))
      s11[i]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
      s12[i]=egam2[i]*exp(-lambda1[i])
      s13[i]=(exp(-lambda1[i]-lambda2[i]))*dg[i]
      s[i]=s11[i]+s12[i]+s13[i]
```

```
      A13[i,]=egam2[i]*exp(-lambda1[i])*((-
lambda1[i]*x[i,])+((lambda1[i])^2*psi1*x[i,]))
      A23[i,]=exp(-lambda1[i]-lambda2[i])*((-lambda1[i]*x[i,])-
((lambda1[i])^2*psi1*x[i,]))*t*dg1beta1[i]
      A3[i,]=1/s[i]*(A13[i,]+A23[i,])*w[gw,i]
      pers.13[i,]=(1-b[i]-c[i]-d[i])*A3[i,]

      # Persamaan2
      dg1beta1[i,]=((t-1)*1/(1-(psi1*t*(-
lambda1[i]*x[i,])*(1+(psi1*lambda1[i])))))*g1[i])
      u[i]=1+(eta*(1-g1[i])*((exp(-y2[i]))-g2[i]))
      B13[i,]=exp(-lambda1[i])*u[i]*((-lambda1[i]*x[i,])-
((lambda1[i])^2*x[i,]*psi1))
      B23[i,]=egam1[i]+(exp(-lambda1[i])*u[i])
      B3[i,]=B13[i,]/B23[i,]*dg1beta1[i]*w[gw,i]
      pers.23[i,]=(b[i])*B3[i,]

      # Persamaan3
      dg1beta1[i,]=((t-1)*1/(1-(psi1*t*(-
lambda1[i]*x[i,])*(1+(psi1*lambda1[i])))))*g1[i])
      C13[i,]=y1[i]*(x[i,]-(lambda1[i]*psi1*x[i,]))*w[gw,i]
      C23[i,]=-
(lambda1[i])*x[i,]*(1+(psi1*y1[i]))*(1+(lambda1[i]*psi1))*w[gw,i]
      C330[i,]=egam2[i]+(exp(-lambda2[i])*(1+(eta*(1-g2[i])*(exp(-
y1[i])-g1[i]))))
      C33[i,]=1/C330[i,]*dg1beta1[i]*w[gw,i]
      C3[i,]=C13[i,]+C23[i,]+C33[i,]
      pers.33[i,]=(c[i])*C3[i,]

      #persamaan4
      D13[i,]=y1[i]*x[i,]*w[gw,i]
      D23[i,]=-(lambda1[i])*y1[i]*psi1*x[i,]*w[gw,i]
      D33[i,]=(-(lambda1[i]*(1+(y1[i]*psi1)*x[i,])))*(1-
(lambda1[i]*psi1))*w[gw,i]
      z[i]=1+(eta*(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i]))
      D43[i,]=1/z[i]*dg1beta1[i]*w[gw,i]
      D3[i,]=D13[i,]+D23[i,]+D33[i,]+D43[i,]
      pers.43[i,]=(d[i])*D3[i,]

    }
    }
  dbeta1=pers.13+pers.23+pers.33+pers.43
  dbeta1=ifelse(is.nan(dbeta1),0,dbeta1)

  ##### turunan beta2 ###################################
  A14=matrix(0,n,p)
  A24=matrix(0,n,p)
  A4=matrix(0,n,p)
  B14=matrix(0,n,p)
  B24=matrix(0,n,p)
  B34=matrix(0,n,p)
  B440=matrix(0,n,p)
  B44=matrix(0,n,p)
  B4=matrix(0,n,p)
  C14=matrix(0,n,p)
  C24=matrix(0,n,p)
  C4=matrix(0,n,p)
  D14=matrix(0,n,p)
```

```
  D24=matrix(0,n,p)
  D34=matrix(0,n,p)
  D44=matrix(0,n,p)
  D4=matrix(0,n,p)
  dg=matrix(0,n,1)
  dh=matrix(0,n,1)
  z=matrix(0,n,1)
  u=matrix(0,n,1)
  pers.14=matrix(0,n,p)
  pers.24=matrix(0,n,p)
  pers.34=matrix(0,n,p)
  pers.44=matrix(0,n,p)
  dbeta2=matrix(0,n,p)
  sumdbeta2=matrix(0,p)
  for (i in 1:n)
    for (j in 1:p)
    {{
      # Persamaan1
      dg[i]=1+(eta*(1-g1[i])*(1-g2[i]))
      s11[i]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
      s12[i]=egam2[i]*exp(-lambda1[i])
      s13[i]=(exp(-lambda1[i]-lambda2[i]))*dg[i]
      s[i]=s11[i]+s12[i]+s13[i]
      A14[i,]=egam1[i]*exp(-lambda2[i])*((-
lambda2[i]*x[i,])+((lambda2[i])^2*psi2*x[i,]))
      A24[i,]=exp(-lambda2[i]-lambda1[i])*((-lambda2[i]*x[i,])-
((lambda2[i])^2*psi2*x[i,]))*t*dg2beta2[i]
      A4[i,]=1/s[i]*(A14[i,]+A24[i,])*w[gw,i]
      pers.14[i,]=(1-b[i]-c[i]-d[i])*A4[i,]

      # persamaan2
      u[i]=1+(eta*(1-g1[i])*((exp(-y2[i]))-g2[i]))
      B14[i,]=y2[i]*(x[i,]-(lambda2[i]*psi2*x[i,]))*w[gw,i]
      B24[i,]=-
(lambda2[i])*x[i,]*(1+(psi2*y2[i]))*(1+(lambda2[i]*psi2))*w[gw,i]
      B34[i,]=egam1[i]+(exp(-lambda1[i])*u[i])
      B44[i,]=1/B34[i,]*dg2beta2[i]*w[gw,i]
      B4[i,]=B14[i,]+B24[i,]+B44[i,]
      pers.24[i,]=(b[i])*B4[i,]

      #persamaan3
      dh[i]=1+(eta*(1-g2[i])*((exp(-y1[i]))-g1[i]))
      C14[i,]=exp(-lambda2[i])*u[i]*((-lambda2[i]*x[i,])-
((lambda2[i])^2*x[i,]*psi2))
      C24[i,]=egam2[i]+(exp(-lambda2[i])*dh[i])
      C4[i,]=C14[i,]/C24[i,]*dg2beta2[i]*w[gw,i]
      pers.34[i,]=c[i]*C4[i,]

      #persamaan4
      z[i]=1+(eta*(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i]))
      D14[i,]=(y2[i]*x[i,])*w[gw,i]
      D24[i,]=-lambda2[i]*y2[i]*psi2*x[i,]*w[gw,i]
      D34[i,]=(-(lambda2[i]*(1+(y2[i]*psi2)*x[i,])))*(1-
(lambda2[i]*psi2))*w[gw,i]
      D44[i,]=(1/z[i]*dg2beta2[i])*w[gw,i]
      D4[i,]=D14[i,]+D24[i,]+D34[i,]+D44[i,]
      pers.44[i,]=(d[i])*D4[i,]
```

```
    }
    }
  dbeta2=pers.14+pers.24+pers.34+pers.44
  dbeta2=ifelse(is.nan(dbeta2),0,dbeta2)
  dbeta2

  #### turunan psi1 #####################################
  A15=matrix(0,n,1)
  A25=matrix(0,n,1)
  A35=matrix(0,n,1)
  A5=matrix(0,n,1)
  B15=matrix(0,n,1)
  B25=matrix(0,n,1)
  B5=matrix(0,n,1)
  C15=matrix(0,n,1)
  C25=matrix(0,n,1)
  C35=matrix(0,n,1)
  C415=matrix(0,n,1)
  C425=matrix(0,n,1)
  C45=matrix(0,n,1)
  C5=matrix(0,n,1)
  D15=matrix(0,n,1)
  D25=matrix(0,n,1)
  D35=matrix(0,n,1)
  D45=matrix(0,n,1)
  D5=matrix(0,n,1)
  dg=matrix(0,n,1)
  dh=matrix(0,n,1)
  z=matrix(0,n,1)
  u=matrix(0,n,1)
  pers.15=matrix(0,n,1)
  pers.25=matrix(0,n,1)
  pers.35=matrix(0,n,1)
  pers.45=matrix(0,n,1)
  dpsi1=matrix(0,n,1)
  sumdpsi1=matrix(0,1)
  for (i in 1:n)
  {
    # Persamaan1
    dg[i]=1+(eta*(1-g1[i])*(1-g2[i]))
    z[i]=1+(eta*(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i]))
    s11[i]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
    s12[i]=egam2[i]*exp(-lambda1[i])
    s13[i]=(exp(-lambda1[i]-lambda2[i]))*dg[i]
    s[i]=s11[i]+s12[i]+s13[i]
    A15[i]=1/s[i]
    A25[i]=egam2[i]*(-lambda1[i])*(-lambda2[i]^2)
    A35[i]=exp(-lambda1[i]-
lambda2[i])*dg[i]*lambda1[i]^2*dg1psi1[i]
    A5[i]=(A25[i,]*A35[i,])/A15[i]
    pers.15[i]=(1-b[i]-c[i]-d[i])*A5[i]*w[gw,i]

    # Persamaan2
    u[i]=1+(eta*(1-g1[i])*((exp(-y2[i]))-g2[i]))
    B15[i]=exp(-lambda1[i])*u[i]*(eb1[i]^2)
    B25[i]=egam1[i]+(exp(-lambda1[i])*u[i]*(1+(psi1*eb1[i])^2))
    B5[i]=B15[i]/B25[i]*dg1psi1[i]
    pers.25[i]=(b[i])*B5[i]*w[gw,i]
```

```
  # Persamaan3
  dh[i]=1+(eta*(1-g2[i])*((exp(-y1[i]))-g1[i]))
  C15[i]=(y1[i]*lambda1[i])
  C25[i]=((y1[i]-1)/(1+(psi1*y1[i]))*y1[i])
  C35[i]=-lambda1[i]*(y1[i]+(lambda1[i]*(1+(psi1*y1[i]))))
  C415[i]=egam2[i]+(exp(-lambda2[i])*dh[i])
  C425[i]=exp(-lambda2[i])*dh[i]
  C45[i]=C425[i]/ C415[i]*dg1psi1[i]
  C5[i]=C15[i]+C25[i]+C35[i]+C45[i]
  pers.35[i]=(c[i])*C5[i]*w[gw,i]

  # Persamaan4
  z[i]=1+(eta*(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i]))
  D15[i]=-lambda1[i]*y1[i]
  D25[i]=(y1[i]-1)*pi10[i]*y1[i]
  D35[i]=-lambda1[i]*(y1[i]-(lambda1[i]*(1+(psi1*y1[i]))))
  D45[i]=1/z[i]*dg1psi1[i]
  D5[i]=D15[i]+D25[i]+D35[i]+D45[i]
  pers.45[i]=(d[i])*D5[i]*w[gw,i]

  dpsi1[i]=pers.15[i]+pers.25[i]+pers.35[i]+pers.45[i]
  dpsi1=ifelse(is.nan(dpsi1),0,dpsi1)
  sumdpsi1<-sum(dpsi1)
}

#### turunan psi2 ##################################
A16=matrix(0,n,1)
A26=matrix(0,n,1)
A36=matrix(0,n,1)
A6=matrix(0,n,1)
B16=matrix(0,n,1)
B26=matrix(0,n,1)
B36=matrix(0,n,1)
B46=matrix(0,n,1)
B6=matrix(0,n,1)
C16=matrix(0,n,1)
C26=matrix(0,n,1)
C6=matrix(0,n,1)
D16=matrix(0,n,1)
D26=matrix(0,n,1)
D36=matrix(0,n,1)
D46=matrix(0,n,1)
D6=matrix(0,n,1)
dg=matrix(0,n,1)
dh=matrix(0,n,1)
z=matrix(0,n,1)
u=matrix(0,n,1)
pers.16=matrix(0,n,1)
pers.26=matrix(0,n,1)
pers.36=matrix(0,n,1)
pers.46=matrix(0,n,1)
dpsi2=matrix(0,n,1)
sumdpsi2=matrix(0,1)
for (i in 1:n)
{
  # Persamaan1
  dg[i]=1+(eta*(1-g1[i])*(1-g2[i]))
```

```
    s11[i]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
    s12[i]=egam2[i]*exp(-lambda1[i])
    s13[i]=(exp(-lambda1[i]-lambda2[i]))*dg[i]
    s[i]=s11[i]+s12[i]+s13[i]
    A16[i]=1/s[i]
    A26[i]=egam1[i]*exp(-lambda2[i])*-(lambda2[i]^2)
    A36[i]=exp(-lambda1[i]-
lambda2[i])*dg[i]*lambda2[i]^2*dg2psi2[i]
    A6[i]=(A26[i]+A36[i])/A16[i]
    pers.16[i]=(1-b[i]-c[i]-d[i])*A6[i]*w[gw,i]

    # Persamaan2
    u[i]=1+(eta*(1-g1[i])*((exp(-y2[i]))-g2[i]))
    B16[i]=-y2[i]*lambda2[i]
    B26[i]=(y2[i]-1)/(1+(psi2*y2[i]))*y2[i]
    B36[i]=-lambda2[i]*(y2[i]-(lambda2[i]*(1+(psi2*y2[i]))))
    B46[i]=exp(-lambda1[i])*u[i]*dg1psi1[i]
    B6[i]=B16[i]+B26[i]+B36[i]+B46[i]
    pers.26[i]=(b[i])*B6[i]*w[gw,i]

    # Persamaan3
    dh[i]=1+(eta*(1-g2[i])*((exp(-y1[i]))-g1[i]))
    C16[i]=exp(-lambda2[i])*dh[i]*eb2[i]^2
    C26[i]=egam2[i]+(exp(-lambda2[i])*dh[i]*(1+(psi2*eb2[i])^2))
    C6[i]=C16[i]/C26[i]*dg2psi2[i]
    pers.36[i]=(c[i])*C26[i]*w[gw,i]

    # Persamaan4
    z[i]=1+(eta*(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i]))
    D16[i]=-lambda2[i]*y2[i]
    D26[i]=(y2[i]-1)*y2[i]/(1+(psi2*y2[i]))
    D36[i]=-lambda2[i]*(y2[i]-(lambda2[i]*(1+(psi2*y2[i]))))
    D46[i]=1/z[i]*dg2psi2[i]
    D6[i]=D16[i]+D26[i]+D36[i]+D46[i]
    pers.46[i]=(d[i])*D6[i]*w[gw,i]

    dpsi2[i]=pers.15[i]+pers.26[i]+pers.36[i]+pers.46[i]
    dpsi2=ifelse(is.nan(dpsi2),0,dpsi2)
    sumdpsi2<-sum(dpsi2)
  }
  dpsi2
  sumdpsi2

### turunan eta ######################################
  A17=matrix(0,n,1)
  A27=matrix(0,n,1)
  A7=matrix(0,n,1)
  B17=matrix(0,n,1)
  B27=matrix(0,n,1)
  B7=matrix(0,n,1)
  C17=matrix(0,n,1)
  C27=matrix(0,n,1)
  C7=matrix(0,n,1)
  D7=matrix(0,n,1)
  dg=matrix(0,n,1)
  dh=matrix(0,n,1)
  z=matrix(0,n,1)
  u=matrix(0,n,1)
```

```
s=matrix(0,n,1)
pers.17=matrix(0,n,1)
pers.27=matrix(0,n,1)
pers.37=matrix(0,n,1)
pers.47=matrix(0,n,1)
deta=matrix(0,n,1)
sumdeta=matrix(0,1)
for (i in 1:n)
{
  # Persamaan1
  dg[i]=1+(eta*(1-g1[i])*(1-g2[i]))
  s11[i]=egam1[i]*(egam2[i]+exp(-lambda2[i]))
  s12[i]=egam2[i]*exp(-lambda1[i])
  s13[i]=(exp(-lambda1[i]-lambda2[i]))*dg[i]
  s[i]=s11[i]+s12[i]+s13[i]
  A17[i]=1/s[i]
  A27[i]=exp(-lambda1[i]-lambda2[i])*dg[i]
  A7[i]=A17[i]*A27[i]
  pers.17[i]=(1-b[i]-c[i]-d[i])*A7[i]*w[gw,i]

  # Persamaan2
  u[i]=1+(eta*(1-g1[i])*((exp(-y2[i]))-g2[i]))
  B17[i]=exp(-lambda1[i])*(1-g1[i])*(exp(-y2[i])-g2[i])
  B27[i]=egam1[i]+(exp(-lambda1[i])*u[i])
  B7[i]=B17[i]/B27[i]
  pers.27[i]=(b[i])*B7[i]*w[gw,i]

  # Persamaan3
  dh[i]=1+(eta*(1-g2[i])*((exp(-y1[i]))-g1[i]))
  C17[i]=exp(-lambda2[i])*(1-g2[i])*(exp(-y1[i])-g1[i])
  C27[i]=egam2[i]+(exp(-lambda2[i])*dh[i])
  C7[i]=C17[i]/C27[i]
  pers.37[i]=(c[i])*C7[i]*w[gw,i]

  # Persamaan4
  z[i]=1+(eta*(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i]))
  D7[i]=(exp(-y1[i])-g1[i])*(exp(-y2[i])-g2[i])/z[i]
  pers.47[i]=(d[i])*D7[i]*w[gw,i]

  deta[i]=pers.17[i]+pers.27[i]+pers.37[i]+pers.47[i]
  deta[i]=ifelse(is.nan(deta[i]),0,deta[i])
  sumdeta<-sum(deta)
}
deta
sumdeta

b111=rep(NA,p)
for (l in 1:p)
  for(i in 1:n)
  {
    b111[l]=sum(dbeta1[,l])
  }
b112=rep(NA,p)
for (l in 1:p)
  for(i in 1:n)
  {
    b112[l]=sum(dbeta2[,l])
  }
```

```
  g111=rep(NA,p)
  for (l in 1:p)
    for(i in 1:n)
    {
      g111[l]=sum(dgamma1[,l])
    }
  g112=rep(NA,p)
  for (l in 1:p)
    for(i in 1:n)
    {
      g112[l]=sum(dgamma2[,l])
    }
  #parameter
  I<-cbind(dgamma1,dgamma2,dbeta1,dbeta2,dpsi1,dpsi2,deta)
  g<-as.matrix(c(g111,g112,b111,b112,sumdpsi1,sumdpsi2,sumdeta))
  Hinv<-ginv(t(I)%*%(I))
  teta1<-tetaawal-(Hinv%*%g)

  eps<-(norm(teta1-tetaawal,type=c("2")))
  pgamma1<-teta1[1:8]
  pgamma2<-teta1[9:16]
  #gama1<-ifelse(gama1>0,gama1,0.1)
  pbeta1<-teta1[17:24]
  pbeta2<-teta1[25:32]
    teta1=as.matrix(c(pgamma1,pgamma2,pbeta1,pbeta2))
  error=abs(teta1-tetaawal)
  tetaawal<-teta1
  eps=sqrt(sum(error^2))


  iterasi<-iterasi+1
}

teta1
se<-sqrt(diag(Hinv))
Z<-teta1/se
pval=2*pnorm(abs(Z), lower.tail = FALSE)
pb1x[gw]=x[gw,]%*%pbeta1
pb2x[gw]=x[gw,]%*%pbeta2
ppgamma1x[gw]=x[gw,]%*%pgamma1
ppgamma2x[gw]=x[gw,]%*%pgamma2

{{

cat("============================================================","
\n")
  cat(" Estiamsi Parameter GWBZIGPR di bawah H1 Lokasi ke-
",gw,"\n")

cat("============================================================","
\n")
}
table=cbind(teta1=round(teta1,5),se=round(se,5),Z=round(Z,4),pval=
round(pval,4))
table1=data.frame(table,stringsAsFactors=FALSE)
rownames(table1)=c("gamma1.0","gamma1.1","gamma1.2","gamma1.3","ga
mma1.4","gamma1.5","gamma1.6","gamma1.7","gamma2.0","gamma2.1","ga
mma2.2","gamma2.3","gamma2.4","gamma2.5","gamma2.6","gamma2.7","in
```

```
tercept.b1","b1.1","b1.2","b1.3","b1.4","b1.5","b1.6","b1.7","inte
rcept.b2","b2.1","b2.2","b2.3","b2.4","b2.5","b2.6","b2.7")
colnames(table1)=c("Estimate","Std Error","Z value","P-value")
{cat("==========================================================",
"\n")
}}
}
y1had=exp(pb1x)
y2had=exp(pb2x)
MSE1=mean((y1had-y1)^2)
MSE2=mean((y2had-y2)^2)
MSE1
MSE2
############### ln likelihood di bawah H1 ##################

  y=as.matrix(data[,1:2])
  y1=as.matrix(data[,1])
  y2=as.matrix(data[,2])

  peb1=exp(pb1x)
  peb2=exp(pb2x)
  pegam1=exp(ppgamma1x)
  pegam2=exp(ppgamma2x)

  plambda1=peb1/(1+(ppsi1*peb1))
  plambda2=peb2[i]/(1+(ppsi2*peb2))
  ppi1=pegam1/(1+pegam1)
  ppi2=pegam2/(1+pegam2)
  ppi10=(1+pegam1)
  ppi20=(1+pegam2)
  t=exp(-1)
  p1g1=exp(plambda1*(t-1))
  p1g2=exp(plambda2*(t-1))
  b=a[,1]
  c=a[,2]
  d=a[,3]
  f1=matrix(0,n,1)
  f2=matrix(0,n,1)
  f31=matrix(0,n,1)
  f32=matrix(0,n,1)
  f33=matrix(0,n,1)
  f34=matrix(0,n,1)
  f3=matrix(0,n,1)
  h1=matrix(0,n,1)
  h2=matrix(0,n,1)
  h3=matrix(0,n,1)
  h4=matrix(0,n,1)
  h5=matrix(0,n,1)
  h6=matrix(0,n,1)
  h7=matrix(0,n,1)
  h8=matrix(0,n,1)
  k1=matrix(0,n,1)
  k2=matrix(0,n,1)
  k3=matrix(0,n,1)
  k3=matrix(0,n,1)
  k4=matrix(0,n,1)
  k5=matrix(0,n,1)
  k6=matrix(0,n,1)
```

```
  k7=matrix(0,n,1)
  k8=matrix(0,n,1)
  m1=matrix(0,n,1)
  m2=matrix(0,n,1)
  m3=matrix(0,n,1)
  m4=matrix(0,n,1)
  m5=matrix(0,n,1)
  m6=matrix(0,n,1)
  m7=matrix(0,n,1)
  m8=matrix(0,n,1)
  m9=matrix(0,n,1)
  m10=matrix(0,n,1)
  m11=matrix(0,n,1)
  m12=matrix(0,n,1)
  m13=matrix(0,n,1)
  f=matrix(0,n,1)
  h=matrix(0,n,1)
  k=matrix(0,n,1)
  m=matrix(0,n,1)
  pers.1=matrix(0,n,1)
  pers.2=matrix(0,n,1)
  pers.3=matrix(0,n,1)
  pers.4=matrix(0,n,1)
 for (i in 1:n)
{
  f1[i]=-log(ppi10[i])
  f2[i]=-log(ppi20[i])
  f31[i]=pegam1[i]*pegam2[i]
  f32[i]=pegam1[i]*exp(-plambda2[i])
  f33[i]=pegam2[i]*exp(-plambda1[i])
  f34[i]=exp(-plambda1[i]-plambda2[i])*(1+(peta*(1-p1g1[i])*(1-
p1g2[i])))
  f3[i]=log(f31[i]+f32[i]+f33[i]+f34[i])
  f[i]=f1[i]+f2[i]+f3[i]
  pers.1[i]=(1-b[i]-c[i]-d[i])*f[i]

  h1[i]=-log(ppi20[i])
  h2[i]=y2[i]*pb2x[i]
  h3[i]=-y2[i]*log(1+(ppsi2*peb2[i]))
  h4[i]=(y2[i]-1)*log(1+(ppsi2*y2[i]))
  h5[i]=-log(factorial(y2[i]))
  h6[i]=-lambda2[i]*(1+(ppsi2*y2[i]))
  h7[i]=-log(ppi10[i])
  h8[i]=log(pegam1[i]+(exp(-plambda1[i])*(1+(peta*(1-
p1g1[i])*(exp(-y2[i])-p1g2[i])))))
  h[i]=h1[i]+h2[i]+h3[i]+h4[i]+h5[i]+h6[i]+h7[i]+h8[i]
  pers.2[i]=b[i]*h[i]

  k1[i]=-log(ppi10[i])
  k2[i]=y1[i]*pb1x[i]
  k3[i]=-y1[i]*log(1+(ppsi1*peb1[i]))
  k4[i]=(y1[i]-1)*log(1+(ppsi1*y1[i]))
  k5[i]=-log(factorial(y1[i]))
  k6[i]=-plambda1[i]*(1+(ppsi1*y1[i]))
  k7[i]=-log(ppi20[i])
  k8[i]=log(pegam2[i]+(exp(-plambda2[i])*(1+(peta*(1-
p1g2[i])*(exp(-y1[i])-p1g1[i])))))
  k[i]=k1[i]+k2[i]+k3[i]+k4[i]+k5[i]+k6[i]+k7[i]+k8[i]
```

```
  pers.3[i]=c[i]*k[i]

  m1[i]=-log(ppi10[i])
  m2[i]=-log(ppi20[i])
  m3[i]=y1[i]*pb1x[i]
  m4[i]=-y1[i]*(log(1+(ppsi1*peb1[i])))
  m5[i]=(y1[i]-1)*(log(1+(ppsi1*y1[i])))
  m6[i]=-log(factorial(y1[i]))
  m7[i]=-plambda1[i]*(1+(ppsi1*y1[i]))
  m8[i]=y2[i]*pb2x[i]
  m9[i]=-y2[i]*log(1+(ppsi2*peb2[i]))
  m10[i]=(y2[i]-1)*log(1+(ppsi2*y2[i]))
  m11[i]=-log(factorial(y2[i]))
  m12[i]=-plambda2[i]*(1+(ppsi2*y2[i]))
  m13[i]=log(1+(peta*(exp(-y1[i])-p1g1[i])*(exp(-y2[i])-p1g2[i])))

m[i]=m1[i]+m10[i]+m2[i]+m3[i]+m4[i]+m5[i]+m6[i]+m7[i]+m8[i]+m9[i]+
m10[i]+m11[i]+m12[i]
  pers.4[i]=d[i]*m[i]
}
  LOMEGA=sum(pers.1)+sum(pers.2)+sum(pers.3)+sum(pers.4)

################ uji serentak GWBZIGPR #################
LOMEGA
LLH0_BETA
LLH0_GAMMA
p=7
n=91
df=2*n*p
csqtbl=qchisq(0.05,df)
Gsquare_Beta=2*(LOMEGA-LLH0_BETA)
Gsquare_Gamma=2*(LOMEGA-LLH0_GAMMA)

AIC=(-2*LOMEGA)+(2*df)
AICc=AIC+((2*df*(df+1))/(n-df-1))
```

(Halaman ini sengaja dikosongkan)

PEMERINTAH PROVINSI JAWA TENGAH

DINAS KESEHATAN

Jl. Piere Tendean No. 24 Telp. (024) 3511351 (Hunting) Fax. 3517463
Website : www.dinkesjatengprov.go.id E-mail : mi_jateng@yahoo.co.id
Kode Pos 50131 Kotak Pos 026 Semarang

SURAT PERNYATAAN

Yang bertandatangan di bawah ini:

Nama : Mufti Agung Wibowo, S.Kom, MIT

NIP : 19731117 199803 1 007

Jabatan : Kepala Seksi Manajemen Informasi Kesehatan

Instansi : Dinas Kesehatan Provinsi Jawa Tengah

Dengan ini menyatakan bahwa data yang telah digunakan oleh:

Nama : Qurotul Aini

NRP : 06211850017001

Status : Mahasiswa Pascasarjana ITS

benar-benar berasal dari Dinas Kesehatan Provinsi Jawa Tengah dan diperkenankan untuk digunakan sebagai bahan pendukung tugas akhir.

Semarang, 24 Januari 2020
Kepala Seksi MIK

PEMERINTAH PROVINSI DINAS KESEHATAN

Mufti Agung Wibowo, S.Kom, MIT
NIP19731117 199803 1 007

SURAT PERNYATAAN

Saya yang bertanda tangan di bawah ini, mahasiswa Departemen Statistika Fakultas Sains dan Analitika Data ITS:

Nama : Qurotul Aini

NRP : 06211850017001

Menyatakan bahwa data yang digunakan dalam Tesis/ ~~Disertasi~~ ini merupakan data sekunder yang diambil dari ~~penelitian/ buku/ Tugas Akhir/ Thesis/ Disertasi/~~ publikasi lainnya, yaitu:

Sumber : Dinas Kesehatan Provinsi Jawa Tengah

Keterangan : Data Kesehatan Menurut Kecamatan di Karesidenan Pekalongan

Surat pernyataan ini dibuat dengan sebenarnya. Apabila terdapat permalsuan data, maka saya siap menerima sanksi sesuai aturan yang berlaku.

Surabaya, 3 Februari 2020

Mengetahui,
Pembimbing Tesis

Dr.Purhadi M.Sc
NIP. 19620204 198701 1 001

Mahasiswa

Qurotul Aini
NRP. 06211850017001

Qurotul Aini lahir di Demak 27 Juli 1985 merupakan anak pertama pasangan Bapak Hariyanto dan Ibu Sundari. Penulis memasuki pendidikan formal pertama kali di SDN Pilangsari pada Tahun 1992, kemudian mealnjutkan pendidikan di SLTP Negeri 2 Demak pada Tahun 1997, selanjutnya penulis melanjutkan pendidikan di SLTA Negeri 1 Demak dan lulus pada Tahun 2003. Pada Tahun 2004 penulis berkesempatan menempuh pendidikan di salah satu perguruan tinggi kedinasan yaitu Sekolah Tinggi ilmu Statistik (STIS) dengan jurusan statistika peminatan sosial kependudukan dan lulus bulan Oktober Tahun 2008. Pada bulan Februari 2009, penulis diangkat menjadi CPNS di Badan Pusat Statistik dan ditempatkan di Kabupaten Bangka Selatan Provinsi Kepulauan Bangka Belitung. Penulis menikah pada bulan Februari Tahun 2010 dan telah dikarunai 3 anak. Setelah 8 tahun bekerja, penulis berkesempatan menerima beasiswa APBN dari BPS untuk melanjutkan pendidikan S2 di Institut Teknologi Sepuluh Nopember (ITS) di Surabaya. Selama berada di ITS, penulis menyelesaikan program pascasarjana di jurusan statistika selama 3 semester (Agustus 2018-Januari 2020). Jika ingin mendiskusikan lebih lanjut mengenai tesis ini, penulis dapat dihubungi melalui email qurotulaini.2707@gmail.com atau qurotul@bps.go.id.